Ref. No.: EX/ARCH/MATH/T/115/2017(OLD)

BACHELOR OF ARCHITECTURE EXAMINATION – 2017 (OLD)

(1^{ST} YEAR 1^{ST} SEMESTER)

MATHEMATICS – IA

FULL MARKS : 100 TIME : 3 HOURS

ANSWER ANY 10 QUESTIONS:

1. (a) Find y_n when $y = x^4 \cos 3x$.

   (b) If $y = \log\left(x + \sqrt{x^2+1}\right)$, then show that

   $$(x^2 + 1)y_{n+2} - (2n+1)xy_{n+1} + n^2 y_n = 0.$$ 4+6

2. (a) State and prove Leibnitz's theorem on successive differentiation.

   (b) Verify Rolle's theorem for the function $f(x) = |x|$ in the interval [-1, 1]. 5+5

3. (a) Prove that $x \le \sin^{-1}x \le \frac{x}{\sqrt{1-x^2}}$, if $0 \le x \le 1$.

   (b) Expand the function $\cos^3 x$ in the neighbourhood of $x = 0$ to three terms plus remainder in Lagrange's form. 5+5

4. (a) Show that sinx(1 + cosx) is a maximum when $x = \frac{1}{3}\pi$.

   (b) Evaluate $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$. 5+5

5. (a) If $u = f(r)$, where $r^2 = x^2 + y^2$, prove that

   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{f'(r)}{r}.$$

   (b) If If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^3}$$ 5+5

6. (a) Evaluate $\lim_{x\to 0}(\operatorname{cosec} x)^{\frac{1}{\log x}}$.

   (b) Find the maximum and minimum values of $x^3y^2(1 - x - y)$. 5+5

7. (a) Evaluate $\int_1^4 e^{kx}\,dx$, using integration as a limit of a sum.

   (b) Evaluate $\lim_{n\to\infty}\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2\left(1+\frac{3^2}{n^2}\right)^3 \ldots\left(1+\frac{n^2}{n^2}\right)^n\}^{2/n^2}$ 5+5

8. (a) Prove that $\int_0^1 e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$.

   (b) Evaluate $\int_0^1 \frac{x^2}{\sqrt{(1-x^5)}}dx$. 5+5

9. (a) Prove that $\int_0^{\pi/2}\sqrt{tan\theta}\,d\theta = \frac{\pi\sqrt{2}}{2}$.

   (b) Show that $\int_0^{\infty}\frac{dx}{x^4+a^4} = \frac{\pi\sqrt{2}}{4a^3}$, $a > 0$. 5+5

5+5

10. (a) Show that $\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}.\Gamma(2n+1)}{2^{2n}\,\Gamma(n+1)}$.

   (b) Show that $B(m, 1/2) = 2^{2m-1} B(m, m)$. 5+5

11. (a) Calculate by Simpson's one-third rule the value of the integral

$$\int_0^5 \frac{dx}{4x+5}$$

   Correct upto two decimal places, by taking ten intervals.

   (b) If sinu = $(x^2 + y^2)/(x + y)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = tanu$. 5+5

12. (a) Evaluate

$$\int_0^6 \frac{dx}{1+x^2}$$

   by Trapezoidal rule and compare the result with its actual value.

   (b) Show that the height of the closed cylinder of given surface and greatest volume is equal to its diameter. 5+5