Ex/ARCH/MATH/T/114/2017

# Bachelor of Architecture Examination, 2017

(First Year, First Semester)

## Mathematics

Paper – I

Time : 3 hours Full Marks : 100

(Use a seperate answer-script for each part)

(Symbols have usual meanings, if not mentioned otherwise)

## PART–I (35 marks)

Attempt Q. 1 and any **three** from the rest of this part.

1. (a) If $F$ and $G$ are both continuous on $[a, b]$, that they are both differentiable on $(a, b)$ and that $F'(x) = G'(x), \forall x \in (a, b)$, prove that $F(x)$ and $G(x)$ differ by a constant in this interval, i.e. $F(x) = G(x) + c$ on $[a, b]$.

   (b) Is MVT valid for $f(x) = x^2 + 3x + 2$ in $[1, 2]$ ? Find $c$ if the theorem is applicable.

   $7 + 4 = 11$

2. (a) State Leibnitz's theorem on successive derivatives.

   (b) If $y = a\cos(log x) + b\sin(log x)$, show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (1+n^2)y_n = 0.$$

   $2 + 6 = 8$



3. Evaluate the limit

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x}$$

   8

4. (a) Given a function

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0), \end{cases}$$

   show that $f$ has a directional derivative at $(0, 0)$ in any direction $\beta = (l, m)$ where $l^2 + m^2 = 1$.

   (b) Examine the extreme values of the function $f(x) = 2x^3 - 15x^2 + 36x + 10$.

   $3 + 5 = 8$

5. (a) Expand $log_e(1 + x)$ using Maclaurin's series with Lagrange's form of remainder.

   (b) Verify Rolle's theorem for $f(x) = 2x^3 + x^2 - 4x - 2$.

   $3 + 5 = 8$

## PART–II (65 marks)

Attempt Q. 6 and any ***five*** from the rest of this part.

6. (a) State the fundamental theorem of integral calculus.

   (b) Evaluate the integration $\int_2^5 \frac{(3x^2+5x)}{(x-1)(x+1)^2}dx$ using partial fractions.

   $3 + 7 = 10$

7. (a) Evaluate the integral $\int_0^{\pi/2} \cos x dx$ using the definition of integration as limit of sum.

(b) Show that $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}} = \pi/4$ 6 + 5 = 11

8. (a) Evaluate $\int_0^1 x^3(1-x^2)^{5/2}dx$ using Beta or Gamma function whichever necessary.

(b) Show that $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$. 6 + 5 = 11

9. (a) Calculate the integral $\int_0^3 \sqrt{x^3+1}\, dx$ using Simpson's $\frac{1}{3}$-rule with 6 sub-intervals.

(b) Find the length of the following curve between the indicated points.

$x = e^{\theta} \sin\theta, y = e^{\theta}\cos\theta, \theta = 0$ and $\theta = \pi/2$.

8 + 3 = 11

10. (a) Find the area of the entire surface formed when the cardioide $r = a(1+\cos\theta)$ is revolved about the initial line.

(b) Show that the volume of a sphere of radius $a$ is $\frac{4}{3}\pi a^3$.

7 + 4 = 11

11. (a) Evaluate the double integral $\iint_R (x^2+y^2)\, dx\, dy$ over



$R$ bounded by $y = x^2, x = 2, y = 1$.

(b) Show that $\iint_D e^{y/x}\, dxdy$ where $D$ is a triangle bounded by $y = x, y = 0$ and $x = 1$ is $(e-1)/2$

5 + 6 = 11

12. (a) Evaluate $\int_0^{\infty} e^{-ax} \sin bx dx$.

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(b) Examine the convergence of $\int_a^{\infty} \frac{\sin^2 x}{x^2}\, dx,\ a > 0$.

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