BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2019

(3rd Year, 2nd Semester)

INTRODUCTION TO FINITE ELEMENT METHOD FOR MECHANICAL ENGINEERS

Time : Three hours | Full Marks : 100

Answer any five questions

Question 1

a. Two bar elements as shown in Figure Q1 are held between two rigid walls. The system can have only axial displacement. The temperature of the whole system is raised by ΔT. Find out the displacement at point 2 taking two finite elements.

b. Next assume that $A_1 = A_2, E_1 = E_2, L_1 = L_2, \alpha_1 = \alpha_2$

Find out the thermal stress in the two elements

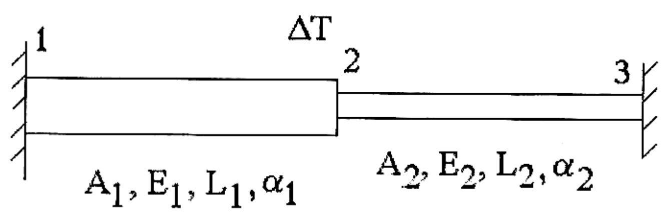

Figure Q1

Question 2

a) Derive the expression for any **one element** of the stiffness matrix of a two-dimensional beam element from minimization of potential energy. Consider a transverse displacement and a rotation at each of the two nodes.

The shape function expressions are given below:-

$$N_1 = \left(1 + 2\frac{x^3}{l^3} - 3\frac{x^2}{l^2}\right)$$

$$N_2 = \left(x - 2\frac{x^2}{l} + \frac{x^3}{l^2}\right)$$

$$N_3 = \left(-2\frac{x^3}{l^3} + 3\frac{x^2}{l^2}\right)$$

$$N_2 = \left(\frac{x^3}{l^2} - \frac{x^2}{l}\right)$$

b) A concentrated force is applied at the mid-span of a simply supported beam. Find out nodal equivalent force-moment.

Question 3

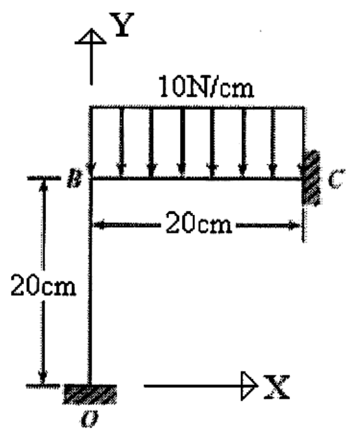

Figure Q3

Find the equivalent nodal loads for the uniformly distributed load shown in Figure Q3

For both the elements b=h=4cm. The modulus of elasticity is $2 \times 10^7\ N/cm^2$

The expression for element stiffness matrix is

$$\underline{k} = \frac{E}{L} \times \begin{bmatrix} AC^2+\frac{12I}{L^2}S^2 & \left(A-\frac{12I}{L^2}\right)CS & -\frac{6I}{L}S & -\left(AC^2+\frac{12I}{L^2}S^2\right) & -\left(A-\frac{12I}{L^2}\right)CS & -\frac{6I}{L}S \\ & AS^2+\frac{12I}{L^2}C^2 & \frac{6I}{L}C & -\left(A-\frac{12I}{L^2}\right)CS & -\left(AS^2+\frac{12I}{L^2}C^2\right) & \frac{6I}{L}C \\ & & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ & & & AC^2+\frac{12I}{L^2}S^2 & \left(A-\frac{12I}{L^2}\right)CS & \frac{6I}{L}S \\ & & & & AS^2+\frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ \text{Symmetry} & & & & & 4I \end{bmatrix}$$

Derive the final three simultaneous equations after incorporation of boundary conditions. Refer Figure Q3

Question 4

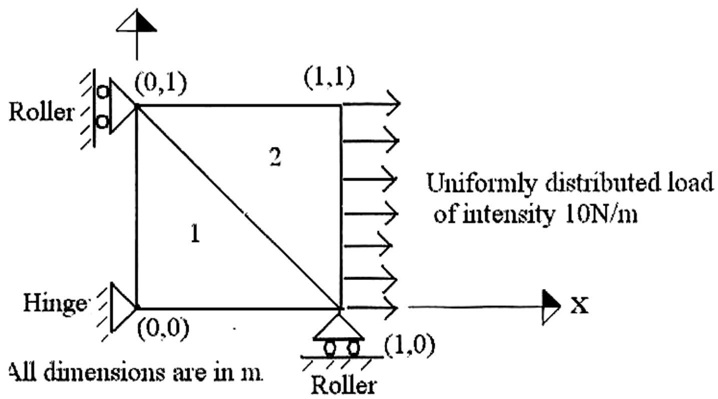

Figure Q4

An assembly of two constant-strain triangles is shown in Figure Q4. Assume **plane stress** conditions. Take thickness as $t = 0.01m$. **<u>All dimensions are in meters</u>**.

For the sake of calculation take $\dfrac{E}{1-\mu^2} = 200GPa$ and $\mu = 0.25$

Use the relation $N_i = \dfrac{1}{2\Delta}(a_i + b_i x + c_i y)$

Where, $a_1 = x_2 y_3 - x_3 y_2 \quad b_1 = y_2 - y_3 \quad c_1 = x_3 - x_2$

a) How many degrees of freedom does this system have after elimination of the boundary conditions?
b) Assemble the element stiffness and the force vector only for the effective (free) degrees of freedom

Question 5

a) When can we use an axisymmetric (ring) element?

b) What are the displacement variables for such problems? Do the displacement variables vary in the circumferential direction?

c) What are the stress components for an axisymmetric problem?

d) Write down the stress-strain matrix [D]

e) Starting from the shape function of CST ($N_i = a_i + b_i x + c_i y$) write down the [B] matrix for a 3 node axisymmetric element.

f) Mention a simple process for integrating $\int_{A^e} [B]^T [D][B]\, r dA^e$ for the above element.

Question 6

(a) Derive the shape functions for a 4-node quadrilateral isoparametric finite element

(b) Sketch the shape functions

(c) Describe the process of forming the stiffness matrix for this element

(d) Consider a **4 node** quadrilateral isoparametric element as shown in Figure Q6. Find out the Jacobian matrix.

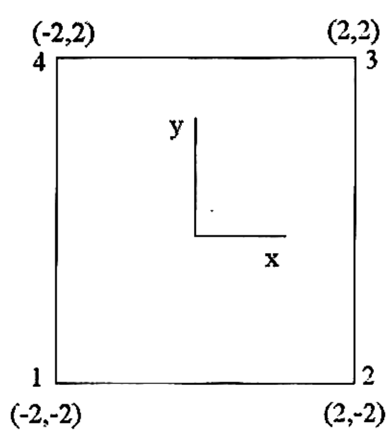

Figure Q6

Question 7

a) Write down the shape functions of a nine-node isoparametric quadrilateral using Lagrange interpolation function

b) Sketch the above shape functions.

c) Evaluate the integral $\int_{-1}^{1}\int_{-1}^{1} r^3 s^3 drds$. Use 2 point and 3 point Gauss quadrature rule. Use the data given in Table 1. Are the results same? Explain your answer.

Table 1. Data for 2 point and 3 point Gauss quadrature rule

Number of points	Locations	Weights
2	±0.57735 02691 89626	1.00000 00000 00000
3	±0.77459 66692 41483 0.00000 00000 00000	0.55555 55555 55556 0.88888 88888 88889

Question 8

(a) Mention the assumptions for Kirchoff plate theory

(b) What are the stress and strain components to be considered in a Kirchoff plate bending element? What are stress resultants?

(c) For a plate bending element prove the moment curvature relation –

$$M_x = -D(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2})$$

(d) Write down the expression for strain energy using stress resultant and curvatures.