B.E. MECH ENGG SECOND YEAR SECOND SEMESTER EXAM, 2019

Time 3.0 Hrs. | **ENGINEERING MECHANICS – IV** | **Full Marks:100**

[**Answer Any Eight (8) Questions taking at least Three (3) Questions from each Group**. Assume any missing data with suitable justifications. Each Question carries **12** marks. **4** Marks are allotted for neat and precise answers]

Group - 1

Q1. For any vector $\vec{A}$ as seen from a rotating reference frame x-y-z with angular velocity vector $\vec{\Omega}_{xyz}$, prove the following relations:-

i) $$\left(\frac{d\vec{A}}{dt}\right)_{XYZ} = \left(\frac{d\vec{A}}{dt}\right)_{xyz} + \vec{\Omega}_{xyz} \times \vec{A}$$

ii) $$\left(\frac{d^2\vec{A}}{dt^2}\right)_{XYZ} = \left(\frac{d^2\vec{A}}{dt^2}\right)_{xyz} + \frac{d\vec{\Omega}_{xyz}}{dt} \times \vec{A} + 2\vec{\Omega}_{xyz} \times \left(\frac{d\vec{A}}{dt}\right)_{xyz} + \vec{\Omega}_{xyz} \times \left(\vec{\Omega}_{xyz} \times \vec{A}\right)$$

Q2. Refer to **FigQ2**. The electric motor shown in the figure carries a disk which rotates at the constant rate of **120 rev /min** (this angular velocity is represented as ω_0 in the fig). Simultaneously, the supporting platform is rotating at a constant rate of N= **30 rev/min.** Assuming the system is not rotating about the x- axis at the instant shown, find the expressions of **velocity** and **acceleration vectors** of a point **A** as marked on the disk. Also, determine the **angular acceleration** of the disk with its proper sense.

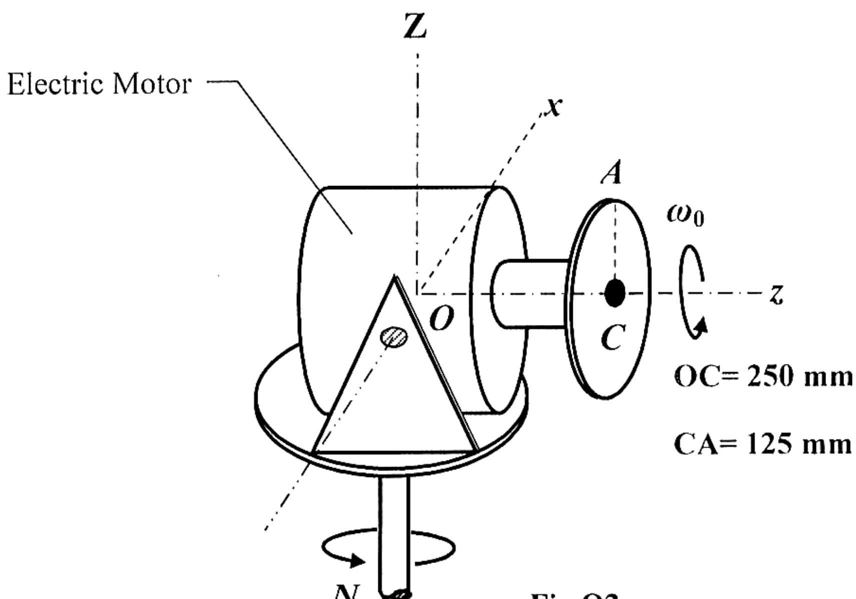

Fig Q2

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Q3. Refer to **FigQ3**. The circular disk of **100 mm** radius rotates about its z-axis at the constant speed $\boldsymbol{p}$ = **240 rev/min**, and the arm **OCB** carrying the bearing **B** of the disk rotates about the ***Y***-axis at the constant speed N= **30 rev/min**. Determine the velocity and the acceleration vectors of a point marked as **A** on the disk as it passes the position shown. **Use reference axes *x-y-z* attached to the arm OCB**.

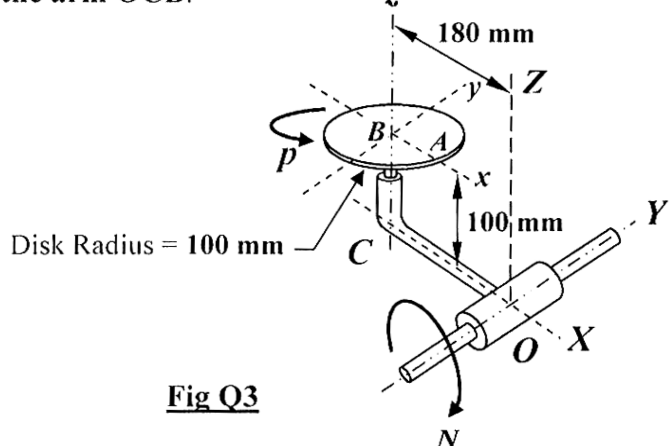


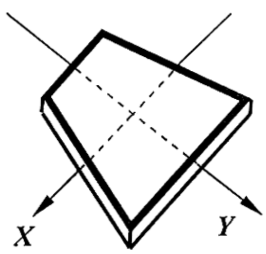
Fig Q3

Q4a) Prove the following transformation theorem of the mass-moment of inertia matrix of a rigid body:-

$$[I]_{x'y'z'} = [T][I]_{xyz}[T]^T$$; where $[T]$ represents the axis transformation matrix.

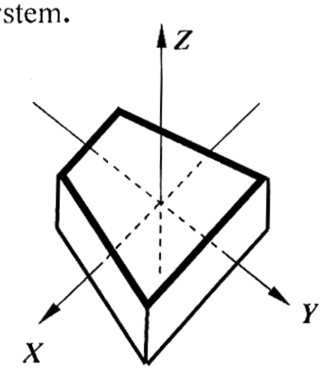
b) For the bodies shown in the following figure, write the **only the structure** of the **mass moment of inertia matrix** about the given co-ordinate system.

i)

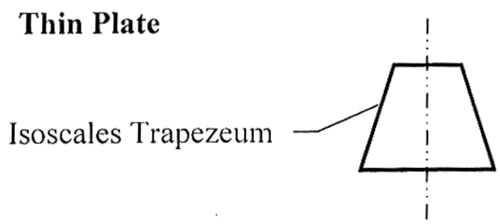


Thin Plate

ii)

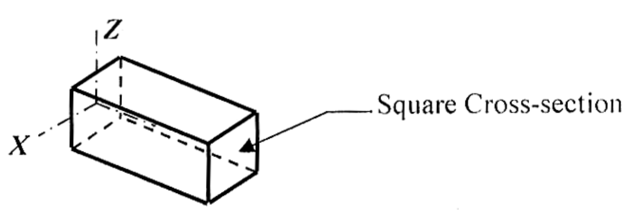


Thick Plate



Top View of both Bodies in **i)** and **ii)**

iii)



Q5. Stating clearly all necessary **assumptions**, derive Euler's equations of motion of rigid body under rotation.

Q6. Refer to **FigQ6**. The **5.0 kg** disk and hub **A** have a radius of gyration of **85 mm** about the z_0 axis and spinning at the rate of $\boldsymbol{p}$= **1250 rev/min**. Simultaneously, the assembly rotates about the vertical z-axis at the rate of $\boldsymbol{\Omega}$ = **400 rev/min**. Applying **Euler's theorem** calculate the bending moment vector $\vec{\boldsymbol{M}}_0$ developed at point **O** in the bent shaft **OBC**. Neglect the mass of the bent shaft but otherwise account for all forces acting on it.

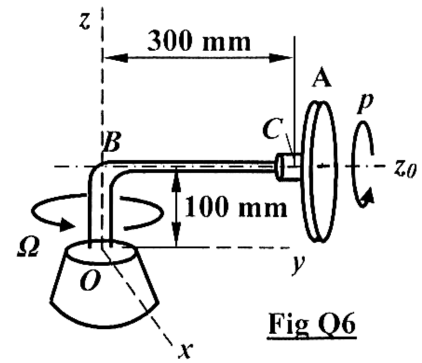
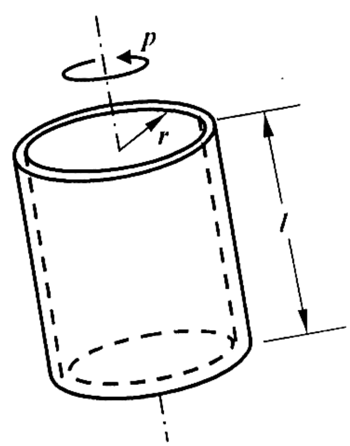


Fig Q6

Q7. What do you mean by "**Torque Free Motion**" or "**Precession with Zero Torque**"? Deduce the expressions of **rate of precession** and the **rate of spin** of an axi-symmetric body moving under zero moment (torque) in terms of **angular momentum**, **nutation angle** and the **mass momenta of inertia** of the body.

The cylindrical shell is rotating in space about its geometric axis as shown in **FigQ7**. If the axis of the shell has a slight wobble, for what ratios of l/r will the motion be **direct** or **retrograde precession**?

Fig Q7



Q8(a) State and explain the conditions for complete balance of a rotor. Point out which condition is for static balance and which is for dynamic balance.

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(b) Refer to **FigQ8**, where the shaft shown is massless and is carrying two identical square plates each of mass m. Calculate at the instant shown, the **dynamic reactions (i.e. neglecting any static load)** developed in bearings **A** and **B**. Assume that the shaft is rotating with a constant speed of ω rad/s.

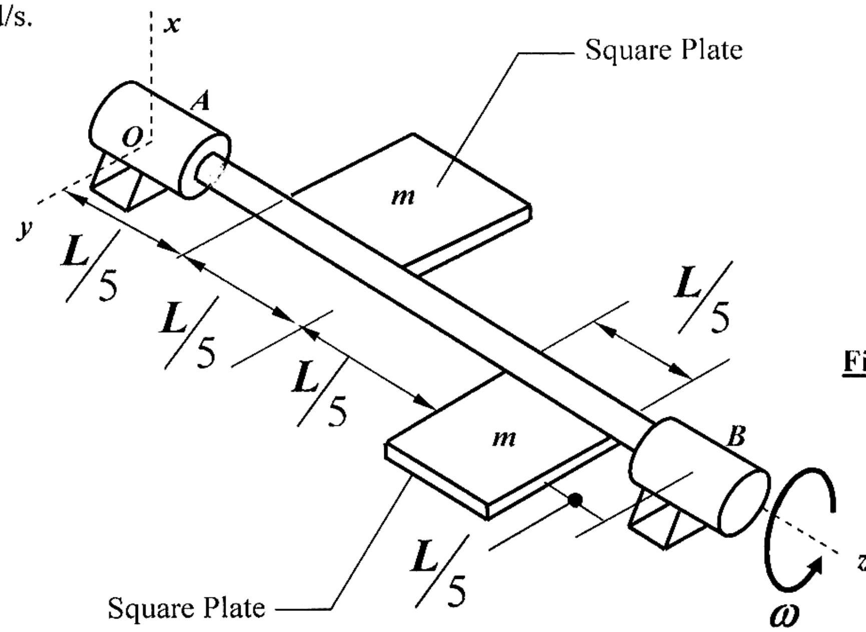
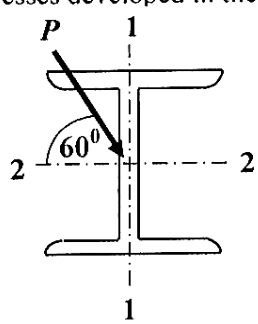


Fig Q8

Group – 2

Q9. Refer **Fig Q9**. A simply-supported beam of length **4.0 m** carries a load of P = **10KN** at its mid span. The cross section of the beam is **ISLB 150** whose relevant properties are given in the adjoining table in the figure. Find the orientation of the neutral axis and draw it. Specify in the diagram which side of the neutral axis is in tension and which in compression. Also calculate the maximum tensile and compressive stresses developed in the section.



View "A-A"

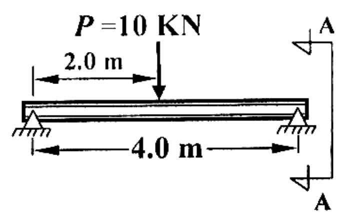


Fig Q9

Depth of Beam (mm)	Flange Width (mm)	Area of Section (mm^2)	Area Moment of Inertia about **1-1** axis (mm^4)	Area Moment of Inertia about **2-2** axis (mm^4)
150	**80**	**180.8**	**55.2×10^4**	**668.2×10^4**

Q10. Show from the **first principles** that the stress at the centre of a solid rotating disc is given by:–

$$\sigma_r = \sigma_\theta = \frac{3+\mu}{8}\rho\omega^2 b^2$$. Symbols have been used to carry their usual meanings.

Q11. Prove that for a curved beam – neutral axis is not the centroidal axis. From **first principles**, derive the Winkler's stress equation for a curved beam:

$\sigma = \dfrac{M(R - R_n)}{AeR}$. In the equation R_n is the radius of curvature of the neutral line and $e = R_n - \bar{R}$ where $\bar{R}$ is the centroidal radial distance.

Q12. Derive the expressions of Octahedral normal stress and Octahedral shear stress developed at a point of a deformable body. Assume the three-dimensional stress state at the point. Also write the expressions of the above stresses in terms of the Cartesian stress components:- $\sigma_{xx}, \tau_{xy}, \tau_{xz}, \tau_{yx}, \sigma_{yy}, \tau_{yz}, \tau_{zx}, \tau_{zy}, \sigma_{zz}$.

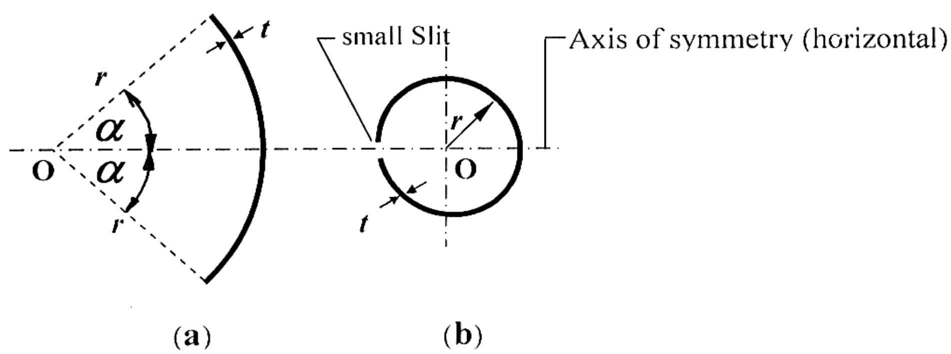
Q13. Refer **Fig Q13.** Part (**a**) shows a cross-section of a thin walled beam having its mean radius r and wall thickness t ($r >> t$). Prove that the **area moment of inertia** of the section about the horizontal axis of symmetry line is given by:

$$\frac{r^3 t}{2}(2\alpha - \sin 2\alpha)$$

Further, prove that the **shear centre** (**SC**) of the section is at a distance e from the centre of curvature **O** of the section, where:

$$e = 4r\frac{\sin\alpha - \alpha\cos\alpha}{2\alpha - \sin 2\alpha}$$

Part (**b**) of **FigQ13** shows the thin-walled section of the beam with an infinitesimally small slit in it. Using the above result, locate the shear centre from the centre point **O** of the section.



Q14.A thick cylinder having internal diameter **1000 mm** and external diameter **1200 mm** is subjected to **uniform external pressure**. If the stress developed in the cylinder is limited to **160 MPa** both in tension and compression and the shear stress is not to exceed **90 MPa**, then calculate the maximum permissible pressure intensity that can be applied. Also, find the percentage change in wall thickness of the cylinder. Assume Young's modulus of elasticity and Poisson's ratio of the cylinder material be **200 GPa** and **0.25** respectively. **Consider only radial and circumferential stresses in your calculation**.