b) State Tchebycheff's inequality. If X is a non negative random variable having mean 'm', then prove that

$$P(X \geq \tau m) \leq \frac{1}{\tau}, \quad T > 0.$$ 2+2

14. a) Find the condition for two variables $U = aX + bY$, $V = cX + dY$ to be uncorreleated. 5

b) If the regression lines are $3x + 2y = 5$ and $2x + 4y = 7$ for two random variables X and Y then find the means of the two variables and the correlation coefficient between the two random variables. 5

Bachelor of Engineering in Mechanical Engineering Examination, 2019

(2nd Year, 2nd Semester)

Mathematics - IV

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each group.

PART - I

Notations and Symbols have their usual meanings

Answer ***any five*** questions :

1. a) Show that, a non empty subset W of a vector space V over a field F is a subspace of V iff $a\alpha + b\beta \in W$, when $a, b \in F$ and $\alpha, \beta \in W$.

b) Define direct sum of two vector subspaces U and W of a vector space V. Then prove that, it is the smallest subspace of V containing U and W. 5+5

2. a) Find a basis and dimension of the subspace W of $\mathbb{R}^3$ where

$$W = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0, \ 2x + y + 3z = 0\}$$

b) Check whether $\{(1, 2, 3), (4, 5, 6), (7,8, 9)\}$ is a basis of $\mathbb{R}^3$ or not. 5+5

[Turn over

3. Let U and W be two subspaces of a finite dimensional vector space V over $\mathbb{R}$, then prove that

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$$ 10

4. a) Determine the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$, which maps the basis vectors $\{(0, 1,1), (1, 0, 1), (1, 1, 0)$ of $\mathbb{R}^3$ to $\{(1, 1, 1), (1, 1, 1), (1, 1, 1)\}$.

   b) Define orthogonal vectors in a Euclidean space V. If α, β are two orthogonal vectors in V, then prove that

$$\| \alpha + \beta \|^2 = \| \alpha \|^2 + \| \beta \|^2$$ 5+5

5. Obtain an orthonormal basis of $\mathbb{R}^3$ using Gram-schmidt process for the vectors $\{(1, 0, 1), (1, 0, -1), (0, 3, 4)\}$ 10

6. a) Let V and W be vector spaces over a field F. Let $T: V \to W$ be a linear mapping. Then prove that T is injective if and only if $\text{Ker } T = \{\theta\}$.

   b) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^2$ is defined by

$$T(x_1, x_2, x_3) = (3x_1 - 2x_2 + x_3, x_1 - 3x_2 - 2x_3),$$

$$\forall (x_1, x_2, x_3) \in \mathbb{R}^3.$$

   Find the matrix of T relative to the ordered bases

   ((0, 1, 1), (1, 0, 1), (1, 1, 0)) of $\mathbb{R}^3$ and ((1, 0), (0, 1)) of $\mathbb{R}^2$. 5+5

   b) Derive the Poisson approximation of Binomial distribution, clearly stating the assumptions. 4

11. a) A man is given 'n' keys of which one fits the lock. He tries them successively without replacement. Find the probability that at the n-th trial the lock will open. 5

   b) Thousand tickets are sold in a lottery in which there is one prize of Rs. 500, four prizes of Rs. 100 each and five prizes of Rs. 10 each. A ticket costs Re 1. If X is your net gain when you buy one ticket, then find E(X). 5

12. a) Let f(x) be the p.m.f. of a discrete random variable X for $x = 0, 1, 2, \ldots$.

   Given that $f(x) = \frac{\lambda}{x} f(x-1)$, for $x = 1, 2, \ldots$

   Deterimne f(x). 5

   b) Among n coins, (n – 1) are of the usual type, while one has head on both sides. A coin is chosen at random and tossed K times. If the coin falls head each time, what is the probability that it is the unusual coin ? 5

13. a) Find the mean and variance of the Binomial β (n, p) variate. 6

PART - II

The figures in the margin indicate full marks.

Answer ***any five*** questions 5×10

(Notations have their usual meanings)

8. a) State the axioms of probability. Write down the law of addition of probability for any three events. 3+2

   b) There are 'r' red balls and 'b' black balls in each of n boxes. One ball is transferred from the 1st to the 2nd box. Then one ball is transferred from the 2nd to the 3rd urn and so on. Finally one ball is drawn from the nth box. Find the probability that the ball is a black ball. 5

9. a) Define Bernoulli trials. In 'n' Bernoulli trials with probability of success 'p' p_r denotes the probability of 'r' success. Find the value of 'r' for which p_r is maximum. 2+5

   b) In a Bernoullian sequence of n trials with probability of success p, find the probability that the i th success occurs at the n th trial. 3

10. a) Prove that the probability of at least k successes in 'n' Bernoulli trials is

$$\int_0^p x^{k-1}(1-x)^{n-k}\,dx \Big/ \int_0^1 x^{k-1}(1-x)^{n-k}\,dx$$ 6

7. a) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

   b) A is a 3×3 real matrix having the eigen values 2, 3, 1. $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ are the eigen vectors of A corresponding to the eigen values 2, 3, 1 respectively. Find the matrix A.