Bachelor of Engineering in Mechanical Engineering Examination, 2019

(1st Year, 1st Semester)

MATHEMATICS - I

Use a separate Answer Script for each Group
(Symbols/ Notations have their usual meanings)

Group – A (50 marks)
Answer any five questions

1.(a) If $y = \sin^{-1} x$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.
(b) Verify Rolle's theorem for the function $f(x) = x(x+2)e^{-x/2}$ in [-2, 0].

5+5

2.(a) State Lagrange Mean Value theorem.
Use Lagrange Mean Value Theorem to prove that when $x > 0$,

$$0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1.$$

(b) Interpret Rolle's theorem geometrically.

6+4

3.(a) Expand $f(x) = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$ in series by Maclaurin's theorem.
(b) Evaluate

$$\underset{x \to 0}{Lt} \frac{e^x - e^{x \cos x}}{x - \sin x}.$$

5+5

4 (a) Find the radius of curvature of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at the point (x, y).
(b) Find the asymptotes of the curve $x^3 + 2x^2 y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$.

5+5

5(a) Expand $(x^2 y + 3y - 2)$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem for several variables.

(b) If u is a homogeneous function of degree n in two independent variables x and y, show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

5+5

6(a) If $u = f(r)$ where $r^2 = x^2 + y^2$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

(b) If $\frac{x^2}{a^2+u^2} + \frac{y^2}{b^2+u^2} + \frac{z^2}{c^2+u^2} = 1$ then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right).$$

5+5

7(a) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm.

(b) Test the convergence of the series

$$\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \ldots\ldots\ldots\ldots$$

5+5

Ref. No.: EX/ME/MATH/T/111/2019(Old)

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester)

Mathematics-I

Time: Three hours | Full Marks: 100

(**50** marks for each Group)

(Symbols and notations have their usual meanings)

Use a separate Answer-Script for each Group

GROUP-B (50 Marks)

Answer ***Q. No. 8*** and any ***three*** from the rest.

8. Evaluate $\int_0^a x^9 \sqrt[3]{a^6 - x^6}$ dx, where a (>0) is a constant. 5

9. a) State and proved fundamental theorem for integral calculus. 8

   b) Show that $\int_0^{\frac{\pi}{2}} e^{-\sin x}\, dx < \frac{\pi}{2}\,(1 - e^{-1})$. 7

10. a) Find the volume of the solid obtained by revolving the cardioide $r = a(1 + \cos\theta)$ about x-axis. 8

    b) Evaluate $\iint_D \sqrt{4a^2 - x^2 - y^2}\; dxdy$, where region D is the upper half of the circle $x^2 + y^2 - 2ax = 0$. 7

11. a) Evaluate $\iint_R \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}}\, dxdy$, where R is the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 7

    b) A function f defined on $[a, b]$ by $f(x) = x^2$. Find $\underline{\int_a^b} f\, dx$ and $\overline{\int_a^b} f\, dx$. Examine whether f is Riemann integral in $[a, b]$. 8

12. Examine the convergence of the following improper integrals:

    a) $\int_0^1 \frac{dx}{\sqrt{(1-K^2x^2)(1-x^2)}}$, $K^2 < 1$ b) $\int_0^\infty \frac{\sin x}{x} dx$ c) $\int_0^\infty \left(\frac{1}{1+x} - \frac{1}{e^x}\right)\frac{dx}{x}$ 15

13. a) Changing the order of integration, evaluate $\int_0^\infty \int_0^\infty e^{-xy} \sin nx\; dxdy$. Hence deduce that $\int_0^\infty \frac{\sin nx}{x}$ dx $= \frac{\pi}{2}$. 7

    b) Calculate the value of $\int_0^1 \sin x^2\; dx$ using (i) Trapezoidal rule, (ii) Simpson's $\frac{1}{3}$ rule by taking ten sub- intervals. 8

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