BACHELOR OF ENGINEERING IN (MECHANICAL ENGINEERING) EXAMINATION, 2019

(1st Year, 2nd Semester)

MATHEMATICS - II

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five*** questions. 10×5=50

All questions carry equal marks.

1. a) Express the following determinant as a square of another determinant and hence find its value :

$$\begin{vmatrix} a^2-bc & b^2-ca & c^2-ab \\ c^2-ab & a^2-bc & b^2-ca \\ b^2-ca & c^2-ab & a^2-bc \end{vmatrix}$$

b) Evaluate the determinant

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} \text{ where } 2s = a+b+c.$$

5+5

[Turn over

2. a) Define symmetric and skew-symmetric matrices with examples.

   Show that every square matrix can be uniquely expressed as a sum of a symmetric and a skew-symmetric matrices.

   b) Let Δ be a determinant of order n and Δ^1 is the adjugate of the determinant Δ. Prove that $\Delta^1 = \Delta^{n-1}$ (Include the case when $\Delta = 0$). 5+5

3. a) Prove that $(AB)^t = B^t A^t$, (where A^t is the transpose of A). Verify the formula for the following matrices :

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

   b) Find the inverse of the matrix

$$S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and if } A = \frac{1}{2}\begin{bmatrix} 4 & -1 & 1 \\ -2 & 3 & -1 \\ 2 & 1 & 5 \end{bmatrix},$$

   show that SAS^{-1} is a diagonal matrix diag (2, 3, 1).

6+4

4. a) Determine the values of p such that the rank of

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ p & 2 & 2 & 2 \\ 9 & 9 & p & 3 \end{bmatrix} \text{ is } 3.$$

   b) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

5+5

5. a) Show that the lines whose direction cosines are given by the equations $l + m + n = 0,\ al^2 + bm^2 + cn^2 = 0$ are

   i) perpendicular, if $a + b + c = 0$,

   ii) parallel if $a^{-1} + b^{-1} + c^{-1} = 0$.

   b) Find the magnitude and equations of the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

   Find also the points where it intersects the lines. 5+5

6. a) Find the equation of a plane such that the direction Cosines of the normal of the plane are a, b, c and its perpendicular distnace from the origin is p.

b) Show that the lines $x+2y-5z+9=0=3x-y+2z-5$, $2x+3y-z-3=0=4x-5y+z+3$ are coplanar. 5+5

7. a) Find the surface generated by the lines which intersect the lines $y=mx,\ z=c$; $y=-mx,\ z=-c$ and x-axis.

b) Find the equation of the sphere for which the circle $x^2+y^2+z^2+2z-4y+2z+5=0$, $x-2y+3z+1=0$ is a great circle. 5+5

PART - II

Answer ***any five*** questions. 10×5=50

All questions carry equal marks.

8. a) i) Prove that the area of the triangle whose vertices are $\vec{A},\vec{B},\vec{C}$ is $\frac{1}{2}|\vec{B}\times\vec{C}+\vec{C}\times\vec{A}+\vec{A}\times\vec{B}|$

ii) Calculate the area of the triangle whose vertices are A(1, 0, –1), B(2, 1, 5) and C(0, 1, 2).

b) Prove that the perpendicular bisectors of the sides of a triangle concur at its circumcentre. Prove it vectorially.

4+6

9. a) A line makes angles $\alpha,\beta,\gamma,\delta$ with diagonals of a cube. Show that

$$\text{Cos}^2\alpha+\text{Cos}^2\beta+\text{Cos}^2\gamma+\text{Cos}^2\delta=\frac{4}{3}$$

b) Find the curvature and torsion of the curve

$x=a\cos t,\ \ y=a\sin t,\ \ z=bt$ 4+6

8. a) What is the directional derivative of at the point (2, –1, 1) in the direction of the normal to the surface $x\log z-y^2=-4$ at (–1, 2, 1) ? Also calculate the magnitude of the maximum directional derivative.

b) Suppose and are two times partialy differentiable vector point functions. Show that

i) Curl Curl $\vec{F} =$ grad div $\vec{F} - \nabla^2\vec{F}$

ii) $\text{grad}(\vec{F}\cdot\vec{G}) = (\vec{F}\cdot\nabla)\vec{F} + \vec{F}\times \text{Curl}\,\vec{G} + \vec{G}\times\text{Curl}\,\vec{F}$.

4+6

11. a) If r is the distance of a point (x, y, z) from the origin, prove that

$$\text{Curl}\left(\hat{k}\times\text{grad}\frac{1}{r}\right)+\text{grad}\left(\hat{k}\cdot\text{grad}\frac{1}{r}\right)=0$$

where $\hat{k}$ is the unit vector in the direction of z-axis.

b) State and prove Stoke's theorem. 4+6

12. a) Use Stoke's theorem to evaluate

$$\int_C [(x+y)dx + (2x-z)dy + (y+z)dz]$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

b) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2+1,\ y = 2t^2,\ z = t^3$ from t = 1 to t = 2. 5+5

13. a) Evaluate $\iint_S \vec{F}\cdot\hat{N}ds$

where $\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$, S is the part of the plane $2x + 3y + 6z = 12$ which is located in the first octant, and is the unit normal vector to the surface S.

b) Verify Green's theorem in the plane for

$$\int_C (xy + y^2)dx + x^2dy$$

which C is the closed Curve of the region bounded by $y = x^2$, and $y = x$. 5+5

14. State Gauss divergence theorem.

Verify this theorem for $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube $0 \le x, y, z \le 1$ 10