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M.E. MECH. ENGG. 1ST YR. 2ND SEM. EXAM., 2019

Theory of Plasticity

Duration 3 hrs **Full Marks 100**

Assume a reasonable value for any missing data.

Group A (*Answer any two*)

1. State Hill's principal of maximum dissipation. From this derive Prandtl-Reuss equations. Derive the expression for the ratio of the components of the plastic strain rate in uniaxial tensile loading. [3+10+7=20]

2. Derive the expression for equivalent stress from the Distortion energy theory. Determine the expression for the maximum equivalent stress for a shaft undergoing combined bending and torsion where the torsional torque is half the bending moment.

[10+10=20]

3. Derive the instability criteria for a thin pressure vessel clearly stating the assumptions. Determine the tensile force at which a bar of 6 mm dia enters into necking. The constitutive relation given in true stress stains is $\sigma = 500 - \frac{300}{1+\varepsilon}$. [12+8=20]

Group B (*Answer any two*)

4. What are isotropic hardening and kinematic hardening? What is Bauschinger effect? Use radial return algorithm to evaluate stress for the following data. Symbols have their usual meanings.

$E = 200\ Gpa,\ \nu = 0.3,\ \sigma_y = 200\ MPa,\ H = 500\ MPa,\ \sigma^0 = [180,\ 0,\ 0, 0, 0, 0]^T MPa$

$\Delta\varepsilon = [0.001, -0.001\nu, -0.001\nu, 0, 0, 0]^T$. [5+5+10]

5. Derive the radial return algorithm for elasto-plasticity with linear kinematic hardening. Also derive the expression for the consistent tangent. [20]

6. What is deformation gradient? Derive the expression for Green's strain and write the expression of small strain and discuss the correlation between the two. Compare Green's strain and small strain as the true measures of strains through mathematical derivations. Show with derivation how true stress and true strain can be evaluated from their engineering counterparts. [5+5+5+5=20]

Group C (*Answer any one*)

7. Through mathematical derivations for elastic-perfectly plastic deformation in torsion of a solid circular shaft determine the expressions for-
   a. Torque required to initiate plastic deformation.
   b. Maximum torque bearing capacity.
   c. Torque required to turn the radius of the elastic core half the original radius.
   d. The residual stress distribution after unloading from case c.

   A beam of length 1m and a rectangular cross section (width=25 mm, height=5 mm) and in simply supported configuration, is carrying a point load (P) in the middle. The material is elastic-perfectly plastic with $\sigma_y = 120\ MPa$. Determine-
   e. Value of P when plasticity is introduced.
   f. Maximum load bearing capacity.
   g. Value of P for the boundary of the elastic core to be $\pm$ 1 mm off the NA.
   h. The residual stress distribution after unloading from case g. (plot on a graph paper) [10+10=20]

8. A thick pressure vessel has inner and outer radii 50 mm and 63.5 mm respectively. If the yield stress in tension be 180 MPa. Assuming material to be elastic-perfectly plastic determine-
   a. Value of the pressure (p_0) when plasticity is introduced. Plot the stress distributions on a graph paper.
   b. Maximum pressure (p_{max}) that can be applied. Plot the stress distributions on a graph paper.
   c. Size of the elastic shell when $p = \frac{p_{max}+p_0}{2}$. Plot the stress distributions on a graph paper.
   d. The residual stress after unloading from case c. Plot the stress distributions on a graph paper. [20]