Ref. No. : Ex/PG/ME/T/127A/2019

M.E. MECHANICAL ENGINEERING FIRST YEAR SECOND SEMESTER EXAM – 2019

Subject: ROTOR DYNAMICS **Time: 3 Hours** **Full Marks: 100**

Answer any four questions.

Any missing information can be assumed suitably with appropriate justification.

Q1. (a) Derive the equations of motion (in Cartesian coordinate system) for lateral vibrations of a Jeffcott rotor with mass-unbalance, mounted on two identical flexible bearings (**Fig. Q1a**). Consider that m is the mass of the rotor, e is the unbalance eccentricity, k_S is the shaft stiffness, k_Y and k_Z are the stiffness coefficients in each bearing in XY and XZ-planes respectively, c is the constant damping coefficient against the vibration in both Y and Z directions and Ω is the constant spin speed of the rotor about the X axis. The force due to weight can be assumed as negligible compared to the force due to unbalance. Clearly state other assumptions relevant to this derivation. In the process show the equivalent stiffness of the system along Y- and Z-directions. **[8]**

(b) Find out the expressions of the non-dimensional magnification factors (ratio of synchronous response amplitude to unbalance eccentricity) and the relative phase angles between the excitation and response along each of the Y and Z directions in terms of the frequency ratio (ratio of spin frequency to natural frequency of vibration along the corresponding direction) and the damping ratio. Show the variation of the unbalance response in terms of the plot of the magnification factors versus frequency ratio and the phase angle versus frequency ratio for different possible values of damping ratio. **[10]**

(c) Find out the expressions of maximum possible magnification factor along any of these directions and the corresponding frequency ratio as a function of effective damping ratio in that direction. **[7]**

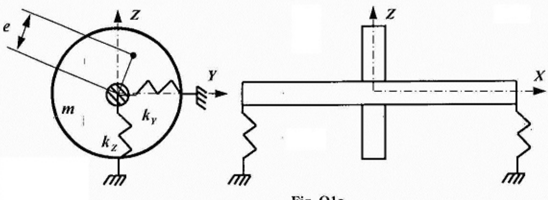

Fig. Q1a

Q2. (a) A rigid symmetric cylindrical rotor is mounted on two identical isotropic undamped flexible bearings at its ends as shown in **Fig. Q2a**. m, J_P, J_T, L and Ω are the mass, principal polar mass moment of inertia, principal transverse mass moment of inertia, length of the rotor between bearings and spin speed of the rotor respectively. k is the stiffness of each bearing along each direction.

Starting from the expressions of angular velocity vector of the system (the derivation of angular velocity vector is not required) in the rotor-fixed reference frame, derive the rotational kinetic energy of system. Consider gyroscopic effect in your derivation. Clearly show with neat sketches different coordinate systems used in the derivation. [8]

(b) Derive the equations of motion for free vibration (for both the cylindrical and the conical mode) for such a system using Lagrange's principle. [8]

(c) How many natural frequencies are possible for this rotor at a given spin speed? Find out their expression and plot them versus spin speed in a Campbell diagram. Present your answer in terms of non-dimensional variables. Hint: you may use complex variables for finding the expressions of natural frequencies. [9]

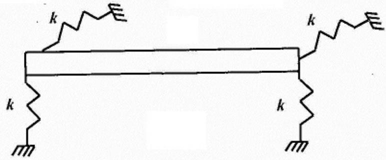

Fig. Q2a

Q3. (a) Consider a Jeffcott rotor mounted on a pair of identical journal bearings at its ends. Write down the relevant equations of motion for such a system under unbalance excitation. Assume standard symbols for mass, shaft stiffness, bearing stiffness and damping coefficients, unbalance eccentricity, spin speed etc. What is the order of the characteristic polynomial of the corresponding system of homogeneous equation? How many eigenvalues are possible for this system? [5]

(b) Derive the expression of the dynamic stiffness matrix for the steady state synchronous vibration of the above-mentioned rotor-bearing system due to unbalance excitation. Given are the mass of the rotor, spin speed, stiffness of the shaft, linearized stiffness and damping coefficients of the bearings. [8]

(c) Find an expression of energy dissipation per cycle of rotation for such a rotor in terms of the bearing stiffness and damping coefficients, spin speed and amplitudes of steady state unbalance response components of journal centre. [12]

Q4. (a) Write the simplified form of Reynolds equation for a long hydrodynamic (plain cylindrical) bearing at steady state. With appropriate substitutions derive an expression of circumferential pressure distribution within such a bearing at steady state assuming full Sommerfeld boundary condition. [15]

(b) From the above expression of circumferential pressure distribution, show that the component of the resultant hydrodynamic force along the line joining the bearing centre and the journal centre is zero for this type of bearing. **[10]**

Q5. (a) Consider a uniform prismatic rotor having a non-circular cross-section area with two non-identical principal area moments of inertia along two principal orthogonal directions of the cross-section. Derive relevant equations of motion for free vibration of such a system rotating at a constant spin speed. Discuss the possibility of instability for such a rotor at a speed in between two natural frequencies of transverse vibration. **[10]**

(b) Derive an expression of **overall transfer matrix for free torsional vibration** of a torsionally compliant rotor having two discs as shown in **Fig. Q5b.** The stiffness and inertia of different parts are same as shown. Clearly indicate in a figure different nodes and sections of the system. Also clearly show the expressions of various field and point matrices during the derivation. From the overall transfer matrix find also the appropriate characteristics equation for free vibration of the system. Find out the lowest torsional natural frequency of the system if $\boldsymbol{K} = 10$ **kN/m** and $\boldsymbol{J} = 2$ $\mathbf{kg{\cdot}m^2}$. **[15]**

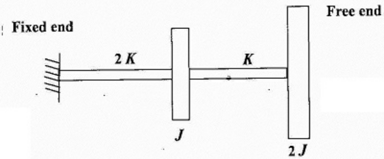

Fig. Q5b

Q6. (a) Write short notes on oil-whirl and oil-whip phenomena in case of a rotor supported on oil-film bearings. **[10]**

(b) With relevant sketches explain the **single-plane balancing method without phase measurement** in case of a short rigid rotor. **[10]**

(c) Describe briefly **two-plane balancing** of a rigid rotor with unknown unbalance using **complex influence coefficients.** **[5]**