MA 1ST YEAR 1ST SEM 2017

Econometrics I

(Ref.: EX/PG/ECO/14/6/2017)

Time: 2 Hours | Full Marks: 30

Answer any five of the following questions. $6 \times 5 = 30$

1. Consider the least square estimates of the model: $\mathbf{y}_{N\times 1} = \mathbf{X}_{N\times K}\boldsymbol{\beta}_{K\times 1} + \mathbf{u}_{N\times 1}$, where $\mathrm{E}(\mathbf{u}|\mathbf{X}) = \mathbf{0}$ and $\mathrm{E}(\mathbf{uu}'|\mathbf{X}) = \boldsymbol{\Sigma} = \sigma^2(\mathbf{I} + \mathbf{AA}')$, where $\mathbf{A}$ is an $N \times m$ matrix with $K < m < N$. Assume, for simplicity, that σ^2 and $\mathbf{A}$ are known.

   (a) Obtain the variance of the OLS estimator of $\boldsymbol{\beta}$.

   (b) Compare your answer in (a) with default OLS variance $\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$. Are the default OLS standard errors biased/inconsistent in any particular direction?

   (c) Determine the variance of the GLS estimator of $\boldsymbol{\beta}$, using the result $(\mathbf{I} + \mathbf{AA}')^{-1} = \mathbf{I}_N - \mathbf{A}(\mathbf{I}_m + \mathbf{A}'\mathbf{A})\mathbf{A}'$.

   (d) Compare the default variance $\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$ of OLS with the true variance of GLS. Does your finding violate that fact that GLS must be BLUE when disturbances are non-spherical?

2. Let $y = \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_K x_K + u$, where $\mathrm{E}(u) = 0$, $\mathrm{cov}(x_j, u) = 0\ \forall j = 1, 2, \ldots, K-1$, but x_K is correlated with u. Let $x_1 = 1$ to admit an intercept in the model. Let $\mathbf{x} = (x_1, x_2, \ldots, x_K)$ and $\mathbf{z} = (x_1, x_2, \ldots, x_{K-1}, z_1, z_2, \ldots, z_M)$. Assume $\mathrm{E}(\mathbf{z}'\mathbf{z})$ is non-singular. Let the linear projection of x_K on $\mathbf{z}$ be $x_K^* = \delta_1 x_1 + \ldots + \delta_{K-1}x_{K-1} + \theta_1 z_1 + \ldots + \theta_M z_M$. Prove that rank $\mathrm{E}(\mathbf{z}'\mathbf{x}) = K$, iff at least one θ_j is different from zero.

3. Derive attenuation bias clearly stating assumptions of the classical error in variables model.

4. Let the model be

$$y_t = \mathbf{x}_t\boldsymbol{\beta} + u_t,$$

   where $\mathbf{x}$ is $1 \times K$ and $u_t = \rho u_{t-1} + \varepsilon_t$, $|\rho| < 1$ and $\varepsilon_t \overset{iid}{\sim} (0, \sigma_\varepsilon^2)$. What is the autocorrelation of u in this model? Now consider the model

$$y_t - y_{t-1} = (\mathbf{x}_t - \mathbf{x}_{t-1})\boldsymbol{\beta} + v_t,$$

   where $v_t = u_t - u_{t-1}$. Compare autocorrelation of v with that of u.

5. Consider the model $y_i = \beta + u_i$, where y_i and u_i are random scalar variables and β is a scalar unknown parameter. u_i are iid with $E(u_i) = 0$, $E(u_i^2) = \beta^2$, $E(u_i^3) = 0$ and $E(u_i^4) = m$. What is the limiting distribution of the vector $\begin{pmatrix} \frac{1}{N}\sum_{i=1}^N y_i^2 \\ \frac{1}{N}\sum_{i=1}^N y_i \end{pmatrix}$?

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6. Let $\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, \hat{\theta}_2)'$ be $\sqrt{N}$-asymptotically normal estimator of $\boldsymbol{\theta} = (\theta_1, \theta_2)'$, with $\theta_2 \neq 0$. Let $\hat{\gamma} = \hat{\theta}_1/\hat{\theta}_2$ be an estimator of $\gamma = \theta_1/\theta_2$.

   (a) Find $Avar(\hat{\gamma})$ in terms of $\boldsymbol{\theta}$ and $Avar(\hat{\boldsymbol{\theta}})$.

   (b) If, for a sample of data, $\widehat{Avar(\hat{\boldsymbol{\theta}})} = \begin{pmatrix} 1 & -0.4 \\ -0.4 & 2 \end{pmatrix}$ and $\hat{\boldsymbol{\theta}} = (-1.5, 0.5)'$, find the asymptotic standard error of $\hat{\gamma}$.

7. Suppose family i chooses annual consumption c_i (in dollars) and annual contribution to a charitable fund q_i (in dollars) to solve the problem

$$\max_{c,q} \; c + a_i log(1+q)$$

subject to the constraint $c + p_i q \leq m_i$; $c,\ q \geq 0$, where m_i is the annual income of family i, p_i is the price of one dollar of charitable fund (where $p_i < 1$ because of tax-deductibility of charitable contributions) and this price differs across families because of different marginal tax rates and different state tax codes, $a_i \geq 0$ determines marginal utility of charitable contributions. Consider m_i and p_i to be exogenous to the family in this problem.

   (a) What is the optimal solution for q_i?

   (b) Define $y_i = 0$ if $q_i = 0$ and $y_i = 1$ if $q_i > 0$. Suppose $a_i = exp(\mathbf{z}_i\boldsymbol{\gamma} + v_i)$, where $\mathbf{z}_i$ is a $J \times 1$ vector of observable family traits and v_i is unobservable. Assume that v_i is independent of $(\mathbf{z}_i, m_i, p_i)$ and v_i/σ has symmetric distribution function $G(.)$, where $var(v_i) = \sigma^2$. Show that,

$$P(y_i = 1|\mathbf{z}_i, m_i, p_i) = G[(\mathbf{z}_i\boldsymbol{\gamma} - logp_i)/\sigma].$$

8. Let the loglikelihood function be $\ell(\boldsymbol{y}, \boldsymbol{\theta}) = \sum_{t=1}^{N} \ell_t(\boldsymbol{y}^t, \boldsymbol{\theta})$, where $\boldsymbol{y}^t$ is the vector $(y_1, y_2, \ldots, y_t)'$, $t = 1, 2, \ldots, N$. Prove that

$$E_\theta\left[\mathbf{g}(\boldsymbol{y},\boldsymbol{\theta})\mathbf{g}'(\boldsymbol{y},\boldsymbol{\theta})\right] = \sum_{t=1}^{N} E_\theta\left[\mathbf{G}_t'(\boldsymbol{y}^t,\boldsymbol{\theta})\mathbf{G}_t(\boldsymbol{y}^t,\boldsymbol{\theta})\right],$$

where E_θ represents expectation with respect to DGP characterized by $\boldsymbol{\theta}$, a typical element of $\mathbf{g}(\boldsymbol{y},\boldsymbol{\theta})$ is $g_i(\boldsymbol{y},\boldsymbol{\theta}) = \dfrac{\partial \ell(\boldsymbol{y},\boldsymbol{\theta})}{\partial \theta_i} = \sum_{t=1}^{N} \dfrac{\partial \ell_t(\boldsymbol{y}^t,\boldsymbol{\theta})}{\partial \theta_i}$, $i = 1, 2, \ldots, K$ and $\mathbf{G}_t(\boldsymbol{y}^t,\boldsymbol{\theta})$ is the t-th row of the matrix $\mathbf{G}(\boldsymbol{y},\boldsymbol{\theta})$ with typical element $G_{ti}(\boldsymbol{y}^t,\boldsymbol{\theta}) = \dfrac{\partial \ell_t(\boldsymbol{y}^t,\boldsymbol{\theta})}{\partial \theta_i}$.