Master of Arts Examination 2017
1st year, 1st sem
Economics
Mathematical Economics

Time: Two hours | Full Marks 30

Section A

Answer question 1 and either question 2 or question 3 from Section A.

1. Solve Cass- Koopman's optimal growth model using any of the three methods of dynamic programming you have learnt. 8

$$Max \sum_{t=0}^{\infty} \beta^t \ln C_t$$

subject to the constraints

$$(i) k_{t+1} = Ak_t^{\alpha} - C_t$$
$$(ii) k_0 = \bar{k}_0$$

2. Solve the optimal control problem. 7

$$Max \int_0^{10} (1 - 4x - 2u^2)$$
$$subject \quad to \quad \dot{x} = u$$
$$x(0) = 0, x(10) free$$
$$u \in \mathbf{R}$$

3. Write a note on Maximum Principle. 7

SECTION B

Answer any three questions. Each question carries 5 marks.

1) Use the principle of mathematical induction to show that the sum of the cubes of the first *n* natural numbers is $\frac{n^2(n+1)^2}{4}$. (5 marks)

2) Prove that every open interval, (a, b), where a and b are real numbers, contains a rational number. Use the above result to show that the set of rational numbers contained in (0,1) has neither a maximum not a minimum, an infimum at zero and supremum at 1. (3+2=5 marks)

3) What do you mean by "a sequence converging to a limit l"? Give an example of a sequence which does not converge to any limit. What is the limit that the following sequence converges to: $\frac{5n^3-2n^2+4}{5n^3-4n^2+2n}$? Explain by referring to the relevant theorems and showing all relevant steps.

(1+1 +3 = 5 marks)

4) Give examples (one each) of strictly increasing and strictly decreasing convergent sequences. Show that if a sequence is increasing and bounded above then it converges to its smallest upper bound. Also show that if a sequence is decreasing and bounded below then it converges to its largest lower bound. (2+1.5+1.5=5 marks)