BACHELOR OF ARTS EXAMINATION, 2017

(2nd Year, 4th Semester)

ECONOMICS (HONOURS)

ECONOMETRICS

Time : Two hours Full Marks : 30

Answer any five of the following questions. $6 \times 5 = 30$

1. A researcher obtains data on household annual expenditure on books, B, and annual household income, Y, for 100 UK households in 2003. He hypothesizes that B is related to Y and the average cognitive ability of adults in the household, IQ, by the relationship

$$\log B = \beta_1 + \beta_2 \log Y + \beta_3 \log IQ + u, \quad (1)$$

where u satisfies Gauss-Markov properties. He also considers the possibility that $\log B$ may be determined by $\log Y$ alone:

$$\log B = \beta_1 + \beta_2 \log Y + u. \quad (2)$$

It may be assumed that Y and IQ are both nonstochastic. In the sample the correlation between $\log Y$ and $\log IQ$ is 0.86. Assuming that equation (1) is the correct specification explain with a mathematical proof whether you expect the coefficient of $\log Y$ to be higher in regression (2).

2. For the regression model $Y_i = \beta X_i + u_i$, where $u_i \overset{i.i.d.}{\sim} (0, \sigma^2)$ and X is non-stochastic, let the two estimators of β be $\hat{\beta}_1 = \overline{Y}/\overline{X}$ and $\hat{\beta}_2 = \sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) / \sum_{i=1}^{n}(X_i - \bar{X})^2$. Show that the optimal (minimum variance) combination of $\hat{\beta}_1$ and $\hat{\beta}_2$ is given by $\hat{\beta} = \rho^2 \hat{\beta}_1 + (1 - \rho^2)\hat{\beta}_2$, where ρ is the correlation coefficient between $\hat{\beta}_3 = \sum_{i=1}^{n} X_i Y_i / \sum_{i=1}^{n} X_i^2$ and $\hat{\beta}_1$.

3. A researcher has data from the National Longitudinal Survey of Youth for the year 2000 on hourly earnings, Y, years of schooling, S, and years of work experience, X, for a sample of 1,774 males and 1,468 females. She performs the following regressions (standard errors in parentheses): Y on S and X on (1) the whole sample, (2) for males only and (3) for females only.

	(1)	(2)	(3)
S	0.094	0.099	0.094
	(0.003)	(0.004)	(0.005)
X	0.046	0.042	0.039
	(0.002)	(0.003)	(0.002)
constant	5.165	5.283	5.166
	(0.054)	(0.083)	(0.068)
R^2	0.319	0.277	0.363
RSS	714.6	411.0	261.6
Obs	3,242	1,774	1,468

[Turn over

The researcher hypothesizes that the earnings function is different for males and females. Perform a test of this hypothesis. $F^{crit}_{99\%}(3, 1000) = 3.80$.

4. Consider the multiple regression $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u$, along with the following auxiliary regressions $X_{2i} = \hat{a} + \hat{b}X_{3i} + \hat{v}_{2i}$ and $X_{3i} = \hat{c} + \hat{d}X_{2i} + \hat{v}_{3i}$. Now consider the regression $Y_i = \delta_1 + \delta_2 \hat{v}_{2i} + \delta_3 \hat{v}_{3i} + w_i$. Find the relationship between $\hat{\beta}$s and $\hat{\delta}$s.

5. Consider the regression model $Y_i = \beta X_i + u_i$, where $u_i \overset{i.i.d.}{\sim} (0, \sigma^2)$ and X is non-stochastic. The minimum mean-squared estimator of β is given by $\tilde{\beta} = \beta^2 \sum_{i=1}^n X_i Y_i / (\beta^2 \sum_{i=1}^n X_i^2 + \sigma^2)$. MSE of an estimator $\hat{\theta}$ is given by $bias^2(\hat{\theta}) + var(\hat{\theta})$, where $bias(\hat{\theta}) = E(\hat{\theta} - \theta)$. Show that MSE($\tilde{\beta}$) is less than MSE($\hat{\beta}_{OLS}$)

6. Consider the linear regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$, where $\mathbf{u} \sim (0, \sigma^2 \mathbf{I})$ and

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{21} & x_{31} \\ 1 & x_{12} & x_{22} & x_{32} \\ 1 & x_{13} & x_{23} & x_{33} \end{bmatrix}, \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}.$$

What is an unbiased estimator of σ^2?

7. Consider the following model

$$Y_i = \mu + \varepsilon_i,$$

where $E(\varepsilon_i | X_i) = 0$, $cov(\varepsilon_i,\ \varepsilon_j | X_i, X_j) = 0\ \ \forall\, i \neq j$, but $var(\varepsilon_i | X_i) = \sigma^2 X_i^2$. Given a random sample of observations on Y_i and X_i, what is the most efficient estimator of μ? What is its variance? What is the OLS estimator of μ and the variance of the OLS estimator? Compare the two variances.

8. Consider the model $Y_i = \alpha + u_i$, where u_is are independent with zero mean and $var(u_i) = \sigma_1^2$ for $i = 1, 2, \ldots, n_1$ and $var(u_i) = \sigma_2^2$ for $i - n_1 + 1, n_1 + 2, \ldots, n$ with $n = n_1 + n_2$. Find $\hat{\alpha}_{OLS}$ along with its variance. Find $\hat{\alpha}_{GLS}$ along with its variance. Compute the relative efficiency of GLS with respect to OLS.