

BACHELOR OF ARTS EXAMINATION, 2017

(1st Year, 1st Semester)

ECONOMICS (HONOURS)**MATHEMATICS FOR ECONOMICS**

Time : Two hours

Full Marks : 30

Answer question no.1 and any two from the following:

1. a) Prove that $P \leftrightarrow Q$ iff $Q \leftrightarrow P$

b) Show the validity of the following argument: If all cats are black then tiffin is black; it is not the case that all cats are black. Therefore tiffin is not black. Name the logic law applied or the nature of the error.

c) Given the function:

$$f(x) = \frac{x^2 + 4x - 2}{x^2 - 2x + 1}$$

Can it be said that $f(x)$ is bounded in $[1,4]$?

d) Let f be a function whose domain contains $-x$, whenever it contains x . If f is integrable on $[0,b]$ prove that:

i. $\int_{-b}^b f(x) dx = 2 \int_0^b f(x) dx$ whenever $f(x)$ is even function

ii. $\int_{-b}^b f(x) dx = 0$ whenever, $f(x)$ is an odd function.

e) Find the asymptotes & holes of the function $f(x) = \frac{(x-r)^2(x-k)}{(x-r)(x+k)}$ 2 x 5

2. a) Sketch the graph of the function: $f(x) = 5x^3 - 5x^2 + 5x - 5$. Does the function have an extremum?

b) State Roll's theorem and prove mean value theorem using Roll's theorem. 7+3

3.a) Find the domain & range of the function $f(x) = \frac{x^2 - a}{(x-b)(x-c)}$ where $a, b, c \in \mathbb{R}$ & $a > b > c$.

b) Prove the function $f(x) = ax + d$ is continuous using the definition of continuity.

c) Let $f(x) = 2x$ and $g(x) = x^2$, Check whether $f \circ g$ is surjective & / or injective

d) Use a truth table to prove the law of disjunctive syllogism

e) Suppose R is a relation on the set of integers such that xRy implies $2x - y = 1$. Is it true that the relation is neither reflexive nor irreflexive? 2 x 5

4.a) Assume f is continuous on $[a, b]$. If $\int_a^b f(x) dx = 0$ prove that $f(c) = 0$ for at least one c in $[a, b]$.

b) Is the function $f(x) = \frac{x-1}{x+1}$ continuous in the interval $(-2, 1)$? Explain your answer?

c) Answer without solving the equation between which of the following two values does the equation $3x^3 + 5x - 11 = 0$ have a solution?

a. Between -1, -2; b. between -1, 0; c. between 0, 1; d. between 1, 2

d) Given $K > 0$ prove that among all positive numbers x & y with $x + y = K$ the sum $x^2 + y^2$ is smallest when $x = y$.

e) Show that : $\neg(P \vee \neg Q) \vee (\neg P \wedge \neg Q) \Leftrightarrow \neg P$. (You can use either propositional logic laws or truth table) 2 x 5