i) Consider the production function Z = f(x,y) = x^3/y^2 defined over the domain x>0 and y > 0. Also, consider the functions G(Z) = InZ and J(Z) = Z^2 + b

Is G(Z) a homoethetic function ? Is G(Z) a homogeneous function in the arguments ? If so, what is its degree ? Is J(Z) = Z^2 + b a homothetic function or homogeneous function ? [2]

Bachelor of Arts Examination, 2017

(1st Year, 2nd Semester)

Economics (Honours)

Mathematical Economics I

Time : Two hours Full Marks : 30

1. a) Find the domain of the following functions :

$$f(x,y)=\sqrt{x^2-y^2}+\ln(x^2+y^2-1)$$

$$f(x,y)=\frac{1}{\sqrt{xy-3}}+\ln(\sqrt{y-x}-x)$$ [2]

b) Prove that a sequence can have at most one limit. [2]

c) The following functions are continuous at (0, 0). True or False ?

i) $$f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } (x,y)\neq 0\end{cases}$$

ii) $$f(x,y)=\begin{cases}\dfrac{4x^2y}{x^2+y^2} & \text{if } (x,y)\neq 0\\ 0 & \text{if } (x,y)=0\end{cases}$$ [2]

d) True or False : The function $3x^4-2x^3-6x^2+6x+1$ does not have any optima in the inverval [0. 2]. [2]

e) Find the value of 'b' for which the following cost function is continuous :

$$c(x) = \begin{cases} 10 \text{ for } x \le 20 \\ 5 + bx^2 \text{ for } > 20 \end{cases}$$

Find MC and when is the average cost minimum ? [2]

f) i) Draw and find the slope of the level curves of the following function [1+2]

$$f(x, y) = 2x^2 - 2y^2 + 78x + 66y - 2xy.$$

When the level curve will consist a single point. [1]

ii) Is this function quasi concave ? [2]

g) Suppose a consumer consumers three goods. The utility function is $U = (1+x)^{\alpha} y + Z^{\beta}$. Utility is maximized subject to the constraint $M = px + qy + z$.

i) Find the utility maximising levels of commodities. [2]

ii) Is the utility function concave ? Also check for quasi concavity (Using Bordered Hessian matrix). [3]

iii) Find price elasticity of demand for the commodities for variation in p and q. [1]

iv) How the consumption levels will vary for any variation in M. [1]

$$F(x, y) = -xy + 100x - x^2/2 - 2y^2 + 310y.$$ [1]

ii) Is this function quasi concave ? [2]

g) Suppose the utility function of a consumer consuming three goods is given by

i) $U(x, y, M) = 100 - e^{-2x} - e^{-m} - e^{-3y}$. The consumer maximizes utility subject to the constraint $x + y + m = V$

ii) Find the utility maximising levels of commodities. [2]

iii) Is the utility function concave ? Also check for quasi concavity. (Using Bordered Hessian matrix) [3]

iv) Find the restriction on the value for V for which the optimal commodities will be positive. [1]

v) How the consumption levels will vary for any variation in V. [1]

h) Solve the following constrained optimization problem.

Minimize $f = 3x + \sqrt{3}y$ [5]

subject to

$$3 - \frac{18}{x} - \frac{5\sqrt{3}}{y} \ge 0$$
$$x \ge 5 \cdot 73$$
$$y \ge 7 \cdot 17$$

[Turn over

2. a) Find the domain of the following functions :

$$f(x,y)=\sqrt{\frac{\ln[(x-1)^2+y^2-4}{y-x}}$$

$$f(x,y)=\sqrt{\frac{\ln[(x-y)]}{\sqrt{18-2x^2-y^2}}}$$ [2]

b) Prove that open balls are open sets. [2]

c) The limit will always exist at the origin. True or false ?

$$f(x,y)=xy\frac{x^2-y^2}{x^2+y^2}$$

$$f(x,y)=\frac{x+y}{4y^2-x^2}$$ [2]

d) True or false : The function $y=x^3-6x^2+9x+1$ will have only no maxima in the interval [0, 5] [2]

e) Can a profit maximizing output exist for the given production function with commodity price denoted by p and price of input x denoted w.

$$f(x)=\frac{1}{(1+e^{-3(z-1)})^2}$$ [2]

f) i) Draw and find the slope of the level curves of the following function. [1+2]

h) Consider the constrained maximization problem

Consider a profit maximizing Company who faces two demand curves

$P_1=D_1(Q_1)=22-10^{-5}Q_1$ in the day time (Peak period)

$P_2=D_2(Q_2)=18-10^{-5}Q_1$ in the night time (off-peak period)

to operate the firm must pay b = 6 per unit of output, whether it is day or night. Furthermore, the firm must install capacity at a cost of c = 8 per unit of output capacity installed. Let K denote total capacity measured in units of Q. The firm must pay for capacity, regardless if it operates in the off peak period. Using Kuhn Tucker condition find out the profit maximizing solution. Who should be charged for the capacity cost - Peak, off-peak, or both sets of customers ? [5]

i) Consider the production function $Z=f(x,y)=x^{\alpha}y$ defined over the domain x > 0 and y > 0. Also, consider the functions G(Z) = InZ and J(Z) = Z^2 + b is Z a homogeneous function ? If so, of what degree ? Is G(Z) a homothetic function ? Is G(Z) a homogeneous function in the arguments ? If so, what is its degree ? Is J(Z) = Z^2 + b a homothetic function ? [2]

[Turn over