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A Study Of Some Integral Equations And Applications To Water Wave Propagation Problems

Abstract

The thesis is concerned with a study of numerical solution of integral equations with regular and singular kernel by boundary element method and their applications to water wave scattering problems by thin curved barrier and rectangular thick barrier present in water region. The work in the present thesis is based on following problems.

[1] Boundary element approach of solving Fredholm and Volterra integral equations.

[2] Line element method of solving singular integral equations.

[3] Hypersingular integral equation formulation of the problem of water wave scattering by a circular arc shaped impermeable barrier submerged in a water of finite depth.

[4] Water wave interaction with a circular arc shaped porous barrier submerged in a water of finite depth.

[5] Scattering of water waves by thick rectangular barriers in presence of ice cover.

[6] Numerical approach on oblique wave scattering by a wide rectangular impediment with a vent placed under a finite depth water body with ice covered surface.

The problems in [1] and [2] illustrate the application of boundary element method (BEM) to solve Fredholm and Volterra integral equations of second kind and also singular integral equations of first kind with weakly singular kernel, viz, Abel integral equation and integral equation with log kernel and hypersingular equations of first and second kind. In this approach, the the range of integration is divided into finite number of line elements. Next, discretizing the interval of definition of the integral equation in to same number of line elements, it is then converted into a system of linear algebraic equations over the line elements. It may be noted that for Volterra and Abel integral equation, a lower triangular matrix associated with the system of linear algebraic equations is obtained. The unknown function is then evaluated on each line element by solving the system of linear algebraic equations. increasing the number of line elements the convergence of the solution was studied. Some examples are considered

here for illustrating the method. It is observed that use of this method produces very accurate results. The error analysis of the boundary element method is also discussed here.

Under the assumption of linearised theory of water waves, the problem of scattering of water waves by a thin circular arc shaped barrier, rigid and porous, submerged in ocean of finite depth, are studied in problems [3] and [4] respectively. By judicious application of Green's integral theorem, the corresponding boundary value problem is reduced to a hypersingular integral equation of first kind for the rigid barrier and of second kind for the porous barrier. This hypersingular integral equation in each problem is solved by using two methods. The first method is based on the Boundary Element Method (BEM) as described in problems [1] and [2]. The second method is a collocation method where the unknown function satisfying the integral equation is approximated by an infinite series involving Chebyshev polynomials. Choosing the collocation points suitably the integral equation is reduced to a system of linear equations. This system of linear equations is solved numerically to obtain the solution the the integral equation. It was observed that the solution of the integral equation by the two methods agree with each other. Using the solution of the integral equation, the quantities of physical interest, ie, reflection and transmission coefficients are studied graphically in each problem in [1] and [2].

In problem [5], the problem of water wave scattering of a normally incident wave train by a thick rectangular barriers present in water with ice cover is studied. The four basic configurations of thick rectangular barrier viz, partially immersed, bottom standing, submerged to a finite depth and wall with a gap are considered. The problem [6] is concerned with the study of the problem of scattering of an obliquely incident wave by a thick rectangular wall with a gap submerged in water of finite depth with ice cover. Multi term Galerkin approximations involving ultraspherical Gegenbauer polynomials is used for solving the integral equations arising in the mathematical analysis for the problems in [5] and [6]. In problem [8], the corresponding integral equation is also solved by using boundary element method described apriori and the solution by both the methods are found to be in good agreement. Numerical estimates for the reflection and transmission coefficients are obtained for various values of different parameters and are studied graphically.

Sudeshna Banerjea.
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Anushree Samanta
28/11/2022

Professor
DEPARTMENT OF MATHEMATICS
Jadavpur University
Kolkata – 700 032, West Bengal

Rumpa Chakraborty

Dr. RUMPA CHAKRABORTY
Assistant Professor
Department of Mathematics
Diamond Harbour Women's University
South 24 Parganas, West Bengal-74336