ABSTRACT

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Title of the Thesis: Study of Ecological and Epidemiological Models in Presence of Noise

Submitted by: Shri Abhijit Majumder

Under the joint supervision of: Prof. Nandadulal Bairagi

The present thesis aims to understand the dynamic behavior of various biological systems by applying stochastic calculus. More precisely, this thesis is devoted to capturing the role of stochasticity by using two methods: adding white noise perturbation into system parameters and adding white noise perturbation around the equilibrium in different ecological and epidemiological models. We analytically studied stochastic models to achieve various statistical and biological phenomena, including extinction, persistence, and stability. Numerically, we would give insights into this mathematical analysis. Additionally, to deepen our understanding of the dynamics of the model, we will study some intriguing phenomena numerically, such as the extinction time of diseases and the distribution of extinction times. We intend to use real-world data to validate the model's output. We will consider the available data on phytoplankton-zooplankton interaction in ecology to verify how our model and its output can capture the natural system dynamics. In an eco-epidemiological case, we will utilize the well-known red grouse data to validate the viability of our model. In this thesis, we are also interested in formulating stochastic models of Covid-19 taking into account the epidemiological status of individuals of a given geographical region and then analyzing it to provide various insights into the persistence and eradication of the disease. The SARS-CoV-2 infection has kept the world under pressure for the last two years. We, therefore, like to propose mathematical models to predict the course of the Covid-19 pandemic considering the uncertainty in the disease transmission & recovery rates. Motivated by the fact that there is uncertainty in vaccine-induced immunity loss, we would like to propose a stochastic epidemic model to decipher the interrelationship between vaccine efficacy and disease transmissibility. Our objective is to demonstrate how disease persistence and eradication are affected due to the uncertainty in the rate parameters like immunity loss and disease transmission coefficient. In all cases, we would like to present a simple but tractable method for calibrating the stochastic model output with the actual Covid-19 pandemic data to estimate system parameters and provide insightful conclusions from the obtained results.

The whole thesis is divided into several chapters. **Chapter 1** contains the origin and development of stochastic calculus, and the thesis ends with the future direction. Various mathematical tools & techniques which are used throughout this thesis are also discussed in the first chapter.

In **Chapter 2**, we study a minimal deterministic model of phytoplankton-zooplankton (prey and predator, respectively) interaction and compare its dynamics with its stochastic version formulated by two different stochastic perturbation techniques. In the first method, two parameters of the deterministic system are replaced by its average value plus an error term. In the second method, the stochastic perturbation is considered proportional to the distance of state variables from their deterministic equilibrium value. We analyze both stochastic models and explore their dynamics. In particular, we determine sufficient conditions for extinction probability, stochastic persistence of populations, and the existence of stationary distribution

of the first stochastic system. We determine sufficient conditions for the asymptotic mean square stability for the other system by defining a suitable Lyapunov function. Different analytical results are illustrated numerically and interpreted biologically. The stochastic behaviors of the system are compared with their deterministic counterpart. A case study has also been done considering the 24 months of data of phytoplankton-zooplankton interaction in Lake Trasimeno, Italy. It is shown that the stochastic model's output can capture the seasonal fluctuations of plankton populations of Lake Trasimeno.

Epidemic models are used to understand the dynamics of disease transmission and explore the possible measures for preventing the spread of infection. Disease transmission is intrinsically random and severely affected by (changing) environmental factors. In **Chapter 3**, we study a stochastic SIS (susceptible-infected-susceptible) type model, where infection transmits through horizontal and vertical transmission modes. White multiplicative noise is considered in the horizontal disease transmission term to incorporate stochasticity in the system. We prove that noise intensity, disease transmissibility, and recovery rates are potential routes for eradicating the disease. Furthermore, it is shown that parasites reduce their fitness for some fixed noise if the relative fecundity of infected hosts and the disease transmissibility are low, but observe an enhanced fitness if any of them is increased.

In **Chapter 4,** we consider a Leslie-Gower type prey-predator model with parasitic infection in prey. Here, we consider stochasticity in the growth rate of susceptible prey, the parasite-induced death rate of infected prey, and the growth rate of the predator. Population extinction is a serious issue both from theoretical and practical points of view. We explore how environmental noise influences the persistence and extinction of interacting species in the presence of a pathogen, even when the populations remain stable in their deterministic counterpart. Multiplicative white noise is introduced in a deterministic predator-prey-parasite system by randomly perturbing three biologically important parameters. It is revealed that the extinction criterion of species may be satisfied in multiple ways, indicating various routes to extinction, and disease eradication may be possible with the right environmental noise. Even when its focal prey strongly persists, the predator population cannot survive if its growth rate is lower than some critical value, measured by half of the corresponding noise intensity. It is shown that the average extinction time of the population decreases with increasing noise intensity and the probability distribution of the extinction time follows the log-normal density curve. A case study on red grouse (prey) and fox (predator) interaction in the presence of the parasites Trichostrongylus Tenuis of grouse is presented to demonstrate that the model well fits the field data.

Novel coronavirus has altered the socio-economic condition of the whole world through its devastating effects on the human population. Mathematical models and computation techniques may play an essential role in understanding this epidemic and contribute much to policy-making to control the infection more systematically and effectively. In **Chapter 5,** we have proposed a deterministic mathematical model for the Covid-19 pandemic taking into account the different epidemiological status of individuals of a given geographical region and analyzing it with respect to the basic reproduction number. Uncertainty is obvious in the case of a growing epidemic, and it multiplies if the disease etiology is unknown. Taking into account the uncertainty in the epidemiological parameters, we extended the deterministic system into a stochastic system through random parameter perturbations in three epidemiological parameters. Analyzing the model, we determined the disease persistence and eradication conditions. The stochastic solution's asymptotic behavior around the deterministic model's coexistence equilibrium was also presented. As a case study, we considered the Covid-

19 pandemic in India and estimated the model parameters from the epidemic data for the period 1st March to 6th December, 20. We demonstrated different analytical results and predicted the course of the epidemic.

A massive vaccination program against SARS-CoV-2 infection started at the beginning of 2021. Studies show that vaccinated people are subject to reinfection, and there is uncertainty in the rate of immunity loss, the force of infection, recovery rate, and vaccine efficacy. In **Chapter 6**, we study a six-dimensional stochastic epidemic model with vaccine-induced immunity loss to demonstrate the effect of vaccination in controlling the Covid-19 epidemic. It is shown that the disease persists for a long time if the stochastic basic reproduction number (SBRN) is greater than unity. We have also proved a sufficient condition for disease eradication. Our analysis shows that the disease cannot persist if $R_{0V}^{ext} < 1$. Noticeably, this condition may not hold if the infectivity increases or/and the vaccine-induced immunity loss increases. Two case studies were done: one with Indian Covid-19 data and the other with the data of Italy. Both Indian and Italian Covid-19 case studies are used to estimate the model parameters and noise intensities. It is revealed that the mean extinction time increases with the increasing rate of immunity loss and force of infection. For the case of India, a nontrivial observation is that mass vaccination cannot eradicate the disease if the vaccine-induced immunity loss is higher than 23 %. The case is almost similar if the infectivity is also high. Similarly, for Italy, it is observed that if the vaccine-induced immunity loss is higher than 12 %, the new cases will increase rapidly. It implies that the infection will last long unless a long-lasting vaccine candidate appears or a low infectious variant replaces the highly contagious variant.

The thesis ends with the future direction of research in **Chapter 7**.

List of Papers Included in the Thesis

1. **Abhijit Majumder**, Debadatta Adak, and Nandadulal Bairagi. "Phytoplanktonzooplankton interaction under environmental stochasticity: Survival, extinction and stability." **Applied Mathematical Modelling, 89(2021), 1382-1404.**
2. **Abhijit Majumder**, Debadatta Adak, and Nandadulal Bairagi. "Persistence and extinction of species in a disease-induced ecological system under environmental stochasticity." **Physical Review E , 103:3 (2021), 032412.**
3. **Abhijit Majumder,** Debadatta Adak, and Nandadulal Bairagi. " Persistence and extinction criteria of Covid-19 pandemic: India as a case study." **Stochastic Analysis and Applications, 40:2 (2022), 179-208.**
4. **Abhijit Majumder,** Debadatta Adak, Nandadulal Bairagi, and Adeline Samson. "Persistence and extinction of infection in stochastic SIS host-parasite epidemic model with horizontal and imperfect vertical transmissions." **Communicated.**
5. **Abhijit Majumder**, and Nandadulal Bairagi. "Is large-scale vaccination sufficient for controlling the Covid-19 pandemic with uncertainties? A model-based study." **Communicated.**

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Prof. Nandadulal Bairagi

Nandadulal Bairagi (Ph. D.)
Professor, Dept. of Mathematics
Jadavpur University
Kolkata-700032

Abhijit Majumder
13/09/22

Abhijit Majumder