Non-equilibrium Thermodynamics
In
Cosmological Context

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CERTIFICATE FROM THE SUPERVISOR

This is to certify that the thesis entitled "**Non-equilibrium Thermodynamics In Cosmological Context**" submitted by **Sri. Subhayan Maity**, who got his name registered on November 13, 2019 (Index No: 134/19/Phys/26) for the award of Ph.D. (Science) degree of Jadavpur University, is absolutely based upon his own work under my supervision and that neither this thesis nor any part of it has been submitted for either any degree/diploma or any other academic award anywhere before.

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DECLARATION BY THE AUTHOR

I hereby declare that the thesis is based on my own work carried out at the Department of Mathematics, Jadavpur University, Kolkata- 700032, India. Also, I declare that, no part of it has not been submitted for any degree/diploma/some other qualification at any other University.

All the figures presented in this thesis have been produced by the author using Maple software. The thesis has been checked several times with extreme care to free it from all discrepancies and typos. Even then the vigilant readers may find some mistakes, and, several portions of this thesis may seem unwarranted or mistaken or incorrect. The author takes the sole responsibility for these unwanted errors which have resulted from his inadequate knowledge in the subject or escaped his notice.

Finally, I state that, to the best of my knowledge, all the assistance taken to prepare this thesis have been properly cited and acknowledged.

Subhayan Maity

Subhayan Maity

In memory of my Grandfather

Satish

Chandra

Maity

SYNOPSIS OF THE RESEARCH WORKS BEHIND THIS THESIS

This thesis is mainly based on the different non-equilibrium thermodynamic processes and its role in cosmic evolution process. In order to explain the cosmic evolution process, we have considered the dissipative Universe i.e. the system (Universe) consists of fluids with dissipative pressure. The dissipative fluids exhibit various types of transport phenomena like particle creation-annihilation mechanism, diffusion, convection etc. Under such transport mechanisms, the Universe is continuously evolving and hence these are thermodynamically unstable processes. Therefore we have modeled the Universe as an isolated, non-equilibrium thermodynamic system and tried to investigate different aspects of cosmic evolution. During this research procedure, we have published four articles (papers) in different international journals and I have attended two international conferences.

In one of our four works (*"Universe consists of diffusive dark fluids : thermodynamics and stability analysis"*, **S. Maity**, P. Bhandari and S. Chakraborty, **The European Physical Journal C**79 (2019)1,82 (**arXiv** :1902.08037 [gr-qc])), we have modeled our Universe as a system of diffusive barotropic fluids of both constant and variable barotropic indices. In both cases, we have obtained the restrictions on the diffusion parameter from thermodynamic stability conditions of the cosmic fluids for different ranges of barotropic indices (ω). It has been found that dark energy fluids with constant ω are always thermodynamically unstable. But diffusive dark energy with variable ω can still be thermodynamically stable under certain restrictions.

In the second work (*"Continuous cosmic evolution with diffusive barotropic fluid: First order thermodynamic phase transition"* **S. Maity**, and S. Chakraborty, **Int. J.Mod. Phys. A** 36 (2021)29,215091), we have successfully shown the continuous and complete cosmic evolution under non-equilibrium thermodynamic phenomena. In order to incorporate the dissipative pressure in a single fluid dissipative Universe, we have introduced an extra term containing scalar field (ϕ) in Einstein's field equations in analogy with the cosmological constant. In this scenario, we have obtained the Friedmann equations and matter conservation equation. Then for different phenomenological choices of ϕ, we have found the different cosmic evolution patterns from emergent to present late time acceleration phase. The thermodynamic behaviour of the cosmic fluid while the cosmic phase transition has also been investigated and this corresponds to first order thermodynamic phase transitions.

In another paper (*"Does diffusion mechanism favor the emergent scenario of the universe ?"*, **S. Maity** and S. Chakraborty, **Int. J.Mod. Phys. A** 37 (2022) 03,2250016), we have emphasised on the possibility of emergent scenario of diffusive Universe and successfully modeled in that way with suitable restrictions on the parameters.

At last in a recent work (*"Is cosmic evolution process with diffusive fluid equivalent to a Heat engine?"*, **S. Maity**, and S. Chakraborty, **Annals phys.** 444 (2022)169045), we have successfully modeled a cosmic heat engine by phenomenological choice of scalar field and proper restrictions on different parameters. This cosmic heat engine corresponds to the cyclic, bouncing and non-singular bouncing scenario.

At two international conferences, I have presented a cyclic evolutionary model of two fluid Universe (Arxiv no. 2205.09759[gr-qc]).

These works establish that the non-equilibrium thermodynamic processes can have a key role in explaining the past and present evolution patterns of the Universe and also the different unorthodox model like cyclic

Grant ommision(UGC), Government of India, through Junior and Senior Research Fellowships.

Subhayan Maity

Subhayan Maity

ABSTRACT

This thesis is based on six chapters including an introductory chapter at the beginning and a discussive chapter on the future scope of these works at the end.

(1) In the first chapter, A brief introduction to thermodynamics and its applications in cosmology has been described. In several sections, the basics of thermodynamics and the classical laws of thermodynamics have been discussed, the necessity of non-equilibrium thermodynamics for Universe has been described and the elements of cosmology have been mentioned briefly.

(2) Chapter:2 deals with the thermodynamic stability criteria of the cosmic fluids under diffusion mechanism. The Universe is modeled as a system of diffusive barotropic fluids of both constant and variable barotropic indices. In both cases, the restrictions on diffusion parameter from thermodynamic stability of the cosmic fluids for different ranges of barotropic indices (ω).

(3) In chapter : 3, It is investigated whether the emergent scenario of the universe is possible under diffusion mechanism. For suitable choices of parameters contained in the evolution equation, the possibility of emergent scenario has been established.

(4) Chapter: 4 is the successful exhibition of the continuous and complete cosmic scenario under non-equilibrium thermodynamic phenomena. In order to incorporate the dissipative pressure in a single fluid isolated Universe, an extra term containing scalar field has been introduced in Einstein's field equations in analogy with the cosmological constant. Hence, the Friedmann equations have been obtained from the modified field equations. Then for suitable phenomenological choices of time dependent scalar field, different phases of complete cosmic evolution have been established. The thermodynamic behaviour of the cosmic fluid has also been studied while cosmic phase transition and it is found as first order thermodynamic phase transition.

(5) In chapter: 5, the Universe is modeled as a cosmic heat engine by phenomenological choice of scalar field and proper restrictions on different parameters. This cosmic heat engine corresponds to the cyclic, bouncing and non-singular evolutionary scenario.

(6) Finally, the thesis ends with " BRIEF SUMMARY AND FUTURE PROSPECT " of the work in Chapter: 6.

PREFACE

The work of this thesis has been carried out at the Department of Mathematics, Jadavpur University, Kolkata- 700032, India. The thesis is based on the following published papers:

- **Chapter 2** has been published as *"Universe consists of diffusive dark fluids : thermodynamics and stability analysis"*, **S. Maity**, P. Bhandari and S. Chakraborty, **The European Physical Journal C**79 (2019)1,82 (**arXiv** :1902.08037 [gr-qc]).

- **Chapter 3** has been published as *"Does diffusion mechanism favor the emergent scenario of the universe ?"*, **S. Maity** and S. Chakraborty, **Int. J.Mod. Phys. A** 37 (2022) 03,2250016.

- **Chapter 4** has been published as *"Continuous cosmic evolution with diffusive barotropic fluid: First order thermodynamic phase transition"* **S. Maity**, and S. Chakraborty, **Int. J.Mod. Phys. A** 36 (2021)29,215091 .

- **Chapter 5** has been published as *"Is cosmic evolution process with diffusive fluid equvalent to a Heat engine?"* **S. Maity**, and S. Chakraborty, **Annals phys.** 444 (2022)169045 .

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At first, I would like to express my sincerec gratitude to my advisor Prof. Subenoy Chakraborty, Dept. of mathematics, Jadavpur University for giving me the golden oportunity of doing research under his supervision. I am very grateful to him for his continuous support of my Ph.D. study and related research, for his patience, motivation and immense understanding of the subject. His affectionate guidance helped me in all the time of research as well as learning the subject. No description is enough to express his profound imprint in my academic and over all life.

I would like to express my gratitude towards all the faculties of Dept. of mathematics and Dept. of physics at Jadavpur University for their support. I am also thankful to the non-teaching staff of the Dept. of mathematics, Jadavpur University for various kinds of helps during my research work. I have to acknowledge the staff of research section for their help in fellowship related issues.

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Finally I would like to thank everyone who was beside me during my journey. I am so sorry that I could not mention everyone's name. I am thankful to God almighty for this life.

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Subhayan Maity

“ Do not judge me by my successes, judge me by how many times I fell down and got back up again ”.

Nelson Mandela

LIST OF FIGURES

CONTENTS

CHAPTER 1

INTRODUCTION

"Thermodynamics is a funny subject. The first time you go through it, you don't understand it at all. The second time you go through it, you think you understand it, except for one or two small points The third time you go through it, you know you don't understand it, but it does not bother you any more ".

Arnold Sommerfeld .

This is the perfect description of this particular branch of physics. Although by terminology, it is the study of flow of heat (therm (heat) + dynamics (motion or flow)), but it has broad applications in different aspects of a system. The thermodynamic descriptions are equally compatible in all different scale systems. Starting from very small scaled system like biological cells to the the analysis of our whole universe, the laws of thermodynamics are used conveniently. Especially when there are very few informations about a system, thermodynamics brings very important clues to access that system.

1.1 Basics of Thermodynamics and its application

Thermodynamics is a branch of physics which deals with the changes and transformations of heat into other forms of energy (especially into the mechanical work done) with in a system. The principal objective of thermodynamics is to develop the relations between mechanical energy (work done) , heat , temperature and others form of energy.Lord Kelvin [1] defined thermodynamics as " thermodynamics is the subject of the relation of heat to forces acting between contiguous parts of bodies and the relation of heat to electrical agency".

The laws of thermodynamics state both qualitatively and quantitatively whether and how the change in energy within a system can perform useful work on its surrounding[2]. The thermodynamic variables of a system are basically the macroscopic variables and hence the thermodynamic states are the macrostates representation of a system. The laws of thermodynamics are useful to describe the changes of the macrostate of a system while interacting with external thermal energy and work done. But those can be explained in the light of microscopic dynamics of the constituent particles of the system by statistical mechanics.

In general, the thermodynamics is the study of a system (fragment of universe) while exchanging heat and others form of energy with surrounding (rest of the universe without the system).

SYSTEM + SURROUNDING = UNIVERSE .

The system and surrounding are separated by a boundary. The boundaries can be classified in three types :

(i) **permeable boundary** allows both mass and energy to pass through it.

(ii) **Diathermal boundary** allows only energy to pass through it but mass can be transmitted and

(iii) nothing can be exchanged between system and surrounding through an **adiabatic boundary**.

The thermodynamic systems are also of three types depending on the nature of boundaries.

The systems enveloped under permeable boundary are called **open system**.

A **closed system** is covered under diathermal boundary.

An **isolated system** remains under adiabatic boundary.

1.1.1 Brief review of laws of thermodynamics:

Thermodynamics consists of four laws. These four laws are mainly based on some basic principles of physics : thermal equilibrium , conservation of energy, joule's equivallence principle, heat engine principle , refrigerator principle and changes in chaosness of a system due to interaction with heat and work respectively. In various cases of different systems, laws of thermodynamics may be expressed in seemingly different forms. The universal and generalised forms of thermodynamic laws are as following.

1.1.1.1 Zeroth law of thermodynamics :

It states that[3](page-22),[4, 5] **if two systems are individually in thermal equilibrium with other system then they must be in thermal equilibrium with each other.**

So, thermal equilibrium is an equivalence condition on the set of thermodynamic systems. In general, any two systems are said to be in thermal equilibrium if no net exchange of energy occurs between them. Microscopically there may be differences of kinetic energies of the constituent particles of the systems but the average energy (macroscopic) of each system will be same in thermal equilibrium. Hence the temperature will be identical of the two systems under thermal equilibrium.

1.1.1.2 First law of thermodynamics :

This law is based on the principle of energy conservation[6](pages-11 $-$ 13) and the equivalence of heat(H) and mechanical work done(W) namely **Joule's equivalence**

principle : $W = JH$. J is Joule's equivalence constant. First law of thermodynamics states that [3] (page-79) **if heat is applied to a system then it will be utilised in two ways. One, some fraction of heat will increase the internal energy of the system (U) and other fraction will be used to perform some external work (W).**

$$\Delta Q = \Delta U + \Delta W \tag{1.1}$$

By applying infinitesimal heat dQ, one has the differential form

$$dQ = dU + dW \tag{1.2}$$

Now for a Gas system, the infinitesimal work done by the system can be found interms of thermodynamic variables (P, V) as,

$$dW = PdV. \tag{1.3}$$

Therefore one has the standard form of the first law of thermodynamics as,

$$dQ = dU + PdV \tag{1.4}$$

Hence only temperature T is not sufficient to represent the thermodynamic state of a system. For complete representation of the thermodynamic state of a system, pressure (P), volume (V) and temperature (T) are the essential parameters. These three are called thermodynamic coordinates.

Consequences of first law of thermodynamics : The first law of thermodynamics insists that any system can evolve from initial state (P_i, V_i, T_i) to another thermodynamic state (P_f, V_f, T_f) in infinite number of different procedures. Each different procedure is called thermodynamic path or thermodynamic process. Some well known thermodynamic processes are :

isothermal process (where T remains constant),

adiabatic process (no net exchange of heat with surrounding) ,

isobaric process (P remains constant),

isochoric process (V remains constat) etc.

These thermodynamic processes can be characterised by pressure - volume relation as following:

(a) for isothermal process, $PV =$ constant,

(b) for adiabatic process, $PV^{\gamma} =$ constant ($\gamma = \frac{c_P}{c_V}$),

(c) for isobaric process, $PV^0 =$ constant and

(d) for isochoric process, $PV^{\infty} =$ constant.

Such type of thermodynamic processes can be termed as the polytropic process $PV^n =$ constant where n is the polytropic index of the process.

Secondly any thermodynamic variable f is a function of pressure, volume and temperature. These thermodynamic variables $f(P,V,T)$ can be classified in two categories.

(a) **state variables** of which the change do not depend on the path only depend on the initial and final state of the system. example : internal energy, entropy etc.

(b) **Path variables** ,the change of which varies in different thermodynamic paths. example : work done, heat etc.

The differential of state variables are exact differential and those of path variables are not so.

Thirdly, the first law establishes the existence of two types of specific heats of gasses : Specific heat at constant volume (C_V) and specific heat at constant pressure (C_P).

$$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V . \tag{1.5}$$

Similarly,

$$C_P = \left(\frac{\partial Q}{\partial T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P . \tag{1.6}$$

Here, considering U as a function of volume (V) and temperature (T), one has

$$\left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P . \tag{1.7}$$

Following equation (1.7), one has the relation between C_P and C_V as,

$$C_P - C_V = \left(\frac{\partial V}{\partial T}\right)_P \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \tag{1.8}$$

For ideal gas, $\left(\frac{\partial U}{\partial V}\right)_T = 0$ and hence

$$C_P - C_V = R, \tag{1.9}$$

which is known as Mayer's relation [7].

The effective thermodynamic energy of a system (specially for gas) can be the sum of internal energy and the work term due to its pressure and volume [8](page- 275),[9]. This thermodynamic parameter is called enthalpy (h). Enthalpy is a state variable.

$$h = U + PV. \tag{1.10}$$

One has the relation

$$C_P = \left(\frac{\partial h}{\partial T}\right)_P. \tag{1.11}$$

In general, the specific heat of a system in a particular polytropic process with polytropic index n is given by $C_n = C_V \frac{\gamma - n}{1-n}$.

Besides compressibilities in isothermal and adiabatic processes can be related with these specific heats.

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T. \tag{1.12}$$

$$K_Q = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_Q. \tag{1.13}$$

Limitations of the first law of thermodynamics : Though the first law of thermodynamics is very useful to describe and analyze any system, it suffers from several limitations. Basically it does not state about the direction of the flow of heat. There is no idea whether heat can flow from lower temperature reservoir to higher temperature reservoir or not[10].

Secondly , the first law does not say whether heat can be fully converted into work or not. Thirdly it is indifferent about entropy. But these ideas are important in applications like heat engines (a device which can convert heat into work) and refrigerator (a device which transfer heat from low temperature system to higher temperature surrounding).[8, 10]

1.1.1.3 Second Law of thermodynamics :

"Nothing in life is certain except death,taxes and the second law of thermodynamics.".

Seth Lioyd

The limitations of the first law especially the heat engine principle and refrigerator principle have been resolved in the second law of thermodynamics. The second law is a series of several principles rather than a single statement. Basically the two statements are taken as the second law[10].

(a) Kelvin- Planck Statement (Heat engine principle) : It states that it is impossible to construct any device that operates in a cyclic way to accept heat from a single reservoir and produce an equivalent amount of work[8, 10, 11].

Consequently, it is found that the efficiency of a heat engine is always less than unity.

(b) Clausius Statement (Refrigerator principle) : As per this principle, it is impossible to construct a device which operates on a cycle produces a sole effect of transferring heat from a cold reservoir to a hot one. [8, 10, 11]

It can be shown that the two statements of the second law of thermodynamics are equivalent[10, 11].

Consequences of second law of thermodynamics :

Firstly , Kelvin - Planck statement states about the working principle of a heat engine. Carnot [8, 10, 12, 13, 14] introduced a cyclic thermodynamic process through which a heat engine can be operated between two reservoirs of temperatures $T_1, T_2 (T_1 > T_2)$ with efficiency $\eta = 1 - \frac{T_2}{T_1}$[10, 11, 12, 13, 14]. The steps of a Carnot cycle is as following : Isothermal expansion $\rightarrowtail$ adiabatic expansion $\rightarrowtail$ isothermal contraction $\rightarrowtail$ adiabatic contraction to its initial state.

The thermodynamic processes can also be categorized in two kinds. Some processes can be reversed to its initial state without any dissipation. They are called reversible processes[8, 10, 15, 16, 17]. Other spontaneous processes undergo dissipation while reversing its direction to the initial state. These processes are irreversible processes[8, 10, 17, 18].

A heat engine operated in a reversible process is a reversible heat engine. The Carnot engine is an example of a reversible engine.

In most of the cases, heat engines are operated in an irreversible process.

Carnot also introduced two theorems regarding the efficiency of heat engines.

(i) Working between the two same temperature limits, no engine is more efficient than a reversible one, $\eta_R \geq \eta_I$ [8, 10, 11, 12, 13].

(ii) Working between the two same temperature limits, all reversible engines are equally efficient[8, 10, 11, 12, 13].

Hence, one has the boundary over the efficiency of any heat engine : $\eta \leq 1 - \frac{T_2}{T_1}$.

It reveals the equivalence of all scales of temperature and also the unattainability of absolute zero, $T_2 \geq 0$.

Secondly , Clausius statement[8, 10, 13, 14] tells about the fact that heat can be transferred from a cold reservoir to a hot reservoir only at the expense of external work.

Introduction to entropy and concept of free energies : The important outcome of second law of thermodynamics is the idea of entropy. The entropy of any system is the measure of chaosness [8, 10, 19]of that system. When heat is applied to a system, the internal chaosness of the system will also be increased besides its internal energy. Generally one can not measure the absolute value of entropy , at best the change in entropy can be measured. The quantitative measurement of change in entropy (ΔS) can be measured as the system's thermal energy per unit temperature which is unavailable for doing useful work[8, 10, 20].

$$\Delta S = \frac{\Delta Q}{T_0} \tag{1.14}$$

where T_0 is the operating temperature. If the operating temperature varies with the application of heat, then one has to consider the elementary change in entropy and then this can be integrated over the range of operating temperature.

$$\Delta S = \int_{T_{01}}^{T_{02}} \frac{dQ(T_0)}{T_0} \tag{1.15}$$

where T_{01}, T_{02} are the ranges of operating temperature. Hence the origin of entropy lies in the unavailability of complete thermal energy of any system. To understand the fact, one may consider a heat engine operating between the heat reservoirs of temperature T_1 and T_2 respectively ($T_1 > T_2$). T_0 is the operating temperature of the engine. Let Q amount of heat is being transferred from T_1 to T_2 reservoir. Now the work output for absorbing Q heat to the engine is given by

$$W_1 = Q\left(1 - \frac{T_0}{T_1}\right). \tag{1.16}$$

Similarly, the work input needed for transferring Q amount of heat from the engine to T_2 reservoir will be

$$W_2 = Q\left(1 - \frac{T_0}{T_2}\right). \tag{1.17}$$

Hence the net amount of unavailable energy is given by

$$\Delta Q = W_1 - W_2 = T_0\left(\frac{Q}{T_2} - \frac{Q}{T_1}\right) \tag{1.18}$$

Clearly

$$\Delta Q = T_0 \Delta S, \tag{1.19}$$

is the thermodynamic unavailable energy. The free energy is defined as the net amount of available thermodynamic energy which can be used to perform useful work.

There are two types of free energies used in thermodynamic analysis.

(i) Helmholtz free energy : $F = U - TS$[8, 10, 21, 22, 23]. Hence one has

$$dF = dU - TdS - SdT \tag{1.20}$$

Again from equations (1.3) and (1.14),

$$TdS = dU + PdV. \tag{1.21}$$

Therefore, the differential of Helmholtz energy is given by,

$$dF = -PdV - SdT \tag{1.22}$$

and hence one has the thermodynamic relations

$$S = -\left(\frac{\partial F}{\partial T}\right)_V, P = -\left(\frac{\partial F}{\partial V}\right)_T. \tag{1.23}$$

(ii) Gibbs free energy: $G = h - TS = U + PV - TS$[8, 10, 21, 22, 23] ,
$dG = dU + PdV + VdP - TdS - SdT$. Hence from equation (1.21), one can write

$$dG = VdP - SdT \tag{1.24}$$

So, another two thermodynamic relations are found in the forms,

$$S = -\left(\frac{\partial G}{\partial T}\right)_P, V = \left(\frac{\partial G}{\partial P}\right)_T. \tag{1.25}$$

Chemical potential : The Gibbs free energy per each particle is defined as the chemical potential (μ).

$$\mu = -\left(\frac{\partial G}{\partial N}\right)_{V,T}. \tag{1.26}$$

Hence the first law of thermodynamics (law of energy conservation) for system with particle creation - annihilation mechanism can be written as[24],

$$TdS = dU + PdV - \mu dN \tag{1.27}$$

This portion will be discussed in details in the later sections.

1.1.1.4 Third Law of thermodynamics :

It states that the entropy of a system approaches a constant value as its temperature approaches absolute zero[25].

It is the consequence of the statistical definition of entropy as $S = K_B \ln \Omega$ with K_B , the Boltzman constant and Ω is the number of micro states under given macro state (thermodynamic state). This equation is known as the Boltzman equation of entropy.

At absolute zero, there will be only one possible micro state (i.e. $\Omega = 1$) and

$$S = 0. \tag{1.28}$$

These four laws of thermodynamics stated and discussed above can be derived from the statistical mechanics (the microscopic description of a system). The expression of thermodynamic parameters can be found from the relation between Helmotz free energy (F) and the partition function (Z) of the system. Partition function (Z) depends on the internal distribution of the constituent particles and can be obtained from the relation[10]

$$Z(T, V, N) = \sum_{1}^{n} g_i \exp \frac{-E_i}{K_B T}, \tag{1.29}$$

where g_i is the number of degeneracy at i th energy level of the system. For a system described under canonical ensemble (T, V, N)[10],

$$F = -K_B T \ln Z. \tag{1.30}$$

Hence by following equation (1.23), one may obtain the equation of state of the system (for example, an ideal gas system has the equation of state $PV = NK_BT$).

Thermodynamic potentials The internal energy (U), enthalpy (h), Helmotz free energy (F) and Gibbs free energy (G) - these four different energy scales in thermodynamics are called the thermodynamic potentials[8]. Thermodynamic potentials are state variables. Among the four potentials, any one can be obtained from other three[10].

1.1.2 Thermodynamic relations :

Some important thermodynamic relations are -

(a) TdS equations

(b) Maxwell's thermodynamic equations

(c) The energy equations etc.

1.1.2.1 TdS equations :

The entropy S of a system can be considered as a state variable of two parameters among three thermodynamic coordinates P, V, T[8, 10]. Hence one can find three TdS equations as[10],

$$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV \tag{1.31}$$

$$TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP \tag{1.32}$$

$$TdS = C_V d\left(\frac{\partial T}{\partial P}\right)_V dP + C_P\left(\frac{\partial T}{\partial V}\right)_P dV. \tag{1.33}$$

These relations directly relate the entropy of a system with the measurable quantities.

1.1.2.2 Maxwell's thermodynamic equations :

There are four mMaxwell's equations in thermodynamics[8, 10].

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \tag{1.34}$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \tag{1.35}$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V \tag{1.36}$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \tag{1.37}$$

These relations are very useful especially while obtaining the thermodynamic behaviour of a system in transition points.

1.1.2.3 Energy equations :

There are two energy equations[8, 10]

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P = T\left(\frac{\partial P}{\partial T}\right)_V - P \tag{1.38}$$

$$\left(\frac{\partial U}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial P}\right)_T \tag{1.39}$$

1.1.3 Thermodynamic phase transition :

A phase (thermodynamic phase) is defined as a homogeneous state having defined thermodynamic boundaries[26].

Phase transition is a mechanism where a system changes from one state to another state with a discontinuity of a certain minimum order derivative of the free energy at the transition point. For example, change of physical state (solid $\rightarrow$ liquid, liquid $\rightarrow$ gas, and vice versa) is a thermodynamic phase transition where the first order derivative of Gibbs free energy shows discontinuity at the transition points and the minimum order derivative of the free energy which shows the discontinuity are called **the order of phase transition** [8, 10, 27].

1st order phase transition : Here the first order derivative of Gibbs free energy (G) becomes discontinuous at the transition point. Hence the Gibbs free energy itself remains continuous[8, 10, 27] but entropy ($S = -\left(\frac{\partial G}{\partial T})_P\right)$ and volume ($V = \left(\frac{\partial G}{\partial P})_T\right)$ become discontinuous. Here the entropy of the system changes at a constant transition temperature by emitting or absorbing latent heat. The behaviour of the system at transition points can be described by Clausius-Clapeyron equation[10, 27]

$$\frac{dP}{dT} = \frac{L}{T_0\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)} \tag{1.40}$$

where L is the latent heat of the transition and T_0 is the transition temperature. ρ_2 and ρ_1 are the densities at the two phases of the system. This equation can also be derived from the Maxwell's equations (1.34),(1.35),(1.36) and (1.37). This equation depicts how transition temperature T_0 can be changed by changing pressure on the system(Working principle of a pressure-cooker).

2nd order phase transition : Here the second order derivatives of the Gibbs free energy become discontinuous and hence entropy and volume (first order derivative of G) remain continuous. Clearly the thermodynamic derivatives $(C_P, C_V, K_S, K_T, \alpha)$ of the system show the discontinuity[27]. Hence the different phases can not coexist in equilibrium in second order phase transition. The behaviour of the system at the transition point is governed by Ehrenfest equations.

The first Ehrenfest equation is given by [10]

$$\left(\frac{dP}{dT}\right)_S = \frac{\Delta C_P}{T_0 \Delta \left(\frac{\partial v}{\partial T}\right)_P} = \frac{\Delta C_P}{T_0 v \Delta \alpha} \tag{1.41}$$

where $v = \frac{1}{\rho}$ is the specific volume of the system.

The second Ehrenfest equation can be found in the form[10]

$$\left(\frac{dP}{dT}\right)_V = \frac{\Delta \alpha}{\Delta K_T} \tag{1.42}$$

These equations can also be obtained from Maxwell's equations.

Limitations of Ehrenfest equations : The derivatives of free energies are always not finite, especially the phase transitions of magnetic substances can not be described by these equations[10, 27].

General approach to the phase transition : If one considers a general system having p number of phases and c number of components then the degrees of freedom which governs the behaviour of the system is given by[8, 10]

$$f = c - p + 2 \tag{1.43}$$

This equation is called the **Gibbs phase rule**.

1.1.3.1 Chemical potential :

The change of internal energy due to adding or eliminating one particle of the system is call the chemical potential of that system. Hence the first law of thermodynamics for a system with variable number of particles can be modified as [10]

$$dE = TdS - PdV + \mu dN. \tag{1.44}$$

Hence one has

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \left(\frac{\partial F}{\partial N}\right)_{T,V} = \left(\frac{\partial G}{\partial N}\right)_{T,P}. \tag{1.45}$$

The physical significance of μ is that the complete equilibrium will be achieved not only when two systems possess equal temperature T but also the same value of μ. Otherwise the particle will flow from higher chemical potential system to the system with lower value in order to equalize the internal energy.

Systems with variable number of particles : Let consider an isolated system with total N number of particles with two different phases. The number of particles of phase - I is N_1 and that of phase - II is N_2. So $N_1 + N_2 = N$. Here the number of particles of the individual phase is not constant $\delta N_1 \neq 0, \delta N_2 \neq 0$ but $\delta N = 0$.

For such a system, the first law of thermodynamics can be written as[10],

$$dE = dQ - PdV + \mu_1 dN_1 + \mu_2 dN_2. \tag{1.46}$$

Here μ_1 and μ_2 are the chemical potentials of the two phases of the system respectively. In general for a system with n phases, first law of thermodynamics can be written as

$$dE = dQ - PdV + \sum_1^n \mu_k dN_k, \tag{1.47}$$

with μ_k being the chemical potential of k- th phase and N_k is the number of particles of k-th phase of the system. Effectively, one may write from equation(1.14),

$$TdS = dE + PdV - \sum_1^n \mu_k dN_k. \tag{1.48}$$

Again the Gibbs free energy

$$G = E + PV - TS = \sum_1^n \mu_k dN_k. \tag{1.49}$$

The above equation is known as **Gibbs - Duhem relation**[8, 10].

Notably, the condition for the equilibrium of all phases is

$$T_i = T_j, \mu_i = \mu_j, P_i = P_j. \tag{1.50}$$

The chemical potential for an ideal gas can be estimated in the form,

$$\mu(P,T) = \mu_0(T) + RT \ln \frac{P}{P_0}, \tag{1.51}$$

with $\mu_0(T) = \mu(P_0, T)$.

1.1.4 Different branches of thermodynamics :

The thermodynamic properties of a system are studied in several related branches. The laws of thermodynamics can be applicable on different modeled systems in different approaches. The models have been developed on theoretical and experimental results and also imposing different principles on the system. The related branches are as following.

1.1.4.1 Classical thermodynamics :

"Classical thermodynamics is the only physical theory of universal content which I am convinced and will not be overthrown.".

Albert Einstein

Classical thermodynamics is based on the description of the system near its thermodynamic equilibrium states. The thermodynamic states are represented by the accessible parameters (externally measurable quantities) of the system. Hence classical thermodynamic states are the macrostate representation of the system which are stable for a sufficiently long time. Following the laws of thermodynamics, the external descriptions of exchange of energy, work and heat are modeled in classical thermodynamics. The exchange of heat, energy, work, etc. results in the change in the macrostate of the system and the system transforms from one initial stable macrostate to another stable thermodynamic state. The microscopic interpretation of these concepts are provided in statistical physics.

1.1.4.2 Statistical physics :

It deals with the microscopic description of the system. Under any particular macrostate, there are several different possible microstates and the stability of a macrostate is proportional to the number of microstates under that given macrostate. So, the number of microstates is called the thermodynamic probability of the system.

As per prior probability principle, all the microstates under a macrostate are equally probable. Any thermodynamic parameter(macroscopic) is the average value over all the microstates. Clearly, the number of microstates depends on the internal decoration of the constituent particles of the system. The famous relation between the thermodynamic probability (Ω) and the entropy of the system is given by[10],

$$S = K_B \ln \Omega \tag{1.52}$$

where K_B is the Boltzman constant. This equation is called **Boltzman equation**.

Under a defined macrostate, the collection of all possible microstates is called the **ensemble** of that system. There are three types of ensemble in Statistical mechanics.

Micro-canonical ensemble : All the microstates under the micro-canonical ensemble possess equal number of particles (N), equal energy (E) and equal volume (V). So for such an ensemble, the system must be thermally and mechanically isolated under adiabatic boundary and no particle creation-annihilation process occurs.

Canonical ensemble : The microstates have the same temperature (T), volume (V) and number of particles (N) under the canonical ensemble. So the system must be mechanically isolated but with a metallic contact with a heat reservoir of temperature (T) to keep the temperature constant. Hence the system must be surrounded by a diathermal boundary and energy will be exchanged between the system and surroundings(reservoir).

Grand canonical ensemble : Here the total number of particles and energy are not fixed but the volume (V), temperature (T) and chemical potential (μ) are same in all microstates. So particle creation- annihilation process may occur or may be exchanged with the surrounding. The system is only mechanically isolated under permeable boundary. The energy will also be exchanged between the system and surrounding. It is the most natural ensemble of the Universe.

Following these different ensembles, the outcomes of the macroscopic description of classical thermodynamics, especially the laws of thermodynamics, can be interpreted. Besides, it allows application of quantum mechanics at the microscopic level and to modify the thermodynamic equations of the Bose and Fermi systems. For ideal gas, Classical thermodynamics is a natural result of Statistics.

1.1.4.3 Chemical thermodynamics :

It is a branch of thermodynamics where the thermodynamic behaviour of a chemical reaction is studied. The change of thermodynamic state with progress of a chemical reaction is analyzed in this topic [28].

1.1.4.4 Equilibrium Thermodynamics :

It is the study of the behaviour of a system which is modeled to be transformed from one thermodynamic equilibrium state to another thermodynamic equilibrium state. It is the ideal version of classical thermodynamics.

Concept of equilibrium : We have already discussed about the thermal equilibrium in " zeroth law of thermodynamics". Thermal equilibrium is not the ultimate equilibrium as there may be exchange of particles between two systems under thermal equilibrium if there exists the difference of chemical potential[10]. **Thermodynamic equilibrium** is the state of stability where there will be no flow between two systems. For a system under thermodynamic equilibrium, all the intensive properties will be homogeneous[8, 10]. So there will be no gradient of any quantities which may generate a force, pressure or driving instinct to flow the constituent particles of the system from one local part to another.

Such modeled systems are ideal for application of laws of thermodynamics. In practice, there is no such system on which equilibrium thermodynamics may be applicable[8].

1.1.4.5 Non-equilibrium thermodynamics :

When a system is not in thermodynamic equilibrium, then the laws of thermodynamics can not be applied on that system directly. In such a case, the system can be described in terms of macroscopic parameters (non-equilibrium state variables) and then the laws of thermodynamics can be applicable under some restrictions[28, 29]. This branch of thermodynamics is termed as non-equilibrium thermodynamics. It deals with transport phenomena, chemical reaction rate etc.

Most of the systems in our universe are not found in thermodynamic equilibrium. An evolving system which undergoes continuous conversion from matter to energy or between different forms of energies can never be in thermodynamic equilibrium. But some processes are close enough to the thermodynamic equilibrium to allow the representation in non-equilibrium thermodynamics in a useful sense.

Notably some natural systems are beyond the applicability of non-equilibrium thermodynamics. Those systems follow the non-variational dynamics and the free energies can not be defined[31, 32].

However the thermodynamic description of a non-equilibrium system is a more general approach than an equilibrium one. Especially the thermodynamics of an inhomogeneous system requires the approximation of local equilibrium to define the physical parameters locally at a particular instant of time.

1.1.5 Applications of thermodynamics :

The use of thermodynamic principles are wide in different branches of physics. Besides the practical applications to human civilisation, thermodynamic laws are useful for accessing different systems with unknown nature because thermodynamic aspects of a system are the fundamental feature and can provide some clues about the dynamics of that system too.

The main two applications of thermodynamics are to provide the working principles of a heat engine and a refrigerator. The efficiency of a heat engine ($\eta = 1 - \frac{T_2}{T_1}$ and coefficient of performance of a refrigerator $W = \frac{T_2}{T_1 - T_2}$ determine how to improve these devices.

The Clausius Clapeyron equation is the working principle of pressure-cooker and similarly thermodynamic laws are used to interpret various phenomena in nature.

The nature of heat flow in classical systems follows Maxwell-Boltzman distribution. Hence some other flow (except heat) which are consistent with Boltzman distribution also follow analogous laws of physics identical to the laws of thermodynamics. For example, the radiation from blackhole surface is not Stephen radiation, rather it is Boltzman radiation. So to study blackhole, a different branch of physics is developed namely **blackhole thermodynamics**.

Most importantly as per recent observation, our universe is undergoing accelerated expansion and there is no concrete explanation behind this type of cosmic evolution. In this case, laws of thermodynamics may have some basic role to set up the background theory behind this late time acceleration. This branch of thermodynamics is known as **Universal thermodynamics**.

1.2 General discussion on non-equilibrium thermodynamics :

It is said earlier that non-equilibrium thermodynamics is more general approach to study the thermodynamic behaviour of a system than equilibrium thermodynamics.

When a system is in equilibrium with other systems or surrounding then there is literally no net flow of any flux or any quantity among the different local parts of that system. All the local portions of a system are homogeneous and there is no gradient of any thermodynamic parameter which may produce any driving force or pressure. Then the whole system can be characterized by one sustainable value of thermodynamic parameters. Then the exchange of heat, other forms of relevant energy, external work done may change the state of the system to another equilibrium one. But the system is modeled not to be disturbed from the equilibrium condition. In such a scenario, equilibrium thermodynamics are perfectly fitted and the laws of thermodynamics can ideally describe the behaviour of the system on exchange of external influences.

But in practice the application of external energy, work done, various influences or sometimes several internal mechanisms like particle creation-annihilation processes lead to break the equilibrium condition. For example, suppose a system (example - a gas bowl) is initially in equilibrium condition with temperature (T), volume (V), Pressure (P) and with sustainable values of other thermodynamic parameters. So the system is homogeneous. Now if one tries to apply heat to the system, some local portions of the bowl will be more heated than other portions i.e. the values of thermodynamic parameters will become inhomogeneous and the final state will not be in equilibrium. So the gradient of thermodynamic parameters within the system leads to produce some driving force which drives the flow of constituent molecules from higher gradient region to lower gradient region. It is a continuous process and the thermodynamic variables can never reach a stable value unless it reaches the thermodynamic equilibrium again. While these mechanisms, classical laws of thermodynamics can not be applied on the system.

On the other hand, non-equilibrium thermodynamics allows to consider useful approximation of local and instantaneous equilibrium and the instantaneous values of thermodynamic parameters can be used in laws of thermodynamics for locally equilibrium fragments of the system. Another aspect of non-equilibrium thermodynamics is to study the flow of constituent particles of the system under the gradient of corresponding parameters. These phenomena[10] are known as **transport phenomena**. Transport phenomena is the key mechanism behind the evolution of most of the non-equilibrium systems in nature.

1.2.1 Fundamental differences between equilibrium and non-equilibrium thermodynamics :

The basic differences between equilibrium and non-equilibrium thermodynamics are as following :

(I) In equilibrium thermodynamics[10, 27, 28], the time courses of physical processes are considered to be 0. On the other hand, non-equilibrium thermodynamics allows us to describe the parameters while progressing the process.

(II) In equilibrium thermodynamic description for theoretical development, one consider ideal **quasi-static process** [10, 30, 31] which is a timeless and conceptual process. No process can be quasi-static in reality. Practically equilibrium thermodynamics is an outcome of differential geometry, not an actual process.

Non-equilibrium thermodynamics tries to describe the actual processes that occur with a time course. The state variables of such a system evolves with time during the process.

(III) There is no idea of transport phenomena in equilibrium thermodynamics.

Non-equilibrium thermodynamics deals with transport phenomena.

1.2.2 Physical significance of state variables of non-equilibrium systems :

In case the system remains nearly close to the equilibrium, the state variables are represented by measuring them over a locally equilibrium region, similarly as done in equilibrium thermodynamics over the whole system.

Generally the non-equilibrium thermodynamic[27, 31, 32] systems are inhomogeneous and evolve with time (i.e. spatially and temporally inhomogeneous). But the flow of quantity is assumed to be smooth up to sufficient degree so that one can define the time and space derivatives of state variables.

The state variables must be functionally related to each other so that the laws of thermodynamics can be written as it is done in equilibrium thermodynamics[33, 34]. This condition is essential but in reality, it is difficult to satisfy the interrelation between the state variables of an inhomogeneous system. So far non-equilibrium thermodynamics is still a modeled approach and work is going on this issue to find the suitable practical way of representation of non-equilibrium state variables.

1.2.2.1 Several approaches of non-equilibrium thermodynamics :

Non-equilibrium thermodynamics is not an established theory rather is an ongoing work. In different systems, several different approaches have been adopted for applying laws of thermodynamics and to represent the state variables.

The important ideas in such non-equilibrium description is to incorporate time rate of dissipation of energy, time rate of entropy production[32, 33], thermodynamic fields, dissipative structure and nonlinear dynamical structure.

(I) Generally in some cases, a condition of the system namely **non-equilibrium steady state** is considered under which the production rate of entropy and flows of some quantities are non-zero but still there is no time variation of physical parameters[33, 34, 35].

(II) In some other cases, the evolution of non-equilibrium systems are considered as **irreversible processes**[33, 34]. Hence the mathematical procedure of defining the state variables is almost identical to the classical thermodynamics with a dissipative term. such approaches are known as **classical irreversible thermodynamics**.

Some other approaches like **extended irreversible thermodynamics** , generalized thermodynamics etc. are also found in literature.

1.2.3 Concept of local equilibrium :

The concept of local thermodynamic equilibrium [34, 35] is based on the assumption that a very small amount of volume of the system is in equilibrium i.e. that the elementary portion is homogeneous. So there is no effective flow within that elementary volume. For example, one may think any system to be divided into cells or micro fragments with infinitesimal volume. The classical thermodynamics can be applicable to good approximations here.

Local thermodynamic equilibrium is the concept of microscopic dynamics of a system. The kinematics of the constituent molecules in microstates is responsible for macroscopic evolution of that system. Hence there are two relaxation times [34] of different order of magnitudes. One is the relaxation time for the internal dynamics of the system to be changed and another is the relaxation time for macroscopic change.

If these two relaxation times are of the same order, then the concept of local thermodynamic equilibrium will lose its significance. For the speed of sound in air is much greater than wind speed, the concept of local equilibrium holds good but if the velocities of sound [33] and wind are of the same order then local thermodynamic equilibrium has no physical meaning in such a case.

For the systems violating the criteria of local equilibrium, other concepts like **extended irreversible thermodynamics** can be adopted.

1.2.3.1 Approach of equilibrium in extended irreversible thermodynamics :

It is beyond the scope of the restrictions of local thermodynamic equilibrium. Instead of considering the value of the physical state parameter, here the flux of the quantities is introduced. The relation between the flux and time rate of change of that quantity is the basic equation in such formalism[35]. Generally such procedure is applied in small scale systems.

1.2.4 Transport phenomena :

Transport phenomena are the most important aspects of non-equilibrium systems. When a system is not in equilibrium but still one considers it in steady state i.e. the state variables are time dependent then the surrounding will also be time dependent due to interaction[8, 10].

It happens when the equilibrium condition is disturbed by external causes, then due to inhomogeneity of system parameters, transport of quantities (momentum, mass, energy etc.) occurs from higher gradient region to lower in order to restore the equilibrium state[10]. The equilibrium conditions can be disturbed in different ways which leads to several transport phenomena namely viscous flow, thermal and electrical conduction, diffusion etc. In each case, the common aspect is the generation of dissipative pressure due to gradient of state parameter which leads to flow or transport of quantity.

1.2.4.1 Viscosity :

If the equilibrium condition of a system is broken by flow velocity or motion of mass[10], there will be relative motion between the adjacent layers of a fluid system. Then there will be gradient of momentum between the adjacent layers of the system which produces a dissipative pressure namely **bulk viscous pressure ($\mathcal{P}$)** . In linear viscous systems, this bulk viscous pressure is linearly proportional to the velocity(u) gradient of the layers.

$$\mathcal{P} = \mathcal{N}\vec{u_n}.\vec{\nabla}u, \tag{1.53}$$

where $\mathcal{N}$ is called the coefficient of viscosity and $\vec{u_n}$ is the unit normal to the surface plane.

This viscous pressure leads to transport of momenta from higher velocity layer to lower velocity layer until the system be restored into equilibrium again. This process is the underlying cause of internal friction or viscosity of that fluid. Viscosity is a general property of matter, especially of fluids.

1.2.4.2 Thermal conductivity :

If the stable condition of a system is disturbed by applying heat into it, then the temperature will be inhomogeneous within the system. The temperature inequality among the different parts of the system drives the molecules to transport the thermal energy from higher temperature region to lower. The time rate of transport of heat is linearly proportional to the temperature gradient and cross sectional area (A) of the conductor[10].

$$\frac{dQ}{dt} = -K\vec{A}.\vec{\nabla}T. \tag{1.54}$$

K is the thermal conductivity of the medium.

1.2.4.3 Electrical conductivity :

When the equilibrium state is imbalanced by adding electrical charges, the electrical potential becomes inhomogeneous among different parts of the system. As a result in order to restore the balance of potential, electrical charges are moved from high potential region to low potential region. This creates the electrical current flow. The value of current (I) is proportional to the potential difference ($\Delta\epsilon$). For linear conducting medium,

$$\Delta\epsilon = IR, \tag{1.55}$$

R is called the resistance of the system.

In general it can be written as,

$$I = \frac{dq}{dt} = -\sigma\vec{A}.\vec{\nabla}\epsilon, \tag{1.56}$$

σ is the electrical conductivity of the medium.

1.2.4.4 Diffusion :

Whenever the equilibrium condition is deviated due to inequality of concentration among different parts of a system, the gradient of concentration creates a certain dissipative pressure namely diffusive pressure. This diffusive pressure drives the molecules to move from higher concentration region to lower and hence the system may restore its equilibrium condition back. Diffusive pressure ($\mathcal{P}_d$) is proportional to concentration gradient[10, 35].

$$\mathcal{P}_d = -D\vec{u_n}.\vec{\nabla}\rho, \tag{1.57}$$

D is called the **coefficient of diffusion**.

The motion of constituent particles due to thermal agitation is the key behind all transport phenomena. At a macroscopic level, this can be realized as the flow of liquid from high region to low due to the gradient of gravitational potential energy. Microscopically, such local inequality of state parameters drives the molecules to move. The velocity of movement of molecules are of very high order and one may expect the transport phenomena to occur rapidly. But in reality, these mechanisms are very slow.

The collisions between the moving particles prevent the free motion of them and the rate of transport phenomena conveniently depends on the **free path**.

Free path : The distance between two successive collisions is the free path. The values of the free path are different for different collisions within the fluid. Hence the average value of free paths are considered and termed as mean free path (λ)[8, 10].

The value of mean free path depends on the number density of molecules and the size of molecules.

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma_m^2 \frac{N}{V}}, \tag{1.58}$$

σ_m is the diameter of the molecule. The above equation(1.58) was derived by Maxwell.

Mean free path can be thought of as the characteristic length of collision. In internal motion, the survival equation is written as [10]

$$N(x) = N_0 e^{-\frac{x}{\lambda}}. \tag{1.59}$$

N_0 is the number of uncollided molecules initially and $N(x)$ is the remaining uncollided molecules after traversing x distance. Hence the reciprocal of the mean free path is the collision probability (P_c) or number of collisions for traveling unit length.

$$P_c = \frac{1}{\lambda}. \tag{1.60}$$

Physically if λ is the mean free path of a system then it means a molecule transports its momentum, energy, mass etc. through an average distance λ. For an equilibrium fluid system, the transport is balanced by the same amount of transfer in two opposite directions. But in non-equilibrium cases, there will be a net flow and it will lead to the net transport phenomena.

1.2.4.5 General approach to the transport phenomena from kinetic theory :

Adopting the general approach for a non-equilibrium system, one can develop the mathematical relation for a transport phenomena. Let consider a system is deviated from its equilibrium condition and a state parameter Θ is inhomogeneous. This Θ may be mass, energy, momentum etc. So Θ has different values at different layers of the fluid. Here for simplicity, the gradient of Θ is assumed to be along Z-axis only.

Let consider an elementary volume element dV at (r, θ, ϕ) . The distance of the layer from the origin is $z = rcos\theta$. If the value of Θ is Θ_0 at the origin, then

$$\Theta(r, \theta, \phi) = \Theta_0 + \frac{d\Theta}{dz} rcos\theta. \tag{1.61}$$

Let the number of molecules per unit volume with speed between c to $c + dc$ is dn_c. Hence the number of collisions within the volume dV equals to $\frac{1}{2}c.P_c.dn_c dV$ per unit time. As each collision yields two new paths, the number of molecules escaped from dV in elementary time interval dt is given by $2.\frac{1}{2}.c.P_c.dn_c.dV.dt$. Here $c.P_c = \frac{c}{\lambda}$, is called the collision frequency or number of collisions per unit time.

These molecules will spread out spherically in all possible directions (isotropically). If one considers an elementary area dA at the origin, then the number of molecules moving towards dA can be estimated as,

$$dN_{r,\theta,\phi} = \frac{dAcos\theta}{4\pi r^2}.c.P_c.dn_c.dV.dt. \tag{1.62}$$

Following the survival equation, the number of molecules which reaches to dA can be written as

$$dN_{dA} = dN_{r,\theta,\phi}.e^{-\frac{r}{\lambda}}. \tag{1.63}$$

Hence total transport of the quantity Θ from upper region to dA will be

$$\mathcal{T}\downarrow = \frac{dAdt}{4\pi\lambda} \int_{c=0}^{\infty} \int_{\text{upper}} cdn_c e^{-\frac{r}{\lambda}} \left(\Theta_0 + rcos\theta \frac{d\Theta}{dz}\right) dV. \tag{1.64}$$

Similarly, the total transport of Θ from lower region to dA can be written as,

$$\mathcal{T}\uparrow = \frac{dAdt}{4\pi\lambda} \int_{c=0}^{\infty} \int_{\text{upper}} cdn_c e^{-\frac{r}{\lambda}} \left(\Theta_0 - rcos\theta \frac{d\Theta}{dz}\right) dV. \tag{1.65}$$

So the net transport to the origin per unit area and in per unit time interval is found as[10],

$$\mathcal{P}_\Theta = \frac{\mathcal{T}\downarrow - \mathcal{T}\uparrow}{dAdt} = \frac{1}{3} dAdt \frac{d\Theta}{dz} \lambda n \bar{c} \tag{1.66}$$

where $\bar{c} = \frac{1}{n} \int_{c=0}^{\infty} cdn_c$, is the average speed of molecules.

coefficient of viscosity : For viscosity, momentum ($p = mu$) is transported so $\Theta = mu$. There fore the viscous pressure can be obtained from equation(1.66) as

$$\mathcal{P} = \frac{1}{3}mn\bar{c}\lambda\frac{du}{dz} = \frac{1}{3}\rho\bar{c}\lambda\frac{du}{dz}. \tag{1.67}$$

Now comparing with equation(1.53), one finds the expression of coefficient of viscosity as

$$\mathcal{N} = \frac{1}{3}mn\bar{c}\lambda. \tag{1.68}$$

Thermal conductivity : Thermal conduction occurs due to temperature gradient and the thermal energy is transported. So $\Theta = Q$ in this case.

Hence from equation (1.66), one has

$$\frac{1}{d}\frac{dQ}{dt} = \frac{1}{3}n\bar{c}\lambda\frac{dQ}{dz}. \tag{1.69}$$

Again comparing with equation(1.54), it can be found that

$$K = \frac{1}{3}n\bar{c}\lambda\frac{dQ}{dT}. \tag{1.70}$$

As, the specific heat $c_V = \frac{1}{m}\frac{dQ}{dT}$, one can write

$$K = \mathcal{N}c_V. \tag{1.71}$$

Electrical conductivity : Electrical conduction happens due to potential difference between the two ends of a conductor and here electrical energy is transported. The famous law which describes the features of electrical conduction is namely Ohm's law. The statement of Ohm's law is as

$$\vec{j} = \sigma\vec{E}, \tag{1.72}$$

where $\vec{j}$ is volume current density vector, E is electric field and σ is the electrical conductivity of the medium.

Diffusion : Diffusion is the transport of constituent molecules from higher concentration region to lower. The concentration gradient creates a pressure namely diffusive pressure which drives the molecule to move.

From equation (1.66), one has the form of diffusive pressure

$$\mathcal{P}_D = -\frac{1}{3}\lambda\bar{c}\nabla\rho. \tag{1.73}$$

Hence the diffusion coefficient is found in the form

$$D = \frac{1}{3}\bar{c}\lambda = \frac{\mathcal{N}}{\rho}. \tag{1.74}$$

Notably the transport coefficients depend on the mean free path λ of the molecules. The mean free path is related to the size of the molecules as

$$\lambda = \frac{1}{\sqrt{2}\pi\mathcal{D}^2 n}, \tag{1.75}$$

with $\mathcal{D}$ is the diameter of a molecule.

So one may conclude that the transport phenomena depends on the size of constituent particles and the density of the system.

Microscopically the internal movement of molecules is responsible for such phenomena. The to and fro motion of the molecules is called **Brownian motion**. Brownian motion is totally an internal dynamics and does not depend on external forces applied on the system.

Salient features of Brownian motion : [8, 10]

(a) The motion is continuous, irregular, random and spontaneous.

(b) The motion is independent of external mechanical forces.

(c) The motion of molecules will be greater for smaller size.

(d) The motion increases with temperature.

(e) Brownian motion varies inversely with viscosity of the fluid.

1.3 Elements of cosmology :

By terminology, **COSMOLOGY** is the study of the Universe. Basically it deals with the origin, large scale structure, kinematics and the evolutionary pattern of our Universe. The ideas of cosmology are investigated through physics, mathematics, astronomy and even in philosophy. Many assumptions made by philosophers in cosmological context are still beyond the scope of scientific verifications. Therefore cosmology is also termed as " historical science".

In the words of Stephen Hawking, we are very special that we can understand the Universe.

"We are just an advanced breed of monkeys on a minor planet of a very average star. But we can understand the Universe. That makes us something very special.".

Stephen Hawking to **Der Spigel** in 1988

The observational part of cosmology is astronomy. The journey of modern astronomy started from the ideas of Johannes Kepler, Nicolaus Copernicus and Galileo Galilei when they individually proposed a heliocentric structure instead of a primitive geocentric Ptolemaic system. The first theoretical explanation behind Kepler's laws of planetary motion came from Issac Newton's laws of gravitation(1687). It was the first generalised law which states that the systems on earth and the celestial systems follow the same law. In the large scale structure of the Universe, the gravitational interaction is the most important interaction as it is the interaction of the longest range. A major part of theoretical cosmology is the study of gravity on a large scale.

The modern cosmology is beyond the description of Newtonian gravity where gravitational force is proposed as an attractive central force of inverse square law $\left(\vec{F} = -\frac{Gm_1m_2}{r^3}\vec{r}\right)$. The essential element of modern cosmology is Einstein's general theory of relativity.

1.3.1 General theory of relativity :

"General relativity is at least very close to the truth.".

Sir Roger Penrose

The general theory of relativity is the explanation of gravitation and the interrelation of it with other forces of nature. It is relevant in the cosmological and astronomical realm. Especially when Newtonian gravity is not sufficient to explain some astronomical phenomena like neutron stars, black holes and gravitational waves, Einstein's general relativity is very successful to interpret these. The precession of the perihelion of the Mercury, gravitational lensing and gravitational redshift are the observational evidences of this modern theory of gravity.

In 1915, Albert Einstein published the most beautiful physical theory. As per this theory, gravity is the manifestation of curvature in space-time i.e. the matter component of gravity is related to the curvature of space-time.

In differential geometry, the curvature of space-time is a 4 -rank tensor but the matter component of a source is represented by a two rank tensor namely energy - momentum tensor ($T_{\mu\nu}$). In order to make a mathematically reasonable correlation between curvature and ($T_{\mu\nu}$), the 4- rank Reimannian tensor is contracted to two rank Ricci tensor ($R_{\mu\nu}$) by the metric tensor of space-time $g_{\mu\nu}$. Hence one has the Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{1.76}$$

where, R is the contraction of Ricci tensor and it is called Ricci scalar curvature. As the energy momentum tensor is conserved, the left hand side is also covariantly conserved. This property of curvature is known as Bianchi identity. G is the universal gravitational constant.

1.3.1.1 Principles of general theory of relativity(GTR) :

There are five principles in **GTR** which generalise the independence of choice of frame of reference, in contrast to the privileged inertial frame of reference. Special theory of relativity (STR) is the special case of GTR where only inertial frames of reference are allowed. So STR can only be applied in the absence of gravity which corresponds to flat space-time Minkowski geometry. The principles of GTR are as following :

Mach's principle : This principle is the consequence of the realisation of Newtonian gravity. If there is some mass, then due to gravitational attraction there will be some acceleration a. Under acceleration, the space-time trajectory will always be nonlinear and it corresponds to the curvature in space-time. The postulates of this principle are [36]

(i) The matter distribution determines the geometry of space-time.

(ii) There will be no curvature in space-time if there is no matter.

(iii)In an empty Universe, a body possesses no inertial property.

Equivalence principle : At every point in space-time in an arbitrary gravitational field it is possible to choose a locally inertial coordinate system such that within a sufficiently small region, the laws of physics take the same forms as in an inertial cartesian coordinate system in the absence of gravitation[37].

Correspondence principle : GTR must agree with STR in the absence of gravity. Also GTR must be equivalent to Newton's laws of gravitation in the limit of weak gravitational field and small velocity compared to the velocity of light c i.e. non-relavistic limit $v << c$.

General covariance principle : The laws of physics are covariant in all frames of reference. The physical quantities may not be invariant in all frames but the laws of physics sustain the same forms in all frames.

Principle of minimal gravitational coupling : No term containing explicitly the curvature tensor should be added while transitioning from STR to GTR.

1.3.1.2 Ideas of mass in Newton's theory :

In Newtonian theory, mass is the important aspect of gravitation. Also the inertial property of a body is measured by the measurement of the mass of that system. Mass is defined in three different ways.

Inertial mass : Inertial mass is the measure of the resistance of a body against being accelerated by an applied force.

$$m_I = \frac{F}{a}, \tag{1.77}$$

m_I is the inertial mass of the system. Notably, inertia is the property by virtue of which a body resists being accelerated and Newton's first law of motion is also known as law of inertia.

Passive gravitational mass : It is the measure of the interaction of the body with the gravitational field. If ϕ be the gravitational potential and a body feels the gravitational force $\vec{F}$ under ϕ, then the the passive gravitational mass m_p is given by

$$m_p = -\frac{\vec{F}}{\vec{\nabla}\phi}. \tag{1.78}$$

Active gravitational mass : If a body generates a gravitational potential ϕ at a r distance apart from it, then the active gravitational mass is defined as

$$m_a = -\phi\frac{r}{G}. \tag{1.79}$$

1.3.1.3 Einstein tensor :

The contraction of Reimannian curvature tensor i.e. the Ricci tensor $R_{\mu\nu}$ is not taken as the representation of curvature in the left hand side of Einstein field equations. $R_{\mu\nu}$ itself is not covariantly conserved but the quantity $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is covariantly conserved. This term is called the Einstein tensor $G_{\mu\nu}$.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}. \tag{1.80}$$

$G_{\mu\nu}$ is the measure of curvature in space-time. Asper Mach's principle,

$$G_{\mu\nu} \propto T_{\mu\nu}. \tag{1.81}$$

This leads to Einstei n field equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1.82}$$

$\frac{8\pi G}{c^4}$ is the coupling constant.

1.3.1.4 Einstein field equation from generalisation of dynamics under curved space-time :

Einstein's field equations can also be obtained from variational principle(Hamilton's principle) or principle of least action. The general action under all types of interactions in general curved space-time is given by

$$I = \int \sqrt{-g}[\mathcal{L}_G + \tilde{\mathcal{L}}]d^4x, \tag{1.83}$$

where $\mathcal{L}_G$ is the lagrangian density for the gravitational field and $\tilde{\mathcal{L}}$ is the lagrangian for all other interactions. g is the contraction of metric tensor $g_{\mu\nu}$.

$$g = g^{\mu\nu} g_{\mu\nu}. \tag{1.84}$$

The form of $\mathcal{L}_G$ is chosen as R and hence applying variational principle $\delta I = 0$, one has

$$\int \sqrt{-g}\left[R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{8\pi G}{c^4}T_{\mu\nu}\right]\delta g^{\mu\nu} d^4x = 0 \tag{1.85}$$

with energy-momentum tensor $T_{\mu\nu}$ is given by

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\left[\frac{\partial\sqrt{-g}\tilde{\mathcal{L}}}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial\sqrt{-g}\tilde{\mathcal{L}}}{\partial g^{\mu\nu},\alpha}\right)\right] \tag{1.86}$$

Hence one may find Einstein's field equations as equation (1.76). Similarly one can also write

$$R_{\mu\nu} - \left(\frac{1}{2}R - \Lambda\right) g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{1.87}$$

where Λ is a constant namely cosmological constant. Clearly $(G_{\mu\nu} + \Lambda g_{\mu\nu})$ will be covariantly conserved because of the conservation of the energy - momentum tensor.

1.3.2 Modifications of gravity theory in the context of cosmology :

GTR is excellently well in accord with some cosmological phenomena which were out of scope of Newtonian gravity. But still it is not suitable to interpret the recent observational result regarding the accelerated expansion. The recent day researchers in cosmology are divided into two groups.

First group is in favour of assuming some exotic cosmic elements with negative pressure called dark energy (DE).

On the other hand, Some cosmologists try to modify the Einstein-Hilbert action. They choose arbitrary function of Ricci scalar $f(R)$ as lagrangian density for gravitational field instead of only R. This type of modified gravity is called $f(R)$ gravity. But still there is no gravity theory which is completely in accord with the experimental results.

1.3.2.1 The cosmological principle :

"On a sufficiently large scale, the Universe is both homogeneous and isotropic".

- this is the formal statement of cosmological principle.

Homogeneity and isotropy are the basic symmetries of a system. Asper Noether's theorem, every symmetry is associated with conservation of some parameter.

Homogeneity of the Universe : The lagrangian will be invariant under arbitrary translation in space-time. Consequently every point in space-time is identical and it leads to the conservation of four momenta (p^{μ}).

In cosmological context, homogeneity of space allows us to consider the uniform space-time geometry on a large scale of the Universe.

Isotropy of Universe : The lagrangian will also be invariant under rotation with an arbitrary angle in space. It effectively implies that the nature of space-time geometry will be independent of direction. From the perspective of Noether's theorem, isotropy of space leads to the conservation of angular momenta.

In cosmological context within an isotropic Universe, the cosmic fluids posses the same values of physical parameters in all directions. For example, the radial and transverse pressure are same for an isotropic fluid. On the other hand, anisotropic fluid possess different radial and transverse pressure.

1.3.2.2 Weyl's postulate :

The significance of Weyl's postulate is that it states - **" The Universe on a large scale is spatially homogeneous and spatially isotropic "** .

There is also the idea of **substratum** or fluid pervading space in which the galaxies move like fundamental particles in the fluid. Here it is also assumed that - **"the particles of the substratum lie in the space-time on a congruence of timelike geodesics diverging from a point in the finite or infinite past ".**

The postulate demands that the geodesics do not intersect except a singular point in the past or probably a similar future singular point. Hence there is only one unique geodesic at any space-time point. As a consequence, it confirms that the matter at any space-time point possesses a unique velocity i.e. the substratum can be taken as a perfect fluid.

In practice, the actual motion of galaxies is slightly deviated from the ideas of Weyl's postulate. But on a large scale, one may accept that Weyl's postulate nearly reflects the actual space-time geometry of the Universe.

1.3.2.3 The Robertson Walker metric :

The universe is homogeneous and Isotropic on a scale more than a 100 million light years. Evidently on that scale, the density of cosmic fluids is uniform and identical in all directions.

Using the idea of spatial homogeneity from Weyl's postulate, one may assume the metric of the Universe on large scale as,

$$ds^2 = c^2dt^2 - h_{ij}dx^i dx^j, \tag{1.88}$$

where $h_{ij}, (i, j = 1, 2, 3)$ are the elements of the general metric tensor. Asper Weyl's postulate, only diagonal metric terms will exist and the expansion factor or scale factor a will be independent of spatial coordinates. The ideal metric which can best describe the Universe is given by

$$ds^2 = c^2dt^2 - \frac{a^2(t)}{1+\frac{1}{4}k(x^2+y^2+z^2)}\left[dx^2+dy^2+dz^2\right]. \tag{1.89}$$

In spherical polar coordinate system, this type of metric can be written as

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{1.90}$$

This is known as **Robertson - Walker metric.** k is called the curvature parameter .

(i) $k = -1$ for the Universe with negative spatial curvature i.e. open Universe.

(ii) $k = +1$ for Universe with positive spatial curvature i.e. closed Universe.

(iii) $k = 0$ for a flat Universe with no curvature.

1.3.2.4 Friedmann equations :

The Einstein's field equations for Robertson - Walker space-time (also known as FLRW metric) leads to the two independent equations.

$$3\frac{\dot{a}^2}{a^2} + 3\frac{3k}{a^2} = 8\pi\rho \tag{1.91}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi P. \tag{1.92}$$

The unit system is chosen here as the natural unit system where $G = c = 1$. Here $\dot{x} = \frac{dx}{dt}$. Notably here the energy- momentum tensor (in covariant form) is taken as an isotropic perfect fluid

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu + Pg_{\mu\nu}, \tag{1.93}$$

with $u_\mu, \mu = 0, 1, 2, 3$ are the components of four-velocity. The matrix of contravariant energy-momentum tensor is

$$T^{\mu\nu} = \text{Diagonal}(\rho, P, P, P). \tag{1.94}$$

With the inclusion of cosmological constant, the Friedmann equations can be written in the forms

$$3\frac{\dot{a}^2}{a^2} + 3\frac{3k}{a^2} - \Lambda = 8\pi\rho \tag{1.95}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \Lambda = -8\pi P. \tag{1.96}$$

These two Friedmann equations, one may obtain the conservation equation

$$\frac{d}{dt}(\rho a^3) + p\frac{d}{dt}(a^3) = 0. \tag{1.97}$$

1.3.2.5 Some observational parameters :

For analysing the experimental data, few well known parameters are introduced in cosmology. These parameters are very important to understand the large scale dynamics of the Universe as well as the other aspects associated with evolution. Some of them are as following :

Hubble Parameter : Asper Hubble's law, the recession velocity v_r of a galaxy is proportional to the distance d from us (a supposed frame of reference).

$$v_r = Hd \tag{1.98}$$

The proportionality constant H is called **Hubble parameter**.In terms of scale factor a,

$$H = \frac{\dot{a}}{a} \tag{1.99}$$

The present value of Hubble parameter is $H_0 = 72 \pm 8$ km/s/Mpc asper the project by Friedmann et.al for the Hubble space telescope. The reciprocal of H_0 is an estimation of the present age of our Universe. This means our Universe is at least 13.9 Gyr old.

The density parameter : The density parameter is an important quantity to understand the curvature of space-time by measuring the variation of density. The required density of the Universe to make the geometry flat (i.e. $k = 0$) is called the critical density ρ_c.

From friedmann equations,

$$\rho(t)_c = \frac{3H^2}{8\pi G}. \tag{1.100}$$

Now the curvature in space-time is related to the deviation in density of the Universe from its critical value. It is relatively expressed in terms of density parameter

$$\Omega = \frac{\rho}{\rho_c}. \tag{1.101}$$

As density is a continuous thermodynamic quantity, Ω remains continuous through out the complete cosmic evolution.

Deceleration parameter : As the recent observation says that our Universe is not only expanding but also it is expanding with acceleration. The positive value of Hubble parameter H_0 confirms about the expansion but it is also important that in which rate, the rate of expansion changes. The time rate of change of Hubble parameter or second order derivative of scale factor with respect to time may be used to quantify this. But the more convenient way to determine the nature of expansion is the deceleration parameter.

The Taylor series expansion of the scale factor about any time epoch t_0 is given by

$$a(t) = a(t_0) + (t - t_0)\dot{a}(t_0) + \frac{1}{2}(t - t_0)^2\ddot{a}(t_0) + ... \tag{1.102}$$

So, one has

$$\frac{a}{a(t_0)} = 1 + H_0(t - t_0) - \frac{1}{2}q(t_0)(t - t_0)^2 + ... \tag{1.103}$$

Here $q(t_0)$ is called the deceleration parameter and is given by

$$q = -\frac{1}{H_0^2}\frac{\ddot{a}(t_0)}{a(t_0)} \tag{1.104}$$

Clearly for accelerated expansion, q must be negative.

Cosmological red shift : Asper Doppler effect, the frequency of wave coming from a away going source will be decreasing with velocity of the source i.e. the wavelength will increase. This effect is also found while detecting the electro-magnetic wave coming from the distant stars. The shift of wavelenght of the electro-magnetic ray from the stars is a measure of the expansion of the Universe. In cosmology, red shift is used as time scale.

The red shift $z = \frac{d\lambda}{\lambda}$ is related to the recession velocity as (applying Hubble's law

$dv_r = Hdr$)

$$\frac{d\lambda}{\lambda} = \frac{dv_r}{c} = \frac{H}{c}dr \tag{1.105}$$

where dr is the displacement of the star. Hence

$$\frac{d\lambda}{\lambda} = Hdt = \frac{da}{a} \tag{1.106}$$

Here $dr = cdt$. The solution of the equation (1.106) yields

$$\lambda(a) = a\lambda_{\text{obs}}. \tag{1.107}$$

λ_{obs} is the observed value of wavelength. Similarly,

$$\lambda_e = a(t_e)\lambda_{\text{obs}}, \tag{1.108}$$

where λ_e is the value of wavelength when it is emitted at t_e. The red shift $z = \frac{\delta\lambda}{\lambda_e} = \frac{\lambda_{\text{Obs}} - \lambda_e}{\lambda_e}$.

Hence one has

$$\frac{\lambda_{\text{obs}}}{\lambda_e} = 1 + z = \frac{1}{a} \tag{1.109}$$

1.3.3 Thermodynamics in cosmology :

The thermodynamic property of the Universe reflects the key features of the evolutionary pattern of it. In general, Universe is assumed as an isolated system of one or more perfect fluids. The clausius equation $TdS = dE + PdV$, deals with the entropy production and for a non-dissipative fluid where entropy production is zero ($dS = 0$), first law of thermodynamics leads to the conservation equation in FLRW Universe. But in case of a dissipative fluid, the matter conservation (non-conservation) equation contains a dissipative term Q.

$$\frac{d}{dt}(\rho a^3) + p\frac{d}{dt}(a^3) = Q \tag{1.110}$$

Considering the cosmic fluid as a barotropic type ($P = \omega\rho$) with barotropic index ω, one finds from equation (1.110),

$$\dot{\rho} + 3(1+\omega)H\rho = Q \tag{1.111}$$

This dissipative term arises due to the dissipative pressure $\pi = -\frac{Q}{3H}$. Effectively the matter conservation equation can be written as

$$\dot{\rho} + 3H(P + \pi + \rho) = 0. \tag{1.112}$$

So the total pressure of the fluid is the resultant of its thermodynamic pressure and the dissipative pressure. For a multi-fluid Universe, the conservation equation of i-th fluid is given by

$$\dot{\rho}_i + 3H(P_i + \pi_i + \rho_i) = 0, \tag{1.113}$$

where ρ_i, P_i and π_i are the density, thermodynamic pressure and dissipative pressure respectively of the i-th fluid. Here $\pi_i = -\frac{Q_i}{3H}$, with Q_i is the dissipative term of the i-th fluid. In an isolated Universe, $\sum_i Q_i = 0$. If one looks at the energy - momentum tensor of the i-th fluid, then $T^{(i)}_{\mu\nu,\mu} \neq 0$ but $\sum_i T^{(i)}_{\mu\nu,\mu} = 0$. But in a single fluid Universe, this dissipative nature of cosmic fluid can be compensated by adding an extra term with the energy momentum tensor of the dissipative fluid. In our work, the dissipative term is chosen to be proportional to a scalar field analogous to the cosmological constant.

The physical origin of the dissipative term (also the dissipative pressure) is the disturbance of equilibrium condition and inhomogeneity within the fluid system. Hence all sorts of transport phenomena within the Universe are considered and the Universe is assumed under local thermodynamic equilibrium description of non-equilibrium thermodynamics. Our works mainly deal with the macroscopic analysis of diffusion and particle creation - annihilation process within the Universe especially we have focussed on the different aspects of diffusive Universe and setting up different models of the cosmic evolution pattern with diffusive fluids.

CHAPTER 2

THERMODYNAMIC STABILITY ANALYSIS OF DIFFUSIVE COSMIC FLUIDS

2.1 Prelude

The greatest challenge of standard cosmology today is to accommodate the recent observational predictions [38, 39, 40, 41, 42, 43, 44, 45, 46]. The modern cosmology is facing the challenging issue of explaining the present accelerated expansion of the universe. In the framework of Einstein gravity, cosmologists are speculating some hypothetical exotic matter (known as dark energy (DE) having large negative pressure) to explain this accelerating phase. It is estimated [47] that about 70 percent of the cosmic fluid consists of this unknown DE component. The simplest as well as the common candidate for this dark fluid is the cosmological constant (the zero point energy of the quantum fields). Although a large number of available observational data are in support of this cosmological parameter as a DE candidate but there are severe drawback of it at the interface of Cosmology and Particle Physics : the cosmological constant problem [48, 49] and the coincidence problem [50]. As a result there are several alternative proposed dynamical DE models into the picture. These DE models have been studied for the last several years [51], yet the cosmological constant is still the best observationally supported DE candidate. However, these different dynamic DE models cannot be compared from observational view point as these models try to adjust the data seamlessly. As a result, cosmologists have been trying with interacting DE model to have a better understanding of the mechanism of this cosmic acceleration.

From recent past, interacting dark fluid models have been receiving much attention as they can provide small value of the cosmological constant and due to their ability of explaining the cosmic coincidence problem [52, 53, 54]. Moreover, recent observed data [55, 56, 57, 58, 59, 60] favor interacting dark fluid models and it is possible to have an estimate of the coupling parameter in the interaction term by various observations[58, 59, 60, 61, 62, 63, 64, 65, 66, 67]. Further,the cosmologists are of the opinion that these interacting dark fluid models may have the solution to the current tensions on σ_8 and the local value of the Hubble constant (H_0) [59, 60, 61, 62, 63, 64, 65, 66, 67, 68]. Moreover, it is speculated that cosmological perturbation analysis may be affected by the interaction terms and consequently, the lowest multipoles of the CMB spectrum [69, 70] should have an imprint of it.

On the other hand, the unknown nature of DE may have some clue from the thermodynamic laws which are applicable to all types of macroscopic systems and are based on experimental evidence. However, unlike classical mechanics or electromagnetism, thermodynamical analysis can not predict any definite value for observables, it may only give limit on physical processes. So it is reasonable to believe that thermodynamical study of dark cosmic fluid may indicate some unknown character of it. Investigation in this direction has been initiated recently by Barboza et al [71]. But their stability conditions demand that the DE should have constant (-ve) equation of state and it is not supported by observation.

Subsequently, Chakraborty and collaborators [72, 73, 74, 75] in a series of works have shown the stability criteria for different type of dark fluids and have presented the stability conditions (in tabular form) for different ranges of the equation of state parameter. The present work is an extension of the interacting dark fluid model. Here a particular realization of the non-conservation of the energy momentum tensor is used with a diffusion of dark matter in a fluid of dark energy. The change of the energy-density is proportional to the particle density. The resulting non-equilibrium thermodynamics is studied and stability conditions are explicitly determined.

2.2 Thermodynamic analysis of non-interacting diffusive cosmic fluids having constant equation of state parameter

The universe is assumed to have different types of non-interacting cosmic fluids (including dark energy and dark matter). Here it will be examined whether all kinds of diffusive fluids are thermodynamically stable or not in an adiabatic universe . Let the equation of state of these cosmic fluids are barotropic in nature having explicit form (for i-th fluid)

$$p_i = w_i \rho_i, \tag{2.1}$$

where ω_i ,a constant ,is the barotropic index of the fluid and p_i and ρ_i are the pressure and energy density of the fluid respectively. So the Friedmann equations for the whole system take the form

$$\begin{aligned} 3H^2 &= \textstyle\sum_i \rho_i \qquad (2.2)\\ 2\dot{H} + 3H^2 &= -\textstyle\sum_i p_i \qquad (2.3) \end{aligned}$$

where $H = a^{-1}\dot{a}$ is the Hubble parameter , $a(t)$ is called the scale factor of the universe as all physical distance is scaled with same factor " a " due to homogeneity and isotropy of space-time. Now due to the diffusive nature of the fluids, they do not obey the matter conservation equation ($T_i^{\mu\nu}{}_{;\mu} = 0$), rather they follow,

$$T_i^{\mu\nu}; \mu = 3k^2 N_i^{\nu}, \tag{2.4}$$

where N_i^{ν} is the current of diffusion corresponding to that fluid and k is a constant. But for the whole universe,the total stress-energy tensor is conserved i.e.($\sum_i T_i^{\mu\nu}{}_{;\mu} = 0$)

$$\sum_i N_i^{\nu} = 0. \tag{2.5}$$

In simplest form, the conservation equation(non conservation) takes the form for FLRW universe [76, 77, 78] as,

$$\frac{\partial \rho}{\partial t} + 3H(1 + \omega_i)\rho_i = \gamma_i a^{-3} \tag{2.6}$$

where, γ_i is a constant for a particular fluid but it is different for different fluids with $\sum_i \gamma_i = 0$. Now, in the context of classical thermodynamics,the contribution to the energy of the universe by i-th fluid is given by

$$E_i = \rho_i v \tag{2.7}$$

where, $v = v_0 a^3(t)$ is the physical volume of universe at given time t and at present time t_0,the physical volume is v_0 with $a(t_0) = 1$. According to the 1st law of thermodynamics(which is an energy conservation equation),

$$TdS_i = dE_i + p_i dv. \tag{2.8}$$

Integrating equation (2.6) one can easily find the expression for energy density of the i-th fluid as,

$$\rho_i = a^{-3(1+\omega_i)} \left[\rho_{i0} + \gamma_i \int_{t_0}^{t} a^{3\omega_i} dt \right] \tag{2.9}$$

where ρ_{i0} is the value of ρ_i at $t = t_0$ i.e. at the present time .The energy density of i-th fluid, from equation (2.9) can be arranged as

$$d(\ln \rho_i) = -(1 + \omega_i) d(\ln v) + d[\ln(\rho_{i0} + \gamma_i A_i)] \tag{2.10}$$

where $A_i(t) = \frac{1}{v_0^{\omega_i}} \int_{t_0}^{t} v^{\omega_i} dt$. Now, taking entropy S_i as a function of two independent thermodynamic variables namely volume(v) and temperature (T) and also considering dS_i to be an exact differential, one obtains from equation (2.8),

$$d \ln T = -\omega_i d \ln v + \frac{w_i}{1 + \omega_i} d[\ln(\rho_{i0} + \gamma_i A_i)]. \tag{2.11}$$

Now combining equations (2.10) and (2.11), a new relation among energy density ,temperature and time can be written as

$$\frac{T}{\rho_i v} = \beta [\rho_{i0} + \gamma_i A_i]^{-\frac{1}{1+\omega_i}}, \tag{2.12}$$

where β is an integration constant. Effectively at present time $t = t_0$,

$$\frac{T_0}{\rho_{i0} v_0} = \beta \rho_{i0}^{-\frac{1}{1+\omega_i}}. \tag{2.13}$$

Now equations (2.12) and (2.13) imply the evolution of energy of the i-th fluid with time and temperature as,

$$E_i = E_{i0}\left(\frac{T}{T_0}\right)\left[1+\frac{\gamma_i}{\rho_{i0}}A_i\right]^{\frac{1}{1+\omega_i}}. \tag{2.14}$$

This can be termed as the modified equation of state of this fluid where as before $E_i = \rho_i v$ and $E_{i0} = \rho_{i0} v_0$.

2.2.1 Determination of thermodynamic derivatives : C_p, C_v, K_S, K_T

Now, the motivation of the present work is to find the criteria of thermodynamic stability of the universe . So the conditions which may be satisfied by the parameters , indicate the thermodynamic features of the evolution of the expanding universe. For any system the thermodynamic derivatives i.e. the heat capacities,compressibility and isobaric expansibility determine whether the system is stable or not. So it will be very important to analyze these parameters for each fluid in the present context.

From the 1st law of thermodynamics in equation (2.8), one can determine the two heat capacities : heat capacity at constant volume C_v and heat capacity at constant pressure C_p by the relations ,

$$C_v = \left[\frac{\partial Q}{\partial T}\right]_v = \left[\frac{\partial E}{\partial T}\right]_v \tag{2.15}$$

$$C_p = \left[\frac{\partial Q}{\partial T}\right]_p = \left[\frac{\partial H}{\partial T}\right]_p, \tag{2.16}$$

where $H = E + pv$, is called the enthalpy of the system.

Now the relation between temperature of a system at a constant volume (T_a) and the physical equilibrium temperature is given by (according to [79])

$$T = T_a\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right) \tag{2.17}$$

The temperature at a constant volume T_a also evolves as

$$T_a = T_0 a^{-3\omega_i}. \tag{2.18}$$

So from equations (2.17) and (2.18),

$$E_i = E_{i0} \left(\frac{1}{T_0}\right) T^{\frac{2+\omega_i}{1+\omega_i}} T_a^{-\frac{1}{1+\omega_i}}. \tag{2.19}$$

Now from equations (2.15)and (2.19) one obtains,

$$C_{iv} = \frac{2+\omega_i}{1+\omega_i} \left(\frac{E_{i0}}{T_0}\right) \left(\frac{T}{T_a}\right)^{\frac{1}{1+\omega_i}}, \tag{2.20}$$

and equations (2.1) ,(2.16) and (2.19) yield (using the definition of enthalpy)

$$C_{ip} = (2+\omega_i) \left(\frac{E_{i0}}{T_0}\right) \left(\frac{T}{T_a}\right)^{\frac{1}{1+\omega_i}} \left[1 - \frac{1}{2+\omega_i} \left(\frac{T}{T_a}\right) \left(\frac{\partial T_a}{\partial T}\right)_{p_i}\right]. \tag{2.21}$$

Also equation (2.21) can be rewritten (using 2.17) as

$$C_{ip} = \frac{E_0}{T_0} \left(\frac{T}{T_a}\right)^{\frac{1}{1+\omega_i}} \left[(1+\omega_i) + T \left\{\frac{\partial}{\partial T} \ln \left(1 + \frac{\gamma_i}{\rho_{i0}} A_i\right)\right\}_{p_i}\right]. \tag{2.22}$$

Further assuming temperature and pressure as independent thermodynamic variables , the variation of volume can be expressed as (See ref. [71] and [72])

$$dv = v(\alpha dT - K_T dp) \tag{2.23}$$

where

$$\alpha = \frac{1}{v} \left(\frac{\partial v}{\partial T}\right)_p \tag{2.24}$$

is familiar as thermal expansibility. The isothermal compressibility (K_T) is given by

$$K_T = -\frac{1}{v} \left(\frac{\partial v}{\partial p}\right)_T. \tag{2.25}$$

Similarly the adiabatic compressibility is

$$K_S = -\frac{1}{v} \left(\frac{\partial v}{\partial p}\right)_S. \tag{2.26}$$

In isothermal process,

$$\frac{\alpha}{K_T} = \left(\frac{\partial p}{\partial T}\right)_v. \tag{2.27}$$

Also one has the well established relation amongst heat capacities and compressibilities as [71]

$$\frac{C_p}{C_v} = \frac{K_S}{K_T}. \tag{2.28}$$

So one can find the expression for isothermal expansibility α from equation (2.11) as,

$$\alpha_i = -\frac{1}{\omega_i T} + \frac{1}{1+\omega_i}\left[\frac{\partial}{\partial T}\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}\right]_{p_i}. \tag{2.29}$$

Now equations (2.27) and (2.28) yield

$$K_{iT} = \frac{\alpha_i v}{\omega_i C_{iv}} \tag{2.30}$$

$$K_{iS} = \frac{\alpha_i v}{\omega_i C_{ip}}. \tag{2.31}$$

Hence one obtains the expressions for compressibilities from equations (2.29) , (2.30) and (2.31) containing the dependence of diffusion parameter γ_i as,

$$K_{iT} = \frac{v}{\omega_i C_{iv}}\left[-\frac{1}{\omega_i T} + \frac{1}{1+\omega_i}\left\{\frac{\partial}{\partial T}\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}_{p_i}\right] \tag{2.32}$$

$$K_{iS} = \frac{v}{\omega_i C_{ip}}\left[-\frac{1}{\omega_i T} + \frac{1}{1+\omega_i}\left\{\frac{\partial}{\partial T}\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}_{p_i}\right]. \tag{2.33}$$

2.2.2 Stability Conditions for cosmic fluids :

For thermodynamic stability of any fluid , it must follow the conditions [71, 72] namely $C_p, C_v, K_T, K_S \geq 0$. For the present work, using the expressions of these thermodynamical parameters , the stability conditions are presented in the following table 2.1.

2.3 Stability criteria with variable equation of state parameter

So far we have examined the conditions for stability of cosmic fluids with constant equation of state parameters. In the present section the stability conditions will be

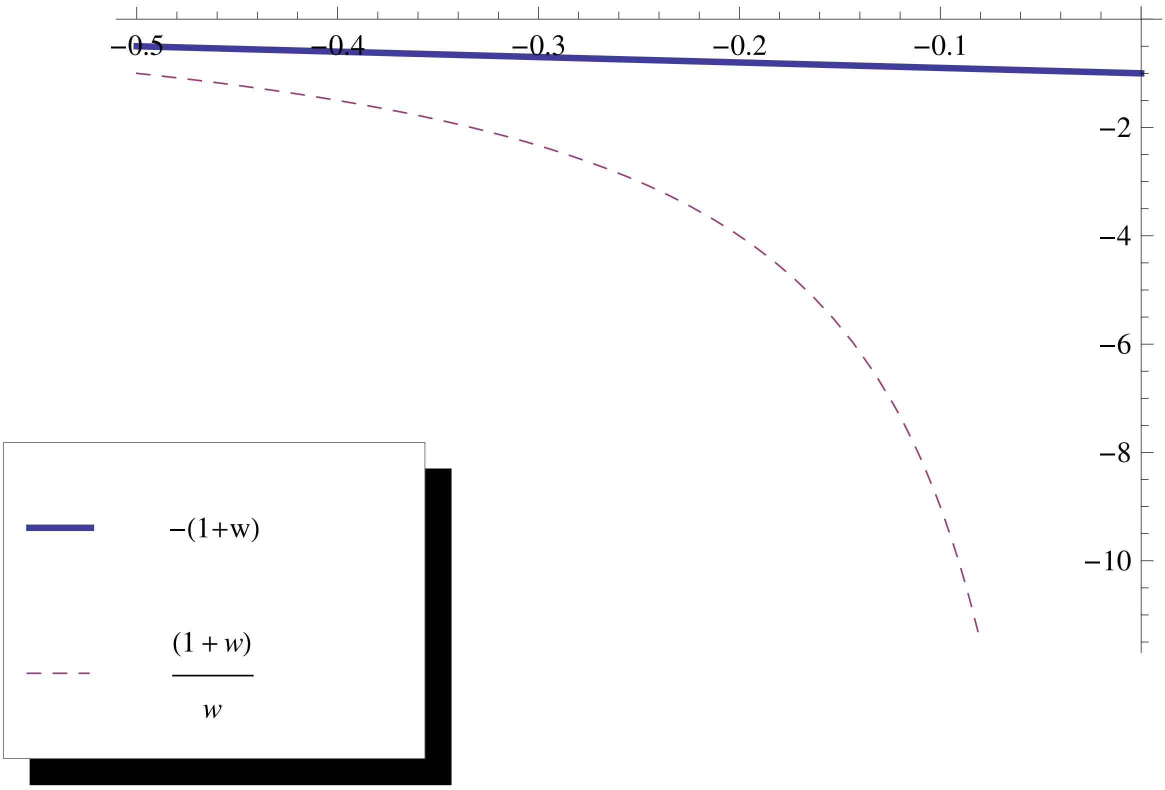

Figure 2.1: Stability region of constant equation of state fluid
: no point having value greater than $-(1+\omega_i)$ and less than $\frac{1+\omega_i}{\omega_i}$ simultaneously.

Table 2.1: Conditions for Stability for constant equation of state parameter fluids

range of ω_i	**Stability Condition**
$\omega_i \geq 0$	$\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial \ln T}\right]_{p_i} \geq \frac{1+\omega_i}{\omega_i}$
$-1 \leq \omega_i \leq 0$	$-(1+\omega_i) \leq \left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial \ln T}\right]_{p_i}$ and $\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial \ln T}\right]_{p_i} \leq \frac{1+\omega_i}{\omega_i}$ simultaneously, which is not possible hence unstable.
$-2 \leq \omega_i \leq -1$	Unstable .
$\omega_i \leq -2$	$\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial \ln T}\right]_{p_i} \geq -(1+\omega_i)$

discussed for cosmic fluids having variable equation of state parameters . The fluids are diffusive as before with no interaction amongst them. In such context, the solution of the conservation equation (2.6) yields,

$$\rho_i = \left[\rho_{i0} + \gamma_i \int_{t_0}^{t} a^{-3} e^{F_i(a)} dt\right] e^{-F_i(a)} \tag{2.34}$$

where,

$$F_i(a) = \int (1+\omega_i)\ d(lnv). \tag{2.35}$$

Taking

$$\int_{t_0}^{t} a^{-3} e^{F_i(a)} dt = A_i(t),$$

one can write equation (2.34) as

$$\rho_i = e^{-F_i(a)}\left[\rho_{i0} + \gamma_i A_i(t)\right]. \tag{2.36}$$

2.3.1 Derivation of the relation between physical equilibrium temperature (T) and temperature at constant volume (T_a):

From 1st law of thermodynamics (i.e. equation (2.8)), by choosing entropy (S) as a function of two independent variables volume (v) and temperature (T), the condition for dS to be an exact differential gives

$$\left(\frac{\partial S}{\partial v}\right)_T = \frac{\rho}{T}(1+\omega) \tag{2.37}$$

$$\left(\frac{\partial S}{\partial T}\right)_v = \frac{v}{T}\frac{d\rho}{dT} \tag{2.38}$$

$$\text{and} \quad \frac{d\rho}{\rho} = \frac{1+\omega}{\omega}\frac{dT}{T} - \frac{d\omega}{\omega}. \tag{2.39}$$

Now, from equation (2.39) one obtains the energy density as a function of temperature as

$$\rho = \frac{B}{\omega}e^{\int \frac{1+\omega}{\omega}\frac{dT}{T}}. \tag{2.40}$$

Again, combining equations (2.38) and (2.40) the expression for entropy can be written as,

$$S = v\,\frac{1+\omega}{\omega T_a}Be^{\int \frac{1+\omega}{\omega}\frac{dT_a}{T_a}}. \tag{2.41}$$

Now, as for an adiabatic system $\Delta S = 0$, so one can determine from equation (2.41), the relation between T_0 and T_a (i.e. evolution of T_a) as

$$T_a e^{-\int_{present}^{t} \frac{1+\omega}{\omega}\frac{dT_a}{T_a}} = T_0\frac{1+\omega}{\omega}\frac{\omega_0}{1+\omega_0}a^3. \tag{2.42}$$

Now combining equations (2.35) , (2.36) and (2.42) one obtains ,

$$\rho_i = \frac{1}{\omega_i}e^{\int_{present}^{t} \frac{1+\omega_i}{\omega_i}\frac{dT_a}{T_a}}[\rho_{i0} + \gamma_i A_i(t)], \tag{2.43}$$

or equivalently

$$\rho_i = \frac{1}{\omega_i}e^{\int_{present}^{t} \frac{1+\omega_i}{\omega_i}\frac{dT_a}{T_a}}3z_i B(A_i){v_0}^{-1} \tag{2.44}$$

where $z_i = \frac{\rho_{i0}v_0}{3T_0}$ and $B(A_i) = T_0 + \frac{\gamma_i T_0}{\rho_{i0}}A_i(t)$.

Now as the adiabatic contribution of entropy for the i-th fluid (excluding the diffusive entropy ($S_{id} = \int \frac{v_o \gamma_i}{T} dT$)) namely,

$$S_{i(ad)} = \frac{(1+\omega_i)\rho_i v}{T} \tag{2.45}$$

does not vary (i.e. $d\ln(S_{i(ad)}) = 0$) , so temperature can be written as,

$$T = \frac{1+\omega_{i0}}{\omega_{i0} L}\left(\frac{T_a}{T_0}\right) B(A_i) \tag{2.46}$$

where , L is a constant . Now combining (2.45) and (2.46) yields

$$\left(1 + \frac{\gamma_i}{\rho_{i0}} A_i\right) = \left(\frac{T}{T_a}\right) Q \tag{2.47}$$

with $Q = \frac{\omega_{i0} L}{1+\omega_{i0}}$, a constant .

2.3.2 Thermodynamic derivatives

Similar to the above section , one can determine the heat capacities and compressibilities of the present thermodynamical system as follows: now the solution of equation (2.6),(or equivalently equation (2.34)) can be expressed as

$$d\ln\rho_i = -(1+\omega_i) d\ln v + d\ln\left[1 + \frac{\gamma_i}{\rho i0} A_i\right]. \tag{2.48}$$

Again from equation (2.39), one finds

$$d\ln\rho_i = \frac{1+\omega_i}{\omega_i} d\ln T - \frac{d\omega}{\omega} \tag{2.49}$$

Combining the above equations (2.48) and (2.49) one can write,

$$\frac{T}{E_i(1+\omega_i)} = Ce^{-\int \frac{1}{1+\omega_i} d\ln\left[1+\frac{\gamma_i}{\rho_{i0}} A_i\right]}. \tag{2.50}$$

Now imposing the initial conditions , one can formulate the modified equation of state in terms of evolution of energy with time and temperature as

$$E_i = E_{i0}\left(\frac{1+\omega_{i0}}{1+\omega_i}\right)\left(\frac{T}{T_0}\right) e^{\int_{present}^{t} \frac{1}{1+\omega_i} d\ln\left(1+\frac{\gamma_i}{\rho_{i0}} A_i\right)}. \tag{2.51}$$

The above equation (2.51) can be written according to equation (2.47) as

$$E_i = E_{i0}\left(\frac{1+\omega_{i0}}{1+\omega_i}\right)\left(\frac{T}{T_0}\right)e^{\int_{present}^{t}\frac{1}{1+\omega_i}d\ln\left(\frac{T}{T_a}\right)}. \tag{2.52}$$

So using equations (2.15) , (2.16), (2.47) and (2.51) ,one can easily express the heat capacities as

$$C_{iv} = \frac{E_i}{T(1+\omega_i)}\left[(2+\omega_i) - T\frac{d\omega}{dT}\right] \tag{2.53}$$

$$C_{ip} = \frac{E_i}{T}\left[(1+\omega_i) + T\left\{\frac{\partial}{\partial T}\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}_p\right] \tag{2.54}$$

and hence one obtains the relation

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{\omega}{v}C_v + \rho\frac{d\omega}{dT}. \tag{2.55}$$

Again equations (2.48) and (2.49) yield

$$d\ln T = \frac{d\omega_i}{1+\omega_i} - \omega_i d\ln v + \frac{\omega_i}{1+\omega_i}d\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right), \tag{2.56}$$

which implies along with equation (2.47) that

$$\left(\frac{\partial \omega}{\partial T}\right)_v = \frac{1+\omega_i+\omega_i^2}{T(1+\omega_i)}. \tag{2.57}$$

So from equations (2.27) , (2.53) , (2.55) and (2.57) one writes

$$K_{iT} = \frac{\alpha_i v[2+\omega_i - T\frac{d\omega_i}{dT}]}{C_{iv}[\omega_i\{1+(2+\omega_i-T\frac{d\omega_i}{dT})\}+1+\omega_i^2]} \tag{2.58}$$

and from (2.28),one obtains

$$K_{iS} = \frac{\alpha_i v[2+\omega_i - T\frac{d\omega_i}{dT}]}{C_{ip}[\omega_i\{1+(2+\omega_i-T\frac{d\omega_i}{dT})\}+1+\omega_i^2]}. \tag{2.59}$$

Now, using equation (2.56) the expression for α_i takes the form

$$\alpha_i = \left[\frac{\partial(lnV)}{\partial T}\right]_{p_i} = \frac{1}{\omega_i(1+\omega_i)}\left[\omega_i\left\{\frac{\partial}{\partial T}\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}_{p_i} + \frac{d\omega_i}{dT} - \frac{1+\omega_i}{T}\right] \tag{2.60}$$

and the expressions for K_T and K_S are given by (using equations (2.58) , (2.59) and (2.60))

$$K_{iT} = \frac{V\left[\omega_i\{\frac{\partial}{\partial_T}ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\}_{p_i}+\frac{d\omega_i}{dT}-\frac{1+\omega_i}{T}\right][2+\omega_i-T\frac{d\omega_i}{dT}]}{\omega_i(1+\omega_i)C_{iv}[\omega_i\{1+(2+\omega_i-T\frac{d\omega_i}{dT})\}+1+\omega_i^2]} \tag{2.61}$$

$$\text{and} \quad K_{iS} = \frac{V\left[\omega_i\{\frac{\partial}{\partial_T}ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\}_{p_i}+\frac{d\omega_i}{dT}-\frac{1+\omega_i}{T}\right][2+\omega_i-T\frac{d\omega_i}{dT}]}{\omega_i(1+\omega_i)C_{ip}[\omega_i\{1+(2+\omega_i-T\frac{d\omega_i}{dT})\}+1+\omega_i^2]}. \tag{2.62}$$

Now according to the conditions of thermodynamic stability , we analyze the equations (2.53) , (2.54) , (2.61) and (2.62) to find the restrictions of thermodynamic stability in different ranges of ω_i in the following Table 2.2 .

Table 2.2: Conditions for Stability Criteria for variable equation of state parameter fluids.

range of ω_i	**Stability Condition**
$\omega_i \geq 0$	$\frac{d\ln\omega_i}{d\ln T} \leq \frac{2+\omega_i}{\omega_i}$ and $\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial\ln T}\right]_{p_i} \geq max\left[-(1+\omega_i), \left(\frac{1+\omega_i}{\omega_i}-\frac{d\ln\omega_i}{d\ln T}\right)\right]$
$-1 \leq \omega_i \leq 0$	$\frac{d\ln\omega_i}{d\ln T} \geq \frac{2+\omega_i}{\omega_i}$ and $\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial\ln T}\right]_{p_i} \geq max\left[-(1+\omega_i), \left(\frac{1+\omega_i}{\omega_i}-\frac{d\ln\omega_i}{d\ln T}\right)\right]$ if $\frac{d\ln\omega_i}{d\ln T} \leq \frac{1+3\omega_i+2\omega_i^2}{\omega_i^2}$. Again $-(1+\omega_i) \leq \left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial\ln T}\right]_{p_i} \leq \left(\frac{1+\omega_i}{\omega_i}-\frac{d\ln\omega_i}{d\ln T}\right)$ if $\frac{d\ln\omega_i}{d\ln T} \geq \frac{1+3\omega_i+2\omega_i^2}{\omega_i^2}$
$\omega_i \leq -1$	$\frac{d\ln\omega_i}{d\ln T} \leq \frac{2+\omega_i}{\omega_i}$ and $\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial\ln T}\right]_{p_i} \geq max\left[-(1+\omega_i), \left(\frac{1+\omega_i}{\omega_i}-\frac{d\ln\omega_i}{d\ln T}\right)\right]$

2.4 Brief discussions

A detailed thermodynamic study of non-interacting cosmic fluids having diffusive nature has been organized in the present work. As a result, individual fluid does not obey the energy conservation relation ,rather the non-conservation is proportional to

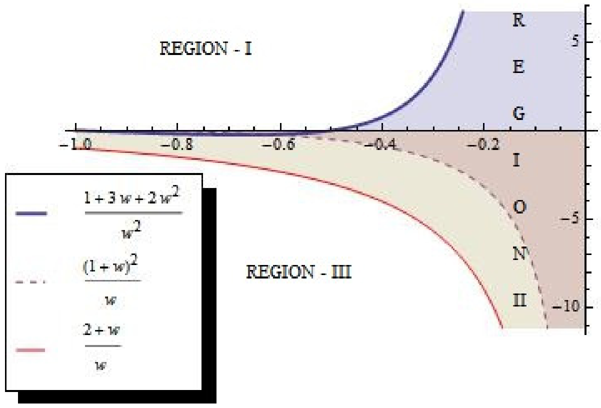

Figure 2.2: Stability region of variable equation of state fluid : for the values of $\frac{d\ln\omega_i}{d\ln T}$ ϵ REGION-I, the stability condition is $-(1+\omega_i) \leq \left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial\ln T}\right]_{p_i} \leq \left(\frac{1+\omega_i}{\omega_i} - \frac{d\ln\omega_i}{d\ln T}\right)$, for $\frac{d\ln\omega_i}{d\ln T}$ ϵ REGION-II, the stability condition is $\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial\ln T}\right]_{p_i} \geq max\left[-(1+\omega_i), \left(\frac{1+\omega_i}{\omega_i} - \frac{d\ln\omega_i}{d\ln T}\right)\right]$ and in REGION-III unstable.

the corresponding diffusive current. Each component of the cosmic fluids is assumed to have barotropic equation of state with constant or variable equation of state parameter. The stability conditions are expressed in the form of inequalities for different ranges of equation of state parameter. Also, the stability conditions for both constant (Table 2.1)and variable (Table 2.2) equation of state parameter in the range $-1 < \omega_i < 0$ (i.e. for dark energy region) have been presented graphically in Figure 2.1 and Figure 2.2 respectively. From Figure 2.2 , different restrictions on diffusion parameter γ_i are found depending on whether the values of $\frac{d\ln\omega_i}{d\ln T}$ (i.e. relative variation of ω_i with respect to the temperature T) belong to REGION-I or REGION-II . However due to expanding nature of the universe, the isobaric expansibility (α_i) should always be positive. This condition restricts the function $\frac{d\ln\omega_i}{d\ln T}$ only in REGION-II,which is acceptable for expanding universe (Table 2.3).

Further, it is interesting to note that, for constant equation of state parameter, (i.e. Table 2.1), the system cannot be thermodynamically stable for $-1 < \omega_i \leq 0$. This result is very much similar to that of [71]. However, the present work is thermodynamically stable in phantom region in contrary with non-diffusive cosmic fluids [71]. Finally, it is found that diffusive fluid with variable equation of state parameter is thermodynamically stable for all possible values of ω_i under some restrictions on the

Table 2.3: Conclusion on the result in Dark energy range ($-0.33 < \omega_i < 0$)

Nature of Dark Energy	**Constraint on diffusion parameter** γ_i	**Restriction on** $\frac{d\ln\omega_i}{d\ln T}$ **from graphs**
Constant equation of state parameter	Always unstable	Always unstable
Variable equation of state parameter	$\left[\frac{\partial\left\{\ln\left(1+\frac{\gamma_i}{\rho_{i0}}A_i\right)\right\}}{\partial\ln T}\right]_{p_i} \geq max\left[-(1+\omega_i), \left(\frac{1+\omega_i}{\omega_i} - \frac{d\ln\omega_i}{d\ln T}\right)\right]$	$\frac{2+\omega_i}{\omega_i} \leq \frac{d\ln\omega_i}{d\ln T} \leq \frac{1+3\omega_i+2\omega_i^2}{\omega_i^2}$

diffusion parameter γ_i and nature of variation of ω_i with respect to temperature.

CHAPTER 3

THE EMERGENT SCENARIO OF THE UNIVERSE UNDER DIFFUSION MECHANISM

3.1 Prelude

Diffusion can be considered as one of the basic macroscopic forces in nature. Several physical and biological processes are caused due to diffusion. Some well known examples of dynamical processes (in physics) are heat conduction, Brownian motion and various transport phenomena [80, 81, 82, 83] in biological systems where diffusion is the driving mechanism . The random collisions between the particles of the system and those of the background is caused due to diffusion mechanism at the microscopic level. On the other hand, random effects are averaged at the macroscopic scale and diffusion is characterised by heat equation or Fokker-Planck equation . Although there is a wide variety of phenomena having diffusive behaviour, still there does not exist a consistent diffusion theory in general relativity. However from cosmological point of view, it is speculated that diffusion may have a basic role in the evolution dynamics of the large scale structure formation of the universe. Further, in standard cosmology, galaxies are assumed as point particles of a fluid, undergoing velocity diffusion [80, 81, 82, 83, 84].

3.2 Modification of GTR for diffusive universe

To consider diffusion in general relativity, one has to consider macroscopic continuum description provided by the Fokker-Planck equation. So, in diffusive process, the energy - momentum tensor is not covariantly conserved (i.e. $\nabla_\mu T^{\mu\nu} \neq 0$), rather it satisfies the Fokker - Planck equation, namely [80, 81, 82, 83, 84]

$$\nabla_\mu T^{\mu\nu} = 3\sigma J^\nu, \tag{3.1}$$

where $\sigma(> 0)$ is the diffusion constant and J^ν , the current density of the matter satisfies

$$\nabla_\mu J^\mu = 0. \tag{3.2}$$

Thus one can not have usual Einstein equations i.e. $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$ for diffusive process, due to Bianchi identity. The simplest modification of the Einstein equation is to introduce two interacting matter components of which one is the usual diffusive fluid having conservation (non-conservation) equation given by equation (3.1) while the simplest choice for the other component (in analogy with cosmological constant) is a cosmological scalar field. so the modified Einstein field equations take the form

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \phi g_{\mu\nu} = T_{\mu\nu}, \tag{3.3}$$

where the scalar field ϕ has the evolution equation [80, 81, 82, 83] (dimension factor in ϕ has been chosen to be unity for convenience.)

$$\nabla_\mu \phi = 3\sigma J_\mu, \tag{3.4}$$

and $T_{\mu\nu}$ satisfies the above Fokker - Planck equation (given by equation (3.1)).

Here 3σ measures the energy transferred from the scalar field to the matter per unit time due to diffusion. Note that in vacuum or in the absence of diffusion, the above modified Einstein field equations (3.3) become Einstein equations with a cosmological constant while in general equation (3.3) may be termed as Einstein equations with variable 'cosmological constant'.

The above diffusion process is usually termed as kinetic model with microscopic velocity of the fluid particles undergoing diffusion. Here the diffusion mechanism takes place on the tangent bundle of the space time and as a result Lorentz invariance of the space-time is preserved. Now choosing the cosmic fluid as perfect fluid, one has the energy-momentum tensor

$$\mathcal{T}_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\mu} + u_\mu u_\nu) \tag{3.5}$$

with current density $J^\mu = nu^\mu$. Here n is the particle number density of the fluid. u^μ is the four velocity of the fluid. ρ and p are the energy density and thermodynamic pressure of the fluid respectively. Now projecting equation (3.1) along the fluid 4-velocity u^μ and on the hyper surface orthogonal to u^μ , one gets [80, 81, 82, 83]

$$\nabla_\mu(\rho u^\mu) + p\nabla_\mu u^\mu = 3\sigma n \tag{3.6}$$

and

$$(p+\rho)u^\mu \nabla_\mu u^\nu + (u^\mu u^\nu + g^{\mu\nu})\nabla_\mu p = 0, \tag{3.7}$$

Here equation (3.7), the Euler equation does not change due to diffusion process as diffusion force acts along the matter flow. It is to be noted that there are several diffusion models in the literature namely for unification of dark energy and dark matter from diffusive cosmology see ref. [85]. Ref. [86] deals with transition between bouncing hyper-inflation to Λ_{CDM} from diffusive scalar fields while unified DE-DM with diffusive interactions and interacting diffusive unified dark energy and dark matter from scalar fields can be found in ref.[87] and [88] respectively. In particular, a Lagrangian formulation of diffusion mechanism can be found in ref. [85].

3.3 Evolution of diffusive universe and its non-equilibrium thermodynamics

In the background of homogeneous and isotropic flat FLRW model, the modified Friedmann equations with diffusion dynamics take the form (taking the coupling constant $\frac{8\pi G}{c^4} = 1$),

$$3H^2 = \rho + \phi \tag{3.8}$$

and

$$2\dot{H} = -(\rho + p) \tag{3.9}$$

Now equation (3.3) for the present geometry simplifies to

$$na^3(t) = \text{constant, i.e. } n = n_0 a^{-3}. \tag{3.10}$$

Hence the modified matter conservation equation (3.1) for the matter field (3.5) takes the form,

$$\dot{\rho} + 3H(p + \rho) = \sigma n_0 a^{-3} = \sigma_0 a^{-3} \tag{3.11}$$

which on integration yields

$$\rho = a^{-3(1+\omega)} \left[\rho_0 + \int_{t_0}^{t} \sigma_0 a^{3\omega} dt \right]. \tag{3.12}$$

Here $\omega = \frac{p}{\rho}$, is the constant equation of state parameter of the fluid. ρ_0 is the energy density at reference epoch of time $t = t_0$ and $a(t_0) = 1$. $n(t_0) = n_0$ is assumed. Now eliminating ρ between equations (3.8) and (3.9), one gets the cosmic evolution equation as

$$2\dot{H} + 3(1 + \omega)H^2 = \phi(1 + \omega) \tag{3.13}$$

On the other hand, the above modified Friedmann equations (i.e. equations (3.8) and (3.9)) for diffusive mechanism can be rewritten as,

$$3H^2 = \rho_d \ , \ 2\dot{H} = -(\rho_d + p_d + \pi_d) \tag{3.14}$$

while the conservation equation(3.11) becomes

$$\dot{\rho}_d + 3H(\rho_d + p_d + \pi_d) = 0, \tag{3.15}$$

with $\rho_d = \rho + \phi$, $p_d = p$ and $\pi_d = -\phi$. Thus interacting two fluid system in diffusion mechanism [89] is equivalent to a single dissipative fluid in Einstein gravity. Here dissipation is chosen in the form of bulk viscous pressure π_d. Further one may consider the above dissipative pressure (i.e. bulk viscous pressure) due to non-equilibrium thermodynamics with particle creation mechanism. In fact, for adiabatic thermodynamic process, the dissipative pressure π_d is related linearly to the particle creation rate Γ_d as [90, 91]

$$\pi_d = -\frac{\Gamma_d}{3H}(\rho_d + p_d). \tag{3.16}$$

Using the 1st friedmann equation in (3.14) of equivalent Einstein gravity into the above equation (3.16) with $p_d = p = \omega\rho$ and $\pi_d = -\phi$, the cosmological scalar field is related to the particle creation rate as

$$\Gamma_d = \frac{3H\phi}{3H^2(1+\omega) - \omega\phi}. \tag{3.17}$$

Hence the present interacting diffusive mechanism [89] with cosmological scalar field can be considered as non-equilibrium thermodynamic description of Einstein gravity with particle creation formalism.

3.4 Emergent scenario in diffusive universe

To overcome the classical singularity of Einstein gravity, cosmologists propose two models namely the bouncing Universe or the emergent Universe. In the present work, for non-singular solution we shall consider the model of emergent scenario as it is very much relevant as pre-inflationary era. An emergent Universe [90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114] is a modeled Universe with no time like singularity having static Einstein era in the infinite past (i.e. $t \to -\infty$). The present work examines whether emergent scenario is possible or not in the present cosmological scalar field diffusion mechanism. In order to **solve the evolution equation (3.13), the cosmological scalar field ϕ should be a function of Hubble parameter H. For simplicity, one may choose phenomenologically the form of ϕ as linear function of H i.e.,**

$$\phi = 3\alpha H, \tag{3.18}$$

with α, a constant. Using this choice of ϕ in the field equations (3.8),(3.9)and (3.11) one gets

$$\sigma_0 a^{-3} = -3\alpha\dot{H}, \tag{3.19}$$

which shows that α and σ_0 are of same sign (due to $\dot{H} < 0$).

For this choice of ϕ, the solutions of the cosmic evolution equation (3.13) yield the form of Hubble parameter and scale factor as,

(i) For $\alpha \geq H_0$:

$$H = \frac{\alpha}{1 + \left(\frac{\alpha}{H_0} - 1\right) e^{-\frac{3}{2}\alpha(1+\omega)(t-t_0)}} \quad , \tag{3.20}$$

$$a = \left[\frac{\left(\frac{\alpha}{H_0} - 1\right) + e^{\frac{3}{2}\alpha(1+\omega)(t-t_0)}}{\left(\frac{\alpha}{H_0} - 1\right) + 1}\right]^{\frac{2}{3(1+\omega)}} \tag{3.21}$$

(ii) For $0 < \alpha < H_0$:

$$H = \frac{\alpha}{1 - \left(1 - \frac{\alpha}{H_0}\right) e^{-\frac{3}{2}\alpha(1+\omega)(t-t_0)}} \quad , \tag{3.22}$$

$$a = \left[\frac{\left(1 - \frac{\alpha}{H_0}\right) - e^{\frac{3}{2}\alpha(1+\omega)(t-t_0)}}{\left(1 - \frac{\alpha}{H_0}\right) - 1}\right]^{\frac{2}{3(1+\omega)}} \tag{3.23}$$

and

(iii) For $\alpha < 0$:

$$H = \frac{|\alpha|}{1 - \left(\frac{|\alpha|}{H_0} + 1\right) e^{-\frac{3}{2}\alpha(1+\omega)(t-t_0)}} \quad , \tag{3.24}$$

$$a = \left[\frac{e^{\frac{3}{2}\alpha(1+\omega)(t-t_0)} - \left(\frac{|\alpha|}{H_0} + 1\right)}{1 - \left(\frac{|\alpha|}{H_0} + 1\right)}\right]^{\frac{2}{3(1+\omega)}} . \tag{3.25}$$

Here H_0 is the value of H at reference epoch of time t_0 .

For $\alpha < 0$, the above cosmological solution (3.25) has a big-bang singularity at the epoch,

$$t_s = t_0 - \frac{2}{3(1+\omega)|\alpha|} \ln\left(1 + \frac{|\alpha|}{H_0}\right) . \tag{3.26}$$

Note that $\alpha < 0$ (i.e. $\sigma_0 < 0$) is not physically realistic, so we shall present the above solution for $\alpha < 0$ only for mathematical completeness.

Again for $0 < \alpha < H_0$, big-rip singularity exists for the cosmological solution (3.23) at the epoch,

$$t_s = t_0 + \frac{2}{3(1+\omega)\alpha} \ln\left[\left(1 - \frac{\alpha}{H_0}\right)\right]. \tag{3.27}$$

In the case $H_0 < \alpha$, the cosmological solution (3.21) has no singularity at any real time.

Clearly, this solution [(3.20) , (3.21)] yields the Emergent scenario as it follows the following criteria [90] :

$$H \to 0\ ,\ a \to \left[\frac{\alpha - H_0}{\alpha}\right]^{\frac{2}{3(1+\omega)}} \text{ when } t \to -\infty \tag{3.28a}$$

$$H \to 0\ ,\ a \to \left[\frac{\alpha - H_0}{\alpha}\right]^{\frac{2}{3(1+\omega)}} \text{ when } t << t_0 \text{ and} \tag{3.28b}$$

$$H \sim \alpha\ ,\ a \simeq \left[\frac{H_0}{\alpha}\right]^{\frac{2}{3(1+\omega)}} \exp\left[\alpha(t - t_0)\right] \quad \text{when } t >> t_0 \tag{3.28c}$$

So evidently the explicit solution for emergent scenario should be in the form (also considering , $\alpha = H_0 + \delta$ with $\delta \geq 0$) :

$$H^{(E)} = \frac{(H_0 + \delta)H_0}{H_0 + \delta e^{-\frac{3}{2}(H_0+\delta)(1+\omega)(t-t_0)}} \tag{3.29a}$$

$$\text{and } a^{(E)} = \left[\frac{\delta + H_0 e^{\frac{3}{2}(H_0+\delta)(1+\omega)(t-t_0)}}{H_0 + \delta}\right]^{\frac{2}{3(1+\omega)}} \tag{3.29b}$$

$$\text{i.e.} H^{(E)} = \frac{(H_0 + \delta)\left(a^{(E)}\right)^{\frac{3}{2}(1+\omega)} - \delta}{\left(a^{(E)}\right)^{\frac{3}{2}(1+\omega)}} \tag{3.29c}$$

under the diffusive non-singular scalar field ,

$$\phi = 3(H_0 + \delta)H. \tag{3.30}$$

Here it can be written in terms of a reference value as, $\phi = \frac{\phi_0}{H_0}H$ **where** $\phi_0 = \phi(t_0)$**.** The nature of corresponding particle creation rate can be found from equation (3.17) as,

$$\Gamma_d = \frac{3H}{1 - (1+\omega)(1 - \frac{H}{\alpha})} = \Gamma_{d0}\frac{1 - (1+\omega)\frac{\delta}{\alpha}}{1 - (1+\omega)(1 - \frac{H}{\alpha})}.\frac{H}{H_0}, \tag{3.31}$$

with $\Gamma_{d0} = \Gamma_d(t_0) = \dfrac{3H_0}{1-(1+\omega)\left(1-\frac{H_0}{\alpha}\right)}$**, a constant.**

Further, one can write down the evolution equation (3.13) as the evolution of Hubble parameter with the scale factor as

$$\frac{dH}{da} + \frac{3}{2}(1+\omega)\frac{H}{a} = \frac{3}{2}\alpha(1+\omega)\frac{1}{a}, \tag{3.32}$$

which on integration gives

$$H = \alpha - \delta(1+z)^{\frac{3}{2}(1+\omega)}, \tag{3.33}$$

where z is the amount of cosmological red shift $\left(z = \dfrac{1}{a} - 1\right)$. Now introducing the dimensionless density parameter, $\Omega = \dfrac{\rho}{\rho_c}$ with $\rho_c = \dfrac{3H^2}{8\pi G}$, the critical density, the above equation can be written as

$$\frac{H^2}{H_0^2} = \Omega_{\Lambda_0} + \Omega_M(1+z)^{3(1+\omega)} + \Omega_{MP}(1+z)^{3(1+\omega_{MP})} \tag{3.34}$$

where $\Omega_{\Lambda_0} = \left(1+\dfrac{\delta}{H_0}\right)^2$, $\Omega_M = \left(\dfrac{\delta}{H_0}\right)^2$, $\Omega_{MP} = 2\dfrac{\delta}{H_0}\left(1+\dfrac{\delta}{H_0}\right)$ and $\omega_{MP} = \left(\dfrac{\omega-1}{2}\right)$ with $\Omega_{\Lambda_0} + \Omega_M + \Omega_{MP} = 1$. From equation (3.34) one can see that as $z \to -1$ i.e. $a \to \infty$, the present model approaches **the De Sitter expansion which** Λ_{CDM} **also has in this limit.** The evolution of the instantaneous equilibrium temperature of a system under non-equilibrium thermodynamic prescription can be written as [91] ,

$$\frac{\dot{T}}{T} + \omega\left(3H - \Gamma_d\right) = 0. \tag{3.35}$$

In the emergent scenario, one has (integrating equation (3.35))

$$T = \frac{T_0}{a^{3\omega}}\left[\frac{a^{\frac{3}{2}(1+\omega)}(H_0+\delta) - \delta(1+\omega)}{H_0 - \omega\delta}\right]^{\frac{2\omega}{1+\omega}} \tag{3.36}$$

where T_0 is the present measured value of temperature (at $t = t_0$) . So, equations (3.34) and (3.36) represent the Hubble parameter and temperature respectively in terms of today's measured value. The time evolution of a and the evolution of H, ϕ and Γ_d **with scale factor** a has been exhibited graphically in Figure 3.1 (a,b,c,d) respectively. Also the variation of thermodynamic parameters namely energy density ρ and temperature

T with **the scale factor** (a) and with equation of state parameter (ω) of the cosmic fluid have been shown in a $3d$ plot in figure 3.2. **From figure 3.1, it is evident that the scale factor a is constant in emergent era, it increases during pre -inflationary era and then there is a sharp increase in the scale factor. The Hubble parameter H is zero at (or near) the emergent scenario, increases in pre-inflationary epoch and again it has a finite, non-zero constant value in the inflationary domain. The 3-d graphs of ρ and T in Figure 3.2 show that ρ increases with the scale factor while temperature decreases as a increases.**

Lastly from the cosmological solutions (3.20) and (3.21), if a_E be the value of the scale factor in the emergent era, then $a_E = \left(\frac{\alpha - H_0}{H_0}\right)^{\frac{2}{3(1+\omega)}}$ is found. Finally, if the present age of the Universe is restricted to be 13 Giga years, then the parameter H_0^{-1} should be larger than this value i.e. $H_0^{-1} \geqslant 13$ Giga years.

3.5 Discussion

The present work is an attempt to examine whether emergent scenario of the Universe is possible under diffusive process. Considering kinetic model of the diffusion process, cosmological scalar field is chosen linearly to the Hubble parameter to obtain emergent scenario of the cosmic evolution and their variations with respect to **scale factor** and equation of state parameter have been shown graphically (3d plot) in Figure 3.2 Different thermodynamic parameters like energy density and temperature also have been determined under emergent scenario.

Further it has been established that such scalar field diffusion process corresponds to the particle creation mechanism [90, 91] in the non-equilibrium thermodynamic description. It is interesting to note that, for the non-singular particle creation process, [see equation (3.31)] the barotropic index of the fluid can be restricted to $-1 < \omega < 0$ in the present scenario. Also for non-singular solution, the cosmological scalar field is chosen phenomenologically as proportional to the Hubble parameter and the proportionality constant is found to be positive. Finally, this work establishes that the dissipative processes like diffusion, particle creation etc. may correspond to the evolution pattern of the universe as per the present observation. For future works, it may be attempted to find the Lagrangian formulation of such non-equilibrium thermodynamic phenomena to study the microscopic behaviour of the universe.

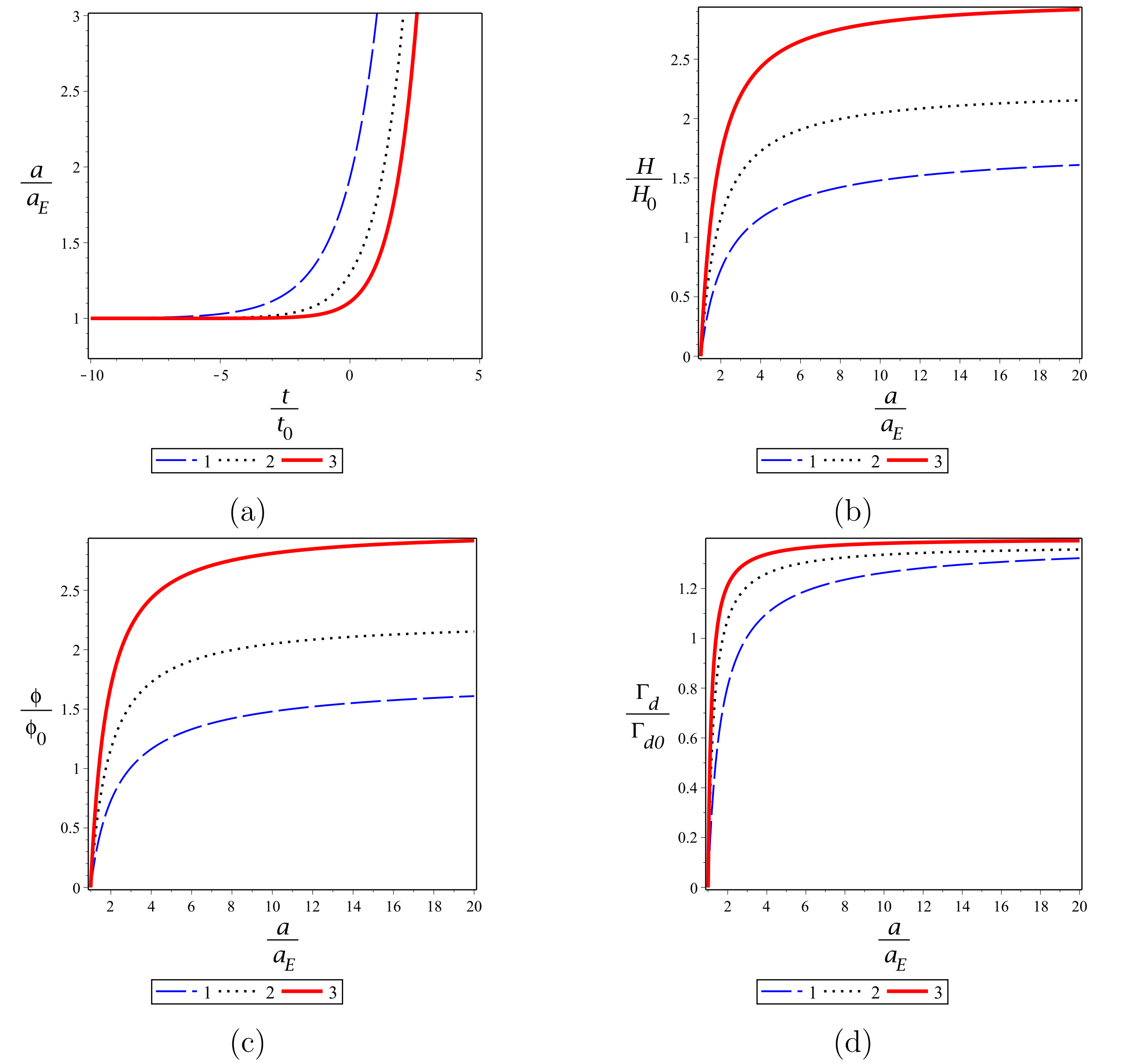

Figure 3.1: Evolution of different physical parameters namely (a) Scale factor a **with time** t (top left), (b) Hubble parameter H **with scale factor** a (top right), (c) Cosmological scalar field ϕ **with scale factor** a (bottom left) and (d) Particle creation rate Γ_d **with scale factor** a (bottom right) for $\alpha = 0.9$, $t_0 = 1$ with three different values of (1) $\delta, H_0, \omega : (0.4, 0.5, -0.5)$ for 1(Blue), (2) $\delta, H_0, \omega : (0.5, 0.4, -0.3)$ for 2(Black) and (3) $\delta, H_0, \omega : (0.6, 0.3, -0.1)$ for 3(Red).

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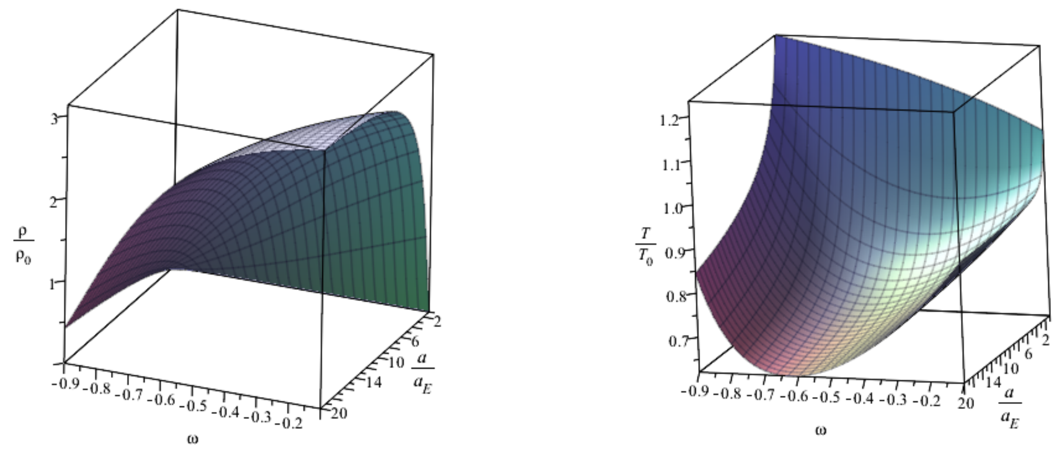


(a) (b)

Figure 3.2: Evolution of different thermodynamic parameters namely (a) Energy density ρ (left) and (b) Temperature T (right) as functions of **scale factor** a and barotropic index of the fluid ω for $H_0 = 0.5$ with $\delta = 0.4$.

CHAPTER 4

1ST ORDER THERMODYNAMIC PHASE TRANSITION OF COSMIC FLUID IN CONTINUOUS AND COMPLETE COSMIC EVOLUTION WITH DIFFUSIVE BAROTROPIC FLUID

4.1 Prelude

The standard cosmology is facing the challenge to estimate the theoretical explanation of the recent observational evidences that our Universe is going through an accelerated expansion[38, 39, 40, 41, 42, 43, 44, 45, 46]. In the framework of Einstein Gravity, Cosmologists are trying for different dynamic exotic matters (known as Dark Energy) to exhibit this present era of evolution. But so far there is no such proposed DE which is well in accord with observation or is acceptable from theoretical point of view. The cosmological constant is so far the best model supporting observational data but theoretically it suffers from two serious problems[48, 49].

Inflationary scenario is a very well accepted theory of cosmic evolution just after big-bang. There are several models for this era of exponential expansion of which the single scalar field (known as inflaton) having self interacting potential is very popular. In the context of cosmology, there are thousands of models which describe only some of the eras of evolution of the Universe. Also there are some models in cosmology which are singularity free (i.e. Emergent or Bouncing Universe). However, there are very few cosmological models which describe the evolution of the Universe starting from inflationary era to present late time acceleration or non-singular model starting from Emergent scenario. The present work is an example of such cosmic model with cosmic fluid as diffusive barotropic fluid. The bulk flow mechanism leads to the evolutionary changes in the Universe or vice-versa. The nature of thermodynamic phase changes of cosmic fluid associated with continuous cosmic evolution is also an important subject to be analysed. Thus, it will be interesting to construct a model which describes a complete and continuous expansion of the Universe, both in cosmic and thermodynamic perspectives.

One can come across diffusive process in various physical and biological processes, for example Brownian motion, transport of materials within cells etc. At microscopic scale, diffusion process is caused due to random collisions between the particles of the system and of the background material. However, at macroscopic level diffusive process is considered as an effective deterministic theory i.e. averaging out all random effects. As a result, physical quantities representing diffusion will satisfy heat equation or Fokker - Planck equation.

4.2 Einstein field equations with a variable cosmological scalar field.

To consider diffusion in general relativity, one has to consider macroscopic continuum description provided by the Fokker-plank equation. In general relativity, for a given matter source the geometry of the space-time is determined through the Einstein field equations (choosing $8\pi G = c = 1$)

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}. \tag{4.1}$$

Now due to Bianchi identity , the above matter component (characterised by the energy-momentum tensor $T_{\mu\nu}$) must be covariantly conserved (i.e. $\nabla_\mu T^{\mu\nu} = 0$). So it can not be the matter part undergoing diffusion. Thus for diffusion process, the energy-momentum tensor associated to a solution of the Fokker-Plank equation satisfies [80, 81, 82, 83, 84, 85, 88, 115]

$$\nabla_\mu T^{\mu\nu} = 3\sigma J^\nu \tag{4.2}$$

where $\sigma(> 0)$ is the diffusion constant and j^μ is the current density of the matter. Due to Bianchi identity, there should be another matter component so that the resulting matter is covariantly conserved. The simplest choice (in analogy to cosmological constant) is to introduce a cosmological scalar field so that modified Einstein field equations become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \phi g_{\mu\nu} = T_{\mu\nu}. \tag{4.3}$$

So the evolution of the scalar field is determined by [80, 81, 82, 83]

$$\nabla_\mu \phi = 3\sigma J_\mu. \tag{4.4}$$

Here 3σ measures the energy transferred from the scalar field to the matter per unit time due to diffusion. Note that in vacuum or in the absence of diffusion, the above modified Einstein field equations (4.3) become Einstein equations with a cosmological constant while in general equation (4.3) may be termed as Einstein equations with variable 'cosmological constant'.

The above diffusion process is usually termed as kinetic model with microscopic velocity of the fluid particles undergoing diffusion. Here the diffusion mechanism takes place on the tangent bundle of the space time and as a result Lorentz invariance of the space-time is preserved. Now choosing the cosmic fluid as perfect fluid, one has the energy-momentum tensor

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu) \tag{4.5}$$

with current density $J^\mu = nu^\mu$. Here n is the particle number density of the fluid. u^μ is the four velocity of the fluid. ρ and p are the energy density and thermodynamic pressure of the fluid respectively. Now projecting the non-conservation equation (4.2) of the dissipative fluid along the fluid 4-velocity u^μ and on the hyper surface orthogonal to u^μ , one gets [80, 81, 82, 83]

$$\nabla_\mu(\rho u^\mu) + p\nabla_\mu u^\mu = 3\sigma.n \tag{4.6}$$

and

$$(p+\rho)u^\mu\nabla_\mu u^\nu + u^\mu u^\nu\nabla_\mu p + \nabla^\nu p = 0. \tag{4.7}$$

As the diffusion force on the velocity vector is acting along the matter flow so expected the Euler equation (4.7) is unaffected in the diffusion mechanism. Although there are other diffusion models in the literature, but still now there is no Lagrangian formulation. The space time geometry is chosen as the maximally symmetric flat FLRW model, so the explicit form of the modified Einstein field equations are

$$3H^2 = \rho + \phi\ ,\ 2\dot{H} = -(\rho + p) \tag{4.8}$$

while the non-conservation of the energy-momentum tensor i.e. equation (4.2) has the explicit form

$$\dot{\rho} + 3H(p+\rho) = 3\sigma n_0\left(\frac{a}{a_0}\right)^{-3} = 3\sigma_0 a^{-3}. \tag{4.9}$$

Now integrating once, ρ has the solution

$$\rho = a^{-3(1+\omega)}[\rho_0 + 3\int_{t_0}^{t}\sigma_0 a^3 dt], \tag{4.10}$$

where $\omega = \frac{p}{\rho}$ is the constant equation of state parameter of the diffusive fluid. Here ρ_0 is the energy density at reference epoch of time $t = t_0$ and $n(t = t_0) = n_0, a(t = t_0) = a_0$. Moreover, eliminating ρ between the modified Einstein field equations (4.8), one has the evolution equation

$$2\dot{H} + 3(1+\omega)H^2 = \phi(1+\omega). \tag{4.11}$$

Further the cosmological scalar field ϕ satisfies the evolution equation

$$\dot{\phi} + 3H\phi = 3\sigma_0 a^{-3} \tag{4.12}$$

which has the solution

$$\phi = \phi_0 \left(\frac{a_0}{a}\right)^3 + \frac{3\sigma_0}{a^3} \int \frac{da}{H}. \tag{4.13}$$

Moreover, the above modified Einstein field equations (i.e. equation (4.8)) for diffusive mechanism can be written as

$$3H^2 = \rho_d, 2\dot{H} = -(\rho_d + p_d + \pi) \tag{4.14}$$

with $\rho_d = \rho + \phi, p_d = p$ and $\pi = -\phi$ as effective energy density, thermodynamic pressure and effective bulk viscous pressure in Einstein formulation respectively. Hence the above diffusive process can be considered as a dissipative phenomena in non-equilibrium thermodynamics. In an open thermodynamic system (as total number of particles are not conserved), the particle number conservation equation can be modified [91] as

$$\dot{n} + 3Hn = n\Gamma_d \tag{4.15}$$

where n is the particle number density and Γ_d is the particle creation (or annihilation) rate in a comoving volume ($v_0 a^3$ say).Then the first law of thermodynamics can also be modified as

$$Tds = d\left(\frac{\rho_d}{n}\right) + p_d\, d\left(\frac{1}{n}\right), \tag{4.16}$$

which can also be simplified as

$$nT\dot{s} = -3H\pi - \Gamma_d(p_d + \rho_d) \tag{4.17}$$

where s is the entropy per particle. By choosing the dissipative process to be adiabatic (i.e. $\dot{s} = 0$), the bulk viscous pressure is related linearly to the particle creation/ annihilation rate as [91]

$$\pi = -\frac{\Gamma_d}{3H}(\rho_d + p_d). \tag{4.18}$$

Using the first modified Friedmann equation in (4.8) the particle creation /annihilation rate has the explicit form in terms of cosmological scalar field as

$$\Gamma_d = \frac{3H\phi}{3H^2(1+\omega) - \omega\phi}. \tag{4.19}$$

Another important thermodynamic quantity namely temperature of the cosmic fluid can be determined [91] as

$$\frac{\dot{T}}{T} + \omega\,(3H - \Gamma_d) = 0. \tag{4.20}$$

4.3 different eras of evolution : role of cosmological scalar field (ϕ)

In this section, the proper form of the time dependent cosmological scalar field ϕ have been estimated to find the several typical cosmological solutions(i.e. emergent scenario, inflation, decelerating expansion and late time acceleration) of complete evolutionary scenario in the present context. The functional forms of ϕ have been taken as $\phi \propto H^m$, m being a constant. The values of m can be taken phenomenologically to match with the physically valid cosmic evolution in different eras.

One has the choices for the form of ϕ as following :

(i) $m = 1, \phi = 3\eta_E H$ for emergent scenario.

(ii) $m = 3, \phi = 3\eta_1 H^3$ for early time acceleration.

(iii) $m = 2, \phi = 3\eta_2 H^2$ for decelerating expansion.

(iv) $m = 0, \phi = 3\eta_3$ for late time acceleration.

Here $\eta_E, \eta_1, \eta_2, \eta_3$ are real constants.

4.3.1 Emergent Scenario

Choosing ϕ to be linearly dependent on Hubble parameter($m = 1$), the evolution equation (4.11) takes the form ,

$$2\dot{H} + 3(1+\omega)H^2 = 3(1+\omega)\eta_E H. \tag{4.21}$$

The above differential equation(4.21) yields the solutions :

(i) For $\eta_E \geq H_0$:

$$H = \frac{\eta_E}{1 + \left(\frac{\eta_E}{H_0} - 1\right) e^{-\frac{3}{2}\eta_E(1+\omega)(t-t_0)}} \tag{4.22}$$

and

$$a = \left[\frac{\left(\frac{\eta_E}{H_0} - 1\right) + e^{\frac{3}{2}\eta_E(1+\omega)(t-t_0)}}{\left(\frac{\eta_E}{H_0} - 1\right) + 1} \right]^{\frac{2}{3(1+\omega)}} \tag{4.23}$$

(ii) For $0 < \eta_E < H_0$:

$$H = \frac{\eta_E}{1 - \left(1 - \frac{\eta_E}{H_0}\right) e^{-\frac{3}{2}\eta_E(1+\omega)(t-t_0)}} \tag{4.24}$$

and

$$a = \left[\frac{\left(1 - \frac{\eta_E}{H_0}\right) - e^{\frac{3}{2}\eta_E(1+\omega)(t-t_0)}}{\left(1 - \frac{\eta_E}{H_0}\right) - 1} \right]^{\frac{2}{3(1+\omega)}} \tag{4.25}$$

(iii) For $\eta_E < 0$:

$$H = \frac{|\eta_E|}{1 - \left(\frac{|\eta_E|}{H_0} + 1\right) e^{-\frac{3}{2}\eta_E(1+\omega)(t-t_0)}} \tag{4.26}$$

and $$a = \left[\frac{e^{\frac{3}{2}\eta_E(1+\omega)(t-t_0)} - \left(\frac{|\eta_E|}{H_0} + 1\right)}{1 - \left(\frac{|\eta_E|}{H_0} + 1\right)} \right]^{\frac{2}{3(1+\omega)}} \tag{4.27}$$

In $\eta_E < 0$ i.e. for outward bulk flow, big-bang singularity occurs at the epoch

$$t_s = t_0 - \frac{2}{3(1+\omega)|\eta_E|} \ln\left(1 + \frac{|\eta_E|}{H_0}\right) \tag{4.28}$$

Again for $0 < \eta_E < H_0$, big-rip singularity exists at the epoch

$$t_s = t_0 + \frac{2}{3(1+\omega)\eta_E} \ln\left[\left(1 - \frac{\eta_E}{H_0}\right)\right] \tag{4.29}$$

But in the case $H_0 < \eta_E$, there is no singularity at any real time as here only the singularity may occur at the imaginary time

$$t_s = t_0 + \frac{2}{3(1+\omega)\eta_E} \ln\left[-\left(\frac{\eta_E}{H_0} - 1\right)\right] \tag{4.30}$$

Clearly, it is found that among above all the solutions, the first one (i.e. for $\eta_E \geq H_0$) yields the Emergent scenario as it follows the following criteria[90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114]

:

$$H \to 0\ ,\ a \to \left[\frac{\eta_E - H_0}{\eta_E}\right]^{\frac{2}{3(1+\omega)}} \text{ when } t \to -\infty \tag{4.31a}$$

$$H \to 0\ ,\ a \to \left[\frac{\eta_E - H_0}{\eta_E}\right]^{\frac{2}{3(1+\omega)}} \text{ when } t << t_0 \text{ and} \tag{4.31b}$$

$$H \sim \eta_E\ ,\ a \simeq \left[\frac{H_0}{\eta_E}\right]^{\frac{2}{3(1+\omega)}} \exp\left[\eta_E(t-t_0)\right] \quad \text{when } t >> t_0 \tag{4.31c}$$

So evidently the explicit solution for emergent scenario should be in the form (also considering $\eta_E > H_0$) :

$$H^{(E)} = \frac{\eta_E H_0}{H_0 + (\eta_E - H_0)e^{-\frac{3}{2}\eta_E(1+\omega)(t-t_0)}} \tag{4.32a}$$

$$\text{and } a^{(E)} = \left[\frac{(\eta_E - H_0) + H_0 e^{\frac{3}{2}\eta_E(1+\omega)(t-t_0)}}{\eta_E}\right]^{\frac{2}{3(1+\omega)}}. \tag{4.32b}$$

Further, one can write down the evolution equation (4.21) as the evolution of Hubble parameter with the scale factor as

$$\frac{dH}{da} + \frac{3}{2}(1+\omega)\frac{H}{a} = \frac{3}{2}\eta_E(1+\omega)\frac{1}{a}, \tag{4.33}$$

which on integration gives

$$H = \eta_E + (H_0 - \eta_E)(1+z)^{\frac{3}{2}(1+\omega)}, \tag{4.34}$$

where z is the amount of cosmological red shift$\left(z = \frac{1}{a} - 1\right)$. Now introducing the dimensionless density parameter, $\Omega = \frac{\rho}{\rho_c}$ with $\rho_c = \frac{3H^2}{8\pi G}$, the critical density, the above equation can be written as

$$\frac{H^2}{H_0^2} = \Omega_{\Lambda_0} + \Omega_M(1+z)^{3(1+\omega)} + \Omega_{MP}(1+z)^{3(1+\omega_{MP})} \tag{4.35}$$

where $\Omega_{\Lambda_0} = (\frac{\eta_E}{H_0})^2, \Omega_M = (1 - \frac{\eta_E}{H_0})^2, \Omega_{MP} = 2\frac{\eta_E}{H_0}(1 - \frac{\eta_E}{H_0})$ and $\omega_{MP} = (\frac{\omega-1}{2})$ with $\Omega_{\Lambda_0} + \Omega_M + \Omega_{MP} = 1$. From equation (4.35) one can see that as $z \to -1$ i.e. $a \to \infty$, the present model approaches Λ_{CDM} model. The evolution of the temperature in the emergent era can be obtained by integrating equation(4.20) as

$$T = T_0(1+z)^{3\omega} e^{\alpha\omega(t-t_0)} \tag{4.36}$$

where T_0 is the present measured value of temperature(at $t = t_0$). So, equations (4.35)and (4.36) represent the Hubble parameter and temperature respectively in terms of today's measured value.

4.3.2 Early time acceleration(Inflation)

For early time acceleration, the choice for ϕ yields the evolution equation(4.11) in the form

$$2\dot{H} + 3(1+\omega)H^2 = 3(1+\omega)\eta_1 H^3. \tag{4.37}$$

So one has the solutions

$$H = \frac{H_1}{\eta_1 H_1 - (\eta_1 H_1 - 1)\left(\frac{a}{a_1}\right)^{\frac{3}{2}(1+\omega)}} \quad \text{for } \eta_1 > \frac{1}{H_1} \tag{4.38a}$$

$$H = \frac{H_1}{\eta_1 H_1 + (1 - \eta_1 H_1)\left(\frac{a}{a_1}\right)^{\frac{3}{2}(1+\omega)}} \quad \text{for } 0 < \eta_1 < \frac{1}{H_1} \tag{4.38b}$$

$$H = \frac{H_1}{(|\eta_1|H_1 + 1)(\frac{a}{a_1})^{\frac{3}{2}(1+\omega)} - |\eta_1|H_1} \quad \text{for } \eta_1 < 0 \tag{4.38c}$$

The equation (4.38b) is taken in this present context for early time inflation which yields the explicit form of Hubble parameter as

$$H^{(1)} = \frac{H_1}{\eta_1 H_1 + (1-\eta_1 H_1)(\frac{a}{a_1})^{\frac{3}{2}(1+\omega)}}, \tag{4.39a}$$

$$\boldsymbol{a^{(1)} = a_1 \left[\frac{\eta_1 H_1}{1-\eta_1 H_1} \text{Lambert } W\left(\frac{1-\eta_1 H_1}{\eta_1 H_1} \exp \frac{2(1-\eta_1 H_1) + 3(1+\omega)H_1(t-t_1)}{2\eta_1 H_1}\right)\right]^{\frac{2}{3(1+\omega)}}} \tag{4.39b}$$

where t_1 marks the end of the inflationary phase. [Here Lambert $W(x)e^{\text{Lambert}W(x)} = x$] It satisfies the condition $\{\ H \sim \frac{1}{\eta_1}(\text{constant})$ when $a \to 0\ \}$ of early time acceleration i.e. exponential expansion. Other two parameters namely energy density (ρ) and temperature (T) are found from the equations(4.10) and(4.20) as

$$\rho^{(1)} = \frac{\rho_1}{\left[\eta_1 H_1 + (1-\eta_1 H_1)(\frac{a}{a_1})^{\frac{3}{2}(1+\omega)}\right]^2} \tag{4.40a}$$

$$T^{(1)} = T_1 \left[\eta_1 H_1 + (1-\eta_1 H_1)(\frac{a}{a_1})^{\frac{3}{2}(1+\omega)}\right]^{-\frac{2\omega}{(1+\omega)}}, \tag{4.40b}$$

$\rho_1 = \rho(a = a_1)$ and $T_1 = T(a = a_1)$ while the deceleration parameter $q^{(1)} = -\eta_1 H^{(1)}$ indicates the accelerated expansion.

4.3.3 Decelerated expansion

For decelerating expansion phase, the scalar field ϕ is found to be quadratically dependent on the Hubble parameter. Then the evolution is governed by the equation

$$2\dot{H} + 3(1+\omega)H^2 = 3(1+\omega)\eta_2 H^2. \tag{4.41}$$

So the solution will be in the form

$$H^{(2)} = H_1 \left(\frac{a}{a_1}\right)^{\frac{3}{2}(1+\omega)(\eta_2 - 1)}, \tag{4.42}$$

with $\boldsymbol{a^{(2)} = a_1 \left[1 + 3\frac{1+\omega}{2} H_1(1-\eta_2)(t-t_1)\right]^{\frac{2}{3(1+\omega)}}}$. The deceleration parameter $q^{(2)} = -\left[\frac{3}{2}(1+\omega)(\eta_2 - 1) + 1\right]$ constrains $\eta_2 < 1 - \frac{2}{3(1+\omega)}$ to have $q^{(2)} > 0$ (i.e. decelerated expansion). One also has energy density and temperature as,

$$\rho^{(2)} = \rho_1 \left(\frac{a}{a_1}\right)^{3(1+\omega)(\eta_2 - 1)} \tag{4.43a}$$

$$T^{(2)} = T_1 \left(\frac{a}{a_1}\right)^{3\omega(\eta_2 - 1)} \tag{4.43b}$$

4.3.4 Late time acceleration

In this evolutionary phase, the scalar field is chosen as independent of Hubble parameter i.e. identical with the cosmological constant and the diffusion process leads to the evolution equation

$$2\dot{H} + 3(1+\omega)H^2 = 3(1+\omega)\eta_3. \tag{4.44}$$

The solutions are given by

$$H=\sqrt{\eta_3-\left(\eta_3-{H_3}^2\right)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}} \quad \text{for} \quad \eta_3>{H_3}^2 \tag{4.45a}$$

or

$$H=\sqrt{\eta_3+\left({H_3}^2-\eta_3\right)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}} \quad \text{for} \quad \eta_3<{H_3}^2 \tag{4.45b}$$

The corresponding deceleration parameter will be

$$q=\frac{3}{2}(1+\omega)\left(1-\frac{\eta_3}{H^2}\right)-1 \tag{4.46}$$

Here the explicit solution is chosen in the form

$$H^{(3)}=\sqrt{\eta_3+({H_3}^2-\eta_3)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}} \quad \text{for} \quad {H_3}^2(1-\frac{2}{3(1+\omega)})<\eta_3<{H_3}^2 \tag{4.47a}$$

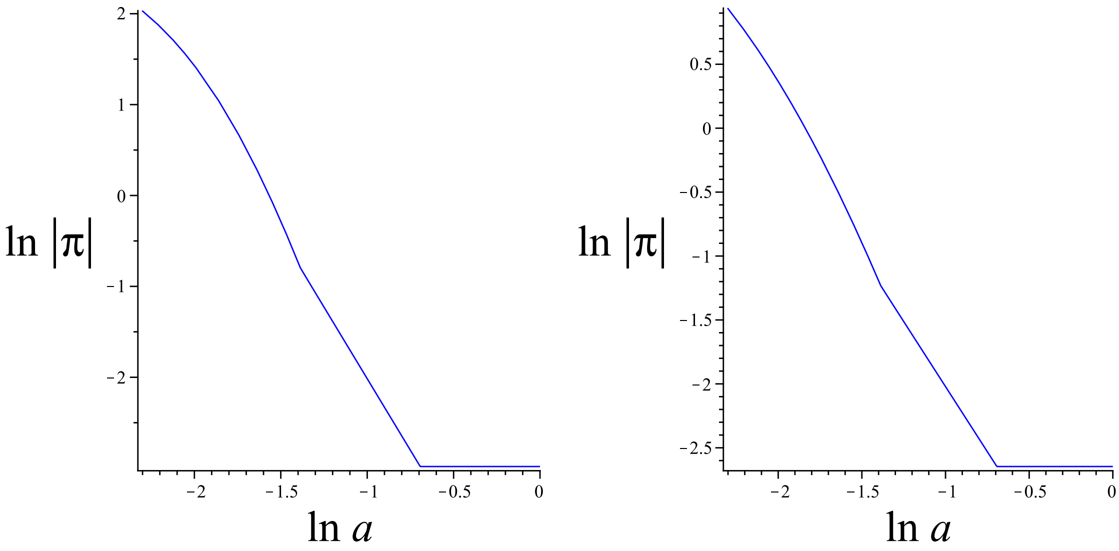
$$\rho^{(3)}=\rho_3\frac{\eta_3+({H_3}^2-\eta_3)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}}{{H_3}^2} \tag{4.47b}$$

$$T^{(3)}=T_3\left[\frac{H_3}{\sqrt{\eta_3+({H_3}^2-\eta_3)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}}}\right]^{-\frac{2\omega}{(1+\omega)}}, \tag{4.47c}$$

$$\boldsymbol{a^{(3)}=a_3\left[\sqrt{\frac{1-\eta_2}{\eta_2}}\sinh\{\frac{3(1+\omega)}{2}\sqrt{\eta_2}H_3(t-t_j)\}\right]^{\frac{2}{3(1+\omega)}}}. \tag{4.47d}$$

Here t_j is the integration constant. $a(t_1)=a_1$ and at the beginning of the late time acceleration phase, $a(t_3)=a_3$. One has, $t_3-t_1=\frac{2}{3(1+\omega)(1-\eta_2)}(\frac{1}{H_3}-\frac{1}{H_1})$ and $\frac{3}{2}(1+\omega)\sqrt{\eta_2}H_3(t_3-t_j)=\sinh^{-1}\sqrt{\frac{\eta_2}{1-\eta_2}}$.

Table (4.1) shows the cosmic solutions for different choices of cosmological scalar field with corresponding restrictions on involved constants for valid solutions. Also the variation of π as a function of a has been shown in Figure 4.1(a) and 4.1(b) in logarithmic scale for two fluids.



(a) $\omega = 0.5$ (b) $\omega = -0.03$

Figure 4.1: Plot of Dissipative pressure as a function of scale factor : $\ln|\pi|$ vs ln a

4.4 Continuous unified evolution of the Universe.

Although the solutions for valid cosmic evolution violate the criteria for thermal stability yet [71, 116] the dissipative diffusion[1] continues in different era as the previous section. However for smooth diffusion process, the cosmological scalar field will be continuous at transition epochs i.e.

$$\phi^{(1)}(a_1) = \phi^{(2)}(a_1) \tag{4.48a}$$

$$\phi^{(2)}(a_3) = \phi^{(3)}(a_3) \tag{4.48b}$$

[1]This dissipative diffusion process is like the convection i.e. the combination of advection(where the transported quantity and substance remains conserved) and non-conservative molecular diffusion (like bulk flow).

Table 4.1: FORM OF ϕ AND RESTRICTION ON CONSTANTS AT DIFFERENT ERA OF EVOLUTION

Form of ϕ	Explicit solution of H	restricted range of constants for valid evolution
$\phi^{(E)} \propto H$	$H^{(E)} = \frac{\eta_E H_0}{H_0+(\eta_E-H_0)e^{-\frac{3}{2}\eta_E(1+\omega)(t-t_0)}}$	$\eta_E > H_0$ for emergent scenario.
$\phi^{(1)} \propto H^3$	$H^{(1)} = \frac{H_1}{\eta_1 H_1+(1-\eta_1 H_1)\left(\frac{a}{a_1}\right)^{\frac{3}{2}(1+\omega)}}$	$0 < \eta_1 < \frac{1}{H_1}$ for early time acceleration.
$\phi^{(2)} \propto H^2$	$H^{(2)} = H_1\left(\frac{a}{a_1}\right)^{\frac{3}{2}(1+\omega)(\eta_2-1)}$	$\eta_2 < 1 - \frac{2}{3(1+\omega)}$ for decelerating expansion.
$\phi^{(3)} \propto H^0$ i.e. cosmological constant.	$H^{(3)} = \sqrt{\eta_3 + ({H_3}^2-\eta_3)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}}$	${H_3}^2(1-\frac{2}{3(1+\omega)}) < \eta_3 < {H_3}^2$ for late time acceleration.

which yields that

$$\eta_2 = \eta_1 H_1 \tag{4.49a}$$
$$\eta_3 = \eta_2 H_3^2 = \eta_1 H_1 H_3^2 \tag{4.49b}$$

where a_1 and a_3 are the values of scale factor as two transition epochs : inflation to decelerated expansion and decelerated expansion phase to late time acceleration respectively. Also from continuity of Hubble parameter, one finds

$$H_3 = H_1\left(\frac{a_3}{a_1}\right)^{\frac{3}{2}(1+\omega)(\eta_2-1)} \tag{4.50}$$

Table (4.2) describes the continuous cosmic evolution showing different era of evolution and also the thermodynamic parameters (namely energy density, temperature)are presented for each era. The continuity of different parameters like Hubble parameter(H),

Table 4.2: UNIFIED COSMIC EVOLUTION UNDER DIFFUSION

Ranges of epochs	Cosmological scalar field ϕ	Hubble parameter H	Different thermodynamic parameters
$a \leq a_1$	$3\eta_1 H^3$	$\frac{H_1}{\eta_1 H_1+(1-\eta_1 H_1)(\frac{a}{a_1})^{\frac{3}{2}(1+\omega)}}$	$\rho^{(1)} = \frac{\rho_1}{\left[\eta_1 H_1+(1-\eta_1 H_1)(\frac{a}{a_1})^{\frac{3}{2}(1+\omega)}\right]^2}$ $T^{(1)} = T_1\left[\eta_1 H_1 + (1-\eta_1 H_1)(\frac{a}{a_1})^{\frac{3}{2}(1+\omega)}\right]^{-\frac{2\omega}{(1+\omega)}}$
$a_1 \leq a \leq a_3$	$3\eta_2 H^2$	$H_1\left(\frac{a}{a_1}\right)^{\frac{3}{2}(1+\omega)(\eta_2-1)}$	$\rho^{(2)} = \rho_1\left(\frac{a}{a_1}\right)^{3(1+\omega)(\eta_2-1)}$ $T^{(2)} = T_1\left(\frac{a}{a_1}\right)^{3\omega(\eta_2-1)}$
$a \geq a_3$	$3\eta_3$	$\sqrt{\eta_3+({H_3}^2-\eta_3)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}}$	$\rho^{(3)} = \rho_3\frac{\eta_3+({H_3}^2-\eta_3)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}}{{H_3}^2}$ $T^{(3)} = T_3\left[\frac{H_3}{\sqrt{\eta_3+({H_3}^2-\eta_3)\left(\frac{a}{a_3}\right)^{-3(1+\omega)}}}\right]^{-\frac{2\omega}{(1+\omega)}}$

energy density (ρ), absolute temperature (T) are demonstrated in the Figure 4.2($a.c,d$) and 4.3 (a,c,d) for both positive ($\omega = 0.5$) and negative ($\omega = -0.03$) pressure fluid respectively for suitable choices of parameters as per the derived conditions in the equations (4.49a),(4.49b) and (4.50). The variation of deceleration parameter q is also shown in Figure 4.2(b) and 4.3(b) for these two fluids respectively.

Here in this context, the deceleration parameter q is not continuous at a_1 though it is continuous near a_1. On the other hand, q is continuous at a_3, but there decelerating phase continues. The transition from deceleration to late time acceleration occurs a little bit later from a_3 and the shift of transition epoch depends on the value of ω. At the transition epoch $\tilde{a_3}(= a_3+\delta a_3)$, the deceleration parameter will be $q_2(\tilde{a_3}) = q_3(\tilde{a_3}) = 0$. So the shift of transition epoch $\delta a_3 = a_3\left[\exp\left\{-\frac{1}{3}\frac{\ln\left(\frac{2\eta_2}{(1+3\omega)(1-\eta_2)}\right)}{1+\omega}\right\}-1\right]$. The variation of shift of a_3 is shown in Figure 4.4. There will be no shift for the value of ω =

$\omega_0 = \frac{2}{3(1-\eta_2)} - 1$. This shifting of transition scale factor leads to the non-simultaneity of evolutionary phase transition and thermodynamic phase transition which will be discussed in the next section.

The solutions for scale factor a as a function of time t for different phases are continuous at the transition epochs t_1 and t_3. Hence the continuity of a indicates the complete cosmic scenario from emergent phase to present late time acceleration phase. The continuous time evolution of the scale factor has been presented graphically in Figure 4.7(b) and 4.7(d) for the two fluids. The value of t_1 is chosen as 0.5 unit.

4.5 Thermodynamic phase transition of cosmic fluids.

The continuity of diffusion process evokes the idea of continuity of some other thermodynamic parameters of cosmic fluids. It will be very interesting to examine whether and how the free energies (particularly Gibbs free energy) of cosmic fluids are varied at transition epochs of evolution. The minimum order of derivative of Gibbs free energy at any particular fixed temperature which shows the discontinuity, determines the order of phase transition.

To determine the expression of Gibbs free energy(G), one starts with the equation

$$G = h - TS \tag{4.51}$$

where T is the absolute temperature of the fluid and $h(= E + PV)$ is the enthalpy of the fluid. So

$$G = E + PV - TS \tag{4.52}$$

Applying the first law of thermodynamics, one finds

$$dQ = TdS = dE + PdV \tag{4.53}$$

$$dG = VdP - SdT \tag{4.54}$$

So, clearly

$$S = -\left(\frac{\partial G}{\partial T}\right)_P \text{ and } V = \left(\frac{\partial G}{\partial P}\right)_T \tag{4.55}$$

Now putting the expression of Entropy S in equation (4.51), one finds the relation after simplification as

$$\left(\frac{\partial G}{\partial T}\right)_P - \frac{G}{T} = -(1+\omega)\frac{E}{T} \tag{4.56}$$

From equation (4.10), it can be found that

$$d\ln\rho = -(1+\omega)d\ln V + d\ln\left[\rho_0 + A(a)\right] \tag{4.57}$$

where $A(a) = 3\int_{t_0}^{t}\sigma_0 a^{3\omega}dt$. Now taking Entropy as a state function (i.e. dS is an exact differential) of two independent variable Volume(V) and temperature (T) one finds

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV. \tag{4.58}$$

Again, one can write from equation(4.53) that (with $E = \rho V$ and $P = \omega\rho$)

$$TdS = (1+\omega)\rho dV + v\frac{d\rho}{dT}dT, \tag{4.59}$$

where ρ is taken as the function of temperature only i.e. $d\rho = \frac{d\rho}{dT}dT$. Now comparing equation(4.52) with equation(4.59), one obtains

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T}\frac{d\rho}{dT}\,,\quad \left(\frac{\partial S}{\partial V}\right)_T = (1+\omega)\rho. \tag{4.60}$$

Now imposing the condition that dS is an exact differential $\left(\frac{\partial^2 S}{\partial V\partial T} = \frac{\partial^2 S}{\partial T\partial V}\right)$, it is found that

$$d(\ln\rho) = \frac{1+\omega}{\omega}d(\ln T). \tag{4.61}$$

Combining the equations (4.57) and (4.61), one finds the relation

$$d\ln T = -\omega d\ln V + \frac{\omega}{1+\omega}d\ln\left[\rho_0 + A(a)\right] \tag{4.62}$$

Now from the equations (4.57) and (4.62), the expression of Energy can be obtained in the form

$$E = E_0\left(\frac{T}{T_0}\right)\left[1 + \frac{A}{\rho_0}\right]^{\frac{1}{1+\omega}} \tag{4.63}$$

with E_0, the value of energy at reference time t_0 . The relation between the temperatures at adiabatic and isochoric diffusion for the same value of the diffusion parameter [116, 79] is given by $\left[1+\frac{A}{\rho_0}\right]=\frac{T}{T_a}$. At a particular epoch a, T_a is constant. So, one finds the explicit form of Gibbs free energy from the equation (4.56) as

$$G=-(1+\omega)E+\frac{T}{T_0}\left[G_0+(1+\omega)E_0\right] \text{ [with } G_0\text{, the Gibbs free energy at the reference time } t_0] \tag{4.64}$$

Now the variation of the equation of state parameter of the cosmic fluid ω are assumed negligible at different eras of evolution [2] i.e. $\omega^{(1)}=\omega^{(2)}=\omega^{(3)}$. As in the present context, both E and T are continuous at transition epochs a_1 and a_3, then it is clear that G is also continuous. The continuity of Gibb's free energy density (g) is shown in Figure 4.7(a) and 4.7(c) for two fluids ($\omega=0.5$ and $\omega=-0.03$) respectively. Now the expression for Entropy is found from equations (4.55) and (4.64) as

$$S=C_P-\frac{G_0+(1+\omega)E_0}{T_0} \tag{4.65}$$

where C_P is the specific heat of the fluid at constant pressure. Now $C_P=(1+\omega)C_V+E\left(\frac{\partial\omega}{\partial T}\right)_P$ with C_V, the specific heat of the fluid at constant volume. From equation (4.62), one can estimate that

$$\left(\frac{\partial\omega}{\partial T}\right)_P=\frac{1}{T}\cdot\frac{d\ln|\omega|}{-d\ln V+\frac{1}{1+\omega}d\ln[\rho_0+A(a)]}\ .$$

Clearly, $\left(\frac{\partial\omega}{\partial T}\right)_P$ will not be continuous because $A(a)$ is different at different eras of evolution i.e. $\left(\frac{\partial\omega}{\partial T}\right)_P^{(1)}\neq\left(\frac{\partial\omega}{\partial T}\right)_P^{(2)}\neq\left(\frac{\partial\omega}{\partial T}\right)_P^{(3)}$. So, clearly S is not continuous at transition epochs. Hence discontinuity of first order derivative of Gibbs free energy with respect to temperature indicates the first order thermodynamic phase transition of cosmic fluid under continuous diffusion process.

For the first order phase transition, one may estimate the expression of latent heat from Clausius-Clapeyron equation :

$$\frac{dP}{dT}=\frac{L_{ij}}{T_{ij}\left(\frac{1}{\rho^{(j)}}-\frac{1}{\rho^{(i)}}\right)} \tag{4.66}$$

where L_{ij} is the latent heat of transition from i-th phase to j-th phase and $\rho^{(i)}$, $\rho^{(i)}$

[2] $P=\omega\rho$, So ω will be variable in isobaric condition even for a constant equation of state parameter fluid, as the energy density ρ always varies with the evolution. But with out such isobaric constraint on universe one may neglect the variation of ω with expansion.

are the energy densities of the fluid at the corresponding phases. T_{ij} is the transition temperature between these two phases. The above equation can also be modified as

$$\omega \lim_{\Delta T\to 0}\left[\frac{\Delta\rho}{\Delta T}\right]_{a=a_{ij}} = -\frac{L_{ij}[\rho^{(i)}]_{a_{ij}}[\rho^{(j)}]_{a_{ij}}}{T_{ij}\,(\Delta\rho)_{a=a_{ij}}} \tag{4.67}$$

and hence one can estimate

$$\frac{\left[\rho^{(j)}(a_{ij}+\Delta a)-\rho^{(i)}(a_{ij}-\Delta a)\right]^2}{T^{(j)}(a_{ij}+\Delta a)-T^{(i)}(a_{ij}-\Delta a)} \approx -\frac{L_{ij}{\rho_{ij}}^2}{\omega T_{ij}} \tag{4.68}$$

where a_{ij} is the scale factor at transition epoch from i-th phase to j-th phase , $\rho_{ij} = [\rho^{(j)}]_{a_{ij}} = [\rho^{(i)}]_{a_{ij}}$, $T_{ij} = [T^{(j)}]_{a_{ij}} = [T^{(i)}]_{a_{ij}}$ and $\Delta\rho = \rho^{(j)} - \rho^{(i)}, \Delta T = T^{(j)} - T^{(i)}$. Δa is a small perturbation in the scale factor i.e. $\Delta a \to 0$ and terms with higher order of Δa can be neglected.

Now one can easily obtain the expression of latent heat by applying Taylor series expansion in the above equation (4.68) as

$$L_{ij} = -\frac{2\omega T_{ij}\left(\frac{\partial\rho}{\partial a}\right)^2_{a_{ij}}\Delta a}{\rho_{ij}^2\left(\frac{\partial T}{\partial a}\right)_{a_{ij}}} \tag{4.69}$$

where $\left(\frac{\partial\rho}{\partial a}\right)^{(j)}_{a_{ij}} = \left(\frac{\partial\rho}{\partial a}\right)^{(i)}_{a_{ij}} = \left(\frac{\partial\rho}{\partial a}\right)_{a_{ij}}$ and $\left(\frac{\partial T}{\partial a}\right)^{(j)}_{a_{ij}} = \left(\frac{\partial T}{\partial a}\right)^{(i)}_{a_{ij}} = \left(\frac{\partial T}{\partial a}\right)_{a_{ij}}$. So in this present context, the latent heats are found in the forms :-

$$L_{12} = -\frac{2\omega T_1\left(\frac{\partial\rho}{\partial a}\right)^2_{a_1}\Delta a}{\rho_1^2\left(\frac{\partial T}{\partial a}\right)_{a_1}} = -6(1-\eta_1 H_1)(1+\omega)^2\frac{\Delta a}{a_1} \tag{4.70}$$

$$L_{23} = -\frac{2\omega T_3\left(\frac{\partial\rho}{\partial a}\right)^2_{a_3}\Delta a}{\rho_3^2\left(\frac{\partial T}{\partial a}\right)_{a_3}} = -6(1-\eta_1 H_1)(1+\omega)^2\frac{\Delta a}{a_3} \tag{4.71}$$

The latent heat is negative for both the transitions. As the Universe is expanding and latent heat is negative, one must conclude that $\left(\frac{dP}{dT}\right)$ is also negative. For a particular epoch of transition i.e. for fixed value of ρ, one has the forms of Clausius Clayperon equation[from equation (4.66)] as

$$\rho_{ij}\lim_{\Delta T\to 0}\frac{\Delta\omega}{\Delta T} = -L_{ij}\frac{[\rho^{(i)}]_{a_{ij}}[\rho^{(j)}]_{a_{ij}}}{(\Delta\rho)_{a_{ij}}}. \tag{4.72}$$

Then one can write by simplifying the equation(4.72),

$$\Delta T = -2\frac{\Delta\omega}{L_{ij}}\left[\frac{\partial(\ln\rho)}{\partial a}\right]_{a_{ij}}, \tag{4.73}$$

with $\Delta\omega = (\omega-\omega_0)$. Here ω_0 is the value of equation of state parameter with shifting of neither transition scale factor nor the transition temperature i.e. $[\Delta T_{ij}]_{\omega_0} = [\delta a_{ij}]_{\omega_0} = 0$. In the present context, the value of ω_0 is also derived in the Section (4) as, $\omega_0 = \frac{2}{3(1-\eta_2)} - 1$.

Then the values of shifted transition temperatures will be in the forms,

$$T_1{}' = T_1\left[1 - \frac{(\omega-\omega_0)}{(1+\omega)}\right] \tag{4.74a}$$

$$T_3{}' = T_1\left(\frac{a_3}{a_1}\right)^{3\omega(\eta_2-1)}\left[1 - \frac{(\omega-\omega_0)}{(1+\omega)}\right] \tag{4.74b}$$

The Shifting of transition temperatures with equation of state parameter ω are shown in Figure 4.5.

4.6 discussions

The unified cosmic evolution under diffusion process is successfully established in this work. While transition from one cosmic era to next adjacent era, the nature of thermodynamic phase transition of the diffusive cosmic fluid is found to be of first order. The negative latent heat for both the transitions indicates the emission of energy in transition epochs. The thermal power associated with emitted latent heat is found as, $P_L \propto T^{\frac{1+\omega}{2\omega}}$. As per Stephen's law of radiation ($P \propto T^4$), the value of ω should be $\frac{1}{7}$ for electromagnetic radiation in transition epochs.

Importantly the limiting value of ω is $\left[\frac{2}{3(1-\eta_2)} - 1\right]$ and hence, $\omega > -\frac{1}{3}$ for valid cosmic evolution. Dark energy with constant equation of state parameter can not be thermodynamically stable [71, 116]. In spite of that, dark energy is still a candidate for explaining the evolution of the Universe though it is not the only candidate for such evolution. Positive pressure fluid which violates the criteria for thermal stability $\left[\frac{\partial\left\{\ln\left(1+\frac{3}{\rho_0}\int_t^{t_0}\sigma_0 a^{3\omega}dt\right)\right\}}{\partial\ln T}\right]_P \geq \frac{1+\omega}{\omega}$ [116], can also be taken for valid expansion too. Moreover, in case of negative ω, temperature increases with expansion and it contradicts the recent experimental results[117]. Non-diffusive, positive pressure fluid always satisfies the condition for thermal stability [71] and can not be responsible for continuous non-inertial evolution. Hence it can be concluded that the bulk flow type diffusion process of positive pressure fluid is the essential mechanism in this evolutionary scenario. In all eras, ϕ is positive and evidently only the inward bulk flow is possible to validate the desired natures of expansion in different epochs.

Moreover, it is to be noted that the present cosmological model describing different cosmological eras as continuous evolution with diffusive barotropic fluid, is not valid at the phantom barrier i.e. $\omega = -1$. Then diffusive nature disappears due to zero enthalpy at the instant and the model reduces to Λ_{CDM} model. The variation of Ω as a function of scale factor in the present model is presented in Figure 4.6.

Finally, it will be very interesting in future works to set up a similar model of the Universe for variable equation of state parameter and analyse the influences on expansion. It may be expected that this work along with some further related research works on this matter will be able to specify the suitable candidates among a large number of options to interpret the past and present nature of evolution.

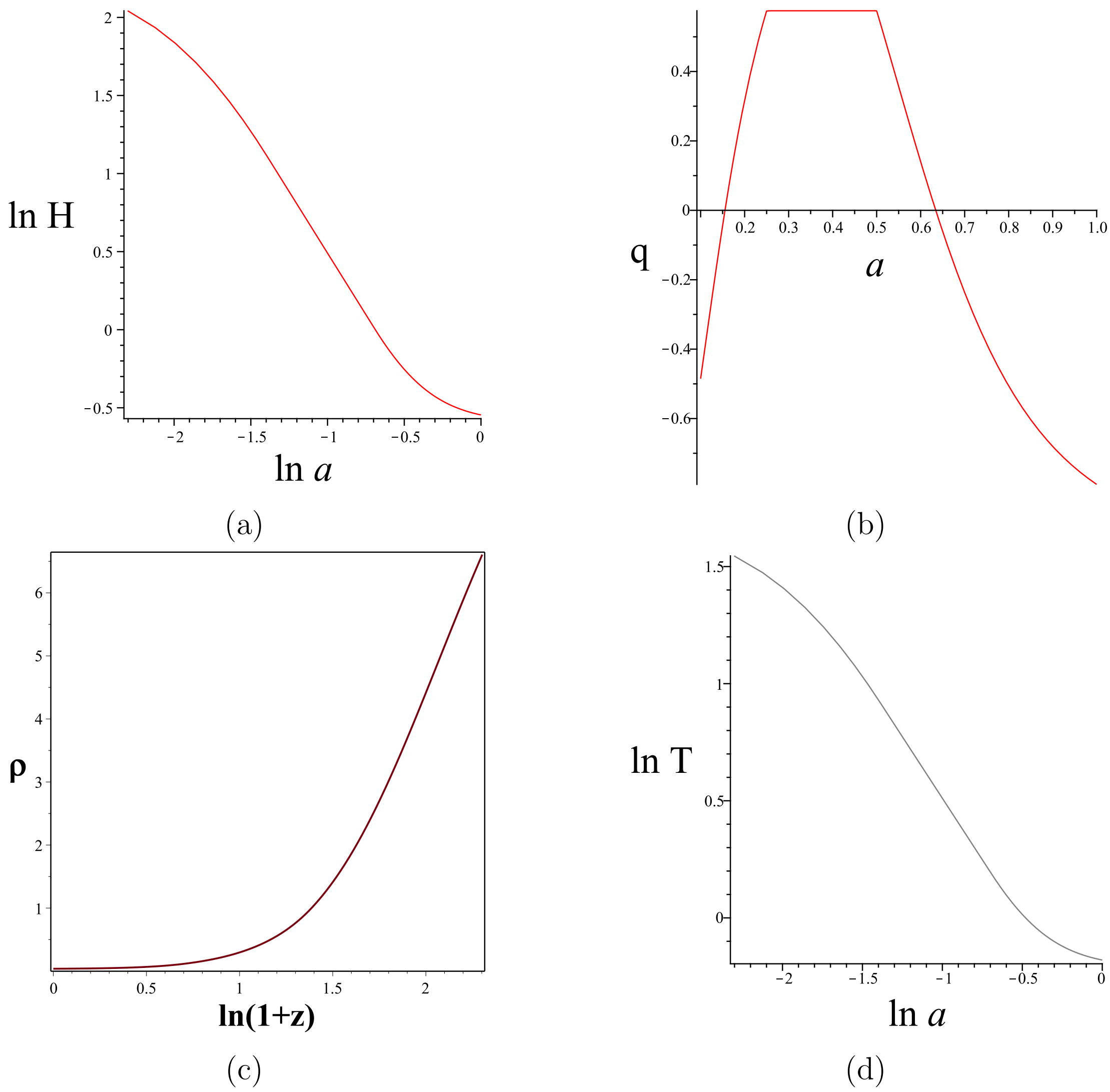


Figure 4.2: Variation of different physical parameters namely (a) Hubble parameter H (top left), (b) Deceleration parameter q (top right), (c) **Energy density** ρ **with** $\ln(1+z)$ **(bottom left)**, (d) Temperature T (bottom right) for $\omega = -0.03$ (positive pressure fluid). Here the best fitted values of model parameters $a_1 = 0.25, a_3 = 0.50, H_1 = 3, \eta_1 = 0.1, T_1 = 2.5, \rho_1 = 1$ and $g_0 = 1$ are chosen.

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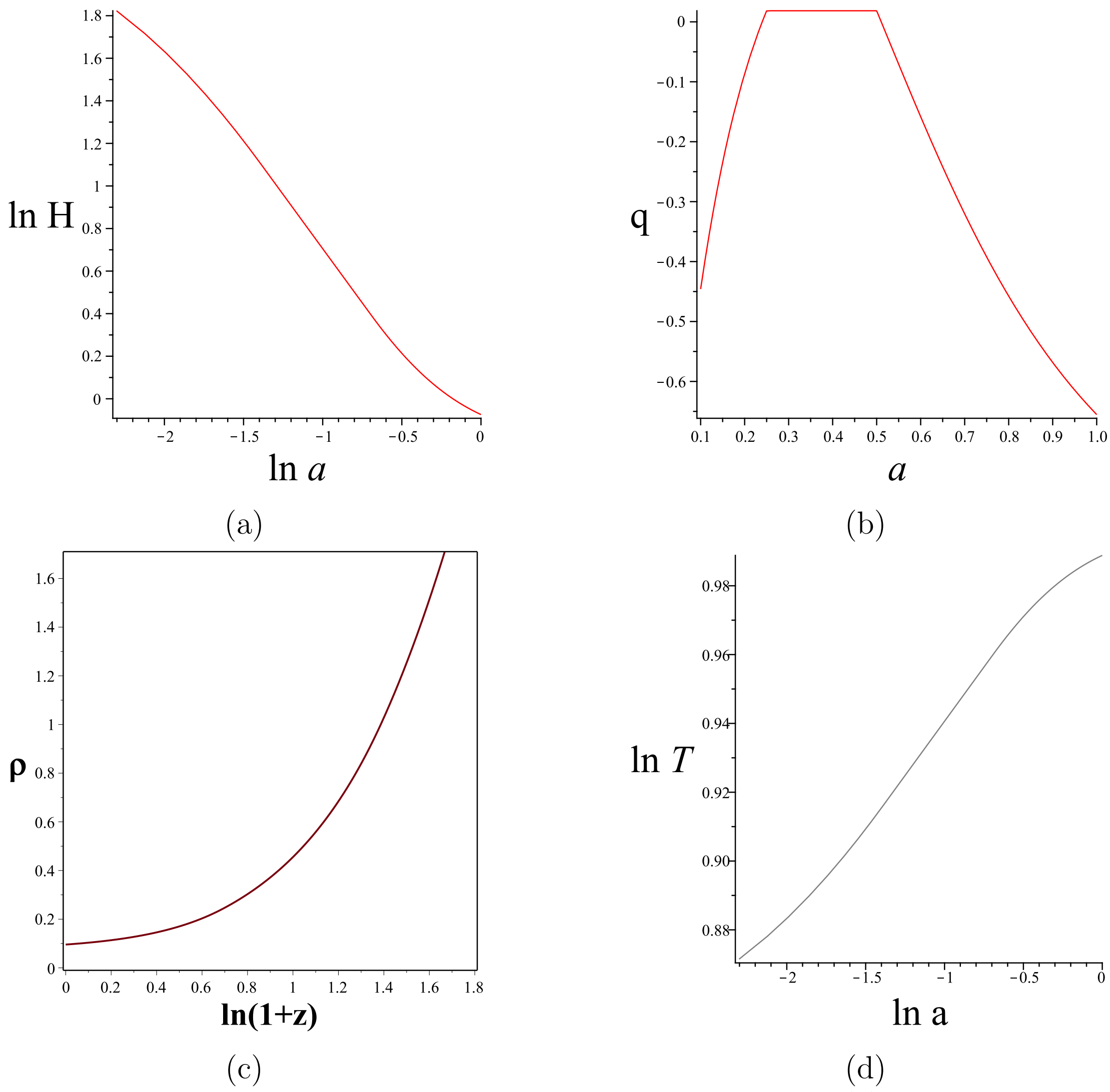

Figure 4.3: Variation of different physical parameters namely (a) Hubble parameter H (top left), (b) Deceleration parameter q (top right), (c) **Energy density** ρ **with** $\ln(1+z)$ **(bottom left)**, (d) Temperature T (bottom right) for $\omega = -0.03$ (Negative pressure fluid). Here the best fitted values of model parameters $a_1 = 0.25, a_3 = 0.50, H_1 = 3, \eta_1 = 0.1, T_1 = 2.5, \rho_1 = 1$ and $g_0 = 1$ are chosen.

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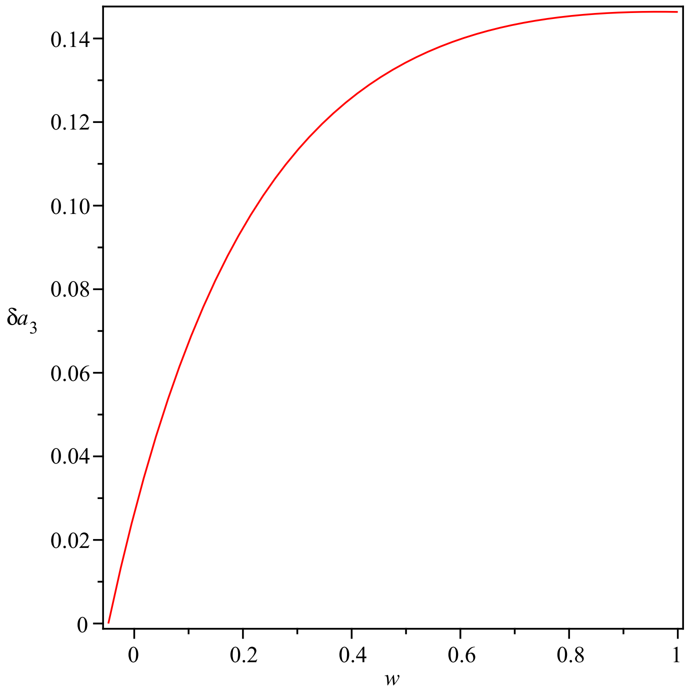
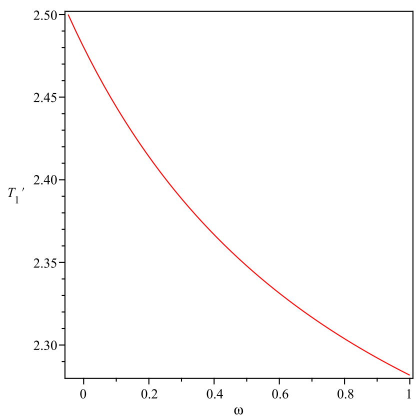
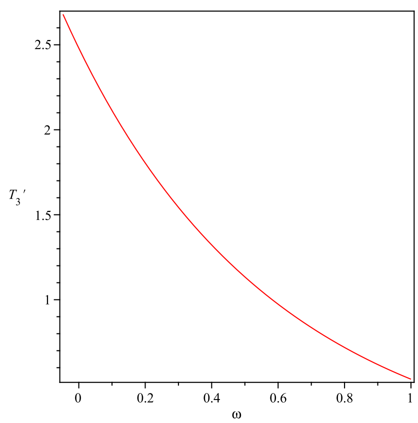


Figure 4.4: Variation of δa_3 with ω



(a) Variation of T_1 with ω



(b) Variation of T_3 with ω

Figure 4.5: Variation of transition temperatures T_α and T_β with ω. $a_1 = 0.25, a_3 = 0.50, H_1 = 3, \eta_1 = 0.1,\ \ T_1 = 2.5, \rho_1 = 1$ and $g_0 = 1$. The value of $\omega_0 = -\frac{1}{21}$ is found.

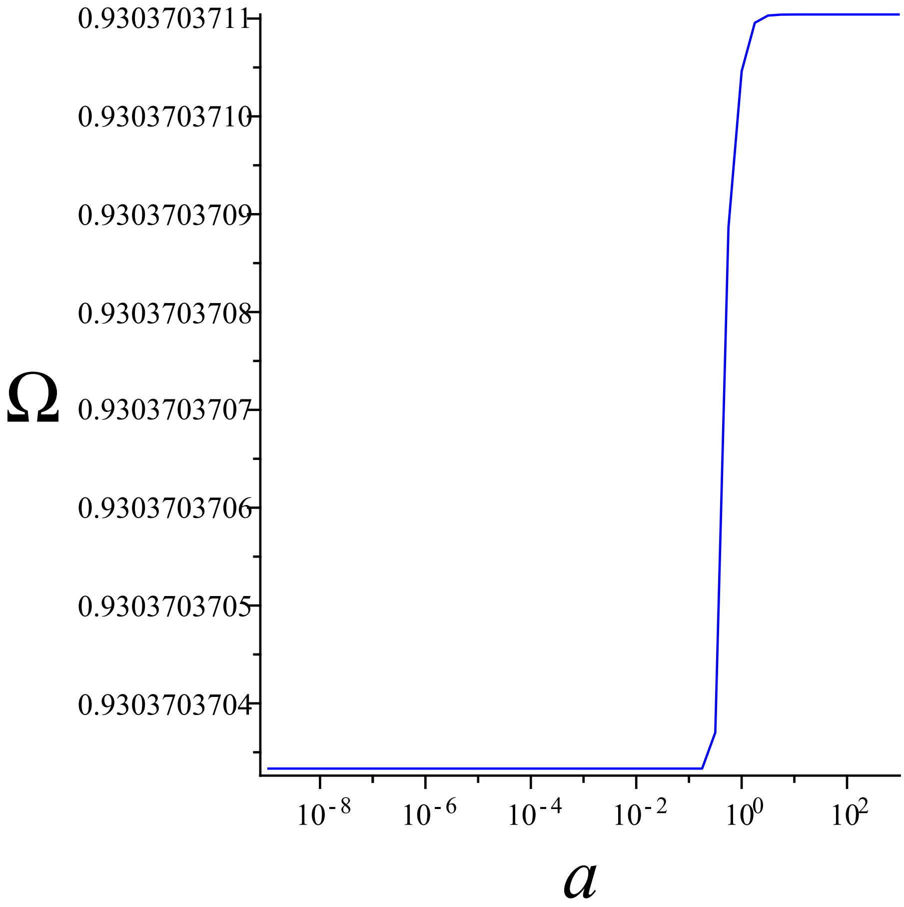


Figure 4.6: Variation of Density parameter Ω with scale factor a

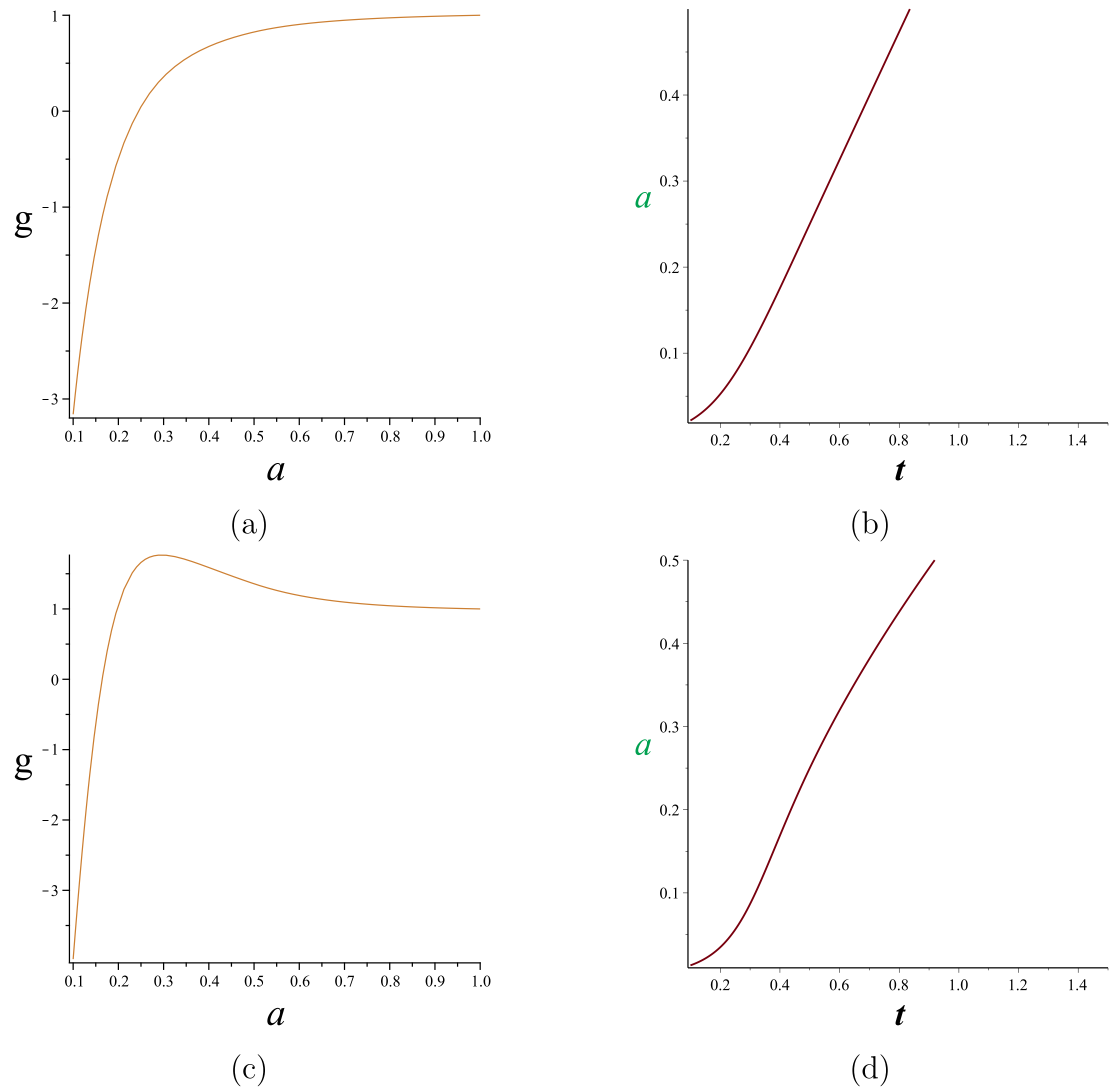


Figure 4.7: Exhibition of continuity of the Gibbs free energy density (g) and the scale factor a for two fluids with $\omega = 0.5$ (top) and $\omega = -0.03$(bottom) respectively. The variation of g reperesents the first order thermodynamic phase transition and the graph of a exhibits the continuous cosmic evolution.
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CHAPTER 5

CYCLIC EVOLUTION OF A DIFFUSIVE UNIVERSE : A PHENOMENOLOGICAL APPROACH TO MODEL A COSMIC HEAT ENGINE

5.1 Prelude

'Bulk flow' or 'advection' is the motion or flow of an entire substance from or to the system due to the pressure gradient (for example water flow out of the tap). On the other hand, 'diffusion' is the gradual transport or dispersion of concentration within a body, due to the concentration gradient with no net movement of the substance from or to the system. The diffusion depends on the random motion of the particles and results in mixing or mass transport without any net bulk motion. Inhomogeneity of concentration within the system (it may be due to various processes like bulk flow, particle creation-annihilation etc.) leads to the disturbance in the equilibrium condition among the different parts of the system. To restore the equilibrium state, a pressure termed as 'diffusive pressure' is generated which drives the flow of molecules from higher to lower concentration regions. Both advection and diffusion processes are the transport phenomena and hence thermodynamically irreversible process [118]. Clearly advection can cause diffusion process even in a single fluid system. The combination of these two irreversible process is called 'convection'. Convection process results in a net transport of mass and it occurs in a non-equilibrium system. So, to apply the laws of thermodynamics, one may assume the quasi-steady state where the process changes with time very slowly and then the thermodynamic laws are applicable instantaneously [118].

The cosmic diffusion mechanism can be considered as the diffusion of a cosmic fluid within a continuously evolving isolated system. Here the effect of diffusion pressure deficiency (resulting from instantaneous inhomogeneity of the fluid) leads to the transport phenomena along with an expansion of the background space-time of the isolated Universe. Hence the usual matter conservation equation of a common fluid needs a modification to incorporate the effect of the dissipation of the fluid and also the effect of continuous evolution of the background space-time.

5.2 Modelling of Universe under diffusion

In the present context, the Universe is assumed to be an isolated system of diffusive barotropic fluid with constant equation of state parameter (ω) i.e. the equation of state of the fluid is $p = \omega\rho$, where p, ρ are respectively the pressure and energy density of the fluid. Due to the bulk viscous pressure (π) in the fluid, the particle creation or annihilation process occurs in the Universe. The particle creation -annihilation mechanism is a bulk flow process which creates the concentration gradient within the system and it leads to generate diffusive pressure ($\mathcal{P}$). Under such diffusion process (irreversible), the energy-momentum tensor will not be conserved. Due to the Fokker-Planck equation, the rate of change of the energy momentum tensor ($T^{(d)}_{\mu\nu}$) of a diffusive fluid is assumed to be proportional to the diffusion current(N^{ν}) i.e. [119, 116]

$$T^{(d)}_{\mu\nu}{}_{;\mu} = 3\sigma N^{\nu}. \tag{5.1}$$

This is the simplest modification of the matter conservation equation for a dissipative fluid and the covariant derivative of the energy - momentum tensor contains the effect of background space-time.

Clearly the energy - momentum tensor of a diffusive fluid is not covariantly conserved and hence it violates the Bianchi identity of the Einstein field equations. So, one needs some modifications in the usual Einstein field equation to incorporate the diffusive fluid in the matter component of GTR. Here, an extra matter component is chosen as a form of a time dependent cosmological scalar field (ϕ) so that the total effective energy - momentum tensor ($T_{\mu\nu}$) becomes covariantly conserved. The suitable form of $T_{\mu\nu}$ has been assumed as,

$$T_{\mu\nu} = T_{\mu\nu}^{(d)} - \phi g_{\mu\nu}. \tag{5.2}$$

Eventually, ϕ and $T_{\mu\nu}^{(d)}$ are mutually interacting. The cosmological scalar field ϕ acts as a matter creation- annihilation field and it causes a bulk flow mechanism. This bulk flow process leads to the pressure gradient and drives the cosmic fluid into diffusion.

Now, following the Bianchi identity ($T_{\mu\nu}, \mu = 0$) one has the evolution equation for ϕ as,

$$\nabla_\mu \phi = 3\sigma N_\mu, \tag{5.3}$$

where ($\sigma > 0$) is a constant. 3σ measures the energy transferred from the diffusive fluid to the scalar field due to diffusion. The diffusion current density of the diffusive fluid satisfies the equation of continuity

$$\nabla_\mu N^\mu = 0. \tag{5.4}$$

The modified Einstein field equations take the form

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \phi g_{\mu\nu} = T_{\mu\nu}^{(d)}. \tag{5.5}$$

It is notable that the modified field equations are identical with the normal field equation with a varying cosmological constant (Λ) i.e. $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with Λ being replaced by a time dependent scalar field ϕ.

In the background of homogeneous and isotropic flat FLRW model, the modified Friedmann equations with diffusion dynamics take the form,

$$3H^2 = \rho + \phi \tag{5.6}$$

and

$$2\dot{H} = -(\rho + p) \tag{5.7}$$

The modified matter conservation equation (5.1) for the matter field (5.2) takes the form,

$$\dot{\rho} + 3H(p + \rho) = \sigma n_0 a^{-3} = \sigma_0 a^{-3} \tag{5.8}$$

which on integration yields

$$\rho = a^{-3(1+\omega)} \left[\rho_0 + \int_{t_0}^{t} \sigma_0 a^{3\omega} dt \right]. \tag{5.9}$$

Here ρ_0 is the energy density at reference epoch of time $t = t_0$ and $a(t_0) = 1$. $n(t_0) = n_0$ is assumed. Now eliminating ρ between equations (5.6) and (5.7), one gets the cosmic evolution equation as

$$2\dot{H} + 3(1 + \omega)H^2 = \phi(1 + \omega) \tag{5.10}$$

On the other hand, the above modified Friedmann equations (i.e. equations (5.6) and (5.7)) for diffusive mechanism can be rewritten as,

$$3H^2 = \rho_d \ , \ 2\dot{H} = -(\rho_d + p_d + \pi_d) \tag{5.11}$$

while the conservation equation(5.8) becomes

$$\dot{\rho_d} + 3H(\rho_d + p + \pi_d) = 0 \tag{5.12}$$

Here, $\rho_d = \rho + \phi, p_d = p$ and $\pi_d = -\phi$ are respectively the effective energy density, effective thermodynamic pressure and effective bulk viscous pressure of the equivalent Einstein gravity model. Thus the cosmological scalar field is the (negative) dissipative pressure in effective Einstein gravity.

Under this convection process in non-equilibrium thermodynamic prescription of the Universe, the evolution of the instantaneous equilibrium temperature (T) is governed by the equation [120],

$$\frac{\dot{T}}{T} + \frac{\partial P}{\partial \rho}(3H - |\Gamma_\phi|) = 0. \tag{5.13}$$

Here Γ_ϕ is the particle creation/annihilation rate of the scalar field which satisfies the particle number (non)conservation equation $\dot{n} + 3Hn = n\Gamma_\phi$.

5.3 Cyclic model of universe and heat engine

Matter creation mechanism involves change in Gibbs free energy (G) of a system. Considering the chemical potential μ of the cosmic fluid particle one can estimate the first law of thermodynamics as,

$$TdS = dE + PdV - \mu dN. \tag{5.14}$$

Further the Second law of thermodynamics demands that any system interacting with thermal energy converts a fraction of heat into the corresponding amount of work i.e. any thermodynamic system has the heat engine property with efficiency[121] $\eta < 1$. A heat engine generally operates between two heat reservoirs (having temperatures T_1 and T_2, $T_1 > T_2$) (say) in a cyclic process and converts thermal energy into mechanical energy. Reversible heat engines are operated in theoretically reversible thermodynamic cycles and they can not generate entropy. Carnot engine is a perfect example of reversible engine with efficiency $\eta = 1 - \frac{T_2}{T_1}$. Again as per Carnot's theorem, a reversible engine possesses a maximum possible efficiency and are equally efficient within the same temperature limit. However in practice, there is no such ideal reversible heat engine. In fact, within a continuously evolving isolated system (like Universe) which produces entropy while its evolution, one may think of at best the chance of irreversible heat engine cycle. The efficiency of an irreversible heat engine is generally less than that of a reversible engine. Hence if one has the motivation to predict the cosmic evolution pattern from a thermodynamic point of view then it may be a natural idea to model the Universe as an irreversible heat engine .

The cosmic heat engine clearly follows the cyclic and bouncing evolution pattern and it can be assumed as : Emergent stage $\rightarrowtail$ Inflation $\rightarrowtail$ Decelerating expansion $\rightarrowtail$ Decelerating contraction $\rightarrowtail$ Inflationary contraction $\rightarrowtail$ Emergent . Now in a bouncing Universe, near emergent state[90, 92, 93, 95, 94], $t \rightarrow \pm\infty, a \rightarrow$ constant, $H \rightarrow 0, \phi \sim H$.

The trivial choice of ϕ is taken as a general power law of H,

$$\phi(H,a,t) = \sum_{0}^{\infty} f_i(a,t)H^i \tag{5.15}$$

Considering the dependence of ϕ on H upto second order, one has

$$\phi(H,a,t) = f_0(a,t) + f_1(a,t)H + f_2(a,t)H^2 \tag{5.16}$$

Now imposing the boundary condition, at $t \to \pm\infty$, $a \to b_0$ (a constant), $H \to 0$, $H^2 \ll H$, $\phi \propto H$, one takes phenomenologically

$$f_0(a,t) = \alpha\left(1 - \frac{b_0}{a}\right) \tag{5.17}$$

and

$$f_1(a,t)H = \alpha\frac{b_0}{a}\left|(t-\tilde{t})H\right|, \tag{5.18}$$

α being a constant and $\tilde{t}$ is the transition time epoch from expansion to contracting phase. Also from equation (5.6) one has, at $t = \tilde{t}, H = 0, \phi = -\rho(\tilde{t})$ (a negative constant) and at $t \to \pm\infty$, $H \to 0^{\mp}$, $\phi \to -\rho_E$. ρ_E is the energy density at emergent epoch. Hence α must be a negative constant. For simplicity of the calculation, one takes $f_2 = 3$ and $\alpha = -\frac{4\beta}{1+\omega}$ with β, a constant and the suitable form of ϕ is found as,

$$\phi = 3H^2 - \frac{4\beta}{1+\omega}\left(\frac{b_0}{a}\left|(t-\tilde{t})H\right| + \left(1 - \frac{b_0}{a}\right)\right) \tag{5.19}$$

Then, the evolution equation (5.10) can be written as,

$$2\dot{H} + 4\beta\left[(1 - \frac{b_0}{a}) + \frac{b_0}{a}|t-\tilde{t}|H\right] = 0. \tag{5.20}$$

The equation (5.20) yields the solution as,

$$H = -2\beta(t-\tilde{t}\ )\left(1 - \frac{b_0}{a}\right) \tag{5.21}$$

$$\text{and} \qquad a = b_0 + b_1 \exp\left[-\beta(t-\tilde{t}\ )^2\right] \tag{5.22}$$

where b_1 is also a constant. Hence the deceleration parameter is found as,

$$q = \frac{a}{a - b_0}\left[\frac{1}{2\beta} \cdot \frac{1}{(t-\tilde{t})^2} - 1\right] \tag{5.23}$$

which suggests that, within the epoch range : $\tilde{t} - \frac{1}{\sqrt{2\beta}} \leq t \leq \tilde{t} + \frac{1}{\sqrt{2\beta}}$, the deceleration occurs and beyond this range, acceleration occurs. The other two thermodynamic parameters are as,

$$\rho \;=\; 3H^2 - \phi \;=\; \frac{4\beta}{1+\omega}\left[\left(1 - \frac{b_0}{a}\right) + \frac{b_0}{a}\left|t - \tilde{t}\right| H\right] \tag{5.24}$$

$$T \;\sim\; \left[\left|t - \tilde{t}\right|\left(1 - \frac{b_0}{a}\right)\right]^{\frac{2\omega}{1+\omega}} . \tag{5.25}$$

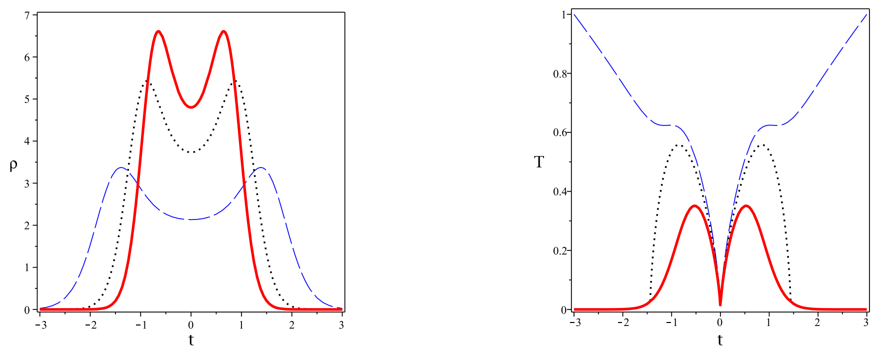

Figure 5.1: Variation of different physical parameters namely (a) energy density ρ as a function of time t (left) and (b)temperature T as a function of t(right), with $\omega = 0.5$ (positive pressure fluid) and $\tilde{t} = 0$ and $b_0 = 0.2$, $b_1 = 0.8$, $\beta = 1$ (for blue, long dash), $b_0 = 0.3$, $b_1 = 0.7$, $\beta = 2$ (for black, dot) and $b_0 = 0.4$, $b_1 = 0.6$, $\beta = 3$ (for red, solid)

Thus, for a thermodynamically well defined cyclic process i.e. a bouncing cosmic evolution follows a valid thermodynamic cyclic process for cosmic fluids except $-1 \leq \omega \leq 0$.

From the above analytic solution the cosmological parameters namely the scale factor (a), Hubble parameter (H), and the acceleration ($\ddot{a}$) as well as the thermodynamic parameters namely temperature (T) and energy density (ρ) have been shown graphically in Figure 5.1 and Figure 5.2 for various choices of the parameters (namely b_0, b_1 and β). The figure for the scale factor shows that in the infinite past the universe

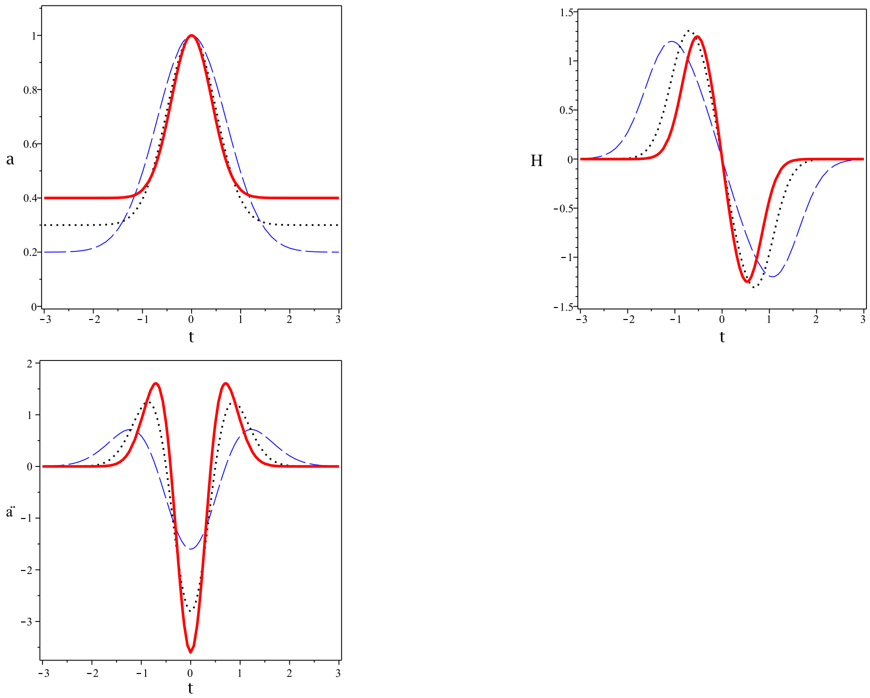

Figure 5.2: Variation of different physical parameters namely (a) Scale factor a as a function of time t (top left), (b) Hubble parameter H as a function of t(top right), and (c) $\ddot{a}$ (bottom) with $\omega = 0.5$ (positive pressure fluid) and $\tilde{t} = 0$ and $b_0 = 0.2$, $b_1 = 0.8$, $\beta = 1$ (for blue, long dash), $b_0 = 0.3$, $b_1 = 0.7$, $\beta = 2$ (for black, dot) and $b_0 = 0.4$, $b_1 = 0.6$, $\beta = 3$ (for red, solid)

was in an emergent scenario and then the Universe expands and reaches a maximum. Then a decreases i.e. universe contracts and finally the Universe enter again in an emergent scenario. The Hubble parameter as expected has zero value in the emergent phase and then it undergoes increasing $\rightarrow$ decreasing $\rightarrow$ increasing behaviour before again it approaches to zero in the future emergent phase. The acceleration is zero in both the emergent phases and it has positive $\rightarrow$ negative $\rightarrow$ positive values between the two asymptotic emergent epochs. The two thermodynamic parameters namely energy density (ρ) and temperature (T) show similar behaviour with the evolution of the Universe namely : at the emergent scenarios both ρ and T vanish and within these two asymptotic phase they have increasing (reaches a local maxima) $\rightarrow$ decreasing (reaches a local minima) $\rightarrow$ increasing (till a local maxima) $\rightarrow$ decreasing behaviour and finally approaches to zero in the future emergent phase. Further the time variation of scalar field ϕ has been shown in the 2-d plot while the variation of ϕ with time and equa-

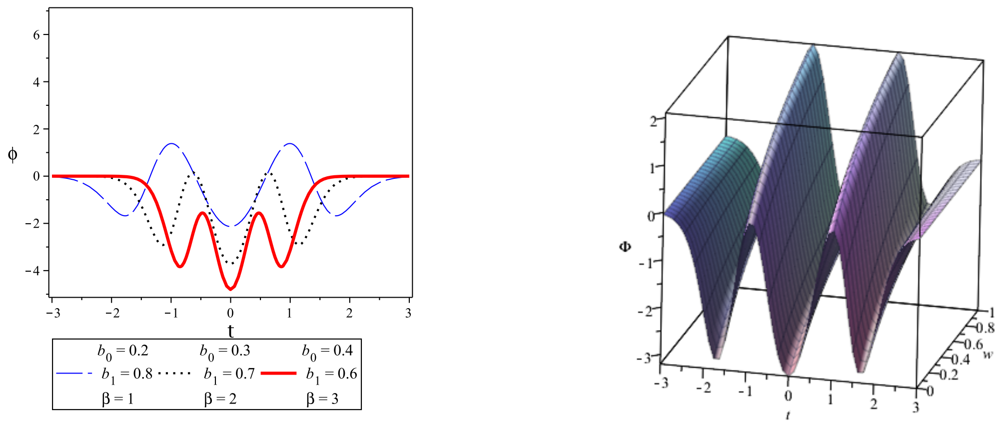

Figure 5.3: Variation of cosmological scalar field ϕ with respect to time t for $\omega = 0.5$ (left) and with respect to t and ω in a 3 d plot with $b_0 = 0.2, b_1 = 0.8, \beta = 1$ (right).

tion of state parameter ω has been presented in the 3-d plotin figure 5.3. Moreover the present cosmic model may be considered as a bouncing model of the Universe for non-fantom cosmic fluid with a valid cyclic thermodynamic process. Therefore, the present diffusive mechanism for cosmic evolution may have some analogy with cyclic thermodynamic process like heat engine.

In a bouncing scenario, the deceleration parameter q will not be continuous at $t = \tilde{t}$ but $\ddot{a}$ will be continuous and it will indicate the acceleration or deceleration phase in different epoch. The variation of $\ddot{a}$ with time is presented in Figure 5.2 . Notably, the cosmic evolution pattern (variation of a, H and $\ddot{a}$) does not depend on the barotropic index ω of the fluid. But the thermodynamic evolution (ρ, T, ϕ) depends on ω.

5.4 Discussion

The present singularity free model of the Universe with diffusive fluid is interesting both from cosmological and thermodynamic point of view. The present cosmic evolution with emergent scenarioes at infinite past and future may be considered as a cyclic model of the Universe if one assumes a wormhole connecting the two asymptotic emergent phases. Similar bouncing model of the Universe is already obtained in the literature(see ref. [122]) with a phenomenological choice of the scale factor. Further the present model resembles with the observations as the model goes through accelerating $\rightarrow$ decelerating $\rightarrow$ accelerating era of evolution between the two emergent epochs in the asymptotic limits.

Finally, from thermodynamic point of view, the present cosmic model may be considered as a non-equilibrium cyclic thermodynamic process and has some analogy with bheat engine. Interestingly, the initial and final emergent phases are thermodynamically identical and the cyclic process will continue for ever with work output as a thermodynamic heat engine. Also this work output supplies the chemical potential for particle creation and hence this model of the Universe may also be considered as one of the underlying mechanism behind the cosmic particle creation process, besides the scalar field interpretation.

CHAPTER 6

BRIEF SUMMARY AND FUTURE PROSPECT:

We shall now briefly describe the summary of the work presented in the thesis ans also some related future works.

Chapter two is basically the description of a model of diffusive universe consists of diffusive barotropic fluids. The thermodynamic analysis of the cosmic fluids are presented and as an outcome, the restrictions are obtained for thermodynamic stability for both constant and variable barotropic index fluid. Most importantly, the acceptibility of dark energy hypothesis has been discussed from thermodynamic point of view.

Chapter three is a work of investigation whether the emergent scenario is possible in a single fluid diffusive Universe. Incorporating the diffusion mechanism in Einstein's field equation by introducing a scalar field ϕ , the evolution equation of the Universe has been set up. Then by a proper phenomenological choice of ϕ, a well behaved emergent model of Universe has been presented and the thermodynamic behaviour of such model is analysed.

Chapter four deals with the successful exhibition of complete and continuous cosmic evolution from emergent era to present late time acceleration phase. This is an ever expanding model of the Universe. The cosmic phase transition from inflation to decelerated expansion and from decelerated expansion to present late time acceleration are found as the first order thermodynamic phase transitions of the cosmic fluid. Different aspects regarding this model have been presented graphically .

The unorthodox cosmological model namely cyclic Universe has been shown in chapter five. Any system undergoing cyclic thermodynamic process corresponds to the heat engine. Here considering the phenomenological choice of the scalar field, a cyclic and Gaussian evolution pattern of the Universe has been presented. It is an introduction of the evolution of a cosmic heat engine.

For future work, It may be possible to study different other models of Universe in the context of diffusive fluid system. Also there is no consistent diffusion theory in GTR. Therefore an attempt may be taken to formulate the field theoretic descrpition of diffusion mechanism. Basically these works presented in this thesis deal with the thermodynamic behaviour of the Universe under diffusion. But still a question may be thought whether the curvature of space-time may be changed continuously with the diffusive evolution of the Universe i.e. the interrelation between the causal structure of the Universe and the transport phenomena parameter.

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