On some issues of supply chain coordination under various uncertainties

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On some issues of supply chain coordination under various uncertainties

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CERTIFICATE FROM THE SUPERVISOR

This is to certify that the thesis entitled **"On some issues of supply chain coordination under various uncertainties"** submitted by Sri. **Joyanta Kumar Majhi** who got his name registered on 8th February, 2018 (INDEX NO : 32/18/ Maths./25) for the award of Ph. D. (Science) degree of Jadavpur University, is absolutely based upon his own work under the supervision of **Prof. Bibhas Chandra Giri**, Department of Mathematics, Jadavpur University, Kolkata-700032 and that neither this thesis nor any part of it has been submitted for either any degree/diploma or any other academic award anywhere before.

11.05.2022

Prof. Bibhas Chandra Giri

(Signature of the Supervisor, date with official seal)

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Dedicated to

My beloved Family

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Chapter 1

Introduction

"Unity is strength....when there is teamwork and collaboration, wonderful things can be achieved." – **Mattie Stepanek**

Globalization, fast changing customer needs, high-speed telecommunications, regulatory requirements, product life cycle shortening, product diversification, and technology innovation are only a few of the causes behind today's businesses complexity. To survive in this complex business environment, organizations have been driven to rely on a specific transaction exclusively, *i.e.,* to decide among being on the supplying or manufacturing or retailing side. Large organisations are in naturally more difficult situation due to the increased number of stakeholders involved in decision-making. Complex organisations' managers and employees must learn to work together and embrace change on a daily basis.

In such a complicated business environment, organisations can no longer compete as isolated entities but must concentrate on collaborative supply chains to enhance their overall performance. Organizations must align their own goals and business practises with the interests of the entire supply chain, which increases customer satisfaction and decreases total system expenditure. This leads to formation of supply chain coordination, with the goal of increasing supply chain efficiency and competitiveness in global marketplaces. It is thus a difficult job for researchers to devise optimal supply chain techniques to address these challenges and ensure that global channels run efficiently and effectively. This thesis aims to develop and analyse optimization-based models for various demanding yet unsolved problems in supply chain coordination, addressing several real-world business issues experienced by supply chain managers. The chapter begins with some background on supply chain, supply chain management, and supply chain coordination, followed by a discussion on supply chain coordination challenges.

1.1 Supply chain and supply chain management

A supply chain comprises of all entities engaged, directly or indirectly, in fulfilling a customer's request. A supply chain involves not just the manufacturer and supplier, but also transporters, warehouses, retailers, and even customers. Within each company, such as a manufacturer, the supply chain includes all operations involved in receiving and fulfilling a customer request. New product creation, marketing, operations, distribution, financing, and customer support are some of the responsibilities that fall under these operations. The context of supply chain starts with the source of supply and stops at the point of consumption.

From the perspective of a product, supply chain activities convert natural resources, raw materials, and components into a final product and delivered to its customer. For instant consider a customer who enters a Wal-Mart store to buy laundry detergent. The supply chain starts with the customer's requirement for detergent. The next stage in this supply chain is the Wal-Mart retail shop that the consumer visits. Wal-Mart stocks its shops with inventory which may have received from a final-goods warehouse or a distributor utilising vehicles provided by a third party. The distributor in turn is supplied by the manufacturer (say, Proctor & Gamble [P&G] in this case). The P&G production plant obtains their raw materials from a variety of suppliers, some of whom may have received their materials from lower-tier suppliers. For example, package materials may arrive from Tenneco Packaging; however, Tenneco acquires raw materials from other suppliers to produce the packages. These examples demonstrate the importance of the customer in the supply chain. In fact, the major goal of every supply chain is to meet consumers' needs while also profiting from the process. The phrase "supply chain" brings up thoughts of goods or services passing through a chain from suppliers to manufacturers to distributors to retailers to customers. Though this is undoubtedly a part of the supply chain, but it is equally significant to visualise information, cash, and product flows in both directions of this supply chain. Fig 1.1 depicts a supply chain, with arrows pointing in the directions of information, physical product, and fund flow.

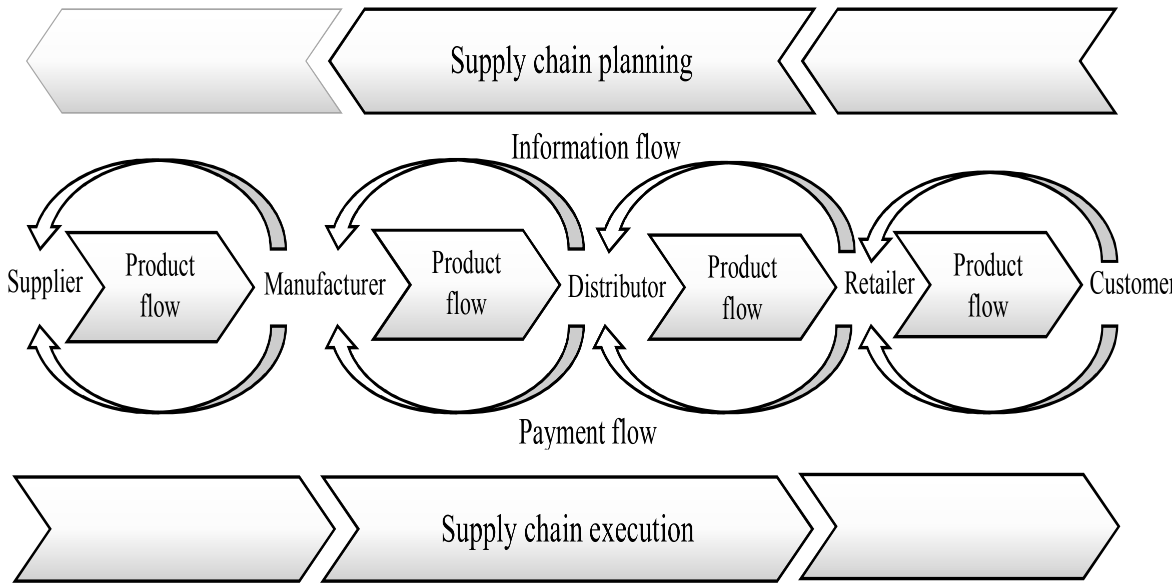

Fig. 1.1: Pictorial representation of supply chain.

The success of a supply chain is intimately connected to the management of the supply chain. Wal-Mart, Dell Computer, and Amazon are just a few examples of organisations that have achieved their success based on outstanding supply chain design, planning, and execution. A well-planned supply chain is essential for optimising asset utilisation, reducing excessive inventories, reducing waste, and meeting or exceeding customers expectations. A supply chain's performance is determined not only by how it tackles external issues, but also by how the chain is constructed to prevent inter-chain conflicts as much as possible. Supply chain management means maintaining control over all involved entities so that the correct quantity is served at the correct time at the correct place, and thus increasing profit or decreasing cost.

1.2 Supply chain strategies

Every supply chain is unique in its basic area of operation. When compared to one another, their modes of operation and strategies are also distinct. Despite the fact that each supply chain has its own set of market needs and operational constraints, the concerns are essentially the same in each situation. In any supply chain, companies must make decisions concerning their strategies both collectively and individually. They must distinguish between competitive strategy and supply chain strategy while deciding on their approaches. Again, supply chain strategy is determined by supply chain structure. Supply chain structures are divided into two categories: centralized and decentralized. Let's take a closer look at the two main structures.

1.2.1 Centralized structure

In this strategy, each member of the supply chain acts as a single decision maker, tries to maximize/minimise total profit/cost by selecting optimal decision variables. This approach is essentially holistic. This means that, in a organisation, the maximum profit level of any supply chain can only be achieved when all of the entities work together rather than thinking only about their individual profits. Of course, such a structure is difficult to implement in practice, but it serves as a benchmark for administrators and strategists who seek to implement different policies across channel members in order to achieve the same total profit as in the centralised one.

A centralized supply chain incorporates joint decision making in which individual entities align their objectives and decisions in the interest of the entire supply chain. It also requires the sharing of information and operational strategies across chain members. In specific, smooth and well controlled flow of information and materials are main ingredients for a centralized supply chain. Though it has been demonstrated that a centralized supply chain can achieve maximum channel profit, it is practically impossible to execute such a policy because it requires the hypothetical existence of a single decision maker who possesses all relevant information as well as the contractual authority to implement such decisions in order to maximize the total profit of the chain rather than the profit of individual entities.

1.2.2 Decentralized structure

On the contrary, a decentralized strategy is totally opposite to a centralised approach. In a decentralized strategy, decision rights are naturally divided across several decision makers or players, and each player seeks to maximize/minimize his or her individual profit/cost by making the possible optimal choices in their favors. Different supply chain members have different personal information and incentives, and they are frequently hesitant to reveal confidential information about cost and demand. Due to lack of collaboration, supply chain performance may be suboptimal. Because each decision maker optimizes a different objective function, a local optimum does not have to be globally optimal for the entire supply chain. In other sense, the decentralized system is inefficient since its total expected profit is less than that of the centralized supply chain.

In vast majority of cases, the upstream entity of a supply chain charges a fixed per unit wholesale price, based on which the downstream entity determines his own wholesale price, and so on, which is referred to as a *'wholesale price-only contract'*. This sub-optimization decreases the overall profit of the chain, and the dilemma is well known as *'double marginalization effect'* (Spengler, 1950). Another issue is the *'bullwhip effect'*, which arises when supply chain participants make decisions without consulting others, resulting in the spread of misleading information about demand, cost functions, *etc.,* (Lee et al., 1997). Individual interests, local perspectives, and opportunistic behaviors among supply chain members lead to supply and demand mismatches (Fisher et al., 1997).

Nash games are used to solve optimization problems in the decentralised model. Nash games have received a lot of attention in Game Theory, both in theory and in practice (Gupta and Weerawat, 2006). The Nash Equilibrium seems to be the solution to a game in which two or more players take decisions. Each player considers an opponent's decision, but has no motive, and nothing to gain, by changing his or her plan. In a Nash Equilibrium, every firm does the best it can, given the actions of its competitors. Nash equilibria are known for producing non-cooperative outcomes. In a specific game, there are decisional states from which no player wants to depart. Nash equilibria are such examples. So when decisions of other players are taken into account, each player's approach in the Nash Equilibrium is optimal. A Nash equilibrium point has the unique property of being attained immediately during the first iteration of the game if the internal models of the players are known. As a result, the existence of Nash equilibrium points reduces the negotiating process to a single information exchange.

1.3 Supply chain coordination

The best performance can be achieved if the companies work together by negotiating on a set of transfer payments, such that each company's goal is aligned with the supply chain's goal. Firms can generate such incentive by adjusting their conditions of trade through a contract that sets a transfer payment mechanism. A contract is said to coordinate the supply chain if the set of optimal actions is in a Nash equilibrium, i.e., no chain member has a beneficial unilateral deviation from the set of optimal activities of the supply chain. The phrases network, channel, and supply chain coordination are all used to describe the same thing. In general, the objective is to propose contracts that induce collaboration through suitable regulations for information and incentives such that supply chain performance is optimised. This strategy can be found in a variety of contexts. Cachon (2003) reviewed various investigations on supply chain contracts. A supply chain contract should typically include the three types of flows that occur between supply chain members, namely, information, financial, and physical flows. However, categorising supply chain contracts is not easy. To date, no commonly accepted classification appears to exist. Tsay et al. (1999) categorised the literature on supply chain contracts by eight contract clauses including specification of decision rights, minimum purchase commitments, pricing, quantity flexibility, allocation rules, buy-back or returns policies, lead time, and quality. According to Cachon (2003), there are various potential contracts that can be used to establish coordination in a newsvendor scenario, including revenue-sharing contract, buy-back contract, quantity flexibility contract, sales-rebate contract, and quantity discount contract *etc.* This contracts contain certain parameters and requirements that the contracting parties have agreed to follow. According to Lee (2000), contract is a vehicle for redesigning decision rights, work flow, and resources among chain participants in order to leverage decent improvement such as higher profit margin, increased customer service performance, and quicker response time.

1.3.1 Criteria of contracts

Certain contract mechanisms are developed among individual decision makers in order to encourage them to pursue channel coordination. The following criteria are extremely essential in evaluating a contract's strengths and limitations (Cachon, 2003):

◇ *Supply chain coordination*

The action in the decentralized supply chain matches all chain partners' decisions with the centralized supply chain. That is, each individual member's decisions should maximize entire supply chain's profit, and also no company should have a unilateral opportunity to deviate from optimal supply chain actions.

◇ *Arbitrary split of supply chain profit*

A supply chain contract should be flexible enough to accommodate any division of entire supply chain's profit. If contract parameters are changed to distribute profits arbitrarily, a contract that Pareto dominates a non-coordinating contract will always exist.

◇ *Administrative costs*

Last but not least, there will be a tradeoff between a supply chain contract's efficiency (in terms of profit) and its administrative expenditures. Administrative costs are often determined by the contract's specification of the kind and extent of material and information flows.

In terms of administrative costs, both the wholesale price contract and the quantity discount contract are equally expensive to operate because they only need a single transaction. Other contracts such as revenue-sharing, buy-back, or quantity flexibility are more expensive to administer since they necessitate additional material or informational transfers between the companies.

1.3.2 Necessity of supply chain coordination

To give a greater quality of service without incurring an unnecessary economic burden, all supply chain activities must be balanced. It is critical to understand that customer service comprises all points of contact between the supplier and the customer and it is the consequence of the combined impact of all activities performed by channel members along the supply chain. They are also interrelated; if one operation fails, the chain is disturbed, resulting in poor performance and disrupting workload in other areas, threatening the supply chain's efficiency.

To maximize the trade-off between entire cost and service excellence, it is vital to think in the context of the entire supply chain rather than particular functional areas. Unfortunately, most members' functional perspectives and aims are in conflict, making integration along the supply chain difficult. The development of supply chain coordination is essential to effectively resolve these disputes and transform the supply chain into a weapon for achieving competitive advantage by synchronizing supply chain activities in line with customer expectations.

1.3.3 Advantages of supply chain coordination

A well-coordinated supply chain increases total profit and enhances customer experience by reducing lead times, preventing stock outs, raising product availability, and increasing the supply network's response in delivering items or services to customers. Except this, other potential advantages, are given below:

◇ *Partnership formation*:

The fundamental advantage of coordinating a supply chain is the development of a partnership. What was previously a buyer-and-customer relationship has now evolved into a partnership. This improves the amount of trust. Suppliers and buyers can save money on transactions by forming long-term business agreements.

◇ *Felicitates forecasting*

Having the right amount of stock on hand is crucial to meet customer demands. Demand forecasting looks at how much of a particular product users expect to demand within a given week, month, or quarter. However, forecasting customer behaviour is much more than just anticipating desires and needs. The more detailed and valuable information assists businesses in planning and executing inventory management, shipping, and manufacturing schedules with the purpose of enhancing system efficiency can be achieved through coordination.

◇ *Inventory management and on-time delivery*

A well-coordinated supply chain enhances inventory management, resulting in fewer overstock and understock situations. Just-in-time delivery is also facilitated by a coordinated supply chain, which ensures that things are supplied as soon as they are ordered and produced.

◇ *Profit margins*

Improving system-wide performance is an essential goal of supply chain coordination. The system's expenses are reduced as a result of operational flexibility and efficient inventory control, which drives profit margins.

◇ *Risk sharing*

Another benefit that is pursued by entering into supply chain contracts is sharing the risk arising from the uncertainty in the supply chain. Given that the firms are supposed to be risk neutral, the concept of risk should be handled with caution; the model maximises expected profit rather than risk hedging.

1.4 Some important issues

Since the major goal of this thesis is to address several key issues in supply chain coordination, we have chosen a few of them to explore in the following sub-sections:

1.4.1 Newsboy framework

The newsvendor model is a mathematical model used to evaluate optimal inventory levels in operations management and applied economics. This model is also known as the "Newsvendor Problem" or "Newsboy Problem" because it is analogous to the dilemma faced by a newspaper vendor who must decide how many copies of the day's paper to stock in the face of unknown demand and knowing that leftover copies will be worthless at the end of the day. When the overage and underage expenses are proportionate to their sizes, determining the optimal quantity of product to make accessible is simple. The newsboy problem is a single-period inventory model in which a single order is placed for a product, and the product is either sold out or an excess of unsold products is sold for a salvage value at the end of the period. This occurs in goods that are seasonal or perishable, such as newspapers, seasonal apparel, and so on. Since we only order once over the period, the only inventory decision we make is how much to order. Of course, if the demand is deterministic and well-known, the solution is straightforward. As a result, demand is supposed to be a random variable with a particular probability distribution.

1.4.2 Demand uncertainty

Demand is the most crucial aspect in any supply chain. Of all the market dynamics, demand is the most unpredictably unpredictable. Management personnel have very little and restricted influence over it. Even a small fluctuation in the demand pattern for a specific item creates lots of new issues for the production unit in concern. Overall, this means that the demand for any item follows a discernible pattern throughout time. Supply chain's demand pattern can be divided into two types: deterministic and stochastic. In reality, most of the demand is essentially random as we can never be sure of the exact size of the market. Many unanticipated factors may enter in the picture and cause demand to rise or fall unexpectedly. Therefore, to retain a constant watch on the marketing behavior of various items, companies try to classify the demand sensitivity factors. Demand has been found to be sensitive to a number of factors including price, time, quality, stock, promotional effort, and so on. Stochastic demand pattern is one major area of concern of this doctoral thesis.Analyzing demand components, customer service level goals, sales and marketing strategies, inventory targets, customer order entry and promising, distribution resource planning, demand forecasting and aggregate levels, new product introduction, product commitments, and so on, we can only acquire an idea of demand pattern.

1.4.3 Supply uncertainty

Supply uncertainty problems in operations have spanned a vast body of research contributions, ranging from perishable goods to exposure obsolescence, from vaccine production (Chick et al., 2008) and blood banks to food and beverage, to semiconductor and wafer processing, market-based commodities processing, and production processes requiring chemical or thermal treatments. The manufacturing process in the industries is marked by significant yield unpredictability. For instance, semiconductor production process is regarded by the complexity and variability. The output of this process is very uncertain and typically less than the initial input, since rework and scrapping are common because of the cutting-edge technology involved in its process (Chao and Sivakumar, 2004). Pharmaceutical production typically encompasses a number of processes including dispensing, mixing, compressing, and coating, each having a number of factors (such as raw material quality and human mistakes*etc.,*) that can have a significant influence on product output. In cultivation, agricultural production are immensely dependent on environmental conditions like temperature, climate, nutrient levels, soil humidity, and availability of water.

Transportation is also subject to supply uncertainty. For example, a European pharmaceutical industry sources medications from Asia. The medicines are transported by sea freight and must keep within a particular temperature range throughout the journey. The medicines must be discarded if the temperature falls outside of the acceptable range during shipment. Temperature hazards aren't the only supply risk that businesses have to deal with. Other risks involve product handling, wrapping, air flow inside container, and many other. Also, manufacturing units that combine a large number of components into finished goods experience significant supply uncertainty because of the present popularity of outsourcing and supply chain extension. For example, in 2012, the manufacturing of a component (in-cell multi-touch display) for the iPhone 5 found to be extremely time-consuming. The production process of this component was stochastic. As a result, the iPhone 5's supply was reduced, and its sales were hampered. The challenge of supply uncertainty has been well explored in the framework of single side i.e., either in random yield production process (for a comprehensive review, see Yano and Lee (1995); Tang and Kouvelis (2014)), or transportation risk (Güler et al., 2013) or supply uncertainty in assembly system (Güler and Bilgiç, 2009). In this thesis, we concentrate on supply uncertainty in the production process, but the approach can be used to analyse random yield in any supply risk scenario.

1.4.4 Sustainable development

The need of maintaining ecological balance in the supply chain has become increasingly obvious as a consequence of sustainable development. Some of the ideal approaches that industry may adopt as significant initiatives are reusing, recycling, and remanufacturing. Corporate social responsibility (CSR) is a self-regulatory business concept that allows an organisation to be socially responsible to its stakeholders, the general public, and even itself. Social responsibility refers to the part of sustainability that are related to people, and it is concerned with ensuring that persons have what they require. A traditional supply chain has an impact on employees, workers, consumers, and local communities, which is why it is crucial to manage such influences proactively. For instance, utilising renewable sources of energy by companies can decrease draughts as they need less energy and water to maintain. The ultimate objective of social sustainability is to improve people's quality of life by providing properly spread health care and addressing ethical issues in the supply chain.

Furthermore, exerting CSR boosts a company's reputation and has a major impact on consumer goodwill (Komodromos and Melanthiou, 2014), that increase market demand (Hsueh, 2014). There are several instances of well-known companies using CSR to gain a competitive advantage and produce sustainable goods, such as technology giant Hewlett-Packard, which has made a significant effort to encourage the recycling of its used products such as laptops and printers. The firm recycled 395,200 tonnes of hardware in 2018. As a result, the goal of a socially responsible firm goes beyond financial value, and it includes fair trade, better employment practices, proper customer relations, ethical brand value recognition among the community, and so on, in order to ensure environmental protection and better health standards for living beings.

An another form of sustainable development is remanufacturing which restores a used product to as-new. Remanufacturing is prevalent in many industries and, for a variety of products including disposable cameras (Kodak, 2008), motor vehicle components (Bosch, 2016a; Ferrer and Whybark, 2001), aerospace equipment (Treat, 2012), medical devices (Hosseini-Motlagh et al., 2020), consumer products Apple (2016); Bosch (2016b), and retreaded tires (Debo et al., 2006) to name a few. Remanufacturing is an opportunity as it incurs relatively low cost of remanufacturing used products compared to producing new ones and its potential environmental benefits. Reusing products or components can also help reduce waste generation and extend the life cycle of products. Dell and Apple collect and recycle old computers when the customer buys a new product (Govindan and Popiuc, 2014). Remanufacturing of waste products decreases both the need for natural sources and generation of waste (Qiang, 2015). Hence the implementations of remanufacturing have economic, social and environmental benefits.

1.4.5 Customer return

A definitive retail return policy can help increase sales because an overwhelming percentage of the consumer population looks for it. Therefore, having a well thought out return policy clearly displayed in store is key to attracting and keeping customers. A return policy is a good business for those stores where the visitors don't get to see and hold the physical product before they buy it. And it is essential to do business online, so e-commerce sites must ensure that their return policies are fair and appealing to their customers. The basic message is, "if for some reason you don't like our product, return it for the full value of what you paid for it." On the outside, it is an unconditional agreement that guarantees the quality of the product. A concise and clear return policy gives consumers a feeling of security that what they are buying is guaranteed to be what it is represented.

Some consumers return products that perform unsatisfactorily while others return products that function satisfactorily, for other reasons such as not meeting expectations or tastes. Managing returns is never quite as simple as putting items back on a shelf to be shipped off to another customer. Returns involve a quality control process. The reason for returning an item should be established as soon as possible once it has been returned. The correct implementation of this process enables management not only to manage the reverse product flow efficiently, but to identify opportunities to reduce unwanted returns and to control the reason why the product was unsatisfactory, or did not meet customers' expectations by product quality improvement.

1.5 Outline of the thesis

The primary goal of this doctoral thesis is to investigate the EOQ (economic order quantity) in the context of the newsvendor problem and examine this in various supply chain models with a focus on supply chain coordination. This thesis is constructed by eight chapters wherein the current chapter is the introductory. Each chapter begins with a motivation for the model under consideration, followed by a description of earlier contributions discussed in the literature. Following that, we develop a mathematical model and examine distribution-free stochastic consumer demand influenced by several factors such as price, effort, CSR activities, quality, and greenness for both centralized and decentralized configurations. Each chain member's optimal decisions are evaluated and compared using centralised and decentralised strategies. Some strategies, statements, and mathematical expressions may be found to be repeated or similar in several chapters. This has been done for the purpose of the model's completeness as well as to retain the independence of each chapter. The research study presented in this thesis is divided into chapters as outlined below.

Chapter 1 is the introductory part which explains the purpose and scope of our work. It explains the fundamental concepts of supply chain networks as well as their various characteristics. Some major concerns in supply chain coordination are examined, including the newsboy framework, the nature of demand, the best coordination technique, CSR activities, and remanufacturing.

In Chapter 2, a brief literature survey is provided so that one can link the problems described in the following chapters.

Chapter 3 demonstrates the aspects of a socially responsible supply chain by providing a better quality of life promoting fairly distributed healthcare and supply chain's ethical issues. In this chapter, we study a two-tier SC comprising of a manufacturer and a retailer where only the manufacturer exhibits corporate social responsibility (CSR) to increase company's goodwill. The retailer sells a single kind of perishable or seasonal product to satisfy price and CSR dependent stochastic market demand in a single period. The production process of the manufacturer is subject to random yield. Though one widely used way to tackle random yield is to use a secondary resource, but there may be a situation where a manufacturer can't access a secondary resource to mitigate his yield risk. Depending on existence of a secondary market, two possible centralized models are provided: one by considering no secondary market, and other one by incorporating a secondary market. We find that, in particular, the secondary resource is detected to have a positive effect on supply chain performance, but we also find a situation where the presence of secondary resource might not be beneficial for a supply chain. We also analyze a centralized scenario where CSR activities are not exhibited in the supply chain as a benchmark model. It is shown that the SC's estimated benefit with CSR is persuaded to be greater than the SC without CSR in terms of profit. We investigate two scenarios in the decentralized supply chain model. One scenario assumes the risk of randomness in both demand and production and the cost of CSR investment is not shared among the chain members. The other scenario assumes that the chain members share both the risk of uncertainty and the cost of CSR investment. In each scenario, the optimal pricing and ordering strategy of the retailer, and CSR investment and production decisions of the manufacturer are analyzed. In the risk sharing decentralized scenario, we show that a simple revenue-sharing contract fails to coordinate such a supply chain. However, a composite contract combining revenue-sharing, and cost-sharing is shown to coordinate the supply chain and allow arbitrary allocation of total channel profit to ensure that both the retailer and the manufacturer are benefited. We further analyze the impact of randomness in production as well as the effect of CSR investment on the performance of the entire supply chain.

Chapter 4 deals with the negative impact of decentralization among the supply chain entities and minimizes double marginalization effect within the chain, especially when the end-customers' demand is not deterministic. This chapter investigates coordination issue in a three-level supply chain with one raw material supplier, one manufacturer, and one retailer. The retailer exerts effort to promote the product as well as his retail shop locally. The customer demand is assumed to be stochastic and dependent on both retail price and sales-effort. Both the supplier and the manufacturer face random yield in production, and the manufacturer cannot access a secondary market due to its brand image and specific configuration or feature. The integrated supply chain is first analyzed as the benchmark case for comparison. In the decentralized setting, aiming at how the risk of uncertainties in both yield and demand can be distributed among the supply chain members, we analyze the wholesale price contract as no risk sharing contract, and develop our risk sharing composite contract which distributes the risk of uncertainties among the parties to enhance the supply chain performance. In both the cases, we determine the optimal ordering, pricing, sales effort, and production decisions. Two different composite contract mechanisms are implemented to outperform the base case in terms of chain's total profit as well as individual profits. We find that a composite contract having two components a contingent buyback with target sales rebate and penalty between the retailer and the manufacturer, and a revenue-sharing contract between the manufacturer and the supplier achieves supply chain coordination and allows arbitrary allocation of total channel profit among all the chain members. The impact of randomness in both demand and production, and the impact of non-existence of emergency resource for the final product on the performance of the entire supply chain are analyzed. Moreover, models under linear, exponential and quadratic demand patterns are developed. A numerical example is provided to illustrate the developed model and draw some important managerial insights.

In Chapter 5, as an extension of our previous work, we consider supply disruption as another form of supply uncertainty besides random yield. We consider a single period three-echelon supply chain with three possible uncertainties, in which a retailer faces an uncertain market demand for a short shelf-life product and sources it from a manufacturer under voluntary regimes. The manufacturer sources the raw materials from two unreliable suppliers without any emergency resource. The manufacturer's main supplier who delivers the order quantity at a cheaper wholesale price is prone to disruption and, therefore, can deliver full order quantity if not disrupted, but it delivers nothing, if disrupted, while the backup supplier who provides similar quality product at a comparatively higher wholesale price is prone to random yield and, therefore, can only fulfill a random fraction of the manufacturer's order. The risks of supply uncertainty at both the suppliers are assumed to be independent. We analyze the integrated model as the centralized benchmark case and the decentralized model with wholesale price-only contract as decentralized benchmark case. Then aiming at how the risk of uncertainties in both supply and demand can be distributed among the supply chain entities, we introduce a spanning revenue sharing contract into the decentralized system. Under spanning revenue sharing mechanism, each supplier decreases its wholesale price, which induces the manufacturer to reduce its wholesale price too at the beginning of the selling season. The compensation for reduced wholesale price of both the supplier and the manufacturer is given in terms of revenue share by the retailer, after the selling season. We explore coordination conditions and elaborate the circumstance under which the contract is desirable to each of the individual members as well as the entire supply chain. From the numerical results, it shown that, if the enhancement in supply uncertainty is mainly due to increase in random yield in production, it is optimal to increase the use of the cheaper supplier. In contrast, if the enhancement in supply uncertainty comes mainly from supply disruption, it is beneficial to over-utilize the expensive supplier.

Chapter 6 examines how customer feedbacks should be used to improve product quality in the context of product returns due to mismatch of customers' needs and expectations. In this chapter, we consider a supply chain in which a manufacturer produces a product with yield uncertainty and sells it to a retailer who offers a full refund return policy to the customers in which the consumers can return the purchased products if the products do not fit their individual needs or tastes. In order to improve the product, the manufacturer invests in product quality improvement according to customers' likes and preferences. Since products are returned as a poor match to customer needs, rather than functioning problems, has a savage value. The customer demand is assumed to be stochastic and depends on quality improvement investment. We investigate both the centralized and decentralized models in order to shed light on how to spread the two uncertainties (demand and yield) as well as quality improvement cost across chain members in the presence of customer returns. We present two contract mechanisms to coordinate the supply chain in the decentralized model. For a manufacturer-led scenario, we combine a buy-back contract in which retailer credits only for unsold product, with a revenue-sharing contract where manufacturer shares the retailer's revenue for her reduced wholesale price. For a manufacturer-retailer-led scenario, we combine a differentiated buyback policy with two buy-back prices - one for unsold product and another for product returned by the customer with a revenue-sharing cost-sharing scheme where the manufacturer shares both the retailer's revenue and the cost of investment for product improvement. We find that the buy-back with revenue sharing contract is unable to coordinate the supply chain, whereas the differentiated buy-back policy with revenue-sharing cost-sharing scheme is able to do so. Apart from SC coordination, we also demonstrate how the manufacturer can motivate the retailer to collect and send customer feedbacks regarding their product expectations and tastes by applying extra rewards for returned products to reduce customer returns rate. We also investigate the effect of demand and supply uncertainties on the optimal decisions as well as how channel partners collaborate on product quality improvement investments. Our research also determines whether the retailer can convince the manufacturer to invest in product quality improvement.

Chapter 7 demonstrates the conflicts of creating two types of uncertainty (demand and supply uncertainties) in the forward flow and the return of two types of products (defective and waste products) in the reverse flow of a closed-loop supply chain (CLSC) for product recycling and improvement. We model a two-tiered close-loop supply chain which consists of two members - one manufacturer and one retailer. In forward logistics, the manufacturer produces a product under production yield and sells to the retailer. Consumers can purchase this item from the retailer with the manufacturer's free-repair warranty as a safeguard from premature failures. In reverse logistics, two types of products are returned from consumers. The first one is the return of premature failure products due to functioning issues. The manufacturer is obligated to accept such products and perform maintenance services to repair the product and returns to their owners during the warranty period. The other type is the returned waste products that have reached the end of their useful life (EOL). The retailer is responsible for collecting waste products from customers and returning them to the manufacturer for recycling. We suppose that the retailer performs CSR activities to raise customers' environmental awareness and encourage them to return their used products and the manufacturer invests in quality improvement in the form of improved product design, updated equipment, higher-quality raw materials, and improved quality control processes to reduce faulty product returns. To coordinate the system, we first propose a buy-back pay-back contract in which buy-back contract is offered by the manufacturer to the retailer and pay-back contract is offered by the retailer to the manufacturer. In this agreement, the manufacturer compensates for the retailer's leftover inventory at the end of the selling season and the retailer compensates for the manufacturer's excess output beyond his order. It is shown that the proposed contract cannot coordinate the supply chain. Then the contract is modified to a buy-back with pay-back-cost sharing contract in which the retailer promises not only to compensate for the manufacturer's excessive output above his order but also to share a portion of the quality improvement expenditure with the manufacturer. We demonstrate that this contract can accomplish coordination and allocate supply chain profit to the manufacturer and retailer in a number of different ways. The model is numerically demonstrated and a sensitivity analysis is performed to investigate the effects of demand and supply uncertainties, quality elasticity coefficient and CSR awareness coefficient on the optimal solution. It is also aimed to explore the impact of defect products return for fixing and quality improvement to reduce defects on optimal decisions of SC as well as how channel partners participate on product quality improvement investment.

In Chapter 8, an overview of the overall conclusion of the study done in this thesis is provided, as well as some future research areas are suggested.

1.6 Goal of the thesis

The goal of this doctoral study is to perform analytical and empirical research in the area of Supply Chain Management while providing mechanisms for organizations to effectively coordinate physical and information flows and enhance the compatibility between upstream and downstream stages of their supply chains. In this thesis, we investigate several models under centralized (integrated) and decentralized (Vertical Nash) scenarios, as well as decentralized scenario using contracts. Operational coordination mechanisms usually involve logistics synchronization and sharing of information.

Throughout the thesis, we explore various tactics for various market scenarios based on the above-mentioned aspects. We also analyze the cases numerically to illustrate the proper application of the models we built. We create several supply chain models and try to coordinate decentralized systems under some contract mechanism. Although the centralized policy is constructed as a benchmark scenario, it is not always feasible to execute the centralized policy in real-life situations. In this context, we look at a few multi-layer supply chain problems and try to enhance decentralized systems using certain contract mechanism. The sensitivity of the key parameters is depicted to assist decision makers in making effective marketing strategy decisions. The final goal of this thesis is to assist corporates to fulfil their sales target, to enhance their public image, to provide the right plans for market demand, to strengthen the relation and coordination among downstream and upstream members of their supply chains.

Chapter 2

Review of literature

This chapter presents a brief literature review to identify some key issues in supply chain coordination that are relevant to this thesis, and explores some limitations of previous research works.

2.1 Demand uncertainty

In recent years, the negative impact of demand fluctuation on the performance of supply chains has been studied under complex scenarios like growing modern technologies, intense global competition, market instability, and short life-cycle products such as vegetables, toys, stylish goods, etc. The situation becomes more complex when the demand is sensitive on few factors like price, promotional effort, time, quality, stock, CSR, etc. Consequently, demand fluctuation for such products gets more attention of a buyer when it makes procurement plans.

Since retail price is one of the main factors for customers to decide about buying a product, joint determination of inventory and pricing decisions has been extensively discussed in the literature. Emmos and Gilbert (1998) developed a supply chain model with price-dependent demand and found that although the return policy fails to coordinate the supply chain, it still performs better than wholesale price contract. Yao et al. (2008) analyzed the impact of price sensitivity factors on return policy under price-dependent stochastic demand, and concluded that the manufacturer has to surrender a part of his profit to the retailer when demand variability is high. Wang and

Chen (2017) framed the pricing and coordination strategies in a supply chain of fresh products with wholesale price and portfolio contracts where losses may arise during transportation. Hu et al. (2018) discussed coordination of order quantity and pricing decisions for a two-level supply chain through option contracts. Yadav et al. (2020) analyzed a single manufacturer multiple buyers model where demand is price-sensitive to each buyer. They suggested how to react to a certain change in some parameter to determine the right inventory policy. Giri and Glock (2021) examined the bullwhip effect in a manufacturing/remanufacturing supply chain in which they incorporated the price of the product in the demand. For this demand model, they employed the order-up-to inventory policy with a minimum mean square error forecasting scheme and measured the bullwhip effect at each echelon of the supply chain.

In the supply chain management literature, sales effort has been used to influence the market demand. For a supply chain where demand is influenced by the retailer's sales effort, Taylor (2002) developed a composite contract combining target sales rebate and buyback contracts to achieve coordination. Xiao et al. (2005) analyzed a model with two competing retailers who can choose to invest in sales effort to influence the demand. They examined how the price subsidy rate contract can achieve supply chain coordination. He et al. (2006) exhibited that the coordination and win-win outcome may be achieved by an augmented revenue-sharing contract based on sales rebate and penalty under effort-dependent stochastic demand. He et al. (2009) further investigated a two-echelon supply chain facing stochastic demand which is sensitive to both retail price and sales effort, and showed that although the buyback contract can't achieve channel coordination, a properly designed composite contract is able to achieve this. Zha et al. (2015) studied the coordination of a supply chain with an effort-induced demand function under Stackelberg game strategy, and showed that a cost sharing contract can coordinate the supply chain. Yan and He (2020) examined effectiveness of cooperative advertising policy in a SC structure in apparel industries in which the retailer provides a price discount offer in the second period. An inventory model for perishable products is formulated by Shah et al. (2022) for price and stock-dependent demand rate along with greening efforts. They developed an algorithm to calculate the retailer's profit function with respect to cycle time, selling price, and greening effort.

2.2 Supply uncertainty

A large number of studies on supply chain has considered that supply quantity equals the order quantity. However, in real business activities, the supply uncertainty is inevitable due to influence of many factors. For example, a supplier may fulfil a random fraction of an order (usually referred to as random yield) or a supplier who is supposed to deliver the full order by in-house production, may suffer from disruption under which nothing will be delivered to the buyer.

In many industries with equal quantity input, the output amount of the production process usually varies due to influence of many factors. In most of the agricultural based industries and any high-tech manufacturing industry like LCD (liquid crystal display), semi-conductor, silicon chips and so on, the production is uncertain. In particular, almost all industries have somewhat the same phenomena with respect to production randomness. Dada et al. (2007) proposed an extension to the newsvendor model where a supplier is either reliable or suffering from yield randomness, having uncertainty in both the amount and per unit cost of product, and showed that a given supplier will be selected only if all less expensive suppliers are selected regardless of the given supplier's reliability level. Choi et al. (2017) offered a concise review of many uncertainty factors related to supply chain, and characterised the works according to an innovative optimization model with various uncertainty factors. Considering a single supplier facing random yield and multiple downstream retailers dealing with random demands, He et al. (2019) presented several analytical models under a game structure in order to investigate the supply risk sharing mechanism within the supply chain. Karim and Nakade (2019) developed a production-inventory model in which production is subject to random yield. They found that the incorporation of safety stock helps the system to mitigate the risk of production uncertainty. Voelkel et al. (2020) modeled a dynamic programming problem with stochastic demand, tracking cost, and random yield, and provided an adjusted value iteration algorithm that finds the optimal solution.

To excel in intense global competition, today's supply chain is becoming more globalised to enjoy cheaper raw material, lower labour cost, tax policy, advance manufacturing technologies and other financial benefits, all of which reduce the per unit production cost of a product. Such globalized supply chain networks frequently experience supply disruption.Among the early researchers in disruption management, Meyer et al. (1979) considered a single production process which is subject to random disruption. A large body of literature concentrates on discovering the optimal number of suppliers to enhance the channel ability in disruption management. Berger et al. (2004) and Berger and Zeng (2006) investigated the issue of supplier selection where a buyer has to choose the optimal number of suppliers that are identical in terms of supply disruption. Ruiz-Torres and Mahmoodi (2007) represented a decision tree which helps to find the optimal number of suppliers, and concluded that the optimal number of utilized suppliers is typically small as long as the suppliers are very unreliable and the cost of failure is very high. Giri et al. (2021) considered a three-level supply chain with price and effort-dependent random demand in which all productions are subject to yield.

2.3 Secondary resource

A lot of studies has emphasised on the issue of dual sourcing under supply uncertainty in the presence of secondary resource. In fact, dual sourcing improves channel performance even when there is no supply uncertainty (Bulinskaya, 1964).

Parlar and Wang (1993) were the first to demonstrate the benefits of emergency sourcing in presence of supply uncertainty for both the EOQ model and the newsvendor model. Chopra et al. (2007) developed a single-period model with dual sourcing to integrate two types of supply uncertainty - supply disruption and random yield. Arcelus et al. (2008) developed a newsvendor model where the manufacturer shares the risk of demand uncertainty with the retailer by offering buyback contract, and mitigated his own risk by the availability of the secondary resource. He and Zhang (2008) studied the effect of random yield in a two-echelon decentralized supply chain under risk sharing contract which was further extended by He and Zhang (2010) by considering the effect of secondary market on the supply chain. Giri and Bardhan (2014) addressed the problem of determining optimal order and reserve quantities at a primary supplier and a secondary supplier, respectively, where the primary supplier is prone to disruption.

The work was further extended by Giri and Bardhan (2015) to incorporate random yield in production of the primary supplier.

Secondary resource as a tool to mitigate supply disruption has been designed by several researchers. Tomlin (2009) studied optimal mitigation strategies for a short life-cycle product where supply base for the buyer is subject to random disruption. He showed that disruption mitigation is not possible only through inventory control; supplier's diversification is an effective mitigation strategy in that case too. In contrast to Tomlin (2009) model, Chopra et al. (2007) considered two suppliers − one is unreliable due to both random yield and disruption uncertainty, and the other one is perfectly reliable. Their mitigation strategy was to reserve a quantity at the reliable supplier and exercise up to that reserved amount if the first supplier can't fulfil the demand due to random yield or supply disruption. Giri and Bardhan (2015) discussed a supply chain model with a retailer and a manufacturer under both random yield in production and disruption risk. The retailer has the option for capacity reservation with a backup supplier. The authors considered a penalty contract and characterized the retailer's joint ordering and reserving decisions and the manufacturer's pricing decision. Majhi et al. (2021) considered a supply chain with perishable or seasonal product where the production process of the manufacturer is subject to random yield. Depending on existence of a secondary market they provided two possible models: one by considering no secondary market, and other one by incorporating a secondary market.

2.4 Quality management

In order to satisfy customers and meet company's goals, quality improvement is a critical component in SCs (Karipidis, 2011; Rong et al., 2011; Franca et al., 2010; Lin et al., 2005; Bernstein and Federgruen, 2007; Xie et al., 2011). Singer et al. (2003) described the strategic behavior about quality inside a supplier-retailer relationship in a disposable product business. Bhaskaran and Krishnan (2009) reviewed various contracts to collaborate in the improvement and introduce a new product within a supply chain. To attract market share, Li et al. (2013) used a return policy and a quality improvement effort as two incentive factors. Under a deterministic demand, they looked at the relationship between quality and return price, as well as their effects on manufacturer profit. Yoo (2014) was one of the first to investigate the relationship between product quality and return policy in a SC with a supplier and a distributor. Under a deterministic demand, they employed Nash equilibrium to find the optimal quality improvement effort, return rate, and price. Yan (2015) assumed a combined pricing and product quality choice problem in a manufacturer-retailer supply chain and analyzed the performance of three distinct contract strategies for this decentralized network. He et al. (2016) designed a single-manufacturer-single-supplier supply chain strategy with reference impacts in supplier quality management problem. Chakraborty et al. (2019) studied a supply chain wherein the retailer has the ability to share in the manufacturer's quality improvement investment. They also looked into the diverse responsibilities that different parties play in quality improvement.

2.5 Sustainable develoment

To sustain in competitive business environment, companies are adopting various modern technologies in their day to day business activities. These activities sometime may affect the environmental and social life of its stakeholders. Therefore, the companies should operate their businesses in a more socially responsible way to buildup a corporate goodwill of their stakeholders.

Many researchers have shown the advantages of CSR activities into business. Carter and Jennings (2002) pointed out the direct and positive impact of CSR activities onto the performance of the supply chain. Cramer (2008) traced out a blueprint for the guidance of managers in selecting their own suitable ways to implement CSR activities into their companies. In the scenario of implementing CSR initiatives, Boyd et al. (2007) claimed that unnecessary monitoring can be inefficient and damage buyer-supplier partnerships and will not actually enhance compliance; rather, visibility, trust and commitment contribute to enhance supply chain performances. Freeman (2010), CSR may help businesses and their stakeholders to improve their overall financial performance. Pino et al. (2016) demonstrated that CSR has a significant influence on consumers' choice of products. Khosroshahi et al. (2019) investigated the effect of manufacturer transparency and CSR on sustainable decision making and profit accumulation of supply chain members, taking into account the impact of transparency and CSR on demand function. Li (2020) investigated the impact of CSR internal cost subsidies on supply chain participants' optimal decision-making and profits.

Remanufacturing waste products by extracting the useful ingredients from low-quality products and blocking the residues of dangerous materials from entering the environment not only helps firms achieve environmental goals, but it can also reduce production costs (Guide Jr and Van Wassenhove, 2006; Wu, 2012; Ferrer and Swaminathan, 2010). Taleizadeh et al. (2017) looked at the consequences of remanufacturing in a CLSC where a third party was in charge of collecting defective goods. They assumed that new and remanufactured products were of different quality, and they examined the influence of quality level on remanufacturing performance using five game theories. Zerang et al. (2018) used the same model, but assumed that remanufactured products were of the same quality as new ones, and that the market demand was predictable and dependent on marketing efforts. To discover the best profit, they used three distinct leaders in the Stackelberg game.

2.6 Customer return policy

In the retail industry, customer return is becoming an increasingly crucial problem (McWilliams, 2012; Chen and Chen, 2017). Toktay et al. (2004) declared that customer return rates for most retailers varied from 5% to 9% of their sales. Mostard and Teunter (2006) predicted the return rate for certain trendy items can be as high as 74%. Many retail stores offer a full-refund returns scheme to their customers to obtain a decent competitive advantage (Vlachos and Dekker, 2003) and customer loyalty (Shulman et al., 2011). However, customer returns have a serious influence on a retailer's decision making and profit, not only through lost profits but also through the expenses of handling returned product. In these articles, consumers are assumed to return a certain fraction of purchased products (Vlachos and Dekker, 2003; Chen and Bell, 2009; Wang, Chen and Chen, 2019). Vlachos and Dekker (2003) focused on a retailer's optimum ordering policy while examining various techniques to handling returned items. Chen and Bell (2009) looked at a retailer's simultaneous pricing and ordering decisions for price-dependent stochastic demand in the event of customer returns. Wang et al. (2020) investigated ordering decisions of a retailer with both wholesale price contract and put option contract for single period problems under the circumstance of customer returns. They demonstrated that the put option contract can help to coordinate the supply chain by minimizing the negative impact of customer returns.

2.7 Supply chain coordination

Due to marginalization phenomenon, supply chain members realize that collaboration is crucial when they seek to maximize their own profits individually. Various types of contract agreement for supply chain coordination have been discussed in the literature. Extensive reviews of supply chain contracts and coordination literature can be found in Lariviere (1999), Tsay et al. (1999) and Cachon (2003). Cachon and Lariviere (2005) showed that the revenue sharing contract is very effective for a wide range of supply chain coordination to align the individual member's objective with the system objective, despite certain limitations. They also compared revenue sharing contract with other popular contracts and found that revenue sharing is equivalent to buyback in newsvendor case, and equivalent to price discount in the price-setting newsvendor case. Jaber and Goyal (2008) investigated the coordination of order quantities amongst the members in a three-level supply chain assuming multiple buyers at the first level, a single vendor at the second level, and multiple suppliers at the third level. Ding and Chen (2008) imposed return policies between each pair of adjacent members in a three-echelon supply chain, and established that the multi-echelon supply chain can be fully coordinated with the above contract with arbitrary profit allocation among the members.

He and Zhang (2008) calculated the impact of random production yield in a two-echelon supply chain under different risk sharing agreements, which was later extended by He and Zhang (2010) by considering the effects of the secondary market on supply chain decisions. Hsueh (2014) first considered socially responsible supply chain under stochastic demand and found that a new revenue sharing contract can coordinate the chain for an exogenous retail price. Zhao and Yin (2018) extended the work of Hsueh (2014) by considering endogenous retail price. They were able to coordinate the supply chain using a modified revenue sharing contract under CSR investment and retail price dependent stochastic customer demand, having a linear CSR investment and retail price dependent deterministic demand component. Although Zhao and Yin (2018) dealt with a random demand, but they assumed deterministic yield in the production process of the manufacturer.

Customer returns provide another dimension to the relationship between the manufacturer and the retailer, emphasizing the importance of collaboration. Li et al. (2012) explored the optimal ordering strategy of a retailer, the optimal wholesale pricing strategy of a manufacturer, and coordination of the supply chain consisting customer return with a buy-back agreement. Xu et al. (2015) analyzed supply chain coordination with a buy-back contract, presuming the return deadline as a decision variable. Heydari, Rastegar and Glock (2017) explored the coordination and Pareto improvement of a supply chain in which buy-back and money-back assurance are considered. They demonstrated that a money-back assurance can cover a larger number of customers. Guo et al. (2017) reviewed supply chain contracts with customer returns.

As the primary focus of closed-loop supply chain is the incorporation of forward supply chain and reverse supply chain for the benefit of the manufacturing plants and the environmental issues, it is hard to establish a coordination mechanism in a CLSC. Zhang et al. (2014) looked at the contract design issue for a CLSC which collects returned products. De Giovanni et al. (2016) used an incentive strategy throughout a supply chain to coordinate a dynamic CLSC and increase consumers' willingness to return used products. Hu et al. (2016) developed five contracts to coordinate an RSC in the context of strategic consumer recycling behaviour. A two-stage pricing contract, wholesale price contract, subsidy contract, cost-pooling contract, and indemnity contract are among these contracts. Panda et al. (2017) also looked into the effects of CSR in a CLSC. They showed that channel coordination is a useful policy for sharing CSR costs among members and improving CSR performance. Heydari and Ghasemi (2018) proposed a revenue sharing contract to coordinate a RSC.

Chapter 3

Coordinating a socially responsible supply chain with random yield under CSR and price-dependent stochastic demand*

3.1 Introduction

One of the most essential concerns of today's supply chain management is to prevent the double marginalization phenomenon (Spengler, 1950) because all the players want to take advantage of both competitive and cooperative relationships. Therefore, they individually seek to optimize their profits that usually lead to a situation where the players have different and sometimes conflicting objectives. For this reason, a supply chain needs collaboration of the members to remove the conflicting objectives among themselves. One of the interesting collaboration instruments to remove the conflictive objectives is a contract mechanism among the channel members. A contract mechanism is a method which removes conflicts among the entities by determining a precise set of actions such that each firm's objective becomes aligned with that of the whole system.

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To create that incentive to share risk and/or reward, the parties can adjust their terms of trade through introducing trade parameters between them, including the rule of transfer payment scheme of money and products. Contracts are effective instruments to get rid of information asymmetry and multiple marginalizations by providing accurate information and incentives to all entities so that the decentralized chain behaves as closer as possible, if not exactly same as the centralized chain. A contract with this efficiency has been called a "perfect coordination contract" (Bernstein and Federgruen, 2005). A great amount of literature has discussed contract-based coordination with the help of popular contracts such as quantity discounts (Jeuland and Shugan, 1983; Mandal and Giri, 2019); quantity flexibility (Tsay, 1999; Xiong et al., 2011); buy-back policy (Pasternack, 1985; Ding and Chen, 2008); and so on. For detail survey on contract mechanism, we refer readers to Cachon (2003) and Tsay et al. (1999). The contracts are structured in such a way that all the members are incentivized to operate as centralized supply chain while maximizing their own expected profits individually.

To sustain in a competitive business environment, companies are adopting various modern technologies in their day to day business activities. These activities sometime may affect the environmental and social life of its stakeholders (Zamanian et al., 2020). Therefore, the companies should operate their businesses in a more socially responsible way to build up corporate goodwill of their stakeholders. Also, an ethical and environmentally conscious customer is willing to buy a product of a CSR company at a higher price. Therefore, CSR activities are becoming popular to both managers and the researchers. However, for companies, it is not easy to exhibit CSR activities into their business strategies. Pre-declared CSR activities may not catch all opportunities to benefit companies. On the other hand, postponed CSR approaches may lead to higher CSR costs when it is found that they have already violated social obligation. Exhibiting CSR activities on the upstream members influences sales and profit of the downstream members. For example, in 1996, sales and image of NIKE dropped down once it was found that a few of its subcontractors had been employed child labor (Gimenez and Tachizawa, 2012). However, the CSR activity is not a problem to a particular supply chain member. Only the morality of the chain members is not enough to exhibit CSR. It is, therefore, required to create an interest among the members to invest in CSR which ensures that each of them will be benefited from these activities.

In many industrial scenarios, the production yield is uncertain due to influence of many uncontrollable factors. Random production can be found in most agriculture-based industries such as egg, vegetable, cereal, *etc.*, where parameters like weather, draught, fertility of the land affect the production yield, and the exact yield quantity can never be anticipated in advance. In any high-tech manufacturing industry like LCD (liquid crystal display), semi-conductor, silicon chips, and so on, the quality of the product is uncertain due to small timing error or the presence of a small amount of dust contained in air of the manufacturing area. In particular, almost all industries have somewhat the same phenomena concerning production randomness.

The randomness may cause under-production or over-production. To deal with such a situation, producers often use a secondary market as an emergency resource to satisfy the unmet demand, and also for salvaging the leftover products. Chopra et al. (2007) reported an incident where a fire took place at the Philips microchip plant in Albuquerque, NM in March 2000 which supplied chips to both Nokia and Ericsson, among whom only Nokia got rid of the shortages in supply with the help of its multi-tiered supplier strategy to obtain chips from other sources. However, the availability of emergence resource in every stage of supply chain is a simplified assumption, particularly when it comes to mitigating demand of the final product of a branded company with specific configuration and features. But also there may be a situation where a manufacturer can't access a secondary resource to mitigate his yield risk. Then the manufacturer's decisions are affected by yield uncertain together with a secondary resource exist or not. However, it is not so clear how this yield randomness and secondary resources impact the decisions of the chain members.

Based on the practices mentioned above, in this chapter, we study a two-tier supply chain comprising of a retailer and a manufacturer who exhibits social responsibility to increase company's goodwill. The retailer sells a single kind of perishable or seasonal product to satisfy stochastic market demand in a single period. The production process of the manufacturer is subject to random yield and the retailer faces a stochastic market demand which is price and CSR sensitive. Because of random production yield, a secondary source is introduced. A composite contract which has two components-revenue-sharing and cost-sharing is proposed to coordinate the supply chain. The primary objective of this study is to find the answer of the following questions:

- How do demand and yield uncertainties affect the supply chain decisions?
- Is only the morality of the chain members enough to exhibit CSR?
- How does the presence of secondary resource effect the performance of the supply chain?
- How to remove the conflictive objectives of the chain members to coordinate the supply chain?

To answer the above questions, we investigate a socially responsible supply chain that consists of a retailer and a manufacturer facing production yield uncertainty. The retailer faces retail price and CSR-dependent stochastic demand. The centralized model of the supply chain is first analyzed as a benchmark model. We investigate two scenarios in the decentralized supply chain model. One scenario assumes the risk of randomness in both demand and production and the cost of CSR investment is not shared among the chain members. The other scenario assumes that the chain members share both the risk of uncertainty and the cost of CSR investment. In both scenarios, the optimal pricing and ordering strategy of the retailer, and CSR investment and wholesale price of the manufacturer are analyzed. The contribution of the chapter with respect to the relevant existing literature is three-fold:

- We incorporate both uncertain demand and random production yield in a socially responsible supply chain.
- We analyze the effects of the CSR activity and the secondary resources, which provide guidance for managers to take action under different market scenarios.
- We also investigate the proposed supply chain's coordination problem. We design a contract mechanism that improves CSR activities as well as the whole supply chain's expected profit.

The remaining chapter is structured as follows: Section 3.2 presents model description, notations and assumptions for developing the proposed model. Section 3.3 discusses the centralized supply chain. Section 3.4 describes the decentralized model under no risk and cost-sharing contract. Section 3.5 illustrates the decentralized model under risk and cost-sharing contract, and discusses two contracts- standard revenue-sharing contract in subsection 3.5.1 and revenue-sharing along with cost-sharing contract in subsection 3.5.2. Section 3.6 is devoted to numerical analysis for theoretical support and gaining more managerial insights. In Section 3.7, the chapter is concluded with some future research directions.

3.2 Problem description and notation

We study a supply chain which consists of a retailer and a manufacturer. The retailer trades a seasonal product in a single period to satisfy uncertain demand. The CSR activities like health and education development, investment for environmental protection, insurance for workers, *etc.* are undertaken by the manufacturer who faces random yield in production. In general, a higher CSR investment results in higher market demand. We consider a non-decreasing function $k\sqrt{\eta}$ as reward market demand where k is the customer's CSR sensitivity which is affected by a huge number of socio-cultural parameters and η is the CSR investment of the manufacturer. The stochastic market demand x, experienced by the retailer is a non-negative continuous random variable with general distribution. Over the region [l,u] $F(\cdot)$ and $f(\cdot)$ are the cumulative distribution function and probability density function, respectively with mean $\bar{x}$ and standard deviation σ_x . After forecasting the market demand x and knowing the manufacturer's contract, the retailer decides to place an order to the manufacturer for Q units of the final product. Because of random yield in production, the manufacturer sets a higher lot size Q_m. Suppose that the produced quantity is yQ_m where y is a random variable having cdf $G(\cdot)$ and pdf $g(\cdot)$ with mean $\bar{y}$ and standard deviation σ_y over the region [a,b], 0$\leq a \leq b \leq 1$. If the produced amount is less than the amount ordered, then there is no emergency resource to fulfill the order. But if the produced amount is more, the excess amount can be salvaged in a secondary market at a lower wholesale price. Therefore, the manufacturer must be very careful when he sets his production lot size Q_m. The manufacturer hands over the produced units to the retailer before the start of the selling season. Based upon the contract agreement, the transfer payment is made. We assume symmetric information *i.e.,* at the starts of the selling season, both the players have the full information. All the members associated in the supply chain are neutral and take a rational decisions. Also, reordering is not possible.

Notations

The notations used in this chapter are listed as given bellow:

x	:	stochastic customer demand with mean $\bar{x}$ and variance σ_x^2
y	:	random yield with mean $\bar{y}$ and variance σ_y^2
c_m	:	unit production cost of the manufacturer
c_r	:	unit handling cost of the retailer
g	:	unit goodwill lost of the retailer for unmet customer demand
v_r	:	unit salvage value of a residual product at the retailer
v_m	:	unit salvage value of leftover at the manufacturer
η	:	CSR expenditure of the manufacturer
p	:	unit retail price of the final product at the retailer
Q	:	order quantity of the retailer
Q_m	:	aimed production lot size of the manufacturer
w_m	:	unit wholesale price of the manufacturer

We will introduce more symbols whenever needed. To avoid trivial cases, the following restrictions are made: $v_m < c_m < w_m$; $v_r < w_m + c_r < p$; $c_r + c_m/\bar{y} < p$. The first two restrictions prevent the manufacturer and the retailer respectively from infinite production and assure that each of them makes a positive profit. The last restriction corresponds that the system's unit selling price is higher than expected unit cost.

3.3 Centralized supply chain model

A centralized supply chain is one in which different members of the supply chain act as a single unit in order to optimize the performance of the supply chain together. Conceptually here only one decision-maker who possesses all the information relevant to make decisions as well as has the contractual power to implement such decisions to maximize the system profit, and the wholesale price charged by the manufacturer to the retailer could be viewed as a transfer of internal revenue. The whole supply chain's expected profit is given by

$$\begin{aligned}
\Pi_c(Q, Q_m, p, \eta) &= pE[\min\{X, Q, yQ_m\}] + v_r E[(\min\{Q, yQ_m\} - x)^+] \\
&\quad -gE[(x - \min\{Q, yQ_m\})^+] - c_r E[\min\{Q, yQ_m\}] \\
&\quad +v_m E[(yQ_m - Q)^+] - c_m Q_m - \eta \\
&= (p + g - v_r)E[\min\{X, Q, yQ_m\}] - (c_r - v_r + v_m)E[\min\{Q, yQ_m\}] \\
&\quad -(c_m - v_m\bar{y})Q_m - \eta - g\bar{x} \qquad (3.1)
\end{aligned}$$

We can rewrite the above profit function as follows:

$$\begin{aligned}
\Pi_c(Q, Q_m, p, \eta) &= (p + g - v_r)\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m - (\alpha - \beta p + k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx \\
&\quad + \int_{yQ_m - (\alpha - \beta p + k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \Big) g(y)dy \\
&\quad + \int_{\frac{Q}{Q_m}}^{b} \Big(\int_l^{Q - (\alpha - \beta p + k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx \\
&\quad + \int_{Q - (\alpha - \beta p + k\sqrt{\eta})}^{u} Q f(x)dx \Big) g(y)dy \Big\} - (c_r - v_r + v_m) \\
&\quad \times \Big\{ \int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Q g(y)dy \Big\} \\
&\quad -(c_m - v_m\bar{y})Q_m - \eta - g\bar{x} \qquad (3.2)
\end{aligned}$$

As observed by Petruzzi and Dada (1999), for a price setting newsvendor problem (PSNP) containing multiple decisions variables in its objective function, it is often difficult to show the joint concavity of the objective function in all of its decisions variables. In the literature (for instance, Wang et al., 2020), it is a common approach to use a repetitive method to show the objective function's concavity. In this chapter, the objective function of the centralized system consists of four decision variables and the exact methods cannot be applied to obtain the optimal solution. So we apply a repetitive method. Let us assume that a finite but not necessarily unique optimal decision set $(Q^c, Q^c_m, p^c, \eta^c)$ exists for the centralized model. The first order partial derivatives of $\Pi_c(Q, Q_m, p, \eta)$ with respect to each of the decision variables are as follows:

$$
\begin{aligned}
\frac{\partial \Pi_c(Q, Q_m, p, \eta)}{\partial Q} &= (p + g_r - v_r) \int_{\frac{Q}{Q_m}}^{b} \Big(\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} f(x)dx \Big) g(y)dy \\
&\quad -(c_r - v_r + v_m) \int_{\frac{Q}{Q_m}}^{b} g(y)dy \\
\frac{\partial \Pi_c(Q, Q_m, p, \eta)}{\partial Q_m} &= (p + g_r - v_r) \int_{a}^{\frac{Q}{Q_m}} \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} y f(x)dx g(y)dy \\
&\quad -(c_r - v_m + v_r) \int_{a}^{\frac{Q}{Q_m}} y g(y)dy - (c_m - v\bar{y}) \\
\frac{\partial \Pi_c(Q, Q_m, p, \eta)}{\partial \eta} &= \frac{k}{2\sqrt{\eta}} (p + g_r - v_r) \Big\{ \int_{a}^{\frac{Q}{Q_m}} \Big(\int_{l}^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \\
&\quad + \int_{\frac{Q}{Q_m}}^{b} \Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \Big\} - 1 \\
\frac{\partial \Pi_c(Q, Q_m, p, \eta)}{\partial p} &= \Big\{ \int_{a}^{\frac{Q}{Q_m}} \Big(\int_{l}^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx \\
&\quad + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \Big) g(y)dy \\
&\quad + \int_{\frac{Q}{Q_m}}^{b} \Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx \\
&\quad + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Q f(x)dx \Big) g(y)dy \Big\} \\
&\quad -\beta (p + g_r - v_r) \Big\{ \int_{a}^{\frac{Q}{Q_m}} \Big(\int_{l}^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \\
&\quad + \int_{\frac{Q}{Q_m}}^{b} \Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \Big\}
\end{aligned}
$$

For given p and η, the second order partial derivatives of $\Pi_c(Q, Q_m, p, \eta)$ with respect to its decision variables Q and Q_m are

$$
\begin{aligned}
\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q^2} &= -\{p + g_r - v_r\}\Big\{\bar{F}\Big(Q - (\alpha - \beta p + k\sqrt{\eta})\Big)\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big) \\
&\quad + f\Big(Q - (\alpha - \beta p + k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q}{Q_m}\Big)\Big\} \\
&\quad + (c_r - v_r + v_m)\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q_m^2} &= -\{p + g_r - v_r\}\Big\{\bar{F}\Big(Q - (\alpha - \beta p + k\sqrt{\eta})\Big)\Big(\frac{Q^2}{Q_m^3}\Big)g\Big(\frac{Q}{Q_m}\Big) \\
&\quad + \int_a^{\frac{Q}{Q_m}} y^2 f\Big(yQ_m - (\alpha - \beta p + k\sqrt{\eta})\Big)g(y)dy\Big\} \\
&\quad + (c_r - v_r + v_m)\Big(\frac{Q^2}{Q_m^3}\Big)g\Big(\frac{Q}{Q_m}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q_m \partial Q} &= \frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q \partial Q_m} = (p + g_r - v_r)\bar{F}\Big(Q - (\alpha - \beta p + k\sqrt{\eta})\Big) \\
&\quad \times\Big(\frac{Q}{Q_m^2}\Big)g\Big(\frac{Q}{Q_m}\Big) - (c_r - v_r + v_m)\Big(\frac{Q}{Q_m^2}\Big)g\Big(\frac{Q}{Q_m}\Big)
\end{aligned}
$$

Now, putting Q^c and Q_m^c in the above second order partial derivatives, we get

$$
\begin{aligned}
\left[\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q^2}\right]_{Q=Q^c} &= -\{p + g_r - v_r\}f\Big(Q^c - (\alpha - \beta p + k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q^c}{Q_m}\Big) \\
\left[\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q_m^2}\right]_{Q_m=Q_m^c} &= -\{p + g_r - v_r\}\int_a^{\frac{Q^c}{Q_m^c}} y^2 f\Big(yQ_m^c - (\alpha - \beta p + k\sqrt{\eta})\Big)g(y)dy \\
\left[\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q_m \partial Q}\right]_{\binom{Q=Q^c}{Q_m=Q_m^c}} &= \left[\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial Q \partial Q_m}\right]_{\binom{Q=Q^c}{Q_m=Q_m^c}} = 0
\end{aligned}
$$

Let H_i denotes the principal minor of order i (i = 1,2) of the associated Hessian matrix

H. To check the positiveness and negativeness of the principal minors we have,

$$H_1 = -\{p + g_r - v_r\} f\Big(Q^c - (\alpha - \beta p + k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q^c}{Q_m}\Big) < 0$$

$$\begin{aligned} H_2 &= \{p + g_r - v_r\}^2 f\Big(Q^c - (\alpha - \beta p + k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q^c}{Q^c_m}\Big) \\ &\quad \times \int_a^{\frac{Q^c}{Q^c_m}} y^2 f\Big(yQ^c_m - (\alpha - \beta p + k\sqrt{\eta})\Big) g(y)dy > 0 \end{aligned}$$

This leads to the following proposition.

Proposition 3.1 *For given retail price p and CSR investment η, the entire system's objective function $\Pi_c(Q, Q_m, p, \eta)$ is jointly concave in Q and Q_m, and the optimal order quantity Q^c and production decision Q^c_m satisfy the following equations:*

$$Q = (\alpha - \beta p + k\sqrt{\eta}) + F^{-1}\Big(\frac{c_r - v_r + v_m}{p + g - v_r}\Big) \tag{3.3}$$

$$\begin{aligned} (p + g - v_r) \int_a^{\frac{Q}{Q_m}} \int_{yQ_m - (\alpha - \beta p + k\sqrt{\eta})}^{u} & y f(x) dx g(y) dy \\ -(c_r - v_m + v_r) \int_a^{\frac{Q}{Q_m}} & y g(y) dy = (c_m - v_m \bar{y}) \end{aligned} \tag{3.4}$$

■

Again, for given R and Q, the second order partial derivatives of $\Pi_c(Q, Q_m, p, \eta)$ with respect to its decision variables p and η are

$$\begin{aligned} \frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial \eta^2} &= -\frac{k^2}{4\eta}(p + g_r - v_r) \times \Big\{ \int_a^{\frac{Q}{Q_m}} f\Big(yQ_m - (\alpha - \beta p + k\sqrt{\eta})\Big) g(y)dy \\ &\quad + \int_{\frac{Q}{Q_m}}^{b} f\Big(Q - (\alpha - \beta p + k\sqrt{\eta})\Big) g(y)dy \Big\} \end{aligned}$$

$$\frac{\partial^2 \Pi_c(Q, Q_m, p, \eta)}{\partial p^2} = -2\beta\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m - (\alpha - \beta p + k\sqrt{\eta})} f(x)dx \Big) g(y)dy$$

$$+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\Big\}-\beta^2(p+g_r-v_r)$$
$$\times\Big\{\int_{a}^{\frac{Q}{Q_m}}f\Big(yQ_m-(\alpha-\beta p+k\sqrt{\eta})\Big)g(y)dy$$
$$+\int_{\frac{Q}{Q_m}}^{b}f\Big(Q-(\alpha-\beta p+k\sqrt{\eta})\Big)g(y)dy\Big\}$$

$$\begin{aligned}\frac{\partial^2\Pi_c(Q,Q_m,p,\eta)}{\partial\eta\partial p} &= \frac{\partial^2\Pi_c(Q,Q_m,p,\eta)}{\partial p\partial\eta}=(p+g_r-v_r)\Big(\frac{\beta k}{2\sqrt{\eta}}\Big)\\ &\times\Big\{\int_{a}^{\frac{Q}{Q_m}}f\Big(yQ_m-(\alpha-\beta p+k\sqrt{\eta})\Big)g(y)dy\\ &+\int_{\frac{Q}{Q_m}}^{b}f\Big(Q-(\alpha-\beta p+k\sqrt{\eta})\Big)g(y)dy\Big\}+\frac{k}{2\sqrt{\eta}}\\ &\times\Big\{\int_{a}^{\frac{Q}{Q_m}}\Big(\int_{l}^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\\ &+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\Big\}\end{aligned}$$

If H_i denotes the principal minor of order i (i = 1,2) of the associated Hessian matrix H, then

$$\begin{aligned}H_1 &= -2\beta\Big\{\int_{a}^{\frac{Q}{Q_m}}\Big(\int_{l}^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\\ &+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\Big\}-\beta^2(p+g_r-v_r)\\ &\times\Big\{\int_{a}^{\frac{Q}{Q_m}}f\Big(yQ_m-(\alpha-\beta p+k\sqrt{\eta})\Big)g(y)dy\\ &+\int_{\frac{Q}{Q_m}}^{b}f\Big(Q-(\alpha-\beta p+k\sqrt{\eta})\Big)g(y)dy\Big\}<0\\ H_2 &= \frac{k^2}{4\eta}\Big\{\int_{a}^{\frac{Q}{Q_m}}\Big(\int_{l}^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\\ &+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\Big\}>0\end{aligned}$$

The above discussion leads to the following proposition.

Proposition 3.2 *For given order quantity Q and production decision Q_m, the entire system's objective function $\Pi_c(Q, Q_m, p, \eta)$ is jointly concave in p and η, and the optimal retail price p^c and CSR investment η^c satisfy the following equations:*

$$\begin{aligned}
&\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx \\
&+ \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \Big) g(y)dy \\
&+ \int_{\frac{Q}{Q_m}}^{b} \Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx \\
&+ \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Q f(x)dx \Big) g(y)dy \Big\} \\
&= \beta(p + g - v_r) \Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \\
&+ \int_{\frac{Q}{Q_m}}^{b} \Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \Big\}
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
\frac{K}{2\sqrt{\eta}}(p + g - v_r) \Big\{ &\int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \\
&+ \int_{\frac{Q}{Q_m}}^{b} \Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \Big\} = 1
\end{aligned} \tag{3.6}$$

■

From equation (3.3), we observe that the order quantity Q of the final product is affected by both exogenous and endogenous parameters related to the final product as well as the raw materials except the distribution of the demand, which is quite natural. After putting the optimal values of the decisions variables in equation (3.2) we get the channel profit $\Pi_c(Q^c, Q^c_m, p^c, \eta^c)$. Although, due to the complexity, we can't find the closed-form solution from $\Pi_c(Q^c, Q^c_m, p^c, \eta^c)$, but with the help of Proposition 3.1, we can show that CSR investment increases the order quantity which leads to a higher expected channel profit.

3.3.1 Centralized model without CSR

We now present the maximal expected profit and optimal decisions of the centralized decision model without CSR for comparison purposes. In this scenario, the expected profit function can be described as

$$\begin{aligned}\Pi_{c0}(Q,Q_m,p) \;=\; & (p+g-v_r)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx \\ & +\int_{yQ_m-(\alpha-\beta p)}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\ & +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx \\ & +\int_{Q-(\alpha-\beta p)}^{u}Qf(x)dx\Big)g(y)dy\Big\}-(c_r-v_r+v_m) \\ & \times\Big\{\int_a^{\frac{Q}{Q_m}}yQ_m g(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\} \\ & -(c_m-v_m\bar{y})Q_m-g\bar{x} \end{aligned} \tag{3.7}$$

Due to complexity of the profit function $\Pi_c(Q,Q_m,p)$ in (3.7), it is difficult to show directly that $\Pi_c(Q,Q_m,p)$ is jointly concave in Q, Q_m and p. So, like previous model, we apply a repetitive method to show concavity of $\Pi_c(Q,Q_m,p)$. We derive the following results to characterize the optimal decisions of the centralized model.

Proposition 3.3 *For given retail price p, the entire system's objective function $\Pi_{c0}(Q,Q_m,p)$ is jointly concave in Q and Q_m, and the optimal order quantity Q^{c0} and production decision Q_m^{c0} satisfy the following equations:*

$$Q \;=\; (\alpha-\beta p)+F^{-1}\Big(\frac{c_r-v_r+v_m}{p+g-v_r}\Big) \tag{3.8}$$

$$\begin{aligned}(p+g-v_r)\int_a^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha-\beta p)}^{u} & yf(x)dxg(y)dy \\ -(c_r-v_m+v_r)\int_a^{\frac{Q}{Q_m}}yg(y)dy \;=\; & (c_m-v_m\bar{y})\end{aligned} \tag{3.9}$$

Proof. Taking partial derivatives of $\Pi_c(Q, Q_m, p)$ with respect to Q and Q_m we get

$$\begin{aligned}
\frac{\partial \Pi_c(Q,Q_m,p)}{\partial Q} &= (p+g_r-v_r)\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{Q-(\alpha-\beta p)}^{u} f(x)dx\Big)g(y)dy \\
&\quad -(c_r-v_r+v_m)\int_{\frac{Q}{Q_m}}^{b} g(y)dy \\
\frac{\partial \Pi_c(Q,Q_m,p)}{\partial Q_m} &= (p+g_r-v_r)\int_{a}^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha-\beta p)}^{u} yf(x)dxg(y)dy \\
&\quad -(c_r-v_m+v_r)\int_{a}^{\frac{Q}{Q_m}} yg(y)dy-(c_m-v\bar{y})
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \Pi_c(Q,Q_m,p)}{\partial Q^2} &= -\{p+g_r-v_r\}\Big\{\bar{F}\Big(Q-(\alpha-\beta p)\Big)\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big) \\
&\quad +f\Big(Q-(\alpha-\beta p)\Big)\bar{G}\Big(\frac{Q}{Q_m}\Big)\Big\}+(c_r-v_r+v_m)\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big) \\
\frac{\partial^2 \Pi_c(Q,Q_m,p)}{\partial Q_m^2} &= -\{p+g_r-v_r\}\Big\{\bar{F}\Big(Q-(\alpha-\beta p)\Big)\Big(\frac{Q^2}{Q_m^3}\Big)g\Big(\frac{Q}{Q_m}\Big) \\
&\quad +\int_{a}^{\frac{Q}{Q_m}} y^2 f\Big(yQ_m-(\alpha-\beta p)\Big)g(y)dy\Big\} \\
&\quad +(c_r-v_r+v_m)\Big(\frac{Q^2}{Q_m^3}\Big)g\Big(\frac{Q}{Q_m}\Big)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \Pi_c(Q,Q_m,p)}{\partial Q_m \partial Q} &= \frac{\partial^2 \Pi_c(Q,Q_m,p,\eta)}{\partial Q \partial Q_m} = (p+g_r-v_r)\bar{F}\Big(Q-(\alpha-\beta p)\Big) \\
&\quad \times\Big(\frac{Q}{Q_m^2}\Big)g\Big(\frac{Q}{Q_m}\Big)-(c_r-v_r+v_m)\Big(\frac{Q}{Q_m^2}\Big)g\Big(\frac{Q}{Q_m}\Big)
\end{aligned}$$

Now, putting Q^c and Q_m^c in the above second order partial derivatives, we get

$$\left[\frac{\partial^2 \Pi_c(Q,Q_m,p)}{\partial Q^2}\right]_{Q=Q^c} = -\{p+g_r-v_r\}f\Big(Q^c-(\alpha-\beta p)\Big)\bar{G}\Big(\frac{Q^c}{Q_m}\Big)$$

$$\begin{aligned}
\left[\frac{\partial^2 \Pi_c(Q,Q_m,p)}{\partial Q_m^2}\right]_{Q_m=Q_m^c} &= -\{p+g_r-v_r\}\int_a^{\frac{Q^c}{Q_m^c}} y^2 f\Big(yQ_m^c-(\alpha-\beta p)\Big)g(y)dy \\
\left[\frac{\partial^2 \Pi_c(Q,Q_m,p)}{\partial Q_m \partial Q}\right]_{\binom{Q=Q^c}{Q_m=Q_m^c}} &= \left[\frac{\partial^2 \Pi_c(Q,Q_m,p)}{\partial Q \partial Q_m}\right]_{\binom{Q=Q^c}{Q_m=Q_m^c}} = 0
\end{aligned}$$

If H_i denotes the principal minor of order i (i = 1,2) of the associated Hessian matrix H, then

$$H = \begin{pmatrix} \frac{\partial^2\Pi_c}{\partial p^2} & \frac{\partial^2\Pi_c}{\partial p \partial e} \\ \frac{\partial^2\Pi_c}{\partial e \partial p} & \frac{\partial^2\Pi_c}{\partial e^2} \end{pmatrix}.$$

We deduce

$$\begin{aligned}
H_1 &= -\{p+g_r-v_r\}f\Big(Q^c-(\alpha-\beta p)\Big)\bar{G}\Big(\frac{Q^c}{Q_m}\Big) < 0 \\
H_2 &= \{p+g_r-v_r\}^2 f\Big(Q^c-(\alpha-\beta p)\Big)\bar{G}\Big(\frac{Q^c}{Q_m^c}\Big) \\
&\quad \times \int_a^{\frac{Q^c}{Q_m^c}} y^2 f\Big(yQ_m^c-(\alpha-\beta p)\Big)g(y)dy > 0
\end{aligned}$$

This shows that the Hessian matrix is negative definite. Then, from the first order optimality conditions $\frac{\partial \Pi_m(Q,Q_m,p)}{\partial Q} = 0$ and $\frac{\partial \Pi_m(Q,Q_m,p)}{\partial Q_m} = 0$, we obtain the optimal solution. ■

Proposition 3.4 *For given order quantity Q and production decision Q_m, the whole supply chain's profit function Π_{c0} in the centralized model without CSR is concave in in p, and the optimal retail price p^c satisfies the following equation:*

$$\begin{aligned}
&\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx+\int_{yQ_m-(\alpha-\beta p)}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx+\int_{Q-(\alpha-\beta p)}^{u}Qf(x)dx\Big)g(y)dy\Big\}
\end{aligned}$$

$$= \beta(p + g - v_r)\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p)} f(x)dx\Big) g(y)dy$$
$$+ \int_{\frac{Q}{Q_m}}^b \Big(\int_l^{Q-(\alpha-\beta p)} f(x)dx\Big) g(y)dy\Big\} \tag{3.10}$$

Proof. Taking partial derivatives of $\Pi_c(Q, Q_m, p)$ with respect to p, we get

$$\begin{aligned}\frac{\partial \Pi_c(Q, Q_m, p)}{\partial p} = & \Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p)} (\alpha - \beta p + x) f(x)dx \\ & + \int_{yQ_m-(\alpha-\beta p)}^u (yQ_m) f(x)dx\Big) g(y)dy \\ & + \int_{\frac{Q}{Q_m}}^b \Big(\int_l^{Q-(\alpha-\beta p)} (\alpha - \beta p + x) f(x)dx \\ & + \int_{Q-(\alpha-\beta p)}^u Q f(x)dx\Big) g(y)dy\Big\} \\ & -\beta(p + g_r - v_r)\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p)} f(x)dx\Big) g(y)dy \\ & + \int_{\frac{Q}{Q_m}}^b \Big(\int_l^{Q-(\alpha-\beta p)} f(x)dx\Big) g(y)dy\Big\}\end{aligned}$$

and
$$\begin{aligned}\frac{\partial^2 \Pi_c(Q, Q_m, p,)}{\partial p^2} = & -2\beta\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p)} f(x)dx\Big) g(y)dy \\ & + \int_{\frac{Q}{Q_m}}^b \Big(\int_l^{Q-(\alpha-\beta p)} f(x)dx\Big) g(y)dy\Big\} - \beta^2(p + g_r - v_r) \\ & \times\Big\{ \int_a^{\frac{Q}{Q_m}} f\Big(yQ_m - (\alpha - \beta p)\Big) g(y)dy \\ & + \int_{\frac{Q}{Q_m}}^b f\Big(Q - (\alpha - \beta p)\Big) g(y)dy\Big\} < 0.\end{aligned}$$

Therefore, $\Pi_c(Q, Q_m, p,)$ is concave with respect to p and from the first order optimality condition $\frac{\partial \Pi_{c0}(Q,Q_m,p)}{\partial p} = 0$, we obtain the optimal retail price p^{c0}. This proves the proposition. ■

The profit functions in equations (3.2) and (3.7) are too complicated to compare their properties. In Section 3.6, we use a numerical example to illustrate this comparison.

3.3.2 Centralized model with CSR in the presence of secondary resource

In this section, we examine the same model in the presence of secondary resource for the manufacturer to mitigate his random production yield. Let $c_m^{'}$ be the purchasing cost of the secondary resource. The expected profit of the entire supply chain in this case is described as

$$\begin{aligned}\Pi_{cs}(Q, Q_m, p, \eta) &= (p + g - c_r - v_m)Q - (p + g - v_r)\int_l^{Q-\left(\alpha+\sqrt{\eta}k-\beta p\right)} F(x)dx \\ &\quad -(c_m^{'} - v_m)Q_m \int_a^{\frac{Q}{Q_m}} g dy - Q_m\left(cm - v\bar{y}\right) - g_r\bar{X} - \eta \qquad (3.11)\end{aligned}$$

Similar to $\Pi_c(Q, Q_m, p, \eta)$, $\Pi_{cs}(Q, Q_m, p, \eta)$ is an objective function of a price setting newsvendor problem containing multiple decisions variables. In the similar manner, for $\Pi_c(Q, Q_m, p, \eta)$, we can obtain the optimal solutions of $\Pi_{cs}(Q, Q_m, p, \eta)$. The entire chain's optimum decisions satisfy the followings equations:

$$(p + g - c_r - v_m) - (p + g - v_r)F(Q) = (c_m^{'} - v_m)G(\frac{Q}{Q_m}) \qquad (3.12)$$

$$(c_m^{'} - v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy = (cm - v\bar{y}) \qquad (3.13)$$

$$Q - \int_l^{Q-\left(\alpha+\sqrt{\eta}k-\beta p\right)} F(x)dx = (p + g - v_r)\beta\int_l^{Q-\left(\alpha+\sqrt{\eta}k-\beta p\right)} f(x)dx \qquad (3.14)$$

$$(p + g - v_r)\frac{K}{2\sqrt{\eta}}\int_l^{Q-\left(\alpha+\sqrt{\eta}k-\beta p\right)} f(x)dx = 1 \qquad (3.15)$$

It is difficult to get explicit analytical solution of the model. Most of the literatures have shown that the secondary resource has a positive impact on the supply chain (Lee and Whang, 2002). It is noted that the accessibility of the secondary resource is helpful to the supply chain in order to obtain a higher profit while production is suffering from random yield. The presence of the secondary market provides the manufacturer with more ways to overcome the production uncertainty and increases the supply chain's performance effectively. Moreover, the double marginalization in the decentralized supply chain decreases in the presence of a secondary resource.

3.4 Decentralized model with price-only contract

Although the integrated model provides the most system potency, it is far from the real business situation. In reality, supply chain entities are freelance decision makers and that they select the most effective decisions to maximize their individual profits. We currently think about a decentralized system wherever there's a price-only contract among the supply chain entities. The method flow is as follows.

The manufacturer simultaneously decides its wholesale price w_m and CSR investment η first. Then, with the knowledge of demand uncertainty and wholesale price offered by the manufacturer, the retailer determines to buy Q units from the manufacturer and the manufacturer decides to produce Q_m units. The amount $min\{Q, yQ_m\}$ is shipped by the manufacturer to the retailer. Lastly, the market demand x occurs and the retailer trades the quantity $min\{X, Q, zQ_m\}$ to the end-customers. We consider a Nash sequence where the manufacturer is the first decision maker, and the system is solved through backward substitution. Therefore, the retailer first determines his optimal decisions. For given Q_m and η, the retailer's profit function $\Pi_r(Q, p)$ can be derived as follows:

$$\begin{aligned}
\Pi_r(Q,p) &= pE[\min\{X,Q,yQ_m\}] + v_rE[(\min\{Q,yQ_m\}-x)^+] \\
&\quad -gE[(x-\min\{Q,yQ_m\})^+] - (w+c_r)E[\min\{Q,yQ_m\}] \\
&= (p+g-v_r)E[\min\{X,Q,yQ_m\}] - (w+c_r-v_r)E[\min\{Q,yQ_m\}] \\
&\quad -g\bar{x} \qquad (3.16)
\end{aligned}$$

We can rewrite the above profit function as follows:

$$
\begin{aligned}
\Pi_r(Q,p) \quad = \quad & (p+g-v_r)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\
& +\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
& +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\
& +\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}-(w+c_r-v_r) \\
& \times\Big\{\int_a^{\frac{Q}{Q_m}}yQ_m g(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\} \\
& -g\bar{x} \qquad (3.17)
\end{aligned}
$$

Due to complexity, the concavity of $\Pi_r(Q,p)$ with respect to Q and p can not be proved analytically. The following proposition explores the retailer's optimal order quantity and retailer price in the wholesale price contract, assuming that Π_r is concave.

Proposition 3.5 *The retailer's optimal order quantity Q and retail price p in the decentralized model with wholesale price-only contract are given by the following equations:*

$$
Q \quad = \quad (\alpha-\beta p+k\sqrt{\eta})+F^{-1}\Big(\frac{w+c_r-v_r}{p+g-v_r}\Big) \qquad (3.18)
$$

and

$$
\begin{aligned}
& \Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\
& +\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
& +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\
& +\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}
\end{aligned}
$$

$$= \beta(p+g-v_r)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} \quad (3.19)$$

Proof. We have

$$\frac{\partial \Pi_r(Q,p)}{\partial Q} = (p+g_r-v_r)\int_{\frac{Q}{Q_m}}^b\Big(\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^u f(x)dx\Big)g(y)dy - (w+c_r-v_r)\int_{\frac{Q}{Q_m}}^b g(y)dy$$

$$\begin{aligned}\frac{\partial \Pi_c(Q,p)}{\partial p} = & \Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\ & + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^u (yQ_m)f(x)dx\Big)g(y)dy \\ & + \int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\ & + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^u Qf(x)dx\Big)g(y)dy\Big\} \\ & -\beta(p+g_r-v_r)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy \\ & + \int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\}\end{aligned}$$

From the first order optimality condition $\frac{\partial \Pi_r(Q,p)}{\partial Q} = 0$ and $\frac{\partial \Pi_r(Q,p)}{\partial p} = 0$, we obtain the retailer's optimal order quantity and retail price. ■

Comparing the retailer's optimal order quantity with that of the centralized system, we find that the retailer orders less. The manufacturer's self-interest bears the double marginalization impact by pricing higher than the production cost, and this is the only way by which he can gain a positive expected profit. Underneath this contract, the manufacturer doesn't have any incentive to cut back his wholesale value because the retailer takes order decisions with no assurance to the manufacturer.

After investigating the retailer's problem and getting the optimum decisions $\left(Q^d, p^d\right)$, we tend to derive the manufacturer's expected profit function $\Pi_m(Q_m, \eta)$ as follows:

$$\begin{aligned}\Pi_m(Q_m, \eta) &= wE[\min\{Q, yQ_m\}] + v_m E[(yQ_m - Q)^+] - c_m Q_m - \eta \\ &= (w - v_m)E[\min\{Q, yQ_m\}] - (c_m - v_m\bar{y})Q_m - \eta\end{aligned}$$

which can be written as

$$\begin{aligned}\Pi_m(Q_m, \eta) &= (w - v_m)\Big\{\int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^b Qg(y)dy\Big\} \\ &\quad -(c_m - v_m\bar{y})Q_m - \eta \end{aligned} \tag{3.20}$$

Similar to the retailer's case, the concavity of the manufacturer's objective function $\Pi_m(Q_m, \eta)$ with respect to Q_m and η can not be proved analytically. The proposition below characterizes the manufacturer's optimal production amount and CSR investment in the decentralized system under wholesale price only contract.

Proposition 3.6 *The optimal production amount Q_m of the manufacturer satisfies the following equation*

$$(w - v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy = (c_m - v_m\bar{y}) \tag{3.21}$$

The CSR investment η in the manufacturer's objective function is in linear form with a negative sign and there is no other term related to η. Therefore, the optimal decision of the manufacturer is not to invest in CSR activities.

Proof. We have

$$\frac{\partial \Pi_m(Q_m, \eta)}{\partial Q_m} = (w - v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy - (c_m - v\bar{y}) \tag{3.22}$$

Solving the first order optimality condition, we can obtain the optimal production. ■

Under the price-only contract, the manufacturer does not have any influence on the retailer's order quantity. The retailer also takes his decision without any promise to the manufacturer. The manufacturer takes the risk of his production uncertainties alone. In this situation, we assume that the wholesale prices are negotiated based on the firm's bargaining powers, keeping the gross margin higher than a desired level of acceptance. Now, we compare the above benchmark models in the following theorem:

Theorem 3.1 *Both the order and production quantities in the decentralised model are strictly less than their counterparts in the centralized model. A lower-order amount results in a lower expected supply chain profit in the decentralised setting.* ■

The above Theorem is a generalization of the finding for the two-level supply chain that can be a step back model (Spengler, 1950), demonstrating that, in the decentralized environment, the total network channel output is not reached to its maximum profit even if all chain participants maximize their respective incomes. In the decentralized scenario where the decision power is distributed across the various chain participants, there is a possible deviation from the optimal decisions achieved under the centralized model. In order to align each member's decision with the entire channel, contract mechanisms come into play to prevent sub-optimization by removing members' rivalry without affecting the structure of the supply chain and the decision making powers of its members.

3.5 Coordination contract

The disadvantages of wholesale price contracts in terms of supply chain performance have already been observed in previous section. The purpose of this subsection is to discover a remedy that improve the performance of a supply chain. Coordination is a crucial assessment factor for measuring supply chain performance. Contract mechanism is a technique to attain coordination by modifying each player's expected profit exploitation in terms of trade parameters like valuation, order amount, quality among the players which also enhance the profit of the whole supply chain. A decentralized supply chain is claimed to be coordinated when it attains an equivalent potency as the centralized scenario in terms of profit.

3.5.1 Revenue-sharing contract

We first analyze the case where the manufacturer decides production quantity, CSR investment, and offers revenue-sharing contract to the retailer. In a revenue sharing contract, the retailer keeps a fixed fraction of his revenue for himself and shares the rest to the supplier, apart from paying the per unit wholesale price. Usually the manufacturer reduces the wholesale price in turn to encourage the retailer to order more. Depending on the manufacturer's decisions, the retailer decides its retail price and order quantity. At the end of the marketing season, the retailer keeps a fraction ϕ of his total revenue and returns $(1-\phi)$ proportion to the manufacturer so as to compensate it's reduced wholesale price. Under this contract, for given retail price and order quantity, the expected profit of the manufacturer can be described as

$$\begin{aligned}
\Pi_m^{ors}(Q_m,\eta) &= (1-\phi)pE[\min\{X,Q,yQ_m\}]+wE[\min\{Q,yQ_m\}]+v_mE[(yQ_m-Q)^+] \\
&\quad -c_mQ_m-\phi\eta \\
&= (1-\phi)pE[\min\{X,Q,yQ_m\}]-(w-v_m)E[\min\{Q,yQ_m\}] \\
&\quad -(c_m-v_m\bar{y})Q_m-\phi\eta
\end{aligned} \tag{3.23}$$

We can rewrite the above profit function as follows:

$$\begin{aligned}
\Pi_m^{ors}(Q_m,\eta) &= (1-\phi)p\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\
&\quad +\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
&\quad +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\
&\quad +\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}-(w-v_m) \\
&\quad \times\Big\{\int_a^{\frac{Q}{Q_m}}yQ_mg(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\} \\
&\quad -(c_m-v_m\bar{y})Q_m-\phi\eta
\end{aligned} \tag{3.24}$$

The optimal production lot size and CSR investment of the manufacturer satisfy the following equations:

$$\begin{aligned}(1-\phi)\times p\int_a^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy \\ -(w-v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy &= (c_m-v_m\bar{y}) \quad (3.25)\end{aligned}$$

$$\begin{aligned}\text{and}\quad \frac{K}{2\sqrt{\eta}}(1-\phi)\times p\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy \\ +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} &= 1 \quad (3.26)\end{aligned}$$

For channel coordination, we take $\eta^{ors}=\eta^c$, which gives $\phi p - v_r + g = 0$. The retailer's expected profit function under this contract is given by

$$\begin{aligned}\Pi_{rc}(Q,p) &= \phi pE[\min\{X,Q,yQ_m\}]+v_rE[(\min\{Q,yQ_m\}-x)^+] \\ &\quad -gE[(x-\min\{Q,yQ_m\})^+]-(w+c_r)E[\min\{Q,yQ_m\}] \\ &= (\phi p-v_r+g)E[\min\{X,Q,yQ_m\}]-(w+c_r-v_r) \\ &\quad \times E[\min\{Q,yQ_m\}]-g\bar{x} \quad (3.27)\end{aligned}$$

If $\eta^{ors}=\eta^c$ holds then the expected profit of the retailer becomes negative. Hence the retailer would not agree to sign up the revenue sharing contract. This leads to the following theorem.

Theorem 3.2 *The revenue-sharing contract fails to coordinate the supply chain.* ■

3.5.2 Revenue-sharing with cost-sharing contract

Now, we intercommunicate the case where the manufacturer offers a cost-sharing contract, in addition to revenue sharing policy with the retailer. During a cost-sharing contract, the retailer is actuated to share the CSR investment of the manufacturer.

Using this contract, the manufacturer influences the retailer to share the CSR investment more, which successively enhances the customer demand. Under this setting, the retailer's profit is given by

$$\begin{aligned}\Pi_{rc}(Q,p) &= \phi p E[\min\{X,Q,yQ_m\}] + v_r E[(\min\{Q,yQ_m\}-x)^+] \\ &\quad -gE[(x-\min\{Q,yQ_m\})^+] - (w+c_r)E[\min\{Q,yQ_m\}] - \phi\eta \\ &= (\phi p - v_r + g)E[\min\{X,Q,yQ_m\}] - (w+c_r-v_r) \\ &\quad \times E[\min\{Q,yQ_m\}] - g\bar{x} - \phi\eta \end{aligned} \tag{3.28}$$

An equal representation of the expected profit of the retailer is given by:

$$\begin{aligned}\Pi_{rc}(Q,p) &= (\phi p - v_r + g)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\ &\quad + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\ &\quad + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\ &\quad + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\} - (w+c_r-v_r) \\ &\quad \times\Big\{\int_a^{\frac{Q}{Q_m}}yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\} - g\bar{x} - \eta\phi \end{aligned} \tag{3.29}$$

Similar to the retailer's and the manufacturer's objective functions in section 3.4, it is difficult to show directly that $\Pi_{rc}(Q,p)$ and $\Pi_{mc}(Q_m,\eta)$ are jointly concave with respect to their decision variables. The next two propositions characterize the optimum decisions of the retailer and the manufacturer under the revenue sharing and cost sharing agreement.

Proposition 3.7 *In the decentralized model under revenue-sharing with cost-sharing agreement, the optimal order quantity Q^* and retail price p^* of the retailer are obtained from the following equations:*

$$Q = (\alpha - \beta p + k\sqrt{\eta}) + F^{-1}\Big(\frac{w+c_r-v_r}{\phi p + g - v_r}\Big) \tag{3.30}$$

$$\phi \times \Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx$$
$$+ \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \Big) g(y)dy$$
$$+ \int_{\frac{Q}{Q_m}}^{b} \Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx$$
$$+ \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Q f(x)dx \Big) g(y)dy \Big\}$$
$$= \beta(\phi p + g - v_r) \Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy$$
$$+ \int_{\frac{Q}{Q_m}}^{b} \Big(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \Big) g(y)dy \Big\} \qquad (3.31)$$

Proof. The proof is similar to the Proposition 3.5. ■

From Proposition 3.7 we find that the retailer's optimal order quantity is an increasing function of the retail price p and a decreasing function of its purchasing cost w_m and treating cost c_r.

Now, taking into account the retailer's optimum responses, we determine the manufacturer's optimal decisions. The manufacturer's expected profit function is given by

$$\begin{aligned}\Pi_{mc}(Q_m, \eta) &= (1-\phi)pE[\min\{X, Q, yQ_m\}] + wE[\min\{Q, yQ_m\}] + v_m E[(yQ_m - Q)^+] \\ &\quad - c_m Q_m - (1-\phi)\eta \\ &= (1-\phi)pE[\min\{X, Q, yQ_m\}] - (w - v_m)E[\min\{Q, yQ_m\}] \\ &\quad -(c_m - v_m\bar{y})Q_m - (1-\phi)\eta \end{aligned} \qquad (3.32)$$

We extract an alternative representation of the manufacturer's profit function as given below:

$$\Pi_{mc}(Q_m, \eta) = (1-\phi)p\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx$$
$$+ \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \Big) g(y)dy$$

$$
\begin{aligned}
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx \\
&+\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Qf(x)dx\Big)g(y)dy\Big\}-(w-v_m) \\
&\times\Big\{\int_{a}^{\frac{Q}{Q_m}} yQ_m g(y)dy+\int_{\frac{Q}{Q_m}}^{b} Qg(y)dy\Big\} \\
&-(c_m-v_m\bar{y})Q_m-(1-\phi)\eta
\end{aligned}
\tag{3.33}
$$

The proposition characterizes the manufacturer's optimal production amount and CSR investment in the decentralized system under the proposed composite contract.

Proposition 3.8 *The optimal input amount Q_m^* and CSR expenditure η^* of the manufacturer satisfy the following equations:*

$$
\begin{aligned}
(1-\phi)p\int_{a}^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy& \\
-(w-v_m)\int_{a}^{\frac{Q}{Q_m}} yg(y)dy &= (c_m-v_m\bar{y})
\end{aligned}
\tag{3.34}
$$

$$
\begin{aligned}
\textit{and}\quad \frac{K}{2\sqrt{\eta}}(1-\phi)p\Big\{\int_{a}^{\frac{Q}{Q_m}}\Big(\int_{l}^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy& \\
+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} &= (1-\phi)
\end{aligned}
\tag{3.35}
$$

Proof. The proof is similar to the Proposition 3.7. ■

In order to find the conditions for a win-win outcome, we now characterize the participation problem of the chain members. In the following theorem, we get the circumstances under which the contract coordinates the supply chain:

Theorem 3.3 *Under the voluntary compliance, the revenue-sharing with cost-sharing contract with the wholesale price of the manufacturer*

$$
w_m = \phi\times v_m-(1-\phi)(c_r-v_r) \tag{3.36}
$$

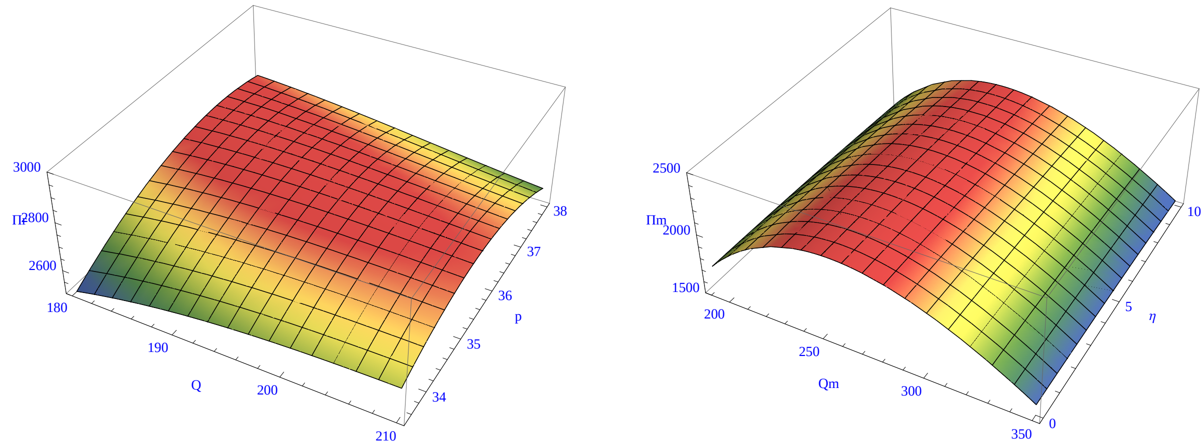
achieves the channel coordination.

Proof. Comparing the supply chain members' optimal decisions of the decentralized model under spanning revenue-sharing contract given in Proposition 3.7 and 3.8 with the optimal decisions of the centralized supply chain in Proposition 3.1 and 3.2 respectively, we find that when the conditions $w_m = \phi \times v_m - (1-\phi)(c_r - v_r)$ holds, the supply chain is coordinated. As a result, the retailer's ordering and pricing decisions align with the centralized model and the manufacturer takes the ordering decisions and CSR investment same as in the centralized model i.e., $Q^* = Q^c, Q^*_m = Q^c_m, \eta^* = \eta^c$ and $p^* = p^c$. Further, using optimal decisions given in Propositions 3.7 and 3.8 and coordination conditions in equation (3.36), the total expected profit of the decentralized SC under spanning revenue sharing contract is $\Pi_{rc}(Q^*, p^*) + \Pi_{mc}(Q^*_m, \eta^*) = \Pi_c(Q^c, Q^c_m, p^c, \eta^*)$ ■

Now, our attention is focused on the issue of individual firm's participation. A situation can occur where a participant of the chain becomes worse by signing the proposed contract. Clearly, the manufacturer wants to receive higher compensation (higher value of ϕ) from the retailer to reduce his wholesale price. On the other hand, the retailer wants to share CSR investment cost as small as possible (lower value of ϕ) to incentive the manufacturer to invest in CSR activities to increase CSR sensitive customer demand. Therefore, the manufacturer wants to increase the share ϕ but the retailer wants to decrease it. Hence, a question arises- how to determine the suitable contract parameters (ϕ, w_m) under which supply chain members are motivated to engage in the proposed coordinating contract mechanism? We will find the answer to this question from the numerical experiment.

3.6 Numerical analysis

In this section, a numerical example is used to explore the effect of various parameters on the decision variables and the expected profit of the whole supply chain as well as individual profits of the channel members. The stochastic factor x of the market demand is uniformly distributed with mean $\bar{x} = 50$ and standard deviation $\sigma_x = 50/\sqrt{3}$. Productions of the supplier is assumed to follow uniform distributions with means σy=0.05 and standard deviations $\bar{y} = 0.8$. The other parameter-values are taken as $\alpha = 500$; $\beta = 10$; S=200; c_m=5; c_r=2.5; g_r=1.5; v=4;v_r=4.5.

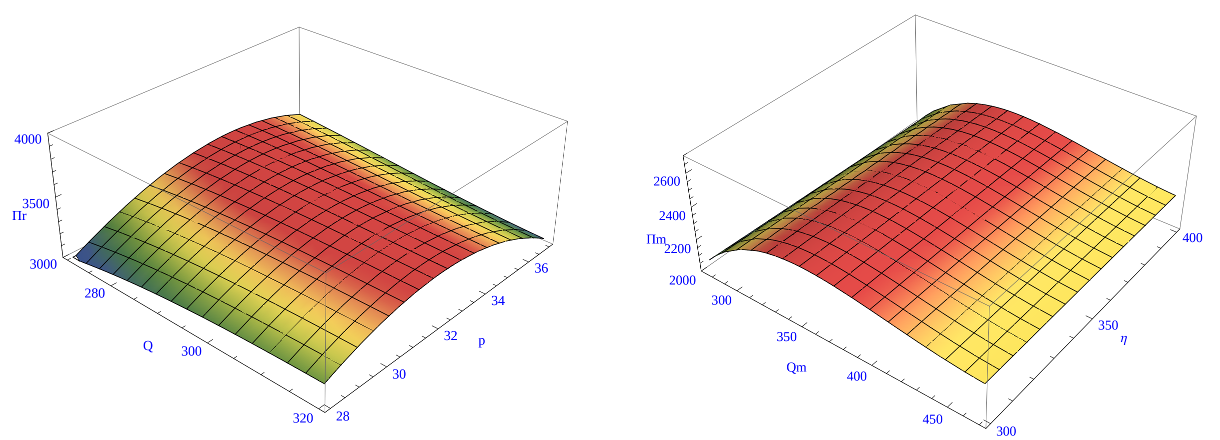


(a) Concavity of the $\Pi_r(Q, p)$.

(b) Concavity of $\Pi_m(Q_m, \eta)$.

Fig. 3.1: Concavity of the objective functions under wholesale price-only contract

For the above set of values, the concavity of the retailer's expected profit function $\Pi_r(Q, p)$ with respect to Q and p and the manufacturer's expected profit function $\Pi_m(Q_m, \eta)$ with respect to Q_m and η under the wholesale price contract are checked graphically as shown in Fig. 3.1. Also, for the same data set, under revenue sharing-cost sharing contract, the retailer's profit function $\Pi_{rc}(Q, p)$ is jointly concave in Q and p and the manufacturer's profit function $\Pi_{mc}(Q_m, \eta)$ is jointly concave in Q_m and η, which can be checked graphically as shown in Fig. 3.2.



(a) Concavity of the $\Pi_{rc}(Q, p)$.

(b) Concavity of $\Pi_{mc}(Q_m, \eta)$.

Fig. 3.2: Concavity of the objective functions under coordinating contract.

In Table 3.1, the optimal decision variables and the expected profits for the SC's centralized decisions with CSR, the SC's centralized decisions without CSR, the SC's centralized decisions with a secondary resource, the decentralized decisions under wholesale price-only contract and SC coordination under profit and cost-sharing contract are compared. Table 3.1 shows that the optimal retail price $p^c = 32.52$ with CSR is higher than the optimal retail price $p^0 = 31.10$. Also, the data in Table 3.1 reveal that the expected profit Π_c=5735.50 is much higher than Π_0=5400.33. It is possible to explain that Π_c is more than Π_0 as follows. The SC with CSR has extra reward demand (S) relative to the SC without CSR and hence has higher expected sales (see EPS in Table 3.1). As the extra revenue arising from additional rewarded demand and the higher selling price ($p^c > p^0$) cover the CSR investment η, the expected profit of the supply chain with CSR is higher than the expected profit of the SC without CSR.

Table 3.1: Optimal decisions and profits of three scenarios

Variables	Centralized model with CSR	Centralized model without CSR	Centralized model with secondary resource	Decentralized model under revenue and cost sharing	Decentralized model under wholesale price
Q	293.036	278.490	287.191	293.036	186.786
Q_m	370.851	350.648	387.987	370.851	253.023
p	32.524	31.102	32.59	32.524	36.482
η	355.225		354.871	355.225	0.010
EPS	251.299	237.111	251.024	251.299	173.312
np	31.110	31.102	31.18	31.110	36.481
w				3.27	18
Π_d	5735.50	5400.33	5717.7	5735.50	4901.30
Π_r				3401.94	2740.20
Π_m				2333.56	2161.10

It is noted that, for the supply chain with CSR, the selling price is greater than that of the supply chain without CSR. At the first glance, it appears that relatively high optimum retail price for the SC with CSR does not help the consumer. However, the CSR activities support the stakeholders including the consumer. For example, Indian company 'P&G' spends a fraction of revenue from all its products for girls education which might give the chance of a healthier and happier life for the girls to meet their own needs. After all, there are also extensive benefits for the community as a whole. An educated woman has the skills, knowledge, and self-confidence to be a better citizen, parent, and employee. In this way, the CSR activity is beneficial to the customers. Here, the manufacturer's CSR activity helps both the members of the SC and other stakeholders. In other terms, the manufacturer's CSR operation accomplishes "win - win" for the supply chain and the community.

For the decentralized decision under revenue and cost-sharing contract, Q, Q_m, η, and p are calculated by equations (3.30), (3.31), (3.34), and (3.35), respectively. The wholesale price w_m is determined following Theorem 3.3 as $w_m = \phi \times v_m - (1-\phi)(c_r - v_r)$. Table 3.1 displays the computational outcome for decentralized scenario under the composite contract. Comparing the results in the second column with those in the fifth column in Table 3.1, we observe that all decision variables and predicted income are the same. Such findings suggest that the composite contract can achieve channel coordination, as described in Theorem 3.3. Making a comparison of the results in the fifth column with those in the sixth column in Table 3.1, we observe that SC's order quantities and CSR's commitment under a composite contract agreement are higher than those under whole-sale price only contract. The expected profits of the SC and its members are greater under channel coordination with the proposed composite contract than those under whole-sale price only contract. Based on the criteria in the empirical case, we consider that if the requirements in Theorem 3.3 are fulfilled, i.e. the supply chain is coordinated, then in turn, for $\phi = 0.59$ all supply chain participants receive the same amount of additional profit due to revenue-sharing and cost-sharing contract relative to the decentralized model with whole-sale price contract only.

Table 3.2: Effects of k on the decision variables and profit functions

k	Π_d	Π_r	Π_m	Q	Q_m	p	η	EPS	np
1.1	5575.72	3306.65	2269.07	286.107	361.225	31.846	180.889	244	31.106
1.3	5648.38	3349.98	2298.40	289.259	365.604	32.154	259.042	247	31.108
1.5	5735.50	3401.94	2333.56	293.036	370.851	32.524	355.225	251	31.110
1.7	5838.29	3463.24	2375.05	297.489	377.041	32.960	472.199	255	31.113
1.9	5958.27	3534.78	2423.48	302.683	384.262	33.469	613.496	260	31.116
2.1	6097.29	3617.69	2479.60	308.696	392.624	34.058	783.641	266	31.119
2.3	6257.64	3713.30	2544.33	315.625	402.263	34.738	988.445	273	31.122

Now, we investigate the effects of CSR-sensitive coefficient k on the expected profits and optimal decisions. To investigate the impact of k under the composite contract, we take ϕ= 0.59. Optimal decision variables and estimated profits for various k are mentioned in Table 3.2. It is evident from the results shown in Table 3.2 that all the decision variables, including order quantity and retail price, increase as k increases. Also the SC's expected profit, each member's expected benefit, and CSR investment increase as k increases. However, the increased rate of CSR investment with k tends to be higher than the expected profit amount. Table 3.2 provides an interesting comparison of the CSR contribution level. Apparently, the ratio of CSR investment to the profit of the supply chain and the ratio of CSR investment to the profit of the manufacturer increase w.r.t. k. This result implies that if the reward demand arising from CSR investment increases, the manufacturer wants to increase its CSR investment. In Table 3.2, it is found that the increased rate of np (31.10-31.12) is very low with k but the CSR investment rises with k at a high rate. A higher value of k is therefore advantageous not only for the SC but also for other SC owners, including customers. A higher value of k is one of the strongest ways to boost the entire channel's revenue and consumers' welfare. The significance of k indicates that when handling the SC with CSR, we strive to choose acceptable CSR practices and boost the income by improving CSR investment. Similar findings can be created on other parameters such as $c_m^{'}$ and σ_y, etc. through a sensitivity analysis.

Most of the researches have shown that there is a beneficial allocation influence of a secondary market for the supply chain. In our numerical analysis, we have quite similar observations. Remember that getting access to the secondary market as an emergency resource is advantageous to the supply chain (Lee and Whang, 2002) to achieve a higher expected profit when producing less. From the numerical example, we see that the profit in the centralized model with a secondary resource is greater than the profit in the centralized model without a secondary resource. The presence of the secondary market offers more alternatives which ultimately increase the supply chain's efficiency. In the presence of a secondary market, the double marginalization effect in the decentralized supply chain is decreased, and the amount moved to the retailer is increased.

It is a little surprising when we find that there may be a situation when a secondary resource may not be favorable for the supply chain. It seems that the manufacturer does not need to invest that much for production to satisfy the same amount Q in the presence of a secondary resource which decreases his production investment as well as the risk of salvaging, compare to the absence of a secondary resource. However, the manufacturer produces less under the expectation that he will purchase from the sec-

Table 3.3: Effects of $c_m^{'}$ on the decision variables and profit functions

$c_m^{'}$	Π_d	Q	Q_m	p	η	EPS	np
10	5738	287.772	367.594	32.5625	355.978	250.367	31.1407
15	5723.47	287.356	382.083	32.585	355.185	250.955	31.1697
20	5717.7	287.191	387.987	32.5939	354.871	251.024	31.1803
25	5714.61	287.102	391.194	32.5987	354.702	251.029	31.1857
30	5712.68	287.047	393.208	32.6017	354.596	251.022	31.1891
35	5711.36	287.009	394.591	32.6038	354.524	251.013	31.1914

ondary resource while producing less. But when the purchasing cost of the secondary resource increases, the performance of the supply chain decreases as shown in Table 3.3. The presence of the secondary resource with high purchasing cost indeed generates more fear on the manufacturer's mind that compels him to increase the amount of his production decision, under the force compliance. The effect on the retailer's decision, when the purchasing cost of the secondary resource is varying, is negligible. Consequently, the expected sales of the retailer almost remain unchanged. Therefore, the risk of salvaging of the final product at the manufacturer level increases as the purchasing cost of the secondary resource increases, which leaves a negative impact on the supply chain performance.

Table 3.4: Effects of σ_y on the decision variables and profit functions

σy	Π_d	Π_r	Π_m	Q	Q_m	p	η	EPS	np
0.01	5763.05	3407.94	2355.11	293.527	359.564	32.484	357.795	252	31.065
0.03	5751.30	3404.67	2346.63	293.315	363.465	32.500	356.567	251	31.084
0.05	5735.50	3401.94	2333.56	293.036	370.851	32.524	355.225	251	31.110
0.07	5717.18	3399.21	2317.97	292.712	379.991	32.551	353.758	250	31.141
0.09	5696.64	3396.32	2300.32	292.349	390.457	32.582	352.147	250	31.175
0.11	5673.86	3393.15	2280.71	291.946	402.103	32.617	350.372	249	31.213

Finally, we explore how the production yield uncertainty impacts on the supply chain under the proposed composite contract. Table 3.4 shows the influence of yield uncertainty on the decisions of the supply chain. For comparison purpose, six different values of yield variance are utilized in Table 3.4. It is shown that the supply chain's benefit rises as the yield uncertainty decreases. This supports the fact that a lower risk leads to the efficiency of the supply chain. Not unexpectedly, the difference between order quantity and planed production amount decreases as the yield uncertainty decreases, i.e., the manufacturer plans to use less input to generate the very same output and satisfy the demand. We also find that the optimal CSR investment increases as the yield uncertainty decreases. This shows that yield uncertainty may encourage the manufacturer to increase the investment in CSR for a higher rewarded demand rather than his other investment, (production investment). When the manufacturer invests more in CSR, it creates more demand and the efficiency of the supply chain is increased in general.

3.7 Conclusion

In this article, we analyze a two-level supply chain coordination problem consisting of a manufacturer and a retailer where only the manufacturer invests in CSR activities. The issue discussed in this article is an extension of the traditional price setting newsvendor model and the existing socio-responsible supply chain coordination problem with demand uncertainty and exogenous retail price. After modeling and addressing SC's centralized decision issue without CSR or with CSR in the presence or absence of a secondary resource, we have solved the problem of SC coordination under the revenue-sharing and cost sharing contract. Because of the difficulty of the problem, we have addressed the problem in the case of linear price-dependent demand, *i.e.,* $d(p) = a - bp$ as taken by Zhao and Yin (2018). Our work is firmly linked to Zhao and Yin (2018) who assumed that stochastic demand follows uniform distribution and the demand is linear in price. We have extended their work by taking arbitrary distribution of demand function and random yield in production. Furthermore, the coordination effect of the modified revenue sharing contract in Zhao and Yin (2018), where only one member exhibits CSR is modified by revenue and cost sharing contract, where both members exhibit CSR.

The following conclusions are drawn from our theoretical analysis and empirical study: First, the SC's estimated benefit with CSR is persuaded to be greater than that of the SC without CSR. Second, the traditional revenue sharing contract is unable to coordinate the SC under the Nash system, but the proposed composite contract can coordinate the SC. Third, in particular, the secondary resource is detected to have a positive effect on supply chain performance, but we also find a situation where the presence of secondary resource might not be beneficial for a supply chain. Ultimately, the SC's expected income, each member's expected benefit, and stakeholder welfare (CSR investment) increase with k. The above results suggest that raising k is crucial for enhancing the SC's profit and its stakeholders' welfare while handling the SC with CSR.

For a socio-responsible two-echelon supply chain faced with price and CSR-dependent random demand, we have concentrated on the coordination issue of the supply chain. Although the revenue sharing and cost sharing contract guarantees that SC's target is met, there are some limitations of this research work. We have assumed that the demand is linear in retail price which can be relaxed. The expansion of this work includes exploring the problems of SC coordination with dual networks for the manufacturer-retailer SC, in the sense of CSR. Extending the revenue sharing and cost sharing contract into a multi-period supply chain is another avenue for future research.

Chapter 4

Coordinating a three-level supply chain with effort and price-dependent stochastic demand under random yield*

4.1 Introduction

Recently fashionable products with higher features have become more favourable for customers than durable and long-lasting products, as customers are searching for variety and they want to use a product for a short time and look for a new one after that. In reality, such behavior can be related to products such as electronic goods (*e.g.,* personal computer, mobile), fashion items, *etc.* To sustain in the competitive business environment, companies therefore have been forced to increase their production rates together with higher efforts on retailing varieties of products. It is, therefore, more acceptable for the companies to focus on a single transaction only, *i.e.*, to choose to be either on the manufacturing side or on the retailing side, and that shows the picture of how today's supply chains have initiated to decentralize.

Since price is one of the main factors for customers to decide whether to buy a

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product or not, joint determination of inventory and pricing decisions is of utmost importance. In addition to retail price, the retailer can also influence demand by sales efforts such as advertising, after-sales service support, providing attractive products' display, and hiring sophisticated sales personnel to influence consumer purchase. For example, in the electronics industry, the market demand depends not only on the brand reputation of the product, but also on retailer's after-sales service support. As reported by Dant and Berger (1996), sales effort programs finance 25-40% of the local advertisements of retailers. In 1970, the cost of sales efforts in US has been estimated at $900 million, whilst the expenditure on sales advertising was approximately $15 billion in 2000 (Nagler, 2006) and $50 billion in 2010 (Yan, 2010).

In the previous chapter, we have discussed about the production yield uncertainty in a two-echelon supply chain where the manufacturer can't access a secondary resource to mitigate his yield risk of the final product. For raw materials shortage, it is possible to access an alternative resource. In that case, raw material producers often use a secondary market as an emergency resource to satisfy the unmet demand, and also for salvaging the leftover products. In this chapter, as an extension of our previous work, we consider a multi-echelon newsvendor problem with random yield and price and sales effort dependent random demand. The proposed supply chain consists of a raw material supplier, a manufacturer and a retailer trading a single product for a single period with short life-cycle. It is also assumed that production of raw materials and production of finished goods are both subject to random yield. The raw-material supplier can mitigate the risk of uncertainty in production by using a secondary market. Similarly, the retailer can access a secondary market for selling the leftover products at a lower price. However, the availability of secondary resource at the manufacturer level is debarred here, which is justified from the perspective of brand value or quality issue of the product produced by the manufacturer.

The main problem in such a multi-echelon supply chain is to decide how to deal with such uncertainties and influences so as to maximize profits of the chain members as well as the whole supply chain. In the literature, various contracts are proposed to coordinate the supply chain under different scenarios. Although most of the existing literatures on contract mechanism focus on channel coordination in two-level supply chain like our previous study, but the real world business supply chains consist of multiple entities. Companies like HP, Lenovo and Lakmé procure raw materials from their suppliers and sell finished products through their retailers. So these supply chains contain more than two entities. The popular contract mechanisms which are designed for two-level supply chains are thus needed to be generalized so as to fit in multi-level supply chains. Many researchers have extended the traditional contracts by establishing contract between pairs of adjacent entities in a multi-echelon supply chain. The present work aims in that direction. However, there are issues related to contract parameters information, simultaneous installation of the contract, etc. when traditional contracts are implemented in a multi-level supply chain. From this perspective, the following research questions arise:

- Can a contract mechanism be designed to coordinate the proposed multi-echelon supply chain by generalizing/combining some existing popular contracts?
- What are the impacts of uncertainties in demand and production as well as non-existence of emergency resource for the final product on optimal decisions and profits of individual members' and the entire supply chain?

To answer these questions, we propose a composite contract by combining three popular contracts viz. a buyback contract, a sales rebate and penalty contract, and a revenue sharing contract. The integrated supply chain is first analyzed as the benchmark case for comparison. In the decentralized setting, aiming at how the risk of uncertainties in both yield and demand can be distributed among the supply chain members, we analyze the wholesale price contract as no risk sharing contract, and develop our risk sharing composite contract which distributes the risk of uncertainties among the parties to enhance the supply chain performance. In both the cases, we determine the optimal ordering, pricing, sales effort, and production decisions. We also analyze the impacts of uncertainties in demand and production as well as non-existence of emergency resource for the final product on optimal decisions of the supply chain.

The contributions of the chapter with respect to the existing literature are as follows. Firstly, our chapter incorporates the risks of random yield, and random demand which is sensitive to both retail price and sales effort in a multi-echelon supply chain.

Secondly, we consider the situation where the manufacturer faces the yield risk but can't access a secondary resource to mitigate yield uncertainty. We suggest that, in this situation, the retailer should set a larger order quantity than the one when the manufacturer has access to the secondary market. We also find that this strategy is beneficial to all the members of the supply chain. Thirdly, this chapter contributes to the multi-echelon supply chain coordination literature by exploring a composite contract. We show that, for a three-echelon supply chain with uncertainty at each stage, a composite contract can ensure both coordination and win-win outcome that overcome the difficulties of contract parameters' estimation and simultaneous installation.

The rest of the chapter is organised as follows. Section 4.2 presents model description and notations of the problem under consideration. Section 4.3 discusses the centralized benchmark model and section 4.4 illustrates the decentralized model with no risk sharing contract. A risk sharing coordination contract is presented in section 4.5. Numerical examples are provided and the optimal results are analyzed in section 4.1. Section 4.7 summarizes the chapter and indicates scopes of future research.

4.2 Model assumptions and notation

Most of the assumptions of this chapter are borrowed from Giri et al. (2016). The proposed model is developed for trading a short life-cycle product such as personal computer, electronic or fashionable good. Since the product life cycle is short, only one-time order is considered. All the entities, namely, the supplier, the manufacturer and the retailer involved in the supply chain are assumed to be risk-neutral, and there is no information asymmetry among them. In view of various uncertainties related to weather, environment, availability of skilled labor, product quality, transportation, etc, it is assumed that productions of the raw material and the finished good are subject to random yield. The supplier can mitigate the risk of uncertainty in production by using a secondary market. Similarly, the retailer can access a secondary market for selling the leftover products at a lower price. However, the availability of secondary market at the manufacturer level has been debarred here, which is justified from the perspective of brand value or quality issue of the product produced by the manufacturer. Following He et al. (2009), the market demand is assumed to be stochastic but sensitive to both retail price and sales effort. The random demand is assumed to be of the form $D = \gamma(p, e) + x$, where $\gamma(p, e) = \alpha - \beta p + ke,\ \alpha, \beta, k > 0$, showing that the actual demand is linear in its deterministic and stochastic components; β and k denote price and sales effort sensitivity parameters, respectively; α is the base demand, and x denotes a particular value of the random variable X. The notations used in this chapter are provided in the following table. More notations will be defined as and when needed.

Notation

c_s : unit production cost at the raw material supplier.

c_s' : unit procurement cost of raw material from the secondary market.

c_m : unit manufacturing cost at the manufacturer.

v : unit salvage value of the final product.

v_s : unit salvage value of excess product at the raw material supplier.

g_r : retailer's goodwill cost for unit unmet demand.

X : a positive random variable with range $[l, u]$, pdf $f(\cdot)$, cdf $F(\cdot)$, mean $\bar{x}$, and variance σ_x^2 representing the stochastic portion of the customer demand.

Y : a random variable with range [c,d], $0 \leq c < d \leq 1$, having pdf $g(\cdot)$ and cdf $G(\cdot)$, denoting the randomness of the production quantity produced by the manufacturer.

Z : a random variable with range $[a, b]$, $0 \leq a < b \leq 1$, with pdf $h(\cdot)$ and cdf $H(\cdot)$, denoting the randomness of the production quantity of the raw material produced by the supplier.

R : planned production quantity of the raw-material supplier, a decision variable.

Q : ordered quantity of the retailer, a decision variable.

p : unit retail price of the final product charged by the retailer, a decision variable.

e : effort level to summarize the retailer's activities to influence market demand, a decision variable. We assume $J(e)$ to be the retailer's cost of exerting an effort level e with $J(0) = 0$, $J'(e) > 0$ and $J''(e) > 0$ when $e > 0$.

w_s : unit wholesale price of the raw material offered by the supplier to the manufacturer, a decision variable.

w_m : unit wholesale price of the finished product charged by the manufacturer to the retailer, a decision variable.

The sequence of events is as follows:

- Firstly the retailer forecasts the market demand D, and negotiates his trade contract with the manufacturer, by placing an order of Q units to the manufacturer.

- The manufacturer negotiates another trade contract with the supplier and passes the order to the supplier. Without loss of generality, we assume that one unit of the final product can be produced from one unit of the raw material.

- After receiving the order quantity Q, the raw material supplier decides his optimal production quantity R, and starts production. The produced amount is zR, where z is a particular value of the random variable Z. If the actual production quantity of the supplier is less than the amount ordered, he buys the difference from the secondary market to fulfill the order; if it is more, the excess amount is salvaged at a lower rate.

- After receiving Q units of raw material from the supplier, the manufacturer starts production. The production output is yQ, y being a particular value of the random variable Y. If the produced amount is less than the amount ordered by the retailer, the manufacturer can't buy the difference from the market due to non-availability of secondary resource of the final product.

- The retailer receives yQ units of the final product. If the demand is less than the on-hand inventory, the excess amount is salvaged; otherwise, the shortage incurs a cost.

In order to avoid trivial cases and ensure positive profit margin for all the chain members, we assume the following:

(i) $v_s < c_s < w_s$, in order to prevent the supplier from producing an infinitely large quantity, and ensure positive profit, (ii) $w_s + c_m < w_m$ and $w_s < c_s'$, in order to ensure that the manufacturer makes positive profit and does not buy raw materials from the secondary market directly, (iii) $v < w_m < p$, in order to ensure that the retailer participates in the business, and does not order infinitely large quantity, (iv) $v < (w_s + c_m)/\bar{y}$, in order to confirm that the salvage value is less than the expected unit production cost of the final product, (v) $p > c_s/\bar{z} + c_m/\bar{y}$, *i.e.*, retailer's unit selling price is higher than the expected unit cost of the product. (vi) The density functions $f(\cdot)$, $g(\cdot)$, $h(\cdot)$, of customer demand, manufacturer's production and supplier's production respectively, are strictly positive on their respective domains. (vii) Also all the probability density functions (f, g, h) along with cumulative density functions (F, G, H) are continuous on their respective domains.

We now formulate the proposed model based on the above assumptions. Let $S(Q, p, e)$ be the expected sales volume which can be expressed as

$$S(Q,p,e) = E[\min\{yQ, D\}] = \bar{y}Q - \int_c^d \int_l^{yQ-\gamma(p,e)} (yQ - x - \gamma(p,e))f(x)g(y)dxdy$$

Using the identity $(a-b)^+ = a - \min\{a, b\}$, the expected leftover inventory $I(Q, p, e)$ can be written in terms of the expected sales as

$$\begin{aligned} I(Q,p,e) &= E[(yQ-D)^+] = \int_c^d \int_l^{yQ-\gamma(p,e)} (yQ - x - \gamma(p,e))f(x)g(y)dxdy \\ &= \bar{y}Q - S(Q,p,e) \end{aligned}$$

The expected lost sales $L(Q, p, e)$ can be written as

$$\begin{aligned} L(Q,p,e) &= E[(D-yQ)^+] = \int_c^d \int_{yQ-\gamma(p,e)}^{u} (x + \gamma(p,e) - yQ)f(x)g(y)dxdy \\ &= \bar{X} - S(Q,p,e) \end{aligned}$$

4.3 Centralized model−The Benchmark case

To establish a performance benchmark, we first analyze the integrated supply chain model, i.e., the centralized model. In this model, conceptually there is only one central decision maker for the whole supply chain. Here, the wholesale price(s) charged by the upstream member(s) to the downstream member(s) may be seen as internal revenue transfer, which will not influence the supply chain performance as a whole. We assume that all the residual products are salvaged and unmet demands are lost. The expected profit of the entire supply chain can then be expressed as

$$\begin{aligned}\Pi_c(R,Q,p,e) \;=\;& pS(Q,p,e)+vI(Q,p,e)-g_rL(Q,p,e)-c_mQ-c_sR\\ &-c_s'E[(Q-zR)^+]+v_sE[(zR-Q)^+]-J(e),\end{aligned} \tag{4.1}$$

where $a^+ = a$ if $a \geq 0$ and 0 if $a < 0$. The first two terms denote revenues earned by selling the final product at the primary and the secondary markets, respectively; the third term indicates the cost for lost sales; the fourth and the fifth terms represent the costs for production of the finished product and the raw material, respectively; the sixth term designates the cost for buying the raw materials from the secondary market; the seventh term represents the revenue generated from selling excess raw materials at the secondary market, and the last term indicates the cost associated with sales effort.

We then have an equivalent representation of the entire supply chain's expected profit function as given below:

$$\begin{aligned}\Pi_c(R,Q,p,e) \;=\;& \{(p+g_r)\bar{y}-c_m-v_s\}Q-(p+g_r-v)\int_c^d\int_l^{yQ-\gamma(p,e)}(yQ-x-\gamma(p,e))\\ &\times f(x)g(y)dxdy-(c_s'-v_s)\int_a^{Q/R}(Q-zR)h(z)dz\\ &-(c_s-v_s\bar{z})R-g_r\bar{x}-J(e).\end{aligned} \tag{4.2}$$

As observed by Petruzzi and Dada (1999), for a price setting newsvendor problem (PSNP) containing multiple decisions variables in its objective function, it is often difficult to show the joint concavity of the objective function in all of its decisions variables. In the literature (for instance, Wang et al., 2020), it is a common approach to use a repetitive method to show the objective function's concavity. In the present article, the objective function of the centralized system consists of four decision variables and the exact methods cannot be applied to obtain the optimal solution. So we apply a repetitive method. Let us assume that a finite but not necessarily unique optimal decision set (R^c, Q^c, p^c, e^c) exists for the centralized model. The first order partial derivatives of $\Pi_c(R, Q, p, e)$ with respect to each of the decision variables are as follows:

$$\begin{aligned} \frac{\partial \Pi_c(R,Q,p,e)}{\partial Q} &= (p+g_r)\bar{y} - (c_m + v_s) - (c_s' - v_s)H\Big(\frac{Q}{R}\Big) \\ &\quad -(p+g_r-v)\int_c^d \int_l^{yQ-\gamma(p,e)} yf(x)g(y)dxdy \qquad (4.3) \\ \frac{\partial \Pi_c(R,Q,p,e)}{\partial R} &= (c_s' - v_s)\int_a^{Q/R} zh(z)dz - (c_s - v_s\bar{z}), \qquad (4.4) \\ \frac{\partial \Pi_c(R,Q,p,e)}{\partial p} &= S(Q,p,e) + (p+g_r-v)\frac{\partial S(Q,p,e)}{\partial p}, \qquad (4.5) \\ \text{and} \quad \frac{\partial \Pi_c(R,Q,p,e)}{\partial e} &= (p+g_r-v)\frac{\partial S(Q,p,e)}{\partial e} - J'(e). \qquad (4.6) \end{aligned}$$

It is easy to observe that, for given p and e,

$$\frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial Q^2} = -\frac{(c_s' - v_s)}{R}h\Big(\frac{Q}{R}\Big) - (p+g_r-v)\int_c^d y^2 f(yQ-\gamma(p,e))g(y)dy < 0$$

since $(c_s' - v_s) > 0$ from assumptions (i) and (ii); $p > v$ i.e., $(p + g_r - v) > 0$ from assumption (iii); f, g, and h are strictly positive from assumption (vi). Further

$$\frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial Q \partial R} = \frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial R \partial Q} = (c_s' - v_s)h(Q/R)\frac{Q}{R^2} > 0$$

since $(c_s' - v_s) > 0$, $h > 0$ and Q and R are strictly positive decision variables and

$$\frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial R^2} = -(c_s' - v_s)\frac{Q^2}{R^3}h\Big(\frac{Q}{R}\Big) < 0$$

Let M be the Hessian matrix of the objective function and M_1, M_2 are the principal minors of M, then

$$M = \begin{pmatrix} \frac{\partial^2 \Pi_c}{\partial Q^2} & \frac{\partial^2 \Pi_c}{\partial Q \partial R} \\ \frac{\partial^2 \Pi_c}{\partial R \partial Q} & \frac{\partial^2 \Pi_c}{\partial R^2} \end{pmatrix},$$

we deduce the principal minors $|M_1| < 0$ and $|M_2| = (c_s' - v_s)\frac{Q^2}{R^3} h\Big(\frac{Q}{R}\Big)(p + g_r - v)\int_c^d y^2 f(yQ - \gamma(p,e))g(y)dy > 0$. This leads to the following proposition.

Proposition 4.1 *For given p and e, $\Pi_c(R,Q,p,e)$ is jointly concave in R and Q, and the optimal values Q^c and R^c satisfy the equations*

$$\int_c^d \int_l^{yQ-\gamma(p,e)} y f(x) g(y) dx dy = \frac{(p+g_r)\bar{y} - (c_m + v_s) - (c_s' - v_s) H\left(\frac{Q}{R}\right)}{(p + g_r - v)} \quad (4.7)$$

$$\text{and} \int_a^{Q/R} z h(z) dz = \frac{c_s - v_s \bar{z}}{(c_s' - v_s)}. \quad \blacksquare \quad (4.8)$$

Again for given R and Q,

$$\frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial p^2} = (-2\beta - \beta^2(p + g_r - v)) \int_c^d f(yQ - \gamma(p,e)) g(y) dy < 0$$

since $\beta > 0$, $p > v$ i.e., $(p + g_r - v) > 0$ from assumption (iii); f and g are strictly positive from assumption (vi). For the same assumptions, we have

$$\frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial p \partial e} = \frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial e \partial p} = k \int_c^d \int_l^{yQ-\gamma(p,e)} f(x) g(y) dx dy$$
$$+ k\beta(p + g_r - v) \int_c^d f(yQ - \gamma(p,e)) g(y) dy > 0,$$

$$\text{and} \quad \frac{\partial^2 \Pi_c(R,Q,p,e)}{\partial e^2} = k^2(p + g_r - v) \int_c^d f(yQ - \gamma(p,e)) g(y) dy - \mu < 0.$$

Therefore, from the Hessian matrix

$$M = \begin{pmatrix} \frac{\partial^2 \Pi_c}{\partial p^2} & \frac{\partial^2 \Pi_c}{\partial p \partial e} \\ \frac{\partial^2 \Pi_c}{\partial e \partial p} & \frac{\partial^2 \Pi_c}{\partial e^2} \end{pmatrix},$$

we deduce the principal minors $|M_1| < 0$ and $|M_2| = 2\beta\mu \int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dxdy -$

$k^2(\int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dxdy)^2 + \beta^2\mu(p+g_r-v)\int_c^d f(yQ-\gamma(p,e))g(y)dy > 0$. This leads to the following proposition.

Proposition 4.2 *For given R and Q, if $2\beta\mu\int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dxdy+\beta^2\mu(p+g_r-v)\int_c^d f(yQ-\gamma(p,e))g(y)dy > k^2(\int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dxdy)^2$, then $\Pi_c(R,Q,p,e)$ is jointly concave in p and e, and the optimal values p^c and e^c satisfy the equations*

$$\int_c^d \int_l^{yQ-\gamma(p,e)} (yQ-x-\gamma(p,e))f(x)g(y)dxdy + \beta(p+g_r-v)\int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dy = \bar{y}Q \tag{4.9}$$

and

$$k(p+g_r-v)\int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dy + \mu e = 0 \quad \blacksquare \tag{4.10}$$

Observation: Since $\gamma(p,e) = \alpha - \beta p + ke$ and the expected sales $S(Q,p,e)$ is positive, therefore from equation (4.5) we have $\frac{\partial S(R,Q,p,e)}{\partial p} < 0$, indicating that the expected sales quantity reduces with higher retail price, and from equation (4.6) we have $\frac{\partial S(R,Q,p,e)}{\partial e} > 0$, indicating that a higher sales effort boosts the expected sales quantity. Putting the optimal values in equation (4.2), the maximum expected profit of the integrated supply chain is obtained as

$$\Pi_c(R^c,Q^c,p^c,e^c) = (p^c+g_r-v)\int_c^d \int_l^{yQ^c-\gamma(p,e)} (x+\gamma(p,e))f(x)g(y)dxdy - g_r\bar{x} - J(e^c) \tag{4.11}$$

4.4 Decentralized model with wholesale price-only contract

In this section, we analyze the supply chain dynamics under wholesale price only contract scenario. Although the centralized model provides the best performance of the supply chain, it is a conceptual benchmark, whereas the decentralized system is more practical and commonly used in real business scenario. Under decentralized setting, all the supply chain members are assumed as independent decision makers, and they negotiate certain contracts specifying money and products transfer. All the firms are risk neutral so that each of them chooses the best decision to maximize its own expected profit, resulting in possible deviation from the optimal solution obtained under centralized setting. Under this strategy, the sequence of events is as follows:

First, the raw material supplier declares his own wholesale price to the manufacturer for the coming selling season. The manufacturer in turn determines his wholesale price and offers it to the retailer. If the retailer accepts this price, he then determines retail price, sales effort, and order quantity simultaneously, and places order at the manufacturer after forecasting the market demand. The manufacturer passes the same amount of order to the supplier, as he does not want to take any risk of overproduction. Receiving the order from the manufacturer, the raw material supplier plans his production quantity and starts production. As there is yield uncertainty associated with raw material production, the supplier might choose a larger production quantity to fulfill the manufacturer's order; otherwise, he has to buy the shortfall quantity from the secondary market at a higher price to deliver the order. Clearly, the manufacturer forces the supplier to bear the random yield risk alone. After receiving the ordered amount from the raw material supplier, the manufacturer produces the final product and delivers a shipment at most equal to the order quantity of the retailer, by which the manufacturer forces the retailer to bear the risk of his production yield.

For any $R > 0$, we define $\Pi_r(Q, p, e)$ as the expected profit of the retailer, given by

$$\begin{aligned}\Pi_r(Q,p,e) &= pS(Q,p,e) + vI(Q,p,e) - g_r L(Q,p,e) - w_m E[yQ] - J(e) \\ &= (p + g_r - w_m)\bar{y}Q - (p + g_r - v)\int_c^d \int_l^{yQ-\gamma(p,e)} (yQ - x - \gamma(p,e)) \\ &\quad \times f(x)g(y)dxdy - g_r\bar{x} - J(e) \qquad (4.12)\end{aligned}$$

Similar to centralized system, we consider a finite but not necessarily unique optimal decision triplet (Q^d, p^d, e^d) exists for the decentralized model. Then the triplet (Q^d, p^d, e^d), *i.e.,* the optimal order quantity, retail price and effort level for the retailer can be derived from the following equations (derived from the first order optimality conditions of the retailer's objective function):

$$\int_c^d \int_l^{yQ-\gamma(p,e)} yf(x)g(y)dxdy = \frac{(p+g_r-w_m)\bar{y}}{(p+g_r-v)}, \quad (4.13)$$

$$S(Q,p,e) + (p+g_r-v)\frac{\partial S(Q,p,e)}{\partial p} = 0, \quad (4.14)$$

$$\text{and} \quad (p+g_r-v)\frac{\partial S(Q,p,e)}{\partial e} - J'(e) = 0. \quad (4.15)$$

Let Π_m be the expected profit of the manufacturer. Then

$$\Pi_m = w_m E[yQ] - (c_m + w_s)Q = (w_m\bar{y} - c_m - w_s)Q. \quad (4.16)$$

For given values of the retailer's decision variables, the expected profit of the raw-material supplier is

$$\begin{aligned}\Pi_s(R) &= w_s Q - c_s' E[(Q-zR)^+] + v_s E[(zR-Q)^+] - c_s R \\ &= (w_s - v_s)Q - (c_s' - v_s)\int_a^{Q/R}(Q-zR)h(z)dz - (c_s - v_s\bar{z})R. \quad (4.17)\end{aligned}$$

The following proposition determines the supplier's optimal production quantity and it's relation with the retailer's ordered quantity.

Proposition 4.3 *(i) The supplier's expected profit is concave in* R*, and the optimal production quantity* R^d *satisfies the equation* $\int_a^{Q/R} zh(z)dz = \frac{c_s - v_s\bar{z}}{(c_s'-v_s)}$*.*
(ii)$R^d(Q) = K_1 Q$, where $K_1(>0)$ is a constant.

Proof. (i) From (4.17) we have $\frac{\partial^2\Pi_s(R)}{\partial R^2} = -(c_s'-v_s)\frac{Q^2}{R^3}H\left(\frac{Q}{R}\right) < 0$ since $(c_s'-v_s) > 0$ from assumptions (i) and (ii); $H > 0$ from assumption (vi), and Q and R are strictly positive decision variables, exhibiting $\Pi_s(R)$ to be concave in R. Also, the first order optimality condition gives

$$\int_a^{Q/R} zh(z)dz = \frac{c_s - v_s\bar{z}}{(c_s'-v_s)}\ . \quad (4.18)$$

(ii) THis follows from the fact that $K(x) = \int_0^x h(t)tdt$ is increasing in x for all x. ■

Proposition 4.3 (i) shows that the relationship between R^d and Q^d depends only on the parameters associated with the supplier's production and it's random yield distribution. Moreover, the supplier's production decision is a linear function of the retailer's ordered quantity. Simple calculation shows that the linear coefficient K_1 is influenced positively by c_s' and negatively by c_s; hence, for higher unit raw material procurement cost at the secondary market, the supplier is inclined to produce more quantity, but for higher unit production cost, the supplier produces lesser quantity, which are quite natural in real market scenario. However, the relationship between K_1 and v_s is not so straightforward, and elementary calculus exhibits that K_1 increases with v_s when $\bar{z} > \frac{c_s}{c_s'}$, and decreases with v_s when $\bar{z} < \frac{c_s}{c_s'}$. From managerial point of view, one can apprehend that when the expected yield is sufficiently low, a higher salvage value would result in lower production, since an extra production would incur an extra loss in addition to the extra cost already present due to lower average yield. After putting the optimal production decision from proposition 4.3, the profit of the supplier reduces to

$$\Pi_s(R^d) = [(w_s - v_s) - (c_s' - v_s)H\left(\frac{Q}{R}\right)]Q \tag{4.19}$$

Here we assume that contract prices are negotiated based on the firm's bargaining power, keeping the profit margin above a desired level of acceptance in the decentralized setting.

Proposition 4.4 *The retailer's order quantity, the supplier's production decision, and the total supply chain's profit under price only contract are less than their counterparts in the integrated supply chain.*

Proof: For reasonable wholesale prices offered by the supplier and the manufacturer, the relations $w_s > v_s + (c_s' - v_s)H\left(\frac{Q}{R}\right)Q$ and $w_m > \frac{c_m + w_s}{\bar{y}}$ must hold simultaneously so as to keep profit margins positive. We have to show $Q^d < Q^c$; however, no explicit form for Q^d or Q^c can be derived. As $K(Q) = \int_c^d \int_l^{yQ-\gamma(p,e)} yf(x)g(y)dxdy$ is strictly increasing in Q, it is sufficient to show $K(Q^d) < K(Q^c)$ in order to establish that $Q^d < Q^c$. Now, we have

$$w_m\bar{y} > c_m + w_s > v_s + c_m + (c_s' - v_s)H\left(\frac{Q}{R}\right)$$

$$\frac{(p+g_r)\bar{y} - w_m\bar{y}}{p+g_r-v} < \frac{(p+g_r)\bar{y} - (v_s+c_m) - (c_s'-v_s)H\left(\frac{Q}{R}\right)}{p+g_r-v}$$

$$K(Q^d) < K(Q^c) i.e., Q^d < Q^c.$$

Also, we have $\frac{Q^d}{R^d} = \frac{Q^c}{R^c}$ which implies that $R^d < R^c$. It is now easy to verify that $\Pi_c(R^c, Q^c, p^c, e^c) > \Pi_d(R^d Q^d, p^d, e^d)$, where Π_d stands for total channel profit under price only contract. ■

Proposition 4.4 is a generalization of the result under wholesale price-only contract for the two-level supply chain, known as double marginalization, showing that the decentralized channel can't reach the maximum efficiency level in terms of generating profit, even if all entities maximize their own expected profits. As coined by He and Zhao (2012), such phenomenon is known as multiple marginalization. Both the supplier's and the manufacturer's individual pricing policies are the reason behind system inefficiency. To overcome such sub-optimization in the decentralized system, contract mechanisms come into play.

4.5 Coordination contracts

A contract mechanism is designed in such a way that the optimal decisions taken by the channel members become identical with those of centralized one and subsequently prevents sub optimization as well as improve the performance of all parties involved in the chain. A contract is said to *coordinate* the supply chain if the sum of profits of all members of the decentralized SC under the contract is equal to the profit of the centralized system. Besides coordination, another desirable feature of a contract mechanism is *win-win* outcome, where each participating firm's profit is strictly better off under that contract compared to the wholesale price contract scenario as discussed in Section 4.4. However, as pointed out by Cachon (2003), implementation of a contract in practice is also an important feature; if adopting a fruitful contract becomes costlier to administer, the contract designer may prefer to design a simpler but leaser effective.

4.5.1 Buyback contract with revenue sharing contract

Some difficulties arise while implementing a traditional contract in a multi-level supply chain. One of the major difficulties occurs when contracts are offered level-by-level from the up-stream entities to the down-stream partners. Since the contract parameters between a pair of adjacent entities depend on the contract parameters between the next adjacent pair, the contract designers may not be able to precisely anticipate the next pair's contract parameters. Another difficulty is simultaneous installation of the contract. As observed by Van Der Rhee et al. (2010), if the contracts are not installed simultaneously, situations may arise where one party may earn benefit without signing the agreement while others have already signed; hence some parties may choose to wait for others' participation in the contract. Unfortunately, if each party chooses to wait for others' move, the coordination can never be established in a multi-echelon supply chain.

To overcome the above mentioned obstacles, following the idea provided by Van Der Rhee et al. (2010), we choose the manufacturer to be in the leading role in installing the contract, since he has significant market power over both the raw material supplier and the retailer due to its market base and popularity among the customers through it's brand name, justifying the existence of a manufacturer dominated supply chain. As mentioned by Güler and Bilgiç (2009), automotive industries are prototype examples of manufacturer dominated supply chain. Since the retailer has private information about the customer demand, and he deals with demand as well as supply uncertainties simultaneously, it appears realistic for the manufacturer to negotiate with the retailer first, and then with the supplier for terms of trade contract. We consider that the manufacturer first offers a buyback contract to the retailer and then a revenue sharing contract to the supplier. Under buyback contract, the manufacturer offers a per unit buyback price b ($v < b < w_m$) to the retailer for every unit of unsold product at the end of the selling season. Clearly, the manufacturer shares the cost incurred by the retailer due to over-stocking. Under a revenue sharing contract, the manufacturer keeps a fraction ϕ of his total revenue for himself, and shares the rest with the supplier; in turn, the supplier reduces per unit wholesale price that induces the manufacturer to increase order quantity.

Under this contract mechanism, the retailer's expected profit function $\Pi_r(Q,p,e)$ is given by

$$\begin{aligned}\Pi_r(Q,p,e) &= pS(Q,p,e) + bI(Q,p,e) - g_r L(Q,p,e) - w_m E[yQ] - J(e) \\ &= (p+g_r-w_m)\bar{y}Q - g_r\bar{x} - J(e) \\ &\quad -(p+g_r-b)\int_c^d \int_l^{yQ-\gamma(p,e)} (yQ-x-\gamma(p,e))f(x)g(y)dxdy \qquad (4.20)\end{aligned}$$

If the above contract coordinates the supply chain then the optimal decisions of the retailer would be same as those obtained in the centralized setting. From the first order conditions for optimality of the retailer's profit function with respect to Q, p and e, we get

$$\frac{\partial \Pi_r(Q,p,e)}{\partial Q} = (p+g_r-w_m)\bar{y} - (p+g_r-b)\int_c^d \int_l^{yQ^*-\gamma(p,e)} yf(x)g(y)dxdy, \qquad (4.21)$$

$$\frac{\partial \Pi_r(Q,p,e)}{\partial p} = S(Q,p^*,e) + (p+g_r-b)\frac{\partial S(Q,p^*,e)}{\partial p}, \qquad (4.22)$$

and $$\frac{\partial \Pi_r(Q,p,e)}{\partial e} = (p+g_r-b)\frac{\partial S(Q,p,e^*)}{\partial e} - J'(e^*) \qquad (4.23)$$

Comparing equations (4.21) with (4.7), (4.22) with (4.9) and (4.23) with (4.10), we find that when $b=v$ and $w_m = [c_m + h_s + (p_s - h_s)H(Q/R)]/\bar{y}$, the manufacturer's pricing strategy will be able to align the retailer's ordering, pricing, and effort decisions with the centralized system i.e., $Q^c = Q^*$, $p^c = p^*$ and $e^c = e^*$. Putting the values of the buyback price $b=v$ and wholesale price w_m in equation (4.20), the retailer's expected profit is seen to be equal to the expected total channel profit under the centralized setting, leaving other channel members at zero profit margin. This leads to the results given in Proposition 4.5.

Proposition 4.5 *A composite contract having buyback between the retailer and the manufacturer as a component with any other contract between the supplier and the manufacturer fails to achieve win-win outcome for all the members.* ■

4.5.2 Contingent buyback with SRP contract and revenue sharing contract

Now, we turn to the case where the manufacturer offers a target sales rebate and penalty (SRP) contract, in addition to contingent buyback policy with the retailer by setting the rules of pricing while postponing the determination of the final contract prices between them. In a target sales rebate and penalty contract, the manufacturer sets up a sales target Q_0 in front of the retailer. If the retailer's sales quantity is beyond the target then he will enjoy a per unit rebate τ for sold products beyond Q_0; otherwise, the retailer has to pay a penalty τ to the manufacturer for each unit of unsold products below the target. Using this contract, the manufacturer influences the retailer to sell more, which in turn enhances the retailer's sales effort level and order quantity. Under this proposed contingent buyback with SRP contract and revenue sharing contract, the retailer's profit is given by

$$\begin{aligned}\Pi_r(Q,p,e) &= pS(Q,p,e) + bI(Q,p,e) - g_r L(Q,p,e) - w_m E[yQ] \\ &\quad +\tau[S(Q,p,j) - Q_0] - J(e) \\ &= (p + g_r - w_m + \tau)\bar{y}Q - \tau Q_0 - (p + g_r - b + \tau) \\ &\quad \times \int_c^d \int_l^{yQ-\gamma(p,e)} (yQ - x\gamma(p,e))f(x)g(y)dxdy \\ &\quad -g_r\bar{x} - J(e) \qquad (4.24)\end{aligned}$$

The optimal order quantity Q_r^*, retail price p^*, and sales effort level e^* for the retailer under this setting are obtained from the first order optimality conditions as

$$\int_c^d \int_l^{yQ_r^*-\gamma(p,e)} yf(x)g(y)dxdy = \frac{(p + g_r + \tau - w_m)\bar{y}}{(p + g_r - b + \tau)} \qquad (4.25)$$

$$\begin{aligned}&\int_c^d \int_l^{yQ-\gamma(p,e)} (yQ - x - \gamma(p,e))f(x)g(y)dxdy \\ &+\beta(p^* + g_r - b + \tau)\int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dy = \bar{y}Q \qquad (4.26)\end{aligned}$$

and $$k(p + g_r - b + \tau)\int_c^d \int_l^{yQ-\gamma(p,e)} f(x)g(y)dy + \mu e = 0. \qquad (4.27)$$

Now we consider the sub-optimal setting where the retailer and the manufacturer work together.The combined profit of the retailer and the manufacturer is

$$\begin{aligned}\Pi_{mr}(Q,p,e) &= pS(Q,p,e) + vI(Q,p,e) - g_r L(Q,p,e) - (w_s + c_m)Q - J(e) \\ &= (p + g_r)\bar{y} - w_s - c_m)Q - g_r\bar{x} - J(e) - (p + g_r - v) \\ &\quad \times \int_c^d \int_l^{yQ-\gamma(p,e)} (yQ - x - \gamma(p,e)) f(x)g(y)dxdy. \end{aligned} \tag{4.28}$$

Hence, the optimal order quantity Q^*_{mr} for the total profit of the retailer and the manufacturer satisfies the following equation:

$$\int_c^d \int_l^{yQ^*_{mr}-\gamma(p,e)} yf(x)g(y)dxdy = \frac{(p\bar{y} + g_r\bar{y} - w_s - c_m)}{(p + g_r - v)} \tag{4.29}$$

Although the manufacturer has random yield in production, the final order quantity of the sub-supply chain is same as that ordered by the retailer, since there is no incentive for the manufacturer to bear the risk of over-production. In order to achieve full coordination between the manufacturer and the retailer, the relation $Q^*_{mr} = Q^*_r$ must hold. Comparing equations (4.25) with (4.29), we have that the optimal order quantity of the retailer and the joint order quantity of the retailer and the manufacturer are same when the optimal contract parameters are as follows:

$$b = \tau + v \tag{4.30}$$

$$\text{and} \quad w_s = w_m\bar{y} - \tau\bar{y} - c_m. \tag{4.31}$$

From equation (4.31), it is easy to derive that $w_s < w_m$, since $\bar{y} < 1$, as expected.

Next, we assume that the manufacturer offers a revenue sharing contract to the supplier. The expected profit of the manufacturer under the transfer payment with the retailer and the supplier is given by

$$\begin{aligned}\Pi_m &= \{(\phi w_m - \tau)\bar{y} - w_s - c_m\}Q - (b - \tau - v)\int_c^d \int_l^{yQ-\gamma(p,e)} (yQ - x - \gamma(p,e)) \\ &\quad \times f(x)g(y)dxdy + \tau Q_0 \end{aligned} \tag{4.32}$$

Also, the supplier's expected profit can be expressed as

$$\begin{aligned}\Pi_s(R) &= w_sQ + (1-\phi)w_mE[yQ] - c_s'E[(Q-zR)^+] + v_sE[(zR-Q)^+] - c_sR \\ &= \{w_s + (1-\phi)w_m\bar{y} - v_s\}Q - (c_s' - v_s)\int_a^{Q/R}(Q-zR)h(z)dz \\ &\quad -(c_s - v_s\bar{z})R \end{aligned} \tag{4.33}$$

Proposition 4.6 *(i)The optimal solution R^* for the supplier satisfies the equation*

$$\int_a^{Q/R^*} zh(z)dz = \frac{c_s - v_s\bar{z}}{(c_s' - v_s)} \tag{4.34}$$

(ii)$R^*(Q) = K_2Q$ where K_2 is a constant.

Proof. The proof is omitted as it is similar to that of Proposition 4.3. ■

The proposed composite contract works as follows. By contingent buyback policy with SRP, we set the rule of pricing while postponing the determination of the final contract prices (*e.g.*, wholesale price, sales rebate and penalty) and the sales target, as the trade contract between the supplier and the manufacturer is yet to be finalized. Once the exact contract parameters are settled between the upstream members, the retailer and the manufacturer would decide their final contract parameters, according to the rule of pricing such that they would be able to secure their own profit shares in presence of the contract signed between the upstream members. In order to achieve supply chain coordination, the manufacturer's composite contract policy (contingent buyback with SRP for retailer and revenue sharing for supplier) can be used to convince the retailer to order and the supplier to produce up to the quantity of the centralized supply chain. Comparing equations (4.25) with (4.7), (4.34) with (4.8), (4.26) with (4.9) and (4.27) with (4.10), we find that when the conditions (4.30), (4.31) and

$$w_m = \tau + \frac{c_m + v_s + (c_s' - v_s)H\left(\frac{Q}{R}\right)}{\bar{y}}. \tag{4.35}$$

hold, we have $R^* = R^c$, $Q^* = Q^c$, $p^* = p^c$, and $e^* = e^c$. This leads to Proposition 4.7.

Proposition 4.7 *The contingent buyback contract with target sales rebate and penalty between the retailer and the manufacturer, and a revenue sharing contract between the manufacturer and the supplier with contract parameters satisfying equations (4.30), (4.31), and (4.35) can fully coordinate the supply chain, and profits may be allocated arbitrarily by varying ϕ, τ, and Q_0.* ■

Equation (4.30) gives $b = v+\tau < \tau+(w_s+c_m)/\bar{y} = w_m$, suggesting that a (w_m, b, Q_0, τ) flexible buyback scheme with SRP prevents the retailer from earning profit by over-ordering. Equation (4.31) implies that, in the (w_s, ϕ) revenue sharing scheme, the wholesale price for raw material depends only on the prices of raw material at the secondary market and the production yield of the raw material. This feature of the (w_s, ϕ) revenue sharing scheme is unique and presents an interesting implementation challenge. We would have rather expected the wholesale price of raw material (w_s) to depend on raw material production cost (c_s) and revenue sharing parameter (ϕ), as suggested in the existing literature. In addition, when equation (4.31) holds, there exists possibility for the supplier to cheat the manufacturer by claiming enhanced c_s' and reduced v_s which would result in coordination failure. Successful coordination between the supplier and the manufacturer therefore needs an additional mechanism of information sharing regarding production yield and emergency resource's prices.

Under the (w_m, b, Q_0, τ) flexible buyback contract with SRP, the retailer is incentivized to sell a product at a retail price even lower than the wholesale price charged by the manufacturer when his actual sale leaves the sales target behind, since selling a product beyond the sales target allows the retailer to earn a per unit profit margin $p - w_m + \tau$, *i.e.*, for a positive profit margin, $p > w_m - \tau$. It is therefore deduced that after reaching the sales target, the retailer may reduce the retail price even lesser than his purchasing cost to influence the market demand, and secure profit when $p > (w_s+c_m)/\bar{y}$. When the retailer's actual sale is below the sales target, even then the retailer is incentivized to enhance sales effort rather than claiming credit for the unsold product from the manufacturer, since selling a product allows him to earn a per unit margin $p-w_m$, while buyback gives him a per unit margin $b-w_m-\tau = v-\tau-(w_s+c_m)/\bar{y}$ which is always negative. Thus the retailer may sell a product at a retail price $p > w_m$ with a positive profit margin or may even reduce retail price to influence the market demand with $p > v - \tau - (w_s + c_m)/\bar{y}$ instead of clamming buyback credit and it is quite easy for the retailer to sell at this retail price which is lesser than it's salvage value. In either way, the incentive of the retailer is to sell the products directly at the market.

However, it is still possible for the supplier to earn more profit by claiming fabricated values of c_s' and v_s. For successful supply chain coordination, the manufacturer therefore requires to monitor the raw material production process and verify the prices of raw material at the secondary market. Although the retailer has no such incentive to earn profit by labelling the leftover inventories as sold out products or vice versa, the manufacturer does possess information regarding salvage value of the product, since he himself sells leftover inventories there. To sum up, the manufacturer must be the dominant party and possess full control over the dynamics of the entire chain, as well as must have access to each and every information related to supply chain functioning and implementation of the contract.

Observation: For a given target sales rebate, if the purchasing price of the raw material at the secondary market increases, the supplier should raise both wholesale price and planed production quantity of raw material and the manufacturer should raise wholesale price whereas the retailer should cut off his order quantity with higher retail price and lower sales effort. ■

Under the composite contract, the supplier's optimal wholesale price is independent of the production cost (c_s). The supplier is incentivised to produce raw material instead of buying from the secondary market because if all the raw materials are brought from the secondary market, his expected profit would be $\{(1-\phi)w_m\bar{y} + (w_s - c_s')\}Q$, where the second term is always negative, and consequently his profit would fall down. We find that the behavioral implications of the composite contract are very much aligned with the manufacturer's objectives. Further, we have the following observations:

- The supplier is motivated to produce raw material on his own instead of mitigating the gap using the secondary market.

- The manufacturer should be dominant enough to have full control over the entire chain.

- The retailer is motivated to sell the product with a higher sales effort.

4.5.3 Implementation of coordination contract

For other contract parameters settled, the manufacturer ensures profit margin at least equal to that obtained in the wholesale price-only scenario by setting sales target

$$Q_0 \geq [(w_m\bar{y} - c_m - w_s)Q^d + (1-\phi)w_{mb}\bar{y}Q^*]/(b-v)$$

in front of the retailer. On the other hand, the manufacturer to earn maximum profit, leaving the profits of all other entities same as those obtained in wholesale price-only contract scenario, sales target Q_0 should be set as

$$Q_0 = [\Pi_b - \Pi_r - \Pi_s + (1-\phi)w_{mb}\bar{y}Q^*]/(b-v)$$

providing the upper limit of Q_0. We conclude that there exists a range of sales target (Q_0) for every combination of τ and ϕ for which the composite contract ensures win-win situation for all the chain members. Clearly, the sales target Q_0 set by the manufacturer to the retailer, depends on the relative bargaining power of the manufacturer and the retailer. The manufacturer, being more powerful, wants to ensure that the retailer earns a desirable enough profit to induce him to accept coordination policy, and capture excess profit by himself through coordination. However, since the retailer has the power of setting the order quantity, retail price and sales effort simultaneously, the manufacturer can't force the retailer for a higher sales target if he wants the retailer to set order quantity, retail price and sales effort to remain same as those in the integrated supply chain. Otherwise, if the manufacturer becomes indifferent about the retailer's power and sets a higher sales target in front of the retailer, the retailer will reduce both order quantity and sales effort and rise retail price, resulting in coordination failure. Therefore, the manufacturer should choose a set of actions (w_s, ϕ) for the supplier and a set of actions (w_m, b, Q_0, τ) for the retailer, which satisfy the equations (4.30), (4.31), and (4.35), in order to coordinate the supply chain and share the additional profit that accrued through supply chain coordination.

4.5.4 Difficulties of other contracts that coordinate the supply chain

We have shown that supply chain coordination is achieved when the manufacturer sets a wholesale price $w_m = [c_m + h_s + (p_s - h_s)H(Q/R)]/\bar{y}$, although the entire supply chain profit is bagged by the retailer. A two-part tariff contract with this wholesale price can also achieve supply chain coordination and allocate the channel profit among the chain members. Under a two-part tariff contract, the manufacturer charges a per unit fee along with a fixed sum as a stocking or licensing fee. The two-part tariff contract has been extensively discussed by Moorthy (1987) and Coughlan and Wernerfelt (1989) who have shown its effectiveness in a manufacturer-controlled supply chain. However, this contract theoretically allows the manufacturer to arbitrarily claim profit for himself. In the current scenario, as the retailer retains the power to control the retail price and the sales effort to influence the market demand, charging a high fixed fee to him might result in difficulty in implementing such a contract. Furthermore, since the manufacturer is well informed about the retailer's power to influence the market demand, the two-part tariff contract is not profitable for the chain members. Since it is impossible to forecast accurately the customer demand, setting the fixed fee without knowing the actual sale, could leave the retailer unprofitable. Similarly, when the retailer tries to raise the retail price and reduce the order quantity, the manufacturer can't anticipate preciously how to react against this. So, a two-part tariff contract with a wholesale price at marginal cost of the manufacturer leaves the manufacturer at worse off.

Cachon and Lariviere (2005) showed that a price-discount sharing contract is effective in the price setting newsvendor case. For successful implementation of the contract, they assumed that the retailer has computer and bar code system to track each sale. So it should not be difficult for the manufacturer to monitor his clients. However, a retailer or a raw material supplier may not agree with such agrement all the time, since the extra investment in installing those technologies would incur extra cost. Our proposed composite contract requires no special monitoring because the supplier must submit the secondary market's documents in revenue sharing contract. Moreover, the manufacturer need not bother about the cost to exert sales effort, since he is not sharing any part of it in any way.

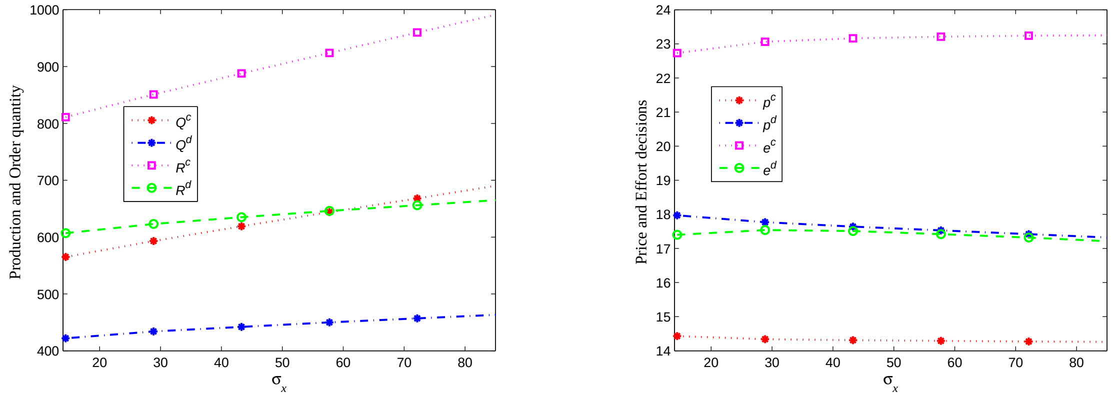
4.6 Numerical illustration

A numerical study is provided to further illustrate the developed model. The random demand is assumed to be of the form $D = \gamma(p, e) + x$, where $\gamma(p, e) = \alpha - \beta p + ke$, $\alpha, \beta, k > 0$, showing that the actual demand is linear in its deterministic and stochastic components; α is the base demand; β and k denote price and sales effort sensitivity parameters, respectively; x is a particular value of the random variable X defined in Notations. As proposed by He et al. (2009), the retailer's cost to exert sales effort level e is assumed to be a convex function of e, specified by $J(e) = \mu\frac{e^2}{2}$, where μ is a parameter representing sales effort cost sensitivity. Following Giri et al. (2016), the parameter-values are set as follows: $v = 4$, $c_m = 1$, $c_s = 2.5$, $v_s = 1$, $c_s' = 7$. Productions of the raw-material supplier as well as the manufacturer are assumed to follow uniform distributions with means and standard deviations ($\bar{z} = 0.7$, $\sigma_z = 0.1$), and ($\bar{y} = 0.8$, $\sigma_y = 0.05$), respectively. In view of the assumptions and optimality conditions provided in Section 3, we further choose remaining parameter-values as: $g_r = 9.5$, $\alpha = 700$, $\mu = 1.5$, $\beta = 25$, $k = 2$, $Q_0 = 580$, $\tau = 5$, $\phi = 0.7$ in appropriate units. The stochastic factor x of the market demand is uniformly distributed with mean $\bar{x} = 50$ and standard deviation $\sigma_x = 50/\sqrt{3}$.

Table 4.1: Optimal decisions under different scenarios

Model	Q	R	p	e	Π_s	Π_m	Π_r	Π	w_s	w_m
Centralized	593	851	14.34	23.05	-	-	-	6670	-	-
Decentralized	434	623	17.77	17.54	1457	1189	3546	6193	7.3	13.80
Coordinated	593	851	14.34	23.05	1592	1282	3795	6670	3.94	11.17

Table 4.1 establishes that the proposed composite contract is able to successfully coordinate the supply chain, paying all the entities better off compared to the wholesale price-only contract. As illustrated in Table 4.1, the retailer's optimal order quantity and sales effort level under the contract are higher than their counterparts in wholesale price only contract whereas the optimal selling price is much lower, resulting in generating more demand and consequently more profit for the entire chain. A higher order quantity also has the possibility to generate more profit for the supply chain by meeting more demand.



(a) Effect of σ_x on Q and R.

(b) Effect of σ_y on the optimal decisions.

Fig. 4.1: Effect of σ_x on decision variables

The standard deviation of the demand distribution is often taken as a measure of uncertainty; the more the uncertainty, the more the deviation. Fig. 4.1 shows that, with higher demand uncertainty, the retailer should set lower selling price and order more products irrespective of the fact whether he is under contract or not. A higher order quantity from the retailer's end in turn enhances production and ordered quantities of the manufacturer and the supplier. It is also an acceptable strategy for the retailer to reduce selling price to attract more demand. However, since there is no inherent risk sharing mechanism present in price only contract, the resulting lower individual expected profit under price only contract has a negative effect on sales effort; the retailer should invest less amount of money in it. Furthermore, as demand uncertainty increases, the supplier should set higher planned production quantity so as to reduce the risk of buying items from spot market at a higher rate.

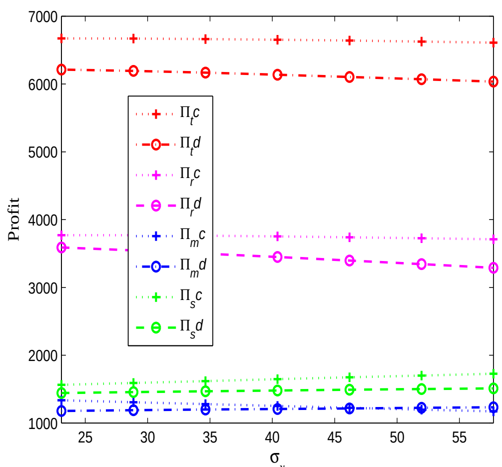
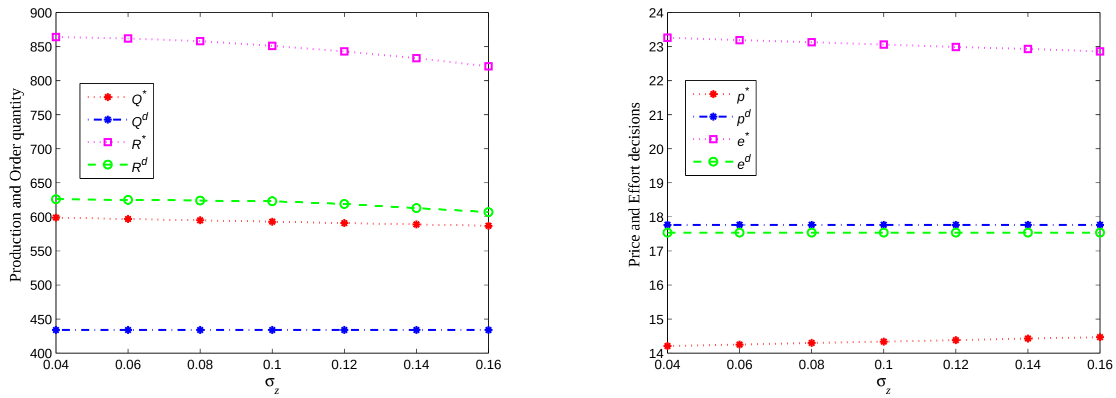


Fig. 4.2: Effect of σ_x on expected profits.

Fig. 4.2 illustrates the changes in the expected profits of the supply chain system and its individual members with varying demand uncertainty (σ_x). It is seen that, under composite contract, the whole chain and its individual members earn more profit.

In general, a higher uncertainty has a negative effect on the system profit due to overstocking and under-stocking risks. It is observed that, for a fixed set of contract parameters, the manufacturer loses his profit share with higher demand uncertainty, indicating that contract parameters should be redesigned in favor of the manufacturer with varying demand uncertainty.



(a) Effect of σ_z on Q and R. **(b)** Effect of σ_z on p and e.

Fig. 4.3: Effect of σ_z on decision variables

Fig. 4.3 exhibits that yield uncertainty at the supplier has no effect on the optimal decisions of the retailer under wholesale price contract, whereas the supplier reduces its planned production quantity to mitigate the risk. On the contrary, the composite contract compels the retailer to order lesser amount under higher yield uncertainty, and raise price and reduce effort level simultaneously to secure per unit profit margin. Nevertheless, the total profit of the system reduces with higher yield uncertainty under both the scenarios (decentralized and coordinated), as is evident from Fig. 4.4. Also, under price only contract, yield uncertainty affects the expected profit of the supplier only. It is also observed that, for a fixed set of contract parameters, the supplier gets benefitted whereas both the manufacturer and the retailer lose their profit shares with higher yield uncertainty.

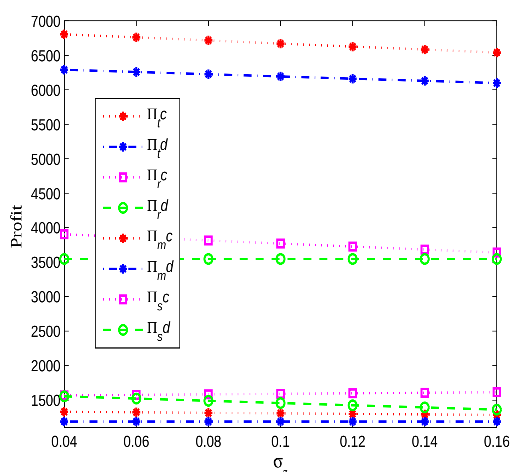
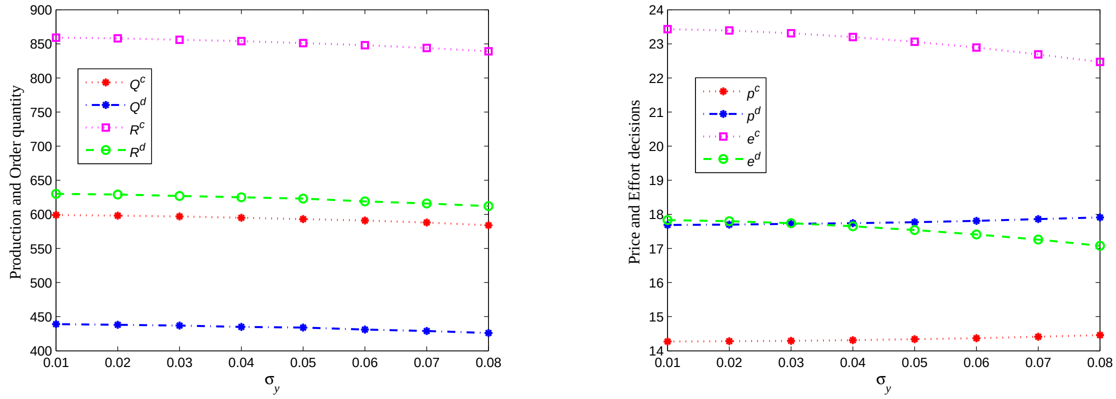


Fig. 4.4: Effect of σ_z on expected profits.



(a) Effect of σ_y on Q and R.

(b) Effect of σ_y on p and e.

Fig. 4.5: Effect of σ_y on decision variables

Figs. 4.5 and 4.6 illustrate the effect of yield uncertainty at the manufacturer on the supply chain performance. It is obvious that a higher yield variability exposes a bigger risk, leading the manufacturer to choose a lower production quantity. The manufacturer reduces his production input quantity to hedge the associated risks, resulting in reduction in profits for all the channel members. The manufacturer should be inclined towards improving production technology and stabilizing the production yield variability as much as possible.

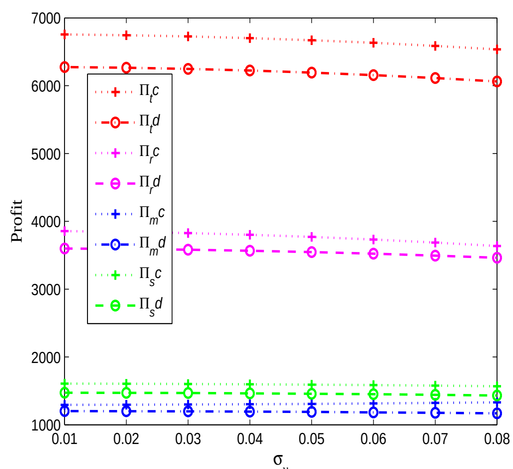


Fig. 4.6: Effect of σ_y on expected profits.

Also, for a fixed set of contract parameters, the manufacturer gets benefitted whereas other members lose their profit shares with higher yield uncertainty.

As observed by Lee and Whang (2002), having access to a secondary market is beneficial for any supply chain member. Table 4.2 suggests that the supplier should aim at reducing dependence on the secondary market by raising his planned production quantity; however, it does not affect the business strategy of the retailer under wholesale price contract. For a predetermined set of contract parameters, the supplier raises wholesale price to prevent additional loss due to tentative salvaging, and consequently enhancing the manufacturer's wholesale price and

Table 4.2: Effect of the procurement cost c'_s on channel performance

	Wholesale price contract				Composite contract									
p_s	Q	R	Π_s	Π_d	Q^*	R^*	w_{sb}	w_{mb}	Π_{sb}	Π_{mb}	Π_{rb}	Π_b	p^*	e^*
5	434	565	1527	6263	597	778	3.78	10.97	1575	1299	3891	6766	14.24	23.20
6	434	598	1487	6223	595	820	3.87	11.09	1584	1290	3836	6711	14.30	23.12
7	434	623	1457	6193	593	851	3.94	11.17	1592	1282	3795	6670	14.34	23.05
8	434	643	1434	6170	591	876	3.99	11.24	1597	1277	3764	6639	14.37	23.01
9	434	659	1416	6152	590	897	4.03	11.29	1601	1273	3739	6614	14.39	22.97

retail price too. The retailer cuts down the order quantity and the investment in sales effort. However, the supplier manages to secure more profit share due to higher wholesale price. Similarly, a higher salvage value of raw material induces the supplier to raise production level and reduce wholesale price to encourage upstream members to order more. A higher order quantity compels the retailer to reduce retail price and raise the sales effort level to generate additional demand, resulting in higher channel profit.

Table 4.3: Effect of the salvage value v_s on channel performance

	Wholesale price contract				Composite contract									
h_s	Q	R	Π_s	Π_d	Q^*	R^*	w_{sb}	w_{mb}	Π_{sb}	Π_{mb}	Π_{rb}	Π_b	p^*	e^*
0	434	599	1434	6170	591	817	3.99	11.24	1597	1277	3764	6639	14.37	23.01
0.5	434	609	1444	6180	592	832	3.97	11.21	1595	1279	3778	6653	14.35	23.03
1	434	623	1457	6193	593	851	3.94	11.17	1592	1282	3795	6670	14.34	23.05
1.5	434	639	1473	6209	594	876	3.90	11.13	1588	1286	3817	6692	14.32	23.09
2	434	662	1493	6229	595	908	3.86	11.07	1583	1291	3844	6719	14.29	23.13

Table 4.3 shows that both the downstream members get benefited under composite contract except the supplier who has to compromise with his own profit share to ensure sale of product through the supply chain rather than selling them at salvage value afterwards. Finally, Table 4.4 shows that, a higher salvage value of the finished product

Table 4.4: Effect of the salvage value v on channel performance

	Wholesale price contract						Composite contract							
v	p	e	Π_s	Π_m	Π_r	Π_d	w_{sb}	w_{mb}	Π_{sb}	Π_{mb}	Π_{rb}	Π	p^*	e^*
2	17.47	17.76	1440	1175	3512	6129	3.94	11.17	1565	1309	3718	6593	14.32	23.00
3	17.75	17.50	1448	1182	3528	6160	3.94	11.17	1578	1296	3755	6630	14.33	23.03
4	17.77	17.54	1457	1189	3546	6193	3.94	11.17	1592	1282	3795	6670	14.34	23.05
5	17.79	17.58	1467	1197	3565	6229	3.94	11.17	1607	1267	3840	6715	14.34	23.08
6	17.80	17.62	1477	1205	3586	6269	3.94	11.17	1625	1249	3889	6764	14.33	23.09

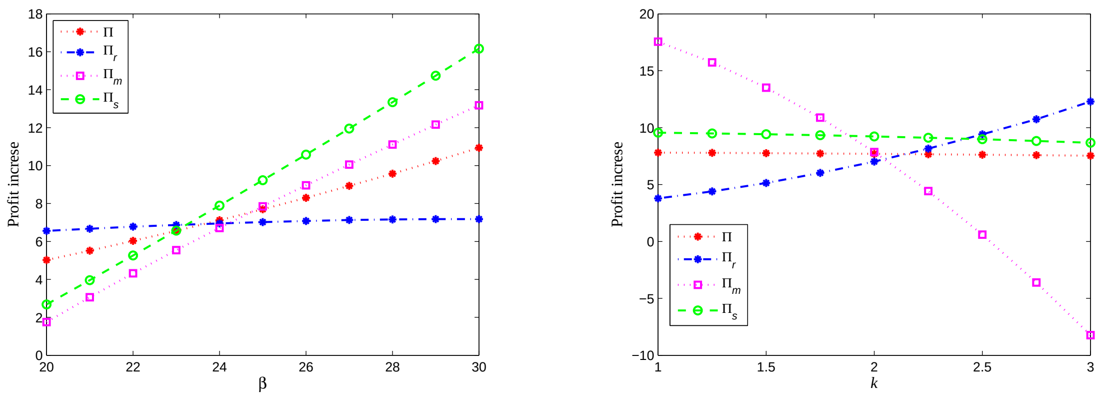
raises profit margin for both the supplier and the retailer due to higher ordered and planned production quantities, but the manufacturer suffers for a predetermined set of contract parameters.

We now aim to study a predetermined market situation from the perspective of different price-demand relationships. Till now the linear relationship between demand and price has been considered which is of the specific form $D = \alpha - \beta p + ke + x$, having parameter-values as $\alpha = 700$, $\beta = 25$, $k = 2$. Keeping the market demand fixed as may be obtained from the optimal values of the decision variables provided in Table 4.1, a simple computer simulation reveals that the same market scenario may also be represented by an exponential price-demand relationship of the form $D = \alpha\beta^{-p}e^k + x$ with parameter-values $\alpha = 899$, $\beta = 1.0659$, $k = 20.00229312$, or by a quadratic relationship as $D = (\alpha p^2 - \beta p + \gamma)\, k^j + x$ with parameter-values $\alpha = 0.1223$, $\beta = 30.3304$, $k = 1.001$, $\gamma = 853$. Table 4.5 exhibits the applicability of the proposed model under various price-demand relationships. It is suggested that the contract parameters are to be set depending on the demand pattern observed from the historical data, since a fixed set of parameter-values allocates different profit shares among the channel members for different demand patterns.

Table 4.5: Channel performance under different demand scenarios

Demand	Q	R	p	e	Π_s	Π_m	Π_r	Π
Linear	593	851	14.34	23.05	1592	1307	3770	6670
Exponential	554	796	13.78	3.02	1488	1411	3408	6308
Quadratic	648	930	14.44	0	1739	1160	4981	7881

Since the market demand depends on both the retail price and the sales effort of the retailer, then the retail price sensitivity parameters β and sales effort sensitivity parameters k are two major factors to influence demand. Figs. 4.7 - 4.9 illustrate the variation in profit under Contingent buyback with SRP contract and revenue sharing contract with different values of such parameters. We plot 'percentage profit increase' of the channel members along with the total supply chain, where $\pi_i = 100 \times (\pi_i^c - \pi_i^w)/\pi_i^w$ for $i = r, m, s$ and T. Since demand decreases linearly with higher values of β, the retailer is forced to reduce the retail price and raise the sales effort to mitigate the negative effects on demand. Fig. 4.8 show that μ has a negative impact on retailer's performance because as μ increase the expenditure regarding sales effort also increase. It is observed that a higher sales rebate τ has a negative impact on retailer's performance but has positive impact on performance of both the manufacturer and the supplier, which are shown in Fig. 4.9.



(a) Effect of β on efficiency of the chain. **(b)** Effect of k on efficiency of the chain.

Fig. 4.7: Performance of coordinated chain with respect to demand parameters

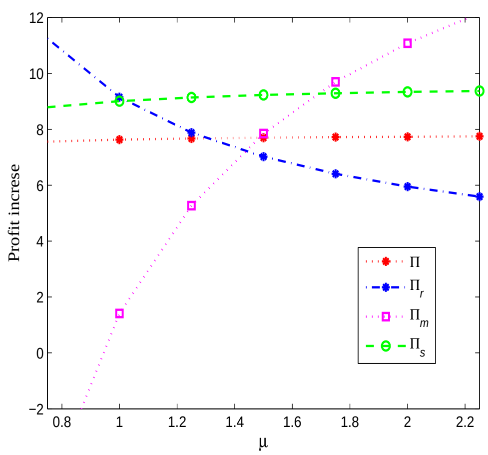


Fig. 4.8: Effect of μ on coordinated channel performance.

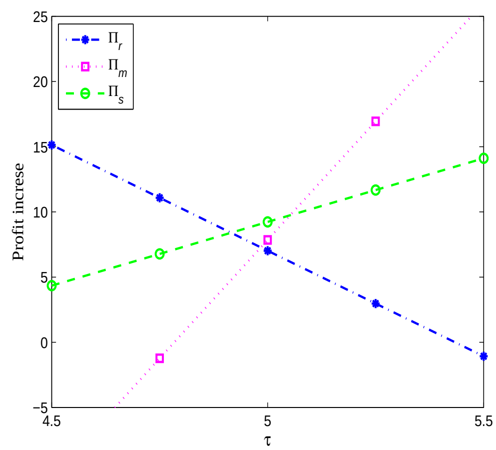


Fig. 4.9: Effect of τ on coordinated channel performance.

4.6.1 Managerial insights

From the numerical study conducted in the previous section, we draw the following managerial insights:

- The proposed composite contract is applicable to a wide range of market scenarios represented by different forms of price-effort relationship. It can reduce the double marginalization effect too.

- Based on available information, if the manager is convinced that the demand is going to be more uncertain for reasons beyond control in a particular business cycle, he should reduce both the sales effort and the retail price, and order more stock under price only contract. Even if the proposed composite contract is established, the retail price should still be lower, but the sales effort should be set at a higher level to induce more demand. Also, contract parameters should be redesigned in favor of the manufacturer with varying demand uncertainty by raising sales target and/or raising per unit penalty.

- Under composite contract, the manager should reduce the production and order quantities, and the retail price to mitigate a higher yield uncertainty at production level; also, more promotional effort should be made. Further, the contract parameters should be redesigned accordingly against the entity with higher yield variation. The observation is consistent with the findings of He and Zhang (2008). However, if the manager fails to implement such a contract, the retailer should keep other decisions unaltered under higher yield at the supplier whereas redesign pricing strategy to reduce demand under higher yield at the manufacturer, and reduce order quantities in both the cases.

- If the manager can sell leftover products, be it raw materials or finished ones, at higher salvage rate, he should raise the order quantity placed at the manufacturer. Both unit penalty cost τ and revenue fraction ϕ should be raised for higher salvage value of the finished product, penalty cost τ should be raised but revenue fraction ϕ should be reduced for higher salvage value of raw material, and penalty cost τ should be reduced but revenue fraction ϕ should be raised for higher price of raw material at the secondary market.

4.7 Conclusion

Random yield is common in many industries. Traditional newsvendor setting allows industries to access a secondary market for emergency resource. This article considers unavailability of secondary market at the manufacturing stage in a supplier-manufacturer-retailer supply chain. The retailer faces both supply and demand uncertainties and makes joint decision on retail price, sales effort and order quantity, and the supplier makes joint decision on wholesale price and production amount. It is proven that perfect supply chain coordination can be achieved by a composite contract offered by the manufacturer. This composite contract consists of two components − one is revenue sharing contract (w_s, ϕ) between the supplier and the manufacturer, and the other one is contingent buyback with target sales rebate and penalty (w_m, b, Q_0, τ) between the retailer and the manufacturer. The applicability of the proposed model is established and valuable managerial insights are provided from numerical results.

In this chapter, we have provided insights and implications of the proposed supply chain contract under various uncertainties and influences. Certainly our work has some limitations and restrictions that can be explored in future research. For instance, we have allowed the manufacturer to control business policies unilaterally. One can study a multi-echelon supply chain model where multiple entities may dominate the supply chain. In our study, we have considered the existence of the secondary market for both the raw-material supplier and the retailer. It would be more realistic and challenging to extend the model by considering complete absence of the secondary market as backup resource anywhere throughout the chain, and develop risk sharing policies accordingly. In addition, the current study considers that the qualities of produced raw materials and the ones bought from the secondary market by the supplier are the same. Consideration of quality difference of the produced raw materials and the ones bought from the secondary market by the supplier, and developing a model where the yield of the manufacturer depends on the produced quantity of the raw material supplier would also be potential future research directions. Finally, the current study focuses on a single-period model. Extending the present model to a multi-period model would be a difficult but challenging task for future research.

Chapter 5

Coordination mechanisms of a three-layer supply chain under demand and supply risk uncertainties*

5.1 Introduction

From the World Trade Center terrorist attack on 11 September, 2001 and blackout on 14 August, 2003 in the U.S. to recent political instability, natural disasters and destructive competitive acts increase the complexity, uncertainty and ambiguity of globalized supply chain. There are mainly two kinds of risk of uncertainty that affect supply chain management and network design. The first risk of uncertainty grows from the matter of demand and supply coordination and the second one grows from supply uncertainty which is emblematically modelled as complete supply disruption where supply halts completely, or yield uncertainty where the supplied quantity can fulfil a random fraction of the placed order size. We incorporate such supply uncertainties with normal demand-supply coordination risks. This chapter builds on the literature regarding the management of the risk of uncertainty, and on the framework of supply

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chain coordination.

To excel in intense global competition, today's supply chain is becoming more globalised to enjoy cheaper raw material, lower labour cost, tax policy, advance manufacturing technologies and other financial benefits, all of which reduce the per unit production cost of a product. Such globalized supply chain networks frequently experience supply disruption arising from operational contingencies (like system failures, equipment shortage, web server error, strikes), natural hazards (like hurricanes, earthquakes, storms and virus), political instability and terrorism. Taiwan earthquake in 1999 halted the production of many semiconductor manufacturing companies causing shipping disruption of 70 percent of word's graphics cards and 10 percent of data hard disk components (Savage, 1999). Emission of fire from Eyjafjllajokull volcano in Iceland on 20th March, 2010 which sent shock waves through several production plants in Europe due to many air transport cancellation throughout the world. A chemical accident in Bhopal in India has driven serious environmental and massive economic consequences (Kleindorfer and Wu, 2003).

To hedge against uncertain supply disruption and to mitigate the devastating effect of supply disruption, many firms are realising the importance of several mitigation strategies. One of the simplest and effective policies adopted by several buyers is 'multi-sourcing policy' which allows the buyer to have more than one resource providing similar quality attributes but may differ in terms of pricing. Another one is 'contract agreement policy' among all the supply chain entities to enhance performance. Using several resources or contracts with them is a critical but crucial decision for buyers, especially for them who produce short life-cycle products. Then how to envision a contract mechanism under multi-sourcing strategies to reduce stock-out risk is an interesting challenge for the buying firms. In this chapter, we investigate risk assessment and risk mitigation strategies to make the supply chain more elastic from the perspective of individual entities as well as the entire channel through implementation of contract mechanism among the chain members.

The supply-demand coordination risk has been the central concern of activities in the previous chapters and studied the impact of demand fluctuation on the performance of supply chains. In the previous chapter, we have discussed about the uncertain supply which takes the form of random yield when the manufacturer sources raw material from only one supplier. In this chapter, as an extension of our previous work, we consider supply disruption as another form of supply uncertainty and considered that the raw materials are procured from two unreliable suppliers without any emergency resource. Our main goal is to design a joint sourcing and contracting policy that enables supply chain entities to mitigate both demand and supply risks of uncertainty under the procurement from two unreliable suppliers.

We consider a single period three-echelon supply chain with three possible uncertainties (independent to each other), in which a retailer faces an uncertain market demand for a short shelf-life product and sources it from a manufacturer under voluntary regimes. The manufacturer sources the raw material from two unreliable suppliers without any emergency resource. The manufacturer's main supplier who delivers the order quantity at a cheaper wholesale price is prone to disruption and therefore may deliver all or nothing of the manufacturer's order, while the backup supplier who provides similar quality attributes at a comparatively higher wholesale price is prone to random yield and therefore can only fulfil a random fraction of the manufacturer's order. The two unreliable sources are usually geographically dispersed so that a supply risk at one supplier will not affect the other supplier *i.e.,* the risks of supply uncertainty at both the suppliers are independent. In this chapter, we assume that both the random yield and disruption risk are unresolved when the manufacturer places its order to both the unreliable suppliers simultaneously.

Under voluntary regime, one can look for a contract mechanism to cope with the demand-supply uncertainty. Available popular contract agreements are limited within only two levels that we considered in Chapter 3. These contracts are extendable in a multi-level supply chain by installing them between any two adjacent pairs of supply chain entities as shown in Chapter 4. But a pair-wise contract in a multi-level supply chain has a drawback from the perspective of simultaneous implementation *i.e.,* there may occur a situation where a firm can be benefitted even without signing the agreement when other entities have already signed (Van Der Rhee et al., 2010). Van Der Rhee et al. (2010) proposed a spanning revenue sharing contract in which one supply chain member takes the lead in making a single contract with all other entities in the supply chain. In this chapter, we assume that the retailer takes the lead in negotiating a revenue sharing contract with all other supply chain members to prevent the two-sided supply-demand uncertainty. Through this agreement, the retailer would take the optimal decisions on the following aspects. Under which condition, the spanning revenue sharing contract is desirable and when the conditions are satisfied, and how much to order from the manufacturer? On the other hand, the manufacturer also needs to decide its optimal order quantities from both the suppliers. In this contract, the manufacturer and both the suppliers are incentivized by the retailer to decrease their wholesale prices so that the retailer can increase its order quantity to satisfy more customer demand such that the expected profit of the entire supply chain increases.

Our work is closely related to the work of Li et al. (2017) who developed a model in which the deterministic demand is met up by two unreliable suppliers: one supplier is subject to random yield and the other one is unreliable due to disruption risk. They investigated three models depending on procurement strategies without any reliable supplier but they didn't consider channel coordination. Our article is also related to the work of Dada et al. (2007) in terms of supplier utilization. In the event of supplier selection for optimal order allocation, we consider a cheaper supplier having the risk of supply disruption and an expensive supplier having random yield in production, and find their relative uses. Our chapter expands on the acumen of Dada et al. (2007) by individually considering whether the risk of supply uncertainty occurs from random yield or supply disruption.

The rest of the chapter is organized as follows: Section 5.2 introduces basic settings of the problem under consideration including model description, assumptions and notation. Section 5.3 presents two benchmarks models: centralized model in subsection 5.3.1 and decentralized model in subsection 5.3.2 and their equilibrium solutions. In Section 5.4, the spanning revenue sharing contract is developed. A win-win situation that ensures chain coordination is also proposed in this section. Numerical results are analyzed in Section 5.5 for best practices in guiding supply disruption and demand uncertainty. We conclude the chapter in Section 5.6, stating conditions for implementation of the proposed model and scopes of future research.

5.2 Problem description and assumptions

We consider a supply chain where a single manufacturer sources raw materials from two unreliable suppliers (main supplier and backup supplier) to respond the order of a retailer who sells a short life-cycle product with uncertain demand over a single period of time. The main supplier who delivers order quantity at a cheaper wholesale price is subject to random disruption and may deliver all or nothing of the manufacturer's order, while the backup supplier who provides similar quality attributes at a comparatively higher wholesale price is subject to random yield *i.e.,* the backup supplier can fulfil only a random fraction of the manufacturer's order. From now onwards, we will term 'main supplier' as the 'cheaper supplier', and 'backup supplier' as the 'expensive supplier'. For mathematical tractability, we consider the following assumptions:

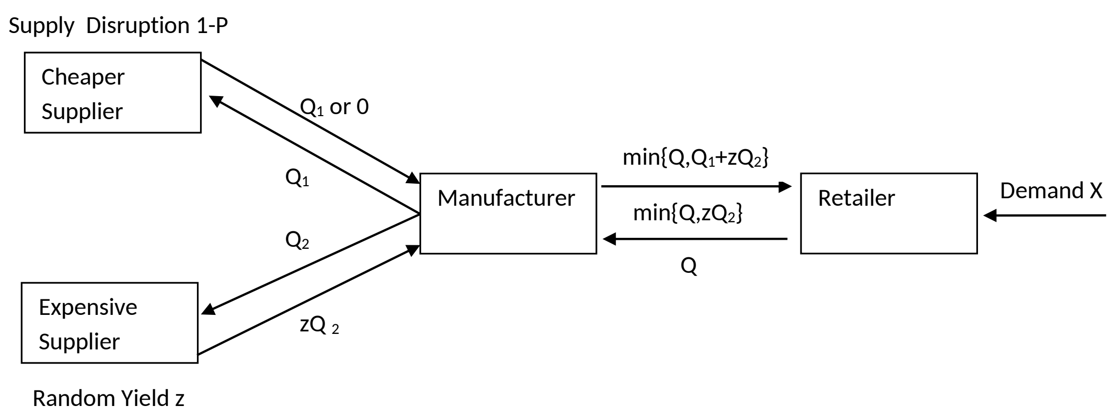


Fig. 5.1: Graphical representation of the proposed supply chain model with two unreliable suppliers.

- Only one order can be placed by a supply chain entity during a single selling season.
- All four entities (manufacturer, retailer, and two unreliable suppliers) involved in the supply chain are risk-neutral and pursue their individual profit maximization.
- All the entities start with zero on-hand inventory *i.e.,* there is no stock from previous period.

- There is a symmetric information flow under which all costs and profit parameters are known to each supply chain entity.

- All costs and profit parameters can be taken as exogenous parameters as all entities determine selling price/wholesale price and salvage value in advance and after that negotiate their order quantities.

The retailer faces a stochastic market demand x which is a positive continuous random variable having a general distribution over interval $[l, u]$ with cumulative distribution function $F(\cdot)$ and probability density function $f(\cdot)$ with mean $\bar{x}$ and variance σ_x^2. The manufacturer submits order Q_1 to the cheaper supplier and Q_2 to the expensive supplier simultaneously before the realization of the supply state of the cheaper supplier and actual demand (Li, 2017). The cheaper supplier can deliver full order quantity Q_1 with a wholesale price w_{s1} if not disrupted, but it delivers nothing, if disrupted. The probability that the cheaper supplier is not disrupted is $\alpha \in [0, 1]$. On the other hand, the expensive supplier can only delivers a random fraction z (having pdf $h(\cdot)$, cdf $H(\cdot)$ and range $[c, d]$, $0 \leq c < d \leq 1$) of order quantity Q_2 with a relatively higher wholesale price w_{s2} than that of the cheaper supplier (*i.e.,* $w_{s2} > w_{s1}$).

Two contract mechanisms are considered in this model viz. price-only contract and spanning revenue sharing contract. In price-only contract, the retailer and the manufacturer decide their order quantities based on their newsvendor problems without considering system wide profit. In this situation, the retailer takes the full risk of demand and supply uncertainty. Meanwhile the cheaper supplier takes the responsibility of its undelivered raw material, if its supply is disrupted. Under spanning revenue sharing mechanism, the supplier and the manufacturer decrease their wholesale prices to provide incentive to the manufacturer and the retailer, respectively. As a consequence, the retailer decides to increase its order quantity so that the availability of the final product to the end customer is increased. In this case, both the supplier and the manufacturer share the risks of demand and supply uncertainty with the retailer. In contrast, the retailer also shares the risk of random yield or disruption with the suppliers and the manufacturer by sharing its revenue with them as a compensation for their decreased wholesale prices.

Notation

Notations used in developing the proposed model are listed below:

x	:	stochastic market demand with mean $\bar{x}$ and standard deviation σ_x
z	:	random yield with mean $\bar{z}$ and standard deviation σ_z
α	:	probability that the cheaper supplier supplies normally
c_{s1}	:	procurement cost of the cheaper supplier for each unit of raw material
c_{s2}	:	production cost of the expensive supplier for each unit of raw material
c_m	:	value-added cost of the manufacturer for unit product
c_r	:	treating cost of the retailer for unit product
p	:	fixed retail price of unit final product
b_r	:	goodwill loss of the retailer for unit unsatisfied demand
h_r	:	salvage value of the unit residual product at the retailer
h_m	:	salvage value of unit raw material leftover at the manufacturer
h_{s1}	:	salvage value of unit undelivered raw material at the cheaper supplier
Q	:	order quantity of the retailer to the manufacturer
Q_1	:	order quantity at the cheaper supplier placed by the manufacturer
Q_2	:	order quantity at the expensive supplier placed by the manufacturer
w_{s1}	:	unit wholesale price of the raw material offered by the cheaper supplier to the manufacturer
w_{s2}	:	unit wholesale price of the raw material offered by the expensive supplier to the manufacturer
w_m	:	unit wholesale price of the finished product charged by the manufacturer to the retailer.

More symbols will be defined whenever needed. To avoid trivial cases, the following assumptions are made: $h_{s1} < c_{s1} < w_{s1}$; $h_m < w_{s1}$; $w_{s2} + c_m < w_m$; $h_r < w_m + c_r < p$; $c_{s2}/\bar{z} < w_{s2}$. These inequalities prevent the supply chain from infinite production and assure that each chain member makes positive profit.

5.3 Benchmark models

In this section, two benchmark models are considered viz. centralized model and decentralized model with price-only contract to measure the performance of the contract mechanism in terms of generating profit and allocation of profit.

5.3.1 *Centralized model*

In reality, although all the supply chain members act independently and take decisions that optimize their respective objective functions, the centralized model is useful to establish a performance benchmark. Conceptually here is only one central decision maker who maximizes the system-wide profit. The expected profit of the entire supply chain is given by

$$\begin{aligned}\Pi_c(Q,Q_1,Q_2) \;=\;& \alpha(p+b_r-h_r)E[\min\{x,Q,Q_1+zQ_2\}]-\alpha(c_m+c_r+h_m-h_r)\\ &\times E[\min\{Q,Q_1+zQ_2\}]-\alpha(c_{s1}-h_m)Q_1+(1-\alpha)(p+b_r-h_r)\\ &\times E[\min\{x,Q,zQ_2\}]-(1-\alpha)(c_m+c_r+h_m-h_r)\\ &\times E[\min\{Q,zQ_2\}]-(1-\alpha)(c_{s1}-h_{s1})Q_1\\ &-(c_{s2}-h_m\bar{z})Q_2-b_r\bar{x} \qquad (5.1)\end{aligned}$$

where the first three terms represent normal working state (with probability α) of the cheaper supplier and the next three terms refer to the scenario where the cheaper supplier is disrupted with probability $(1-\alpha)$ and the final two terms are independent of probability of disruption. We obtain an equivalent representation of the entire system's profit function as follows:

$$\begin{aligned}\Pi_c(Q,Q_1,Q_2) \;=\;& \alpha(p+b_r-h_r)\times\Big\{\int_c^{\frac{Q-Q_1}{Q_2}}\Big(\int_l^{Q_1+zQ_2}xf(x)dx+\int_{Q_1+zQ_2}^u(Q_1+zQ_2)\\ &\times f(x)dx\Big)h(z)dz+\int_{\frac{Q-Q_1}{Q_2}}^d\Big(\int_l^Q xf(x)dx+\int_Q^u Qf(x)dx\Big)h(z)dz\Big\}\\ &+(1-\alpha)(p+b_r-h_r)\Big\{\int_c^{\frac{Q}{Q_2}}\Big(\int_l^{zQ_2}xf(x)dx+\int_{zQ_2}^u zQ_2f(x)dx\Big)h(z)dz\end{aligned}$$

$$+\int_{\frac{Q}{Q_2}}^{d}\Big(\int_{l}^{Q} xf(x)dx+\int_{Q}^{u} Qf(x)dx\Big)h(z)dz\Big\} - \alpha(c_m+c_r+h_m-h_r)$$
$$\times\Big\{\int_{c}^{\frac{Q-Q_1}{Q_2}}(Q_1+zQ_2)h(z)dz+\int_{\frac{Q-Q_1}{Q_2}}^{d} Qh(z)dz\Big\} - (1-\alpha)(c_m+c_r$$
$$+h_m-h_r)\Big\{\int_{c}^{\frac{Q}{Q_2}} zQ_2h(z)dz+\int_{\frac{Q}{Q_2}}^{d} Qh(z)dz\Big\} - \alpha(c_{s1}-h_m)Q_1$$
$$-(1-\alpha)(c_{s1}-h_{s1})Q_1-(c_{s2}-h_m\bar{z})Q_2-b_r\bar{x} \tag{5.2}$$

Due to complexity of the profit function $\Pi_c(Q,Q_1,Q_2)$ in (5.2), it is difficult to show directly that $\Pi_c(Q,Q_1,Q_2)$ is jointly concave in Q, Q_1 and Q_2. We derive the following results to characterize the optimal decisions of the centralized model.

Theorem 5.1 *The expected profit function $\Pi_c(Q,Q_1,Q_2)$ is concave in Q and the optimal order quantity Q^c is given by*

$$Q^c \;=\; F^{-1}\Big(\frac{p+b_r-c_m-c_r-h_m}{p+b_r-h_r}\Big). \tag{5.3}$$

Proof: We have

$$\frac{\partial\Pi_c(Q,Q_1,Q_2)}{\partial Q} = \alpha(p+b_r-h_r)\int_{\frac{Q-Q_1}{Q_2}}^{d}\Big(\int_{Q}^{u} f(x)dx\Big)h(z)dz+(1-\alpha)(p+b_r-h_r)$$
$$\times\int_{\frac{Q}{Q_2}}^{d}\Big(\int_{Q}^{u} f(x)dx\Big)h(z)dz-\alpha(c_m+c_r+h_m-h_r)\int_{\frac{Q-Q_1}{Q_2}}^{d} h(z)dz$$
$$-(1-\alpha)(c_m+c_r+h_m-h_r)\int_{\frac{Q}{Q_2}}^{d} h(z)dz \tag{5.4}$$

and

$$\frac{\partial^2\Pi_c(Q,Q_1,Q_2)}{\partial Q^2} = \alpha\frac{1}{Q_2}h\Big(\frac{Q-Q_1}{Q_2}\Big)\{(c_m+c_r+h_m-h_r)-(p+b_r-h_r)\bar{F}(Q)\}$$
$$+(1-\alpha)\frac{1}{Q_2}h\Big(\frac{Q}{Q_2}\Big)\{(c_m+c_r+h_m-h_r)-(p+b_r-h_r)\bar{F}(Q)\}$$
$$-\alpha(p+b_r-h_r)f(Q)\bar{H}\Big(\frac{Q-Q_1}{Q_2}\Big)-(1-\alpha)(p+b_r-h_r)$$
$$\times f(Q)\bar{H}\Big(\frac{Q}{Q_2}\Big) \tag{5.5}$$

From the first order optimality condition $\frac{\partial\Pi_c(Q,Q_1,Q_2)}{\partial Q}=0$, we obtain the optimal order quantity Q^c in the centralized benchmark setting as

$$Q^c = F^{-1}\Big(\frac{p + b_r - c_m - c_r - h_m}{p + b_r - h_r}\Big) \tag{5.3}$$

Putting optimal order quantity Q^c into the equation (5.5) we get,

$$\begin{aligned}\frac{\partial^2 \Pi_c(Q, Q_1, Q_2)}{\partial Q^2} &= -\alpha(p + b_r - h_r)f(Q^c)\bar{H}\Big(\frac{Q^c - Q_1}{Q_2}\Big) \\ &\quad -(1-\alpha)(p + b_r - h_r)f(Q^c)\bar{H}\Big(\frac{Q^c}{Q_2}\Big) \leq 0. \blacksquare\end{aligned} \tag{5.6}$$

Putting optimal order quantity Q^c in equation (5.3) into equation (5.2), we get

$$\begin{aligned}\Pi_c(Q_1, Q_2) &= \alpha(p + b_r - h_r)\Big\{\int_c^{\frac{Q^c - Q_1}{Q_2}}\Big(\int_l^{Q_1 + zQ_2} xf(x)dx + \int_{Q_1 + zQ_2}^u (Q_1 + zQ_2) \\ &\quad \times f(x)dx\Big)h(z)dz + \int_{\frac{Q^c - Q_1}{Q_2}}^d \Big(\int_l^{Q^c} xf(x)dx + \int_{Q^c}^u Q^c f(x)dx\Big)h(z)dz\Big\} \\ &\quad +(1-\alpha)(p + b_r - h_r)\Big\{\int_c^{\frac{Q^c}{Q_2}}\Big(\int_l^{zQ_2} xf(x)dx + \int_{zQ_2}^u zQ_2 f(x)dx\Big)h(z)dz \\ &\quad + \int_{\frac{Q^c}{Q_2}}^d \Big(\int_l^{Q^c} xf(x)dx + \int_{Q^c}^u Q^c f(x)dx\Big)h(z)dz\Big\} - \alpha(c_m + c_r + h_m - h_r) \\ &\quad \times\Big\{\int_c^{\frac{Q - Q_1}{Q_2}} (Q_1 + zQ_2)h(z)dz + \int_{\frac{Q - Q_1}{Q_2}}^d Qh(z)dz\Big\} - (1-\alpha)(c_m + c_r \\ &\quad + h_m - h_r)\Big\{\int_c^{\frac{Q^c}{Q_2}} zQ_2 h(z)dz + \int_{\frac{Q^c}{Q_2}}^d Q^c h(z)dz\Big\} - \alpha(c_{s1} - h_m)Q_1 \\ &\quad -(1-\alpha)(c_{s1} - h_{s1})Q_1 - (c_{s2} - h_m\bar{z})Q_2 - b_r\bar{x}\end{aligned} \tag{5.7}$$

Proposition 5.1 *The optimal order quantities of the raw materials from the cheaper and expensive suppliers i.e., Q_1^c and Q_2^c satisfy the following equations:*

$$\begin{aligned}\int_c^{\frac{Q^c - Q_1^c}{Q_2^c}} \bar{F}(Q_1^c + zQ_2^c)h(z)dz &= \frac{\alpha(c_{s1} - h_m) - (1-\alpha)(h_{s1} - c_{s1})}{\alpha(p + b_r - h_r)} \\ &\quad + \frac{(c_m + c_r + h_m - h_r)}{(p + b_r - h_r)}H\Big(\frac{Q^c - Q_1^c}{Q_2^c}\Big)\end{aligned} \tag{5.8}$$

$$\begin{aligned}
\textit{and} \qquad & \alpha \int_c^{\frac{Q^c-Q_1^c}{Q_2^c}} z\bar{F}(Q_1^c+zQ_2^c)h(z)dz + (1-\alpha)\int_c^{\frac{Q^c}{Q_2^c}} z\bar{F}zQ_2^c h(z)dz \\
& = \frac{(c_m+c_r+h_m-h_r)}{(p+b_r-h_r)}\Big[\alpha\int_c^{\frac{Q^c-Q_1^c}{Q_2^c}} zh(z)dz + (1-\alpha)\int_c^{\frac{Q^c}{Q_2^c}} zh(z)dz\Big] \\
& + \frac{c_{s2}-h_m\bar{z}}{(p+b_r-h_r)} \qquad (5.9)
\end{aligned}$$

Proof: From equation (5.7), we get

$$\begin{aligned}
\frac{\partial \Pi_c(Q_1,Q_2)}{\partial Q_1} &= \alpha(p+b_r-h_r)\int_c^{\frac{Q^c-Q_1}{Q_2}} \Big(\int_{Q_1+zQ_2}^u f(x)dx\Big)h(z)dz \\
& -\alpha\times(c_m+c_r+h_m-h_r)\int_c^{\frac{Q^c-Q_1}{Q_2}} h(z)dz - \alpha(c_{s1}-h_m) \\
& -(1-\alpha)(h_{s1}-c_{s1}) \qquad (5.10)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi_c(Q_1,Q_2)}{\partial Q_2} &= \alpha(p+b_r-h_r)\int_c^{\frac{Q^c-Q_1}{Q_2}} z\Big(\int_{Q_1+zQ_2}^u f(x)dx\Big)h(z)dz + (1-\alpha)(p+b_r-h_r) \\
& \times\int_c^{\frac{Q^c}{Q_2}} z\Big(\int_{zQ_2}^u f(x)dx\Big)h(z)dz - \alpha(c_m+c_r+h_m-h_r)\int_c^{\frac{Q^c-Q_1}{Q_2}} zh(z)dz \\
& -(1-\alpha)(c_m+c_r+h_m-h_r)\int_c^{\frac{Q^c}{Q_2}} zh(z)dz + (c_{s2}-h_m\bar{z}) \qquad (5.11)
\end{aligned}$$

From the first order optimality conditions $\frac{\partial \Pi_c(Q_1,Q_2)}{\partial Q_1}=0$ and $\frac{\partial \Pi_c(Q_1,Q_2)}{\partial Q_2}=0$, we see that the order quantities Q_1^c and Q_2^c satisfy the equations (5.8) and (5.9). ■

Where $F^{-1}(v) = inf\{u : F(u) = v\}$ and $\bar{F}(v) = 1 - F(v)$, be the survival function. In this model, our main objective is to control the inventory at each stage keeping the other parameters constant, so the entire channel profit depends only on the order quantities of the raw materials and final product. Therefore, by ordering Q^c, Q_1^c and Q_2^c, the expected total profit is maximized and the maximum value is $\Pi_c(Q^c, Q_1^c, Q_2^c)$. Due to model complexity we can not express the explicit form of $\Pi_c(Q^c, Q_1^c, Q_2^c)$. From equation (5.3), we find that the optimal order quantity Q^c is only affected by the exogenous parameters related to the final product, not by parameters (except h_m) related to the raw material, which is quite natural.

5.3.2 Decentralized model with price-only contract

In reality, all supply chain members are independent decision makers and they choose best decisions to maximize their individual profits. We now consider a decentralized system where there is price-only contract among the supply chain entities. The process flow in this decentralized setting is as follows:

First, both the raw material suppliers determine their wholesale prices w_{s1} and w_{s2}. Consequently the manufacturer determines its wholesale price w_m. Then, with the knowledge of disruption probability, demand and yield distributions, the retailer decides to order Q units from the manufacturer. The manufacturer orders Q_1 units of raw material from the cheaper supplier with a disruption risk $(1-\alpha)$ and Q_2 units from the expensive supplier with a production yield risk z. Subsequently, the amount $\min\{Q, Q_1 + zQ_2\}$ or $\min\{Q, zQ_2\}$ is shipped by the manufacturer according to the cheaper supplier's supply state up or down, respectively. Finally, the market demand x occurs and the retailer sells the amount $\min\{x, Q, Q_1 + zQ_2\}$ or $\min\{x, Q, zQ_2\}$ to the end-customer depending on whether there is no disruption or disruption, respectively, at the cheaper supplier.

We consider a Nash sequence where both the suppliers act as the first decision maker and the system is solved through backward induction. Therefore, the retailer first determines its optimal decisions. For given Q_1, Q_2, the retailer's profit function $\Pi_r(Q)$ can be derived as follows:

$$\begin{aligned}\Pi_r(Q) \;=\;& \alpha(p+b_r-h_r)E[\min\{x,Q,Q_1+zQ_2\}] - \alpha(w_m+c_r-h_r)\\ &\times E[\min\{Q,Q_1+zQ_2\}] + (1-\alpha)(p+b_r-h_r)E[\min\{x,Q,zQ_2\}]\\ &-(1-\alpha)(w_m+c_r-h_r)E[\min\{Q,zQ_2\}] - b_r\bar{x} \qquad (5.12)\end{aligned}$$

We can rewrite the above profit function as follows:

$$\begin{aligned}\Pi_r(Q) \;=\;& \alpha(p+b_r-h_r)\Big\{\int_c^{\frac{Q-Q_1}{Q_2}}\Big(\int_l^{Q_1+zQ_2} xf(x)dx + \int_{Q_1+zQ_2}^{u}(Q_1+zQ_2)f(x)dx\Big)h(z)dz\\ &+\int_{\frac{Q-Q_1}{Q_2}}^{d}\Big(\int_l^{Q} xf(x)dx + \int_Q^u Qf(x)dx\Big)h(z)dz\Big\} + (1-\alpha)(p+b_r-h_r)\Big\{\int_c^{\frac{Q}{Q_2}}\Big(\int_l^{zQ_2} x\end{aligned}$$

$$\times f(x)dx + \int_{zQ_2}^{u} zQ_2 f(x)dx\Big)h(z)dz + \int_{\frac{Q}{Q_2}}^{d}\Big(\int_{l}^{Q} xf(x)dx + \int_{Q}^{u} Qf(x)dx\Big)h(z)dz\Big\}$$
$$-\alpha(w_m + c_r - h_r)\Big\{\int_{c}^{\frac{Q-Q_1}{Q_2}}(Q1 + zQ_2)h(z)dz + \int_{\frac{Q-Q_1}{Q_2}}^{d} Qh(z)dz\Big\} - (1-\alpha)$$
$$\times(w_m + c_r - h_r)\Big\{\int_{c}^{\frac{Q}{Q_2}} zQ_2 h(z)dz + \int_{\frac{Q}{Q_2}}^{d} Qh(z)dz\Big\} - b_r\bar{x} \quad (5.13)$$

The optimal decision of the retailer is given in the following theorem.

Theorem 5.2 *The profit function $\Pi_r(Q)$ is concave in Q and the optimal order quantity Q^d is given by*

$$Q^d = F^{-1}\Big(\frac{p + b_r - w_m - c_r}{p + b_r - h_r}\Big) \quad (5.14)$$

Also Q^d increases as the wholesale price (w_m) of the manufacturer and the value-added cost (c_r) of the retailer decrease.

Proof: We have

$$\frac{\partial \Pi_r(Q)}{\partial Q} = \alpha(p + b_r - h_r)\int_{\frac{Q-Q_1}{Q_2}}^{d}\Big(\int_{Q}^{u} f(x)dx\Big)h(z)dz + (1-\alpha)(p + b_r - h_r)$$
$$\times\int_{\frac{Q}{Q_2}}^{d}\Big(\int_{Q}^{u} f(x)dx\Big)h(z)dz - \alpha(w_m + c_r - h_r)\int_{\frac{Q-Q_1}{Q_2}}^{d} h(z)dz$$
$$-(1-\alpha)(w_m + c_r - h_r)\int_{\frac{Q}{Q_2}}^{d} h(z)dz \quad (5.15)$$

and

$$\frac{\partial^2 \Pi_r(Q)}{\partial Q^2} = \alpha\frac{1}{Q_2}h\Big(\frac{Q-Q_1}{Q_2}\Big)\Big\{(w_m + c_r - h_r) - (p + b_r - h_r)\bar{F}(Q)\Big\}$$
$$+(1-\alpha)\frac{1}{Q_2}h\Big(\frac{Q}{Q_2}\Big)\Big\{(w_m + c_r - h_r) - (p + b_r - h_r)\bar{F}(Q)\Big\}$$
$$-\alpha(p + b_r - h_r)f(Q)\bar{H}\Big(\frac{Q-Q_1}{Q_2}\Big) - (1-\alpha)(p + b_r - h_r)$$
$$\times f(Q)\bar{H}\Big(\frac{Q}{Q_2}\Big) \quad (5.16)$$

From the first order optimality condition $\frac{\partial \Pi_r(Q)}{\partial Q} = 0$, we obtain the retailer's optimal order quantity Q^d as

$$Q^d = F^{-1}\Big(\frac{p + b_r - w_m - c_r}{p + b_r - h_r}\Big) \tag{5.14}$$

Putting Q^d in equation (5.14) into equation (5.16) we get,

$$\frac{\partial^2 \Pi_r(Q)}{\partial Q^2} = -\alpha(p + b_r - h_r)f(Q^d)\bar{H}\Big(\frac{Q^d - Q_1}{Q_2}\Big) - (1-\alpha)(p + b_r - h_r)f(Q^d)\bar{H}\Big(\frac{Q^d}{Q_2}\Big) \leq 0$$

To show that Q^d is a decreasing function of w_m, we have to show that $\frac{\partial Q^d}{\partial w_m} \leq 0$. We have, $F(Q^d) = \Big(\frac{p+b_r-w_m-c_r}{p+b_r-h_r}\Big)$. Then $\frac{\partial Q^d}{\partial w_m} = -f(Q^d)(p + b_r - h_r) \leq 0$. By similar argument it can be shown that Q^d is a decreasing function of c_r. ■

Comparing Q^d in equation (5.14) with the order quantity of the centralized system given in equation (5.3), we find that the retailer orders less in the decentralized benchmark model. This indicates that the decentralized system does not run as efficiently as the centralized system. After analyzing the retailer's problem and obtaining the optimal order quantity Q^d, we derive the manufacturer's expected profit function $\Pi_m(Q_1, Q_2)$ as follows:

$$\begin{aligned} \Pi_m(Q_1, Q_2) &= \alpha(w_m - c_m - h_m)E[\min\{Q, Q_1 + zQ_2\}] - \alpha(w_{s1} - h_m)Q_1 \\ &\quad +(1-\alpha)(w_m - c_m - h_m)E[\min\{Q, zQ_2\}] \\ &\quad -(w_{s2} - h_m)\bar{z}Q_2 \end{aligned} \tag{5.17}$$

which can be written as

$$\begin{aligned} \Pi_m(Q_1, Q_2) &= \alpha(w_m - c_m - h_m)\Big\{\int_c^{\frac{Q-Q_1}{Q_2}} (Q_1 + zQ_2)h(z)dz + \int_{\frac{Q-Q_1}{Q_2}}^d Qh(z)dz\Big\} \\ &\quad +(1-\alpha)(w_m - c_m - h_m)\Big\{\int_c^{\frac{Q}{Q_2}} zQ_2h(z)dz + \int_{\frac{Q}{Q_2}}^d Qh(z)dz\Big\} \\ &\quad -\alpha(w_{s1} - h_m)Q_1 - (w_{s2} - h_m\bar{z})Q_2 \end{aligned} \tag{5.18}$$

We characterize the optimal decisions for the manufacturer under the wholesale price only contract, in the following theorem.

Theorem 5.3 *The manufacturer's profit function $\Pi_m(Q_1, Q_2)$ is jointly concave in Q_1 and Q_2, and the optimal order quantities Q_1^d and Q_2^d satisfy the following equations:*

$$Q_1^d = Q - Q_2^d \ H^{-1}\Big(\frac{w_{s1} - h_m}{w_m - c_m - h_m}\Big) \tag{5.19}$$

$$\alpha \int_c^{\frac{Q-Q_1^d}{Q_2^d}} zh(z)dz + (1-\alpha)\int_c^{\frac{Q}{Q_2^d}} zh(z)dz = \frac{(w_{s2} - h_m)\bar{z}}{(w_m - c_m - h_m)} \tag{5.20}$$

Proof: To prove that the manufacturer's profit function is jointly concave in Q_1 and Q_2 in the decentralized benchmark model, we have

$$\frac{\partial \Pi_m(Q_1 Q_2)}{\partial Q_1} = \alpha(w_m - c_m - h_m)\int_c^{\frac{Q-Q_1}{Q_2}} h(z)dz - \alpha(w_{s1} - h_m) \tag{5.21}$$

$$\frac{\partial^2 \Pi_m(Q_1 Q_2)}{\partial Q_1^2} = -\alpha(w_m + c_m - h_m)\frac{1}{Q_2}h\Big(\frac{Q-Q_1}{Q_2}\Big) \tag{5.22}$$

$$\begin{aligned}\frac{\partial \Pi_m(Q_1 Q_2)}{\partial Q_2} &= (w_m - c_m - h_m)\int_c^{\frac{Q-Q_1}{Q_2}} zh(z)dz - (w_{s2} - h_m)\bar{z} \\ &\quad +(1-\alpha)(w_m - c_m - h_m)\int_c^{\frac{Q}{Q_2}} zh(z)dz\end{aligned} \tag{5.23}$$

$$\begin{aligned}\frac{\partial^2 \Pi_m(Q_1 Q_2)}{\partial Q_2^2} &= -\alpha(w_m - c_m - h_m)\frac{(Q-Q_1)^2}{Q_2^3}h\Big(\frac{Q-Q_1}{Q_2}\Big) \\ &\quad -(1-\alpha)(w_m - c_m - h_m)\frac{Q^2}{Q_2^3}h\Big(\frac{Q}{Q_2}\Big)\end{aligned} \tag{5.24}$$

and
$$\frac{\partial^2 \Pi_m(Q_1 Q_2)}{\partial Q_2 \partial Q_1} = -\alpha(w_m - c_m - h_m)\Big(\frac{Q-Q_1}{Q_2^2}\Big)h\Big(\frac{Q-Q_1}{Q_2}\Big) \tag{5.25}$$

The Hessian matrix is

$$H = \begin{pmatrix} \frac{\partial^2 \Pi_m(Q_1,Q_2)}{\partial Q_1^2} & \frac{\partial^2 \Pi_m(Q_1,Q_2)}{\partial Q_2 \partial Q_1} \\ \frac{\partial^2 \Pi_m(Q_1,Q_2)}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \Pi_m(Q_1,Q_2)}{\partial Q_2^2} \end{pmatrix}$$

Let H_i, (i=1,2) be the principal minors of the hessian matrix H. Clearly we have the principal minors $|H_1| = -\alpha(w_m + c_m - h_m)\frac{1}{Q_2}h\Big(\frac{Q-Q_1}{Q2}\Big) < 0$ and $|H_2| = \alpha(1-\alpha)(w_m - c_m - h_m)^2\frac{Q^2}{Q_2^4}h\Big(\frac{Q-Q_1}{Q2}\Big) > 0$. This shows that the hessian matrix is negative definite.

Then, from the first order optimality conditions $\frac{\partial \Pi_m(Q_1Q_2)}{\partial Q_1} = 0$ and $\frac{\partial \Pi_m(Q_1Q_2)}{\partial Q_2} = 0$, we obtain the optimal solution. Therefore, the manufacturer's order quantities Q_1^d and Q_2^d satisfy the following equations:

$$Q_1^d = Q - Q_2^d \times H^{-1}\Big(\frac{w_{s1} - h_m}{w_m - c_m - h_m}\Big) \tag{5.19}$$

$$\alpha \int_c^{\frac{Q-Q_1^d}{Q_2^d}} zh(z)dz + (1-\alpha)\int_c^{\frac{Q}{Q_2^d}} zh(z)dz = \frac{(w_{s2} - h_m)\bar{z}}{(w_m - c_m - h_m)} \quad \blacksquare \tag{5.20}$$

Based on Theorem 5.3, we notice that the cheaper supplier's expected profit function is increasing in Q and decreasing in Q_2. Also, Q_1^d is increasing in w_{s2} and decreasing in w_{s1}, whereas Q_2^d is increasing in w_{s1} and decreasing in w_{s2}. The expected profit Π_{s2} of the expensive supplier is given by

$$\Pi_{s2} = (w_{s2}\bar{z} - c_{s2})Q_2 \tag{5.26}$$

and the expected profit Π_{s1} of the cheaper supplier is given by

$$\Pi_{s1} = \alpha(w_{s1} - c_{s1})Q_1 + (1-\alpha)(h_{s1} - c_{s1})Q_1 \tag{5.27}$$

Under the price-only contract, the manufacturer has to face disruption risk and yield uncertainty of the suppliers. Hence it takes decisions without any promise to anyone. In this scenario, we assume that the wholesale prices are negotiated based on firm's bargaining power, keeping a positive profit margin above a desired level of acceptance (Asian and Nie, 2014). In this setting, to gain a positive expected profit for both the suppliers and the manufacturer, the only way is to set a wholesale price above their relative costs. We compare the above benchmark models in the following theorem:

Theorem 5.4 *All the order quantities in the decentralized model are strictly less than their counterparts in the centralized model. Moreover, a lower order quantity leads to a lower expected profit for the supply chain.*

Proof. We have $w_m > c_m + w_{s2}$ and $w_{s2} > w_{s1}$ which imply $w_m > c_m + w_{s1}$. Since $h_m < w_{s1}$, therefore we have $w_m > c_m + h_m$. Now, comparing equation (5.14) with equation (5.3) and using the fact that $w_m > c_m + h_m$, we find that, in the decentralized benchmark model, the retailer's optimal order quantity is strictly less than that in the centralized benchmark model. As the retailer orders less amount for the final product, therefore, the order quantity of the manufacturer for raw materials in the decentralized benchmark model is also less than that in the centralized benchmark model, which can be realized by comparing equations (5.19) and (5.20) with equations (5.8) and (5.9), respectively. Adding the supply chain members' expected profits, we get $\Pi_r + \Pi_m + \Pi_{s2} + \Pi_{s1} = \Pi_d$. It is easy to verify that $\Pi_c(Q^c, Q_1^c, Q_2^c) > \Pi_d(Q^d, Q_1^d, Q_2^d)$. Both the supplier's and the manufacturer's individual pricing policies are the reason behind the supply chain's inefficiency in the decentralized benchmark setting. ■

Theorem 5.4 is a generalization of the result for the two-echelon supply chain that can be footstep back Spengler (1950) regarding marginalization, showing that the maximum system efficiency is not achieved in the decentralized model with price-only contract even if all chain members maximize their own profits *i.e.*, $\Pi_c(Q^c, Q_1^c, Q_2^c) > \Pi_d(Q^d, Q_1^d, Q_2^d)$ where Π_d denotes the total system profit in the decentralized benchmark model. In the decentralized scenario where the decision power is distributed over the various chain members, there is a possible deviation from the optimal decisions obtained in the centralized model. To align each member's decision with the entire channel, contract mechanisms come into play.

5.4 Spanning revenue sharing contract

A contract or agreement prevents sub-optimization by removing the rivalry among the members without affecting the supply chain structure and its members' decision powers. Available popular contract agreements are limited within only two levels and extended in a multi-level by installing them between any two adjacent pairs of supply chain entities. As pointed out by Van Der Rhee et al. (2010) that a pair-wise contract in a multi-level supply chain has a drawback from the perspective of simultaneous implementation They proposed a new revenue sharing contract mechanism in which one supply chain member takes the lead in making a single contract with all other entities in the supply chain.

In this section, we focus on the decentralized scenario with spanning revenue sharing (SRS) contract where the retailer gives the guidance in negotiating revenue share among the chain members. This arrangement incentivies each supplier to decrease its wholesale price, which influences the manufacturer to reduce its price too at the beginning of the selling season. The compensation for their reduced wholesale prices is given in terms of revenue shares of the retailer with all other entities, as a lower wholesale price gives an opportunity to the retailer to earn more revenue by satisfying more customer. We also examine whether the supply chain is coordinated under SRS contract *i.e.,* the decentralized supply chain achieves the same profit under the SRS contract as in the centralized benchmark model earns. Moreover, we characterize the chain members' participation problem for a win-win outcome. The time-line of events has the following sequence.

First, the retailer forecasts the customer demand, negotiates SRS contract with all upstream entities, decides its optimal order quantity Q, and places its order to the manufacturer before the selling season. Based on the retailer's order quantity, the manufacturer orders Q_1 and Q_2 units of raw materials to cheaper and expensive suppliers, respectively. After realization of the supply uncertainty, the manufacturer receives 0 or Q_1 units from the cheaper supplier depending on whether or not the cheaper supplier is disrupted, and securely receives zQ_2 (a random fraction of its order Q_2) units from the expensive supplier. After receiving raw materials from the suppliers, the manufacturer starts production and delivers to the retailer at once. At the retailer, if the demand is less than the on-hand inventory, the excess amount is salvaged; otherwise, the shortage incurs a goodwill cost. At the end of the selling season, the retailer shares its revenue to all upstream entities according to the rule of pricing under SRS contract. The SRS mechanism is administrated by three wholesale prices w_{s1}, w_{s2} and w_m and three revenue sharing ratios ϕ_{s1}, ϕ_{s2} and $\phi_m (0 \leq \phi_{s1} + \phi_{s2} + \phi_m = \phi \leq 1)$ of the retailer's revenues that are shared with the cheaper supplier, expensive supplier and the manufacturer, respectively.

We first concentrate on the retailer's decision problem to determine the optimal order quantity Q^* subject to any decision (Q_1, Q_2) of the manufacturer. Then, we focus on the manufacturer's decision problem to obtain the optimal order quantities Q_1^*

and Q_2^* to anticipate the retailer's optimal repercussion. The retailer's profit function $\Pi_{rc}(Q)$ under the spanning revenue sharing contract is derived as

$$\begin{aligned}\Pi_{rc}(Q) &= \alpha((1-\phi)(p-h_r)+b_r)E[\min\{x,Q,Q_1+zQ_2\}]-\alpha(w_m+c_r \\ &\quad -(1-\phi)h_r)E[\min\{Q,Q_1+zQ_2\}]+(1-\alpha)((1-\phi)(p-h_r)+b_r) \\ &\quad \times E[\min\{x,Q,zQ_2\}]-(1-\alpha)(w_m+c_r-(1-\phi)h_r)E[\min\{Q,zQ_2\}] \\ &\quad -b_r\bar{x} \qquad (5.28)\end{aligned}$$

The first two terms represent the normal working state of the cheaper supplier and the next two terms refer to the situation where the cheaper supplier is disrupted. The final term is independent of the risk of supply uncertainty. We derive an alternative representation of the retailer's profit function as follows:

$$\begin{aligned}&\Pi_{rc}(Q)=\alpha((1-\phi)(p-h_r)+b_r)\Big\{\int_c^{\frac{Q-Q_1}{Q_2}}\Big(\int_l^{Q_1+zQ_2}xf(x)dx+\int_{Q_1+zQ_2}^u(Q_1+zQ_2)f(x)dx\Big) \\ &\times h(z)dz+\int_{\frac{Q-Q_1}{Q_2}}^d\Big(\int_l^Q xf(x)dx+\int_Q^u Qf(x)dx\Big)h(z)dz\Big\}+(1-\alpha)((1-\phi)(p-h_r)+b_r) \\ &\times\Big\{\int_c^{\frac{Q}{Q_2}}\Big(\int_l^{zQ_2}xf(x)dx+\int_{zQ_2}^u zQ_2f(x)dx\Big)h(z)dz+\int_{\frac{Q}{Q_2}}^d\Big(\int_l^Q xf(x)dx+\int_Q^u Qf(x)dx\Big) \\ &\times h(z)dz\Big\}-\alpha(w_m+c_r-(1-\phi)h_r)\Big\{\int_c^{\frac{Q-Q_1}{Q_2}}(Q_1+zQ_2)h(z)dz+\int_{\frac{Q-Q_1}{Q_2}}^d Qh(z)dz\Big\} \\ &-(1-\alpha)(w_m+c_r-(1-\phi)h_r)\Big\{\int_c^{\frac{Q}{Q_2}}zQ_2h(z)dz+\int_{\frac{Q}{Q_2}}^d Qh(z)dz\Big\}-b_r\bar{x} \qquad (5.29)\end{aligned}$$

The retailer's optimal order quantity Q^* under the spanning revenue sharing contract is characterized in the following theorem.

Theorem 5.5 *The retailer's expected profit function $\Pi_{rc}(Q)$ under the spanning revenue sharing contract is concave in Q and the unique optimal order quantity Q^* satisfy the following equation*

$$Q^* = F^{-1}\Big(\frac{(1-\phi)p+b_r-w_m-c_r}{(1-\phi)(p-h_r)+b_r}\Big) \qquad (5.30)$$

Proof: To prove the concavity of the retailer's profit function in the decentralized benchmark model under spanning revenue sharing contract, we have

$$\begin{aligned}
\frac{\partial \Pi_{rc}(Q)}{\partial Q} &= \alpha((1-\phi)(p-h_r)+b_r)\int_{\frac{Q-Q_1}{Q_2}}^{d}\Big(\int_{Q}^{u} f(x)dx\Big)h(z)dz + (1-\alpha) \\
&\times((1-\phi)(p-h_r)+b_r)\int_{\frac{Q}{Q_2}}^{d}\Big(\int_{Q}^{u} f(x)dx\Big)h(z)dz - \alpha(w_m+c_r \\
&-(1-\phi)h_r)\int_{\frac{Q-Q_1}{Q_2}}^{d} h(z)dz - (1-\alpha)(w_m+c_r-(1-\phi)h_r)\int_{\frac{Q}{Q_2}}^{d} h(z)dz
\end{aligned} \tag{5.31}$$

$$\begin{aligned}
\frac{\partial^2 \Pi_{rc}(Q)}{\partial Q^2} &= \alpha\frac{1}{Q_2}h\Big(\frac{Q-Q_1}{Q_2}\Big)\Big\{(w_m+c_r-(1-\phi)h_r)-((1-\phi)(p-h_r)+b_r)\bar{F}(Q)\Big\} \\
&+(1-\alpha)\frac{1}{Q_2}h\Big(\frac{Q}{Q_2}\Big)\Big\{(w_m+c_r-(1-\phi)h_r)-((1-\phi)(p-h_r)+b_r)\bar{F}(Q)\Big\} \\
&-\alpha((1-\phi)(p-h_r)+b_r)f(Q)\bar{H}\Big(\frac{Q-Q_1}{Q_2}\Big)-(1-\alpha)((1-\phi)(p-h_r)+b_r) \\
&\times f(Q)\bar{H}\Big(\frac{Q}{Q_2}\Big)
\end{aligned} \tag{5.32}$$

From the first order optimality condition $\frac{\partial \Pi_{rc}(Q)}{\partial Q}=0$, we obtain the retailer's optimal order quantity Q^* as

$$Q^* = F^{-1}\Big(\frac{(1-\phi)p+b_r-w_m-c_r}{(1-\phi)(p-h_r)+b_r}\Big) \tag{5.30}$$

Putting Q^* in equation (5.32) we get,

$$\frac{\partial^2 \Pi_{rc}(Q)}{\partial Q^2} \leq 0. \blacksquare$$

To show that Q^* is an increasing function of p, it is sufficient to show that $\frac{\partial^2 \Pi_{rc}(Q)}{\partial Q \partial p} \geq 0$. Now, we have

$$\frac{\partial^2 \Pi_{rc}(Q)}{\partial Q \partial p} = \alpha(1-\phi)\bar{F}(Q)\bar{H}\Big(\frac{Q-Q_1}{Q_2}\Big)+(1-\alpha)(1-\phi)\bar{F}(Q)\bar{H}\Big(\frac{Q}{Q_2}\Big) \geq 0.$$

To show that Q^* is a decreasing function of c_r, it is sufficient to show that $\frac{\partial^2 \Pi_{rc}(Q)}{\partial Q \partial c_r} \leq 0$.

Now, differentiating $\Pi_{rc}(Q)$ twice with respect to Q and c_r we get,

$$\frac{\partial^2\Pi_{rc}(Q)}{\partial Q\partial c_r} = -\alpha\int_{\frac{Q-Q_1}{Q_2}}^{d} h(z)dz - (1-\alpha)\int_{\frac{Q}{Q_2}}^{d} h(z)dz \leq 0 \qquad (5.33)$$

By similar argument it can be shown that Q^* is a decreasing function of w_m. From equation (5.30) we find that the retailer's optimal order quantity is an increasing function of the retail price p and a decreasing function of its purchasing cost w_m and treating cost c_r. Also, the order quantity Q^* does not depend directly on the cheaper supplier's disruption probability.

The manufacturer's expected profit function $\Pi_{mc}(Q_1, Q_2)$ under the spanning revenue sharing contract is given by

$$\begin{aligned}\Pi_{mc}(Q_1, Q_2) &= \alpha\phi_m(p-h_r)E[\min\{x, Q, Q_1+zQ_2\}] + \alpha(w_m - c_m + \phi_m h_r - h_m)\\ &\quad \times E[\min\{Q, Q_1+zQ_2\}] + (1-\alpha)\phi_m(p-h_r)E[\min\{x, Q, zQ_2\}]\\ &\quad +(1-\alpha)(w_m - c_m + \phi_m h_r - h_m)E[\min\{Q, zQ_2\}] - \alpha(w_{s1}-h_m)\\ &\quad \times Q_1 - (w_{s2}-h_m)\bar{z}Q_2 \qquad (5.34)\end{aligned}$$

An equivalent representation of the manufacturer's expected profit function under the proposed contract is given by

$$\begin{aligned}&\Pi_{mc}(Q_1, Q_2) = \alpha\phi_m(p-h_r)\Big\{\int_c^{\frac{Q-Q_1}{Q_2}}\Big(\int_l^{Q_1+zQ_2} xf(x)dx + \int_{Q_1+zQ_2}^{u}(Q_1+zQ_2)f(x)dx\Big)\\ &\times h(z)dz + \int_{\frac{Q^c-Q_1}{Q_2}}^{d}\Big(\int_l^{Q^c} xf(x)dx + \int_{Q^c}^{u} Q^c f(x)dx\Big)h(z)dz\Big\} + (1-\alpha)\phi_m(p-h_r)\\ &\times\Big\{\int_c^{\frac{Q}{Q_2}}\Big(\int_l^{zQ_2} xf(x)dx + \int_{zQ_2}^{u} zQ_2 f(x)dx\Big)h(z)dz + \int_{\frac{Q}{Q_2}}^{d}\Big(\int_l^{Q} xf(x)dx + \int_Q^u Qf(x)dx\Big)\\ &\times h(z)dz\Big\} + \alpha(w_m - c_m + \phi_m h_r - h_m)\Big\{\int_c^{\frac{Q-Q_1}{Q_2}}(Q_1+zQ_2)h(z)dz + \int_{\frac{Q-Q_1}{Q_2}}^{d} Qh(z)dz\Big\}\\ &+(1-\alpha)(w_m - c_m + \phi_m h_r - h_m)\Big\{\int_c^{\frac{Q}{Q_2}} zQ_2h(z)dz + \int_{\frac{Q}{Q_2}}^{d} Qh(z)dz\Big\} - \alpha(w_{s1}-h_m)Q_1\\ &-(w_{s2}-h_m\bar{z})Q_2 \qquad (5.35)\end{aligned}$$

Due to complexity, the concavity of $\Pi_{mc}(Q_1, Q_2)$ with respect to Q_1 and Q_2 can not be proved analytically. The following proposition explores the manufacturer's optimal order quantities assuming that Π_{mc} is concave.

Proposition 5.2 *Under the SRS contract, the manufacturer's optimal order quantities Q_1^* and Q_2^* satisfy the equations:*

$$\int_c^{\frac{Q-Q_1^*}{Q_2^*}} \bar{F}(Q_1^* + zQ_2^*)h(z)dz = \Big(\frac{w_{s1} - h_m}{\phi_m(p - h_r)}\Big) - \frac{(w_m - c_m + \phi_m h_r - h_m)}{\phi_m(p - h_r)} \times H\Big(\frac{Q - Q_1^*}{Q_2^*}\Big) \tag{5.36}$$

$$\text{and} \quad \alpha \int_c^{\frac{Q-Q_1^*}{Q_2^*}} z\bar{F}(Q_1^* + zQ_2^*)h(z)dz + (1 - \alpha)\int_c^{\frac{Q}{Q_2^*}} z\bar{F}zQ_2^*h(z)dz = \frac{(w_{s2} - h_m)\bar{z}}{\phi_m(p - h_r)} - \frac{(w_m - c_m + \phi_m h_r - h_m)}{\phi_m(p - h_r)}\Big[\alpha \int_c^{\frac{Q-Q_1^*}{Q_2^*}} zh(z)dz + (1 - \alpha)\int_c^{\frac{Q}{Q_2^*}} zh(z)dz\Big] \tag{5.37}$$

Proof: From equation (5.35), we have

$$\frac{\partial \Pi_{mc}(Q_1, Q_2)}{\partial Q_1} = \alpha\phi_m(p - h_r)\int_c^{\frac{Q-Q_1}{Q_2}} \Big(\int_{Q_1+zQ_2}^u f(x)dx\Big)h(z)dz - \alpha(w_{s1} - h_m) + \alpha(w_m - c_m + \phi_m h_r - h_m)\int_c^{\frac{Q-Q_1}{Q_2}} h(z)dz \tag{5.38}$$

$$\frac{\partial \Pi_{mc}(Q_1, Q_2)}{\partial Q_2} = \alpha\phi_m(p - h_r)\int_c^{\frac{Q-Q_1}{Q_2}} z\Big(\int_{Q_1+zQ_2}^u f(x)dx\Big)h(z)dz + \alpha(w_m - c_m + \phi_m h_r - h_m)\int_c^{\frac{Q-Q_1}{Q_2}} zh(z)dz + (1 - \alpha)\phi_m(p - h_r)\int_c^{\frac{Q}{Q_2}} z\Big(\int_{zQ_2}^u f(x)dx\Big)h(z)dz + (1 - \alpha) \times (w_m - c_m + \phi_m h_r - h_m)\int_c^{\frac{Q}{Q_2}} zh(z)dz + (w_{s2} - h_m)\bar{z} \tag{5.39}$$

From the first order optimality conditions $\frac{\partial \Pi_{mc}(Q_1,Q_2)}{\partial Q_1} = 0$ and $\frac{\partial \Pi_{mc}(Q_1,Q_2)}{\partial Q_2} = 0$, we obtain the optimal solution. Therefore, the manufacturer's order quantities Q_1^* and Q_2^* satisfy the equations (5.36) and (5.37), respectively. ■

The expected profit Π_{s2c} of the expensive supplier is given by

$$\begin{aligned}\Pi_{s2c} &= \alpha\phi_{s2}(p-h_r)E[\min\{x,Q,Q_1+zQ_2\}]+\alpha\phi_{s2}h_rE[\min\{Q,Q_1+zQ_2\}] \\ &\quad +(1-\alpha)\phi_{s2}(p-h_r)\times E[\min\{x,Q,zQ_2\}]+(1-\alpha)\phi_{s2}h_rE[\min\{Q,zQ_2\}] \\ &\quad +(w_{s2}\bar{z}-c_{s2})Q_2 \end{aligned} \tag{5.40}$$

The expected profit Π_{s1c} of the cheaper supplier is given by

$$\begin{aligned}\Pi_{s1c} &= \alpha\phi_{s1}(p-h_r)E[\min\{x,Q,Q_1+zQ_2\}]+\alpha\phi_{s1}h_rE[\min\{Q,Q_1+zQ_2\}] \\ &\quad +(1-\alpha)\phi_{s1}(p-h_r)\times E[\min\{x,Q,zQ_2\}]+(1-\alpha)\phi_{s1}h_rE[\min\{Q,zQ_2\}] \\ &\quad +\alpha(w_{s1}-c_{s1})Q_1+(1-\alpha)(h_{s1}-c_{s1})Q_1 \end{aligned} \tag{5.41}$$

We now obtain the conditions under which the supply chain is coordinated with the spanning revenue sharing contract.

Theorem 5.6 *Under the voluntary compliance, the SRS contract with the wholesale prices*

$$w_m = c_m+h_m-\phi_m h_r-\frac{c_m+c_r+h_m-h_r}{p+b_r-h_r}\times\phi_m(p-h_r) \tag{5.42}$$

$$w_{s2} = h_m+\frac{c_{s2}-h_m\bar{z}}{\bar{z}(p+b_r-h_r)}\times\phi_m(p-h_r) \tag{5.43}$$

$$w_{s1} = h_m+\frac{\alpha(c_{s1}-h_m)-(1-\alpha)(h_{s1}-c_{s1})}{\alpha(p+b_r-h_r)}\times\phi_m(p-h_r) \tag{5.44}$$

achieves channel coordination.

Proof. Using the supply chain members' optimal decisions of the decentralized model under spanning revenue sharing contract given in Theorem 5.5 and Proposition 5.2, we obtain the conditions such that the supply chain is coordinated. Comparing equation (5.3) with equation (5.30) we get $w_m = \frac{((1-\phi)(p-h_r)+b_r)(c_m+h_m)-\phi_m(b_rh_r+(p-h_r)c_r)}{(1-\phi_{s2}-\phi_{s1})(p-h_r)+b_r}$. As a result, the retailer orders the same amount of the final product that in the centralized model. Also, comparing Proposition 5.2 with Proposition 5.1, we get $w_{s2}=h_m+\frac{c_{s2}-h_m\bar{z}}{p+b_r-h_r}\times\frac{\phi_m(p-h_r)}{\bar{z}}$ and $w_{s1}=h_m+\frac{\alpha(c_{s1}-h_m)-(1-\alpha)(h_{s1}-c_{s1})}{\alpha(p+b_r-h_r)}$. Under these two conditions, the manufacturer takes the ordering decisions for raw material same as in the centralized model. Further, using conditions in equations (5.42) - (5.44) the total expected profit of the decentralized system under spanning revenue sharing contract is $\Pi_{rc}(Q^*) + \Pi_{mc}(Q_1^*, Q_2^*) + \Pi_{s2c} + \Pi_{s1c} = \Pi_c(Q^c, Q_1^c, Q_2^c)$. ■

Although the retailer and the manufacturer are free to make their individual decisions under the SRS contract, if the parameters (w_i, ϕ_i) of the SRS conform to equations (5.42)-(5.44), then the chosen optimal orders Q^*, Q_1^*, and Q_2^* will correspond to Q^c, Q_1^c, and Q_2^c, respectively, that achieves the objective of the centralized SC as studied in section 5.3.1. From equations (5.42)-(5.44), we can observe that the wholesale price w_i are function of the retailer's revenue sharing ratio ϕ_i. So, we concentrate on how to calculate ϕ_i in the SRS contract. Once ϕ_i's are calculated, the wholesale prices w_i can be estimated immediately from Eqs. (5.42)-(5.44) and with these values of w_i, the SRS contact coordinates the supply chain irrespective of the adopted values of ϕ_i *i.e.,* $\Pi_{dc}(Q^*, Q_1^*, Q_2^*) = \Pi_c(Q^c, Q_1^c, Q_2^c)$ where Π_{dc} denotes total system profit in the decentralized model under the SRS contract. These observations are used to solve the problem of individual participation in the next paragraph.

However, the maximum profit of the SC can be assured by adopting the SRS contract with appropriate parameters, this contract is desirable only when the individual profit of all the supply chain entities are higher than those under the decentralized scenario without the SRS contract, as discussed in section 5.3.2. In other words, to participate in the contract agreement, achieving win-win outcomes for all the supply chain entities is a key requirement. Hence under the assumption that the SRS coordinates the SC *i.e.,* equations (5.42)-(5.44) are fulfilled, to achieve a win win outcome, the contract make sure that the following conditions hold:

$$\Pi_{rc} > \Pi_{rd} \quad i.e., \quad \phi_{s1} + \phi_{s2} + \phi_m = \phi < \left(1 - \frac{\frac{\Pi_{rd} + br\bar{x}}{S_1 - S_2 S_3} - br}{p - h_r}\right); \tag{5.45}$$

$$\Pi_{mc} > \Pi_{md} \quad i.e., \quad \phi_m > \frac{\Pi_{md}}{(p - hr)(S_1 - S_2 S_3 - \alpha Q_1 S_4 - \frac{c_{s2} - h_m \bar{z}}{p + b_r - h_r} Q_2)}; \tag{5.46}$$

$$\Pi_{s2c} > \Pi_{s2d} \quad i.e., \quad \phi_{s2} > \frac{\Pi_{s2d} - (w_{s2}\bar{z} - c_{s2}) Q_2}{(p - h_r) S_1 + h_r S_2}; \qquad \text{and} \tag{5.47}$$

$$\Pi_{s1c} \quad > \quad \Pi_{s1d} \quad i.e., \quad \phi_{s1} > \frac{\Pi_{s1d} - \alpha(w_{s1} - c_{s1})Q_1 - (1-\alpha)(h_{s1} - c_{s1})Q_1}{(p - h_r)S_1 + h_r S_2}; \tag{5.48}$$

where $S_1 = \alpha E[\min\{x, Q, Q_1 + zQ_2\}] + (1-\alpha)E[\min\{x, Q, zQ_2\}]$ be the total expected sales, $S_2 = \alpha E[\min\{Q, Q_1 + zQ_2\}] + (1-\alpha)E[\min\{Q, zQ_2\}]$ be the total expected on hand inventory, $S_3 = \frac{(c_m + c_r + h_m - h_r)}{(p + b_r - h_r)}$ and $S_4 = \frac{\alpha(c_{s1} - h_m) + (1-\alpha)(c_{s1} - h_{s1}}{\alpha(p + b_r - h_r)}$.

To sum up, if we adopt the proposed spanning revenue-sharing contract (w_i, ϕ_i) for i $\in \{m, s1, s2\}$, where the revenue sharing ratios ϕ_i's satisfy equations (5.45)-(5.48) and the wholesale prices w_i are calculated by using equations (5.42)-(5.44), then the contract is acceptable to all supply chain entities and the maximum profit of the SC is attained. Because any values of ϕ_i that satisfy Eqs. (5.45)-(5.48) are acceptable, the adaptation of ϕ_i depends on the relative bargaining strength of the SC entities. We find that if the supply chain is coordinated, then for values $\phi_m = 0.45$, $\phi_{s2} = 0.25$, $\phi_{s1} = 0.10$ all the supply chain members earn the same amount of percentage of additional profits compared to the decentralized benchmark setting.

5.5 Numerical example

A numerical study is conducted in this section to illustrate the developed model. The random demand x is assumed to be uniformly distributed over $[l, u]$, $l \leq x \leq u$, with mean $\bar{x} = 500$ and standard deviation $\sigma_x = 500/\sqrt{3}$. The production yield of the expensive raw material supplier is assumed to follow uniform distribution with mean $\bar{z} = 0.77$ and standard deviation $\sigma_z = 0.06$. The exogenous parameter-values are set as follows: $p = 130$, $b_r = 5$, $h_r = 15$, $c_r = 5$, $c_m = 15$, $h_m = 5$, $c_{s1} = 20$, $c_{s2} = 40$, $h_{s1} = 10$ in appropriate units and probability of disruption $1 - \alpha = 0.2$.

For the above set of values, the concavity of the expected profit function $\Pi_c(Q_1, Q_2)$ with respect to Q_1 and Q_2 is checked graphically as shown in Fig. 5.2. Also we have, from Theorem 5.1, $\Pi_c(Q, Q_1, Q_2)$ is concave in Q. Therefore, we can conclude that $\Pi_c(Q, Q_1, Q_2)$ is jointly concave in Q, Q_1 and Q_2 in the centralized model. Now, for the same data set, under spanning revenue sharing contract, the manufacturer's profit function $\Pi_{mc}(Q_1, Q_2)$ is jointly concave in Q_1 and Q_2, which can be checked graphically as shown in Fig. 5.3.

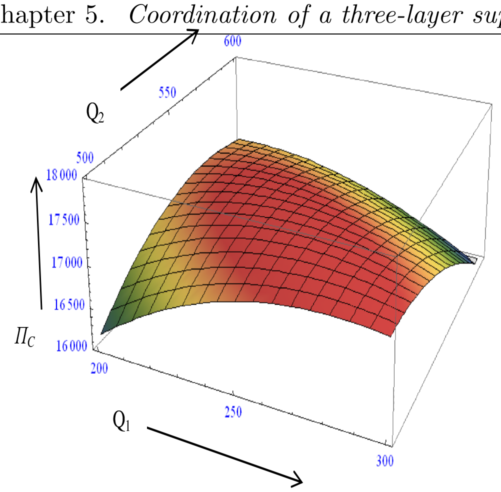


Fig. 5.2: Concavity of $\Pi_c(Q_1, Q_2)$.

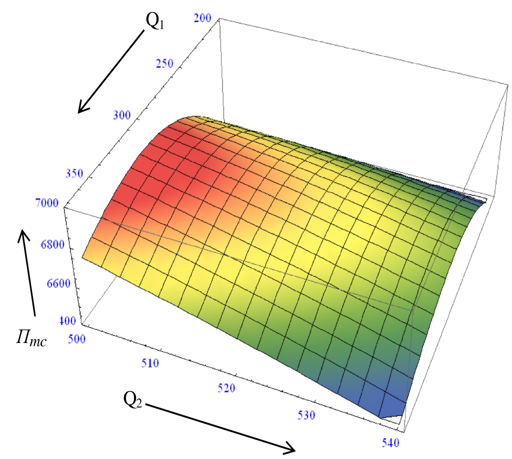


Fig. 5.3: Concavity of $\Pi_{mc}(Q_1, Q_2)$.

Table 5.1: Optimal decisions and expected profits under different scenarios.

Model	Q	Q_1	Q_2	Π_{s1}	Π_{s2}	Π_m	Π_r	Π	w_{s1}	w_{s2}	w_m
Centralized	669	259	528		-	-	-	17234	-	-	-
Decentralized	333	89	317	1247	1573	5362	4047	12230	40	60	90
Coordinated	669	259	528	2498	2824	6613	5299	17234	10.86	20.73	11.40

Optimal decisions and expected profits for the benchmark models and the spanning revenue sharing model are shown in Table 5.1. Here Π stands for the expected total profit of the supply chain irrespective of different kinds of scenario. Table 5.1 shows that the optimal order quantities in the decentralized benchmark model are less than their counterparts in the centralized benchmark model which supports Theorem 5.4. Once the optimal order quantities of the retailer and the manufacturer are derived then we can obtain the minimum values of ϕ_i after calculating equations (5.46)-(5.48) and the maximum value of their sum *i.e.,*$\phi_{s1} + \phi_{s2} + \phi_m = \phi$ from equation (5.45). After the retailers revenue sharing ratios ϕ_i are obtained, the consequent wholesale prices w_i can be derived from Eqs. (5.42)-(5.44).

Theorem 5.6 and related discussion show that the total profit of the SC under SRS contract is same as the centralized model irrespective of the values of ϕ_i. But how to allocate the increased profit among the supply chain entities depends on the values of ϕ_i. From the test results we find that if the conditions in Theorem 5.6 are satisfied *i.e.,* the supply chain is coordinated, then for $\phi_m = 0.3497$, $\phi_{s2} = 0.2844$, $\phi_{s1} = 0.0899$, all the supply chain members earn the same amount of additional profits due to spanning revenue sharing contract, compared to the decentralized benchmarks scenario. From Table 5.1, we find that the optimal order quantities of the retailer and the manufacturer in the spanning revenue sharing contract are equal to the centralized order level. Moreover, we find that the wholesale prices of the suppliers and the manufacturer under the spanning revenue sharing contract are less than their counterparts in price-only contract.

If ϕ_m is specified at its lower bound (0.2835), then the manufacturer's expected profit is the same as that of the decentralized benchmark scenario. Similarly, if $\phi_{s1}$and ϕ_{s2} are specified at its lower bounds (0.2836, 0.0712), the expected profits of both the suppliers are the same as that of the decentralized benchmark scenario. On the contrary, if ϕ *i.e.,* the total revenue sharing ratio of the retailer is specified at its upper bound (0.8826), the retailer's expected profit is the same as that of the decentralized benchmark scenario and all of the additional benefit goes to the manufacturer and the two suppliers depending on the values of ϕ_i. Any values below the lower limits of values of ϕ_i are not acceptable, since there may be a situation where a chain member is worse off by taking part into the proposed contract. Clearly, if the upstream members do not receive appropriate compensation from the retailer, they may not be interested to reduce their wholesale prices. Also the retailer may not propose the revenue sharing contract if the total revenue sharing ratio crosses its upper bound.

Table 5.2: Optimal decisions and expected profits under contract when demand variance varies.

σ_x	Q^*	Q_1^*	Q_2^*	Π_{s1c}	Π_{s2c}	Π_{mc}	Π_{rc}	Π_{dc}
$\frac{425}{\sqrt{3}}$	653	248	520	2349	3282	7594	6128	19354
$\frac{450}{\sqrt{3}}$	659	252	523	2257	3118	7287	5986	18650
$\frac{475}{\sqrt{3}}$	664	255	525	2168	2952	6980	5845	17946
$\frac{500}{\sqrt{3}}$	669	259	528	2079	2783	6669	5702	17234
$\frac{525}{\sqrt{3}}$	674	262	531	1993	2613	6357	5660	16523
$\frac{550}{\sqrt{3}}$	679	265	534	1907	2441	6044	5417	15811
$\frac{575}{\sqrt{3}}$	684	268	537	1823	2268	5730	5274	15097

As expected, profits of all members including the whole supply chain decrease as the demand randomness increases (Asian and Nie, 2014). This is because bad demand forecasting results in the whole system becoming worse off. Due to increase of uncertainty (σ_x) in market demand, the retailer should increase the on-hand inventory level so that the risk of stock-out of the final product decreases. So, the order quantity at all the stages (retailer and manufacturer) will increase if σ_x increases (Asian and Nie, 2014). This may increase the risk of over-stocking and decrease the system profit. The results are summarized in Table 5.2.

Table 5.3: Optimal decisions and total expected profits under both SRS and price-only contract when yield variance σ_z and probability of disruption $(1-\alpha)$ vary.

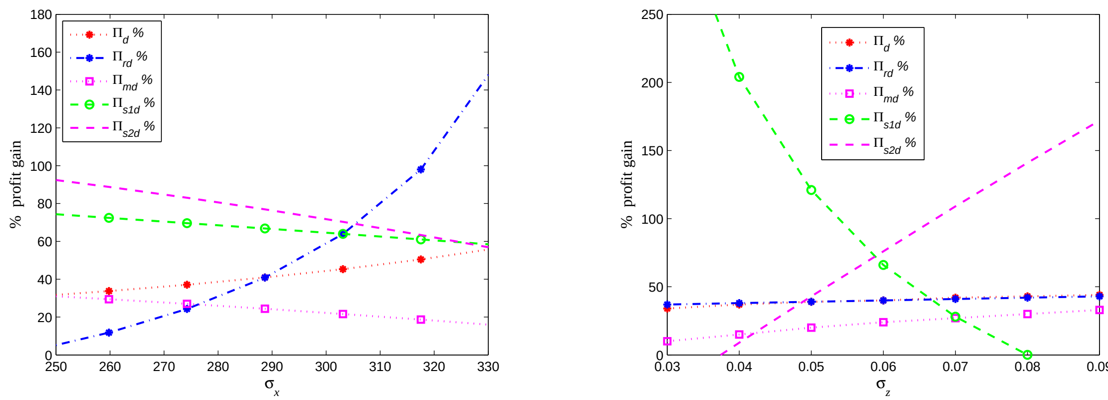
Risk term		SRS Contract				Price-only contract			
$1-\alpha$	σ_z	Q^*	Q_1^*	Q_2^*	Π_{dc}	Q^d	Q_1^d	Q_2^d	Π_d
0.4	0.01	545	30	669	14291	333	10	419	10783
	0.04	594	104	633	14826	333	38	383	10767
	0.07	633	161	601	15202	333	62	352	10717
	0.10	664	207	572	15468	333	82	325	10647
	0.13	691	244	546	15655	333	100	302	10566
0.5	0.01	541	21	673	14178	333	8	421	10469
	0.04	583	75	649	14440	333	31	391	10333
	0.07	618	119	626	14608	333	52	364	10178
	0.10	648	157	604	14704	333	71	340	10013
	0.13	675	189	583	14745	333	87	319	9846
0.6	0.01	538	13	677	14108	333	7	423	10170
	0.04	575	50	662	14192	333	27	397	9948
	0.07	608	80	646	14212	333	45	373	9714
	0.10	637	108	630	14181	333	62	351	9475
	0.13	664	132	613	14105	333	77	332	9236
0.7	0.01	537	6	680	14069	333	6	425	9879
	0.04	571	22	673	14049	333	23	402	9593
	0.07	603	37	664	13979	333	40	380	9298
	0.10	632	51	654	13864	333	55	360	8999
	0.13	658	65	642	13707	333	69	342	8701

In order to examine how the channel members' decisions are affected by supply disruption and yield uncertainty, we change the cheaper supplier's disruption probability $(1-\alpha)$ in the range of [0.4, 0.7]. As the production yield of the expensive raw material supplier is assumed to follow uniform distribution with mean $\bar{z} = 0.77$ then the standard deviation σ_z can vary within the range of [0, 0.1328] and we assume that the standard deviation σ_z takes five different values from its range. The retailer's and the manufacturer's optimal decisions in the decentralized model under both revenue sharing contract and price-only contract are shown in Table 5.3. As mentioned by Chopra et al. (2007), if the growth in supply risk comes from increased yield uncertainty of the expensive supplier then the best mitigation strategy is to increase the use of the cheaper supplier. From Table 5.3 we find that when the growth in supply risk occurs due to increase in yield risk at the expensive supplier, the manufacturer increases the use of the cheaper supplier and decreases the use of the expensive supplier whether the disruption risk $(1 - \alpha)$ of the cheaper supplier is low or high under SRS contract scenario. The manufacturer increases its order quantity at the cheaper supplier more rapidly than it decreases its order quantity at the expensive supplier. As yield risk of one supplier increases, the manufacturer increases utilization of the other supplier. By doing so, the manufacturer is able to mitigate the yield risk of the expensive supplier with the help of the cheaper supplier, which helps the manufacturer to recover from worse off. Similar arguments hold, if the growth in supply risk comes from increased disruption probability of the cheaper supplier.

In the decentralized model with price-only contract too, the manufacturer increases the use of the cheaper supplier and decreases the use of the expensive supplier as the production yield's standard deviation σ_z of the expensive supplier increases. It states that, as the production yield's uncertainty increases, the manufacturer orders extra units from the cheaper supplier and cuts order of the expensive supplier for avoiding risk increase. Since the retailer takes its decision individually in this scenario, it has no incentive to help the manufacturer to mitigate the expensive supplier's yield uncertainty through increasing the order quantity. Therefore, the retailer places unchanged order to the manufacturer to optimize its own objective function. However, the manufacturer realizes this fact and increases its order quantity at the cheaper supplier more slowly than it decreases its order quantity at the expensive supplier. As a result, the expected sales of the final product decreases and it will directly reduce the entire supply chain's profit as shown in Table 5.3

Our general intuition is that the channel performance should increase as the production yield randomness decreases, which has happened for the decentralized benchmark scenario. However, in the decentralized model under SRS contract, we notice a different result. Table 5.3 shows that, for a lower disruption risk of the cheaper supplier ($(1-\alpha)$=0.5, or less), as the yield randomness increases, the entire supply chain's profit also increases. For $(1-\alpha)$=0.6, the expected profit of the system first increases and then decreases as σ_z increases. This result is consistent with the result given in He and Zhang (2008). For a higher disruption risk of the cheaper supplier ($(1-\alpha)$=0.7 or more), as the yield randomness increases, the channel profit decreases. This result begs proper explanation to answer the question: why and how does the increased production yield of an upstream supplier enhance the whole system's efficiency in terms of profit for a lower disruption risk ($(1-\alpha)$=0.5 or less) but follows our general intuition for a higher disruption risk ($(1-\alpha)$=0.7 or more)? From our computational results, we find that, when the disruption risk is lower ($(1-\alpha)$=0.5 or 0.4), the SRS contract can hedge the increased yield uncertainty by decreasing the multiple marginalization effects and incentivizing the retailer to order more to satisfy more customer demand. However, if the disruption risk is too high ($(1-\alpha)$=0.7 or more) then the SRS contract is unable to hedge other risk that comes from increased yield uncertainty. So, as the yield randomness increases, the entire supply chain's profit decreases. For $(1-\alpha)$= 0.6, the SRS contract can hedge the increased yield uncertainty up to a threshold value (σ_z=0.07)). When the yield uncertainty exceeds the threshold value, the SRS contract can't hedge the yield uncertainty any more.

In the decentralized model under both spanning revenue sharing contract and price-only contract, both the retailer's and the manufacturer's optimal ordering policies are shown in Table 5.3 for different values of the cheaper supplier's disruption probability $(1-\alpha)$. For a fixed value of σ_z, we explore that, as the disruption probability $(1-\alpha)$ increases, the retailer orders less from the manufacturer to hedge the associated risk (as the chance of selling a product becomes lesser with higher uncertainty) under spanning revenue sharing contract. Table 5.3 illustrates that an increase in the cheaper supplier's disruption probability $(1-\alpha)$ encourages the manufacturer to increase the use of the expensive supplier to mitigate disruption risk, which is consistent with the results given in Chopra et al. (2007). We also examine the effect of $(1-\alpha)$ on the supply chain's optimal expected profits as shown in Table 5.3. As expected, the entire system's expected profit decreases if probability of disruption $(1-\alpha)$ increases under both revenue sharing contract and price-only contract.



(a) Effect of σ_x on increase in profit under coordination.

(b) Effect of σ_z on increase in profit under coordination.

Fig. 5.4: Performance of coordinated chain w.r.t. demand and supply uncertainties

We now plot '% profit gain' of the retailer, the manufacturer, the cheaper supplier, the expensive supplier and the total supply chain, along vertical axis, by $\Pi_i\%$ which is calculated as $\Pi_i\% = 100 \times (\Pi_{ic} - \Pi_i)/\Pi_i$ for $i \in (r, m, s1, s2, d)$ to show the percentage increase in profit under spanning revenue sharing contract compared to the corresponding profit with wholesale price-only contract as described in He and Zhao (2012). From Figs. 5.4(a) and 5.4(b) we find that the SRS contract is always beneficial to enhance the expected profit of the entire supply chain, whether growth in risk comes from increased demand uncertainty or increased production yield uncertainty. Figure 5.4(a) depicts that, with higher demand uncertainty, the profit gain of the retailer becomes more but for the manufacturer and both the suppliers, it becomes less. This is because a higher demand variance motivates the retailer to increase the on-hand inventory to mitigate demand uncertainty but the other entities have no such incentive as their revenue shares and wholesale prices are independent of demand variance under the proposed contract. Fig. 5.4(b) depicts the effect of production uncertainty of the expensive supplier on the efficiency of coordination. From Fig. 5.4(b), we observe that, with an increase in production uncertainty, the profit gain of all the entities becomes more except the cheaper supplier.

5.6 Conclusion

Supply uncertainty is very common in many companies having global supply networks. Managers are trying to innovate procurement process to mitigate the existing risk in supply chain. They rely on reliable backup resources having sufficient capacity. So far, access of unreliable backup supplier is not used as a mitigation strategy, what we have considered in this study.

In this study, we have analyzed the integrated model as the centralized benchmark case and the decentralized model with wholesale price only contract as decentralized benchmark case. Then aiming at how the risk of uncertainties in both supply and demand can be distributed among the supply chain entities, we have introduced a spanning revenue sharing contract into the decentralized system.

We have explored coordination conditions and elaborated the circumstance under which the contract is desirable to each of the individual members as well as the entire supply chain. Dada et al. (2007) have shown that cost dominates reliability when selecting suppliers. In this chapter, we have expanded their insights by focusing on the relative use of the two unreliable suppliers. Some managerial implications of our model are as follows:

- If the enhancement in supply uncertainty is mainly due to increase in random yield in production, it is optimal to increase the use of the cheaper supplier. In contrast, if the enhancement in supply uncertainty comes mainly from supply disruption, it is beneficial to over-utilize the expensive supplier.

- It is natural that all the supply chain members sign the proposed contract for its successful implementation. Although this does not mean that, for successful implementation, all the members have to sign the agreement simultaneously.

As a future research direction, one can study a multi-echelon supply chain model where more than one member can dominate the chain to control business policies. One can also consider the difference in quality between the raw materials produced by the cheaper supplier and the expensive supplier, and develop a model where the yield of the manufacturer depends on the quality of the raw material.

Chapter 6

Coordination of a supply chain with customer returns and quality improvement through customer feedbacks under demand and supply uncertainties*

6.1 Introduction

In many instances, customers feel confused when they have to choose from a vast product variety as they feel insecure whether specific products fulfill their needs or match their interests. To attract customers in an extremely competitive market and to reduce their hunt for competing alternatives, companies typically provide money-back assurance. That is, they provide assurance that a product can be returned if it does not satisfy customers needs. A return policy is especially good for those stores (e.g., online stores), where the customers don't get to see the physical product before they buy it. Because a large part of the customer population seeks money-back assurance, a retail return policy can help boost sales. Therefore, having a clear well thought out

*This chapter is based on the paper communicated in *European Journal of Operational Research*, (2021).

return policy of store is key to attracting and keeping customers.

Returns management is never as straight forward as putting products back on a shelf and shipping them to another consumer. Returns are subject to a quality check procedure. Once an item has been returned, the cause for its return should be determined. By correctly implementing this process, management is able to not only efficiently manage the reverse product flow, but also discover possibilities to reduce undesired returns and manage the reason why the item was unsatisfactory or did not reach consumers' expectations by enhancing product quality.

Customers' feedback is being used by smart companies like Amazon, Lego, DHL, and others to improve their products. Customer feedback is a useful and vast source of ideas for improving products and ensuring that they are aligned with their present and future demands. Feedback from customers assists in identifying the features and functions that customers are concerned about, as well as provide useful product improvement ideas that will help companies to make modifications to current products/services to fulfill consumers' needs. The world's largest postal and logistics service company DHL is an excellent example of product innovation driven by customer feedback. They collect consumer feedback to design solutions that enhance the delivery experience in order to improve their supply chain.

From this perspective, the manufacturer may see the necessity to collect customer feedback in addition to a return in order to improve the product. As a result, many businesses are now using product quality enhancement as an effective tool to meet consumers' expectations (He et al., 2016). There are two key factors by which quality of product is evaluated-product quality factors and service quality factors (Bergman and Klefsjö, 2010). The first type product quality is the focus of our current research. In reality, the product quality idea involves many more sub-dimensions of products, such as product safety, product reliability, environmental effect, maintenance, product longevity and maintainability. As a result, it is essential for a manufacturer to know customer likes and preferences in order to make design adjustments based on this information. However, improved product quality might result in more expenditures for manufacturers due to increased production costs, which can lead to higher product prices. As a result, quality improvement practices (Banker et al., 1998; Baiman et al.,

2000) influence demand and profitability of a supply chain in the customer returns environment. Hence, it's crucial for retailers to understand how to encourage manufacturers to invest in improving product quality.

The majority of the existing works in the relevant coordination literature has focused on either customer returns (Wang et al., 2020; Ruiz-Benitez and Muriel, 2014; Su, 2009) or product quality improvement (Baiman et al., 2000; He et al., 2016; Chakraborty et al., 2019). However, no attention has been given to the joint impact of these two research directions. The goal of our current study is to contribute to the existing literature by combining these two research directions with the help of customer feedback. We consider a supply chain in which a manufacturer produces a product with yield uncertainty and sells it to a retailer who faces investment dependent stochastic demand. The retailer serves the market with the product under the impact of customer returns. We investigate supply chain coordination issues in order to shed light on how to spread the two uncertainties (demand and yield) as well as quality improvement cost across chain members in the presence of customer returns. The key research questions in our current study are as follows: (1) How can the retailer incentive the manufacturer to invest in quality improvement? Would it be beneficial to all the channel members? (2) How to design appropriate coordinating mechanisms between the manufacturer and the retailer to encourage spontaneous participation? (3) What impact should coordinating contracts have on supply chain equilibrium decisions and profitability? (4) Can manufacturer convert some negative impact of customer returns into a possibility to enhance market demand?

The purpose in this study is to fill in a gap in the literature on supply chain coordination by looking at the combined impact of customer returns and quality improvement. It is also intended to investigate the effect of demand and supply uncertainties on the optimal decisions as well as how channel partners collaborate on product quality improvement investments. Wang et al. (2020) proposed a manufacturer-led game theoretic approach for coordinating a supply chain with consumer returns. However, our model differs from Wang et al. (2020)'s in the sense that instead of just considering a manufacturer-led scenario, we investigate a manufacturer-retailer-led scenario under the circumstance of consumer returns and improvement of product quality in our model. Furthermore, unlike them, we study both demand and production uncertainties simultaneously. Wang et al. (2020) concentrated on the impact of customer returns on supply chain ordering and pricing decisions, while in the present study, we aim to investigate the impact of customer returns, customer feedback for returns, as well as quality improvement investment on both production and ordering decisions. We investigate how the manufacturer incentives the retailer to collect and send customer feedbacks regarding their's product expectations and tastes. Our research determines whether the retailer can convince the manufacturer to invest in product quality improvement, and whether a composite contract can coordinate the supply chain. This study contributes to the literature on supply chain coordination by examining how customer feedbacks should be used to coordinate a supply chain in the context of customer returns and quality improvement.

The rest of this chapter is laid out as follows. In Section 6.2, we provide the description and notation of the proposed model. In Section 6.3, we derive the equilibrium solution of the integrated supply chain model. In Section 6.4, we develop the non-collaborative model under the well known wholesale price contract and investigate the individual decisions of the corresponding supply chain members. In Section 6.5, we consider the decentralized model under two different contracts viz. buy-back with revenue-sharing contract in section 6.5.1, and differentiated buy-back with revenue-sharing-cost-sharing contract in section 6.5.2, and discuss about how these coordinating contracts and customer returns affect equilibrium decisions. We conduct a sensitivity analysis in Section 6.6 to study the effect of a few key-parameters on the optimal decisions and profitability of the supply chain. In Section 6.7, the article concludes with a summary of managerial implications and future research goals.

6.2 Model design and notation

We study a supply chain which consists of a retailer and a manufacturer in the circumstance of product returns and improvement. The supply chain trades a seasonal product in a single period of time. The product quality improvement concept includes many factors of the product like environmental impact, maintenance, reliability, maintainability, safety and durability of the product *etc.* Hence, it is very important for a manufacturer to design appropriate quality improvement strategies. Among many of the earlier studies, Carter and Jennings (2002) showed that quality improvement strategies in the form of environmental sustainibility has a positive effect on supply chain performance by growing customer demand. We consider a strictly non-decreasing concave function $k\sqrt{\eta}$ of the manufacturer's investment η as a reward consumer demand where k is the quality improvement awareness coefficient of the customer for a given investment η. Many empirical studies support this assumption e.g., Zhao and Yin (2018).

To ensure consumers' satisfaction, retailers may offer a full refund return policy in which the consumers can return the purchased products if the products do not fit their individual needs or tastes. This is essentially a 100% money back guarantee offered to consumers. Chen and Bell (2009) and Wang et al. (2020) considered full refund policy in their studies. We also adopt the same assumption in this article. Following Wang, Chen and Chen (2019), in this chapter, we assume that the amount of consumer returns is a fixed proportion (γ) of sold products. The rate of customer return (γ) depends primarily on whether the products will satisfy consumer preferences or tastes rather than functioning defectiveness. In this case, the retailer can predict the rate of consumer returns based on historical data from previous sales or industry reports on related products, prior to the selling season (Yang et al., 2017). A large manufacturer mostly passes all of the expenses associated with the returned products to the retailer. In this situation, all handling expenses (S) relating to the return of the customer, for example, the charges of the workers, processing the returned goods and updating the records, are accompanied only by the retailer and assumed to be fixed.

The stochastic customer demand (D) depends on quality improvement investment. We take $D = \alpha + k\sqrt{\eta} + X$ (Raza, 2014), where α is the potential market share and $k\sqrt{\eta}$ is the deterministic reward demand based on investment and X is a random factor of demand, considered as independent of quality improvement investment η. Over the region $[l, u]$, X follows a continuous twice differentiable cumulative distribution function $F(\cdot)$ and probability density function $f(\cdot)$. The system mechanism of a decentralized supply chain is illustrated in the following:

The retailer places an order of Q units of the final product to the manufacturer. Because of random yield in production, the manufacturer sets a higher production size than Q. Suppose that the production quantity of raw material is Q_m. Then the produced quantity can be expressed as yQ_m (Giri et al., 2016). The production uncertainty rate y is a random variable having cdf $G(\cdot)$ and pdf $g(\cdot)$ over the region $[a, b]$ $0 \leq a \leq b \leq 1$ with mean $\bar{y}$ and standard deviation σ_y. If the produced amount is less than the amount ordered, then there is no emergency resource to fulfil the order. However, if the produced amount is more, then the excess amount after satisfying the retailer's demand can be salvaged in a secondary market at a lower wholesale price. Therefore, the manufacturer must be very careful when he sets his production lot size Q_m. The manufacturer hands over the produced units to the retailer before the start of the selling period. Based on a contract agreement, the transfer payment is made between them. During the selling period, the retailer satisfies consumers' demand at retail price p. Any unsatisfied demand creates a unit goodwill lose g_r and excess inventory can be salvaged at a unit salvage price v. We concentrate on products that are returned as a poor match to customer needs, rather than quality problems as in Chen and Bell (2009). Hence products returned to the retailer by the customer has a savage value $v_r < v$ due to unbox or somehow second hand products.

To better manage a trade off between the customer returns and the product improvement under demand and supply uncertainties, we propose a composite contract, where the manufacturer provides a differentiated buy-back contract to the retailer and the retailer offers a new revenue-sharing-cost-sharing contract to the manufacturer. The composite contract is of the take-it-or-leave-it type.

The distinguishing feature of our study is that we combine consumer returns in a supply chain coordination with product improvement. The existing coordination literature restricts focus to addressing these two areas separately. We also consider the demand uncertainty and the supply uncertainty simultaneously. We presume that the retail price is decided by the marketing team of the manufacturer and is therefore an exogenous parameter as adopted by Wang et al. (2020).

In pursuit of practical design and to prevent trivial cases, the attention is given to the following assumptions:

(i) We consider that one unit of raw material is required to produce one unit of final product.

(ii) We assume symmetric information *i.e.*, at the start of the selling season, both the players have the full information.

(iii) All the members involved in the supply chain are neutral and take rationale decisions to maximize their expected profits. Also reordering is not possible.

(iv) $v < c_m < w$; $v < w + c_r < p$; $c_m/\bar{y} < w$. These restrictions prevent the supply chain from infinite production, and assure that each chain member makes positive profit.

The notations used throughout the chapter are listed bellow:

x	:	stochastic part of customer demand with mean $\bar{x}$ and variance σ_x^2
y	:	random yield with mean $\bar{y}$ and variance σ_y^2
c_m	:	unit production cost of the manufacturer
c_r	:	unit handle cost of the retailer
g_r	:	unit goodwill lost of the retailer for unmet customer demand
v_r	:	unit salvage price of returned product from customer
v	:	unit salvage price of leftover at the manufacturer or retailer
η	:	product improvement expenditure of the manufacturer
p	:	retail price of the unit final product at the retailer
Q	:	order amount of the retailer placed at the manufacturer
Q_m	:	aimed production lot size of the manufacturer
w	:	unit wholesale price charged by the manufacturer to the retailer.

6.3 Centralized supply chain

We first consider the centralized system. Conceptually here only one decision-maker is involved to maximise the system profit. Profit allocation among the members can be viewed as a transfer of internal revenue. The expected profit function of the centralized supply chain can be expressed as

$$\begin{aligned}\Pi_c(Q, Q_m, \eta) &= p(1-\gamma)E[\min\{D, Q, yQ_m\}] + v_r\gamma E[\min\{D, Q, yQ_m\}] \\ &\quad + vE[(\min\{Q, yQ_m\} - D)^+] + vE[(yQ_m - Q)^+] \\ &\quad - g_rE[(D - \min\{Q, yQ_m\})^+] - c_rE[\min\{Q, yQ_m\}] \\ &\quad - c_mQ_m - S - \eta \qquad (6.1)\end{aligned}$$

The first four terms reflect the total revenue from the products that are purchased and retained by customers, purchased products that are returned by consumers, unsold goods selling at salvage price and excessive stock for over-production which are salvaged by the supply chain, respectively. The last five terms cover the total cost of the supply chain. The total cost includes production costs associated with raw material, handling cost of product at the retailing store, goodwill lost when items are out of stock, the cost of exhibiting demand influences activities and the cost of employees who handle returned products. The following is a different representation of the above expression:

$$\begin{aligned}\Pi_c(Q, Q_m, \eta) &= (p + g_r - v - (p - v_r)\gamma)\Big\{ \int_a^{\frac{Q}{Q_m}} \Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})} (\alpha + k\sqrt{\eta} + x) f(x)dx \\ &\quad + \int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \Big) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \Big(\int_l^{Q-(\alpha+k\sqrt{\eta})} (\alpha + k\sqrt{\eta} + x) f(x)dx \\ &\quad + \int_{Q-(\alpha+k\sqrt{\eta})}^{u} Q f(x)dx \Big) g(y)dy \Big\} - c_r\Big\{ \int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Q g(y)dy \Big\} \\ &\quad - (c_m - v\bar{y})Q_m - \eta - g_r\bar{x} - S \qquad (6.2)\end{aligned}$$

The following theorem characterizes the centralized benchmark model's optimal decisions.

Theorem 6.1 *The expected profit function $\Pi_c(Q, Q_m, \eta)$ is jointly concave in Q, Q_m and η. The optimal order quantity Q^c, production decision Q_m^c and quality improvement investment η^c can be determined from the following equations:*

$$Q^c = (\alpha + k\sqrt{\eta}) + F^{-1}\Big(\frac{(1-\gamma)p + v_r\gamma + g_r - c_r - v}{(1-\gamma)p + v_r\gamma + g_r - v}\Big) \qquad (6.3)$$

$$(p+g_r-v-(p-v_r)\gamma)\int_a^{\frac{Q}{Q_m^c}}\int_{yQ_m^c-(\alpha+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy-c_r\int_a^{\frac{Q}{Q_m^c}} yg(y)dy=(c_m-v\bar{y}) \tag{6.4}$$

$$\text{and}\quad \frac{k}{2\sqrt{\eta^c}}(p+g_r-v-(p-v_r)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta^c})} f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta^c})} f(x)dx\Big)g(y)dy\Big\}=1 \tag{6.5}$$

Proof. We have

$$\frac{\partial\Pi_c(Q,Q_m,\eta)}{\partial Q} = (p+g_r-v-(p-v_r)\gamma)\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{Q-(\alpha+k\sqrt{\eta})}^{u} f(x)dx\Big)g(y)dy-c_r\int_{\frac{Q}{Q_m}}^{b} g(y)dy$$

$$\frac{\partial\Pi_c(Q,Q_m,\eta)}{\partial Q_m} = (p+g_r-v-(p-v_r)\gamma)\int_a^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy -c_r\int_a^{\frac{Q}{Q_m}} yg(y)dy-(c_m-v\bar{y})$$

$$\frac{\partial\Pi_c(Q,Q_m,\eta)}{\partial \eta} = \frac{k}{2\sqrt{\eta}}(p+g_r-v-(p-v_r)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\}-1$$

Solving the first order optimality conditions, we can obtain the optimal order quantity Q^c, production decision Q_m^c and product quality investment η^c from the following equations:

$$Q=(\alpha+k\sqrt{\eta})+F^{-1}\Big(\frac{(1-\gamma)p+v_r\gamma+g_r-c_r-v}{(1-\gamma)p+v_r\gamma+g_r-v}\Big) \tag{6.3}$$

$$(p+g_r-v-(p-v_r)\gamma)\int_a^{\frac{Q}{Q_m^c}}\int_{yQ_m^c-(\alpha+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy-c_r\int_a^{\frac{Q}{Q_m^c}} yg(y)dy=(c_m-v\bar{y}) \tag{6.4}$$

$$\text{and}\quad \frac{k}{2\sqrt{\eta^c}}(p+g_r-v-(p-v_r)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta^c})} f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta^c})} f(x)dx\Big)g(y)dy\Big\}=1 \tag{6.5}$$

Second order derivatives of $\Pi_c(Q, Q_m, \eta)$ with respect to its decision variables are

$$\frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial Q^2} = -\{p+g_r-v-(p-v_r)\gamma\}\Big\{\bar{F}\Big(Q-(\alpha+k\sqrt{\eta})\Big)\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big) + f\Big(Q-(\alpha+k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q}{Q_m}\Big)\Big\} + c_r\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big)$$

$$\frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial Q_m^2} = -\{p+g_r-v-(p-v_r)\gamma\}\Big\{\bar{F}\Big(Q-(\alpha+k\sqrt{\eta})\Big)\Big(\frac{Q^2}{Q_m^3}\Big)g\Big(\frac{Q}{Q_m}\Big) + \int_a^{\frac{Q}{Q_m}} y^2 f\Big(yQ_m-(\alpha+k\sqrt{\eta})\Big)g(y)dy\Big\} + c_r\Big(\frac{Q^2}{Q_m^3}\Big)g\Big(\frac{Q}{Q_m}\Big)$$

$$\frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial \eta^2} = -\frac{k}{4\eta^{3/2}}(p+g_r-v-(p-v_r)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} - \frac{k^2}{4\eta}(p+g_r-v-(p-v_r)\gamma) \times\Big\{\int_a^{\frac{Q}{Q_m}} f\Big(yQ_m-(\alpha+k\sqrt{\eta})\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^b f\Big(Q-(\alpha+k\sqrt{\eta})\Big)g(y)dy\Big\}$$

$$\frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial Q_m \partial Q} = \frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial Q \partial Q_m} = (p+g_r-v-(p-v_r)\gamma)\bar{F}\Big(Q-(\alpha+k\sqrt{\eta})\Big)\Big(\frac{Q}{Q_m^2}\Big)g\Big(\frac{Q}{Q_m}\Big) - c_r\Big(\frac{Q}{Q_m^2}\Big)g\Big(\frac{Q}{Q_m}\Big)$$

$$\frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial \eta \partial Q} = \frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial Q \partial \eta} = (p+g_r-v-(p-v_r)\gamma)f\Big(Q-(\alpha+k\sqrt{\eta})\Big)\Big(\frac{k}{2\sqrt{\eta}}\Big) \times\bar{G}\Big(\frac{Q}{Q_m}\Big)$$

$$\frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial \eta \partial Q_m} = \frac{\partial^2 \Pi_c(Q,Q_m,\eta)}{\partial Q_m \partial \eta} = (p+g_r-v-(p-v_r)\gamma)\int_a^{\frac{Q}{Q_m}} \frac{k}{2\sqrt{\eta}}y \times f\Big(yQ_m-(\alpha+k\sqrt{\eta})\Big)g(y)dy$$

Now putting Q^c, Q_m^c and η^c in the above second order derivatives, we get

$$\begin{aligned}
\left[\frac{\partial^2\Pi_c(Q,Q_m,\eta)}{\partial Q^2}\right]_{Q=Q^c} &= -\{p+g_r-v-(p-v_r)\gamma\}f\Big(Q^c-(\alpha+k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q^c}{Q_m}\Big)\\
\left[\frac{\partial^2\Pi_c(Q,Q_m,\eta)}{\partial Q_m^2}\right]_{Q_m=Q_m^c} &= -\{p+g_r-v-(p-v_r)\gamma\}\int_a^{\frac{Q^c}{Q_m^c}} y^2 f\Big(yQ_m^c-(\alpha+k\sqrt{\eta})\Big)g(y)dy
\end{aligned}$$

$$\begin{aligned}
\left[\frac{\partial^2\Pi_c(Q,Q_m,\eta)}{\partial \eta^2}\right]_{\eta=\eta^c} &= -\frac{k^2}{4\eta^c}(p+g_r-v-(p-v_r)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}} f\Big(yQ_m-(\alpha+k\sqrt{\eta^c})\Big)g(y)dy\\
&\quad+\int_{\frac{Q}{Q_m}}^b f\Big(Q-(\alpha+k\sqrt{\eta^c})\Big)g(y)dy\Big\}-\frac{1}{2\eta^c}\\
\left[\frac{\partial^2\Pi_c(Q,Q_m,\eta)}{\partial Q_m\partial Q}\right]_{\binom{Q=Q^c}{Q_m=Q_m^c}} &= \left[\frac{\partial^2\Pi_c(Q,Q_m,\eta)}{\partial Q\partial Q_m}\right]_{\binom{Q=Q^c}{Q_m=Q_m^c}}=0
\end{aligned}$$

If H_i denotes the principal minor of the associated Hessian H of ith order, $i=1,2,3$. Then

$$H_1 = -\{p+g_r-v-(p-v_r)\gamma\}f\Big(Q^c-(\alpha+k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q^c}{Q_m}\Big)<0$$

$$\begin{aligned}
H_2 &= \{p+g_r-v-(p-v_r)\gamma\}^2 f\Big(Q^c-(\alpha+k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q^c}{Q_m^c}\Big)\\
&\quad\times\int_a^{\frac{Q^c}{Q_m^c}} y^2 f\Big(yQ_m^c-(\alpha+k\sqrt{\eta})\Big)g(y)dy>0
\end{aligned}$$

$$\begin{aligned}
H_3 &= -\{p+g_r-v-(p-v_r)\gamma\}^2\frac{1}{2\eta^c}f\Big(Q-(\alpha+k\sqrt{\eta^c})\Big)\bar{G}\Big(\frac{Q}{Q_m}\Big)\\
&\quad\times\int_a^{\frac{Q}{Q_m}} y^2 f\Big(yQ_m-(\alpha+k\sqrt{\eta^c})\Big)g(y)dy<0
\end{aligned}$$

This proves that the expected profit function of the centralized model is strictly concave. ■

The maximum channel profit $\Pi_c(Q^c, Q_m^c, \eta^c)$ is obtained by substituting the optimal values of the decision variables in equation (6.2). We are unable to find a closed form solution due to the model complexity. Theorem 1 reveals that demand and supply uncertainties are both important factors in determining the optimal decisions of the system. When production uncertainty rises, production amount must be increased. Due to rising production uncertainty, the system increases the planned raw material production quantity in order to avoid under-production. On the other hand, the order quantity of the finished product reduces as the system's efficiency drops down in response to increased production uncertainty.

6.4 Decentralized supply chain

Here we consider a decentralized supply chain with a wholesale price-only contract between the retailer and the manufacturer. In this agreement, the manufacturer offers its product to the retailer at a unit wholesale price w and wants to invest a total amount η into its product quality improvement activity to enhance customer demand. We assume the unit wholesale price w is predetermined in order to focus on studying the implications of demand and supply uncertainties on the supply chain coordination (Wang et al., 2020). Getting the approval of the agreement, the sequence of event follows the procedure as defined earlier in section 6.3. We study a Nash sequence in which the manufacturer makes the first decision and the system is solved through backward substitution. As a result, the retailer must first figure out what his optimal decisions would be. For given Q_m and η, the retailer's profit function $\Pi_r(Q)$ can be derived as follows:

$$\begin{aligned}\Pi_r(Q) \quad = \quad & p(1-\gamma)E[\min\{D, Q, yQ_m\}] + w\gamma E[\min\{D, Q, yQ_m\}] + vE[(\min\{Q, yQ_m\} - D)^+] \\ & -g_r E[(D - \min\{Q, yQ_m\})^+] - (w + c_r)E[\min\{Q, yQ_m\}] - S \qquad (6.6)\end{aligned}$$

The first three terms indicate the retailer's total revenue from products that are purchased and retained by consumers, products that are returned to the manufacturer for refund, and unsold products sold at salvage price, respectively. The last three terms are the cost of goodwill lost due to unfulfilled demand, total cost of acquiring and handling the order quantity, and cost of employees who handle returned items and manage returns information data, respectively. A alternate formulation of the above expression is as follows:

$$\begin{aligned}\Pi_r(Q) &= (p+g_r-v-(p-w)\gamma)E[\min\{D,Q,yQ_m\}]-(w+c_r-v)E[\min\{Q,yQ_m\}]\\ &\quad -g\bar{x}-S\\ &= (p+g_r-v-(p-w)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx\\ &\quad +\int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx\\ &\quad +\int_{Q-(\alpha+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}-(w+c_r-v)\Big\{\int_a^{\frac{Q}{Q_m}}yQ_mg(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\}\\ &\quad -g\bar{x}-S \qquad (6.7)\end{aligned}$$

Theorem 6.2 *Under wholesale price-only contract, for given production decision and improvement investment of the manufacturer, the retailer's objective function is concave in Q and the optimal order quantity is given by*

$$Q^d = (\alpha+k\sqrt{\eta})+F^{-1}\Big(\frac{(1-\gamma)(p-w)+g_r-c_r}{(1-\gamma)p+w\gamma+g_r-v}\Big) \qquad (6.8)$$

Proof. We have

$$\begin{aligned}\frac{\partial\Pi_r(Q)}{\partial Q} &= \{p+g_r-v-(p-v_r)\gamma\}\bar{F}(Q-(\alpha+k\sqrt{\eta}))\bar{G}\Big(\frac{Q}{Q_m}\Big)-(w+c_r-v)\bar{G}\Big(\frac{Q}{Q_m}\Big) \qquad (6.9)\\ \frac{\partial^2\Pi_r(Q)}{\partial Q^2} &= -\{p+g_r-v-(p-v_r)\gamma\}\Big(\bar{F}(Q-(\alpha+k\sqrt{\eta}))\frac{1}{Q_m}g\Big(\frac{Q}{Q_m}\Big)\\ &\quad +f(Q-(\alpha+k\sqrt{\eta}))\bar{G}\Big(\frac{Q}{Q_m}\Big)\Big)+(w+c_r-v)\frac{1}{Q_m}g\Big(\frac{Q}{Q_m}\Big) \qquad (6.10)\end{aligned}$$

From the first order optimality condition $\frac{\partial\Pi_r(Q)}{\partial Q}=0$, we find the retailer's optimal order quantity Q^d as

$$Q^d = (\alpha+k\sqrt{\eta})+F^{-1}\Big(\frac{(1-\gamma)(p-w)+g_r-c_r}{(1-\gamma)p+w\gamma+g_r-v}\Big) \qquad (6.8)$$

Putting Q^d presented in equation (6.8) into equation (6.10) we get,

$$\frac{\partial^2\Pi_r(Q)}{\partial Q^2} = -\{p+g_r-v-(p-v_r)\gamma\}f(Q^d-(\alpha+k\sqrt{\eta}))\bar{G}\Big(\frac{Q^d}{Q_m}\Big)\Big) \le 0. \blacksquare$$

It can be shown that the optimal order quantities Q^d is decreasing with respect to handling cost, while it is increasing with respect to customer satisfaction improvement investment. Also, $\frac{\partial Q}{\partial \gamma} < 0$.

It is evident from Theorem 6.2 that the retailer increases his order quantity whenever the manufacturer increases the investment on customer satisfaction level for its product. This corroborates with our existing intuition that a higher quality investment cost impels higher demand that motivates the retailer to order more from the manufacturer. It is clear from equation (6.8) that a higher handling cost and a salvage value have respectively negative and positive effects on the order quantity of the retailer.

Comparing the order quantity in equation (6.8) with that of the centralized system in equation (6.3), we find that the retailer orders less in the decentralized system, resulting in a reduction in profit. Therefore, we see that, in a wholesale price contract, the supply chain is distorted from the retailer's action by ordering less than the optimal order in the centralized system. After investigating the retailer's problem and getting the optimum decision $\left(Q^d\right)$, we now derive the manufacturer's expected profit function $\Pi_m(Q_m, \eta)$ in the following:

$$\begin{aligned}\Pi_m(Q_m, \eta) \;&=\; wE[\min\{Q, yQ_m\}] + vE[(yQ_m - Q)^+] - c_m Q_m \\ &\quad -(w - v_r)\gamma E[\min\{D, Q, yQ_m\}] - \eta \end{aligned} \tag{6.11}$$

The first two terms in (6.11) are sales revenue from the wholesale price and salvage value, respectively; the last three terms are the costs of production, loss due to customers returns and investment on product quality improvement. The above equation can be written as

$$\begin{aligned}\Pi_m(Q_m, \eta) \;&=\; (w - v)E[\min\{Q, yQ_m\}] - (w - v_r)\gamma E[\min\{D, Q, yQ_m\}] - (c_m - v\bar{y})Q_m - \eta \\ &=\; (w - v)\Big\{\int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^b Qg(y)dy\Big\} - (w - v_r)\gamma \\ &\quad \times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})}(\alpha + k\sqrt{\eta} + x)f(x)dx \\ &\quad + \int_{yQ_m-(\alpha+k\sqrt{\eta})}^u (yQ_m)f(x)dx\Big)g(y)dy\end{aligned}$$

$$+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx$$
$$+\int_{Q-(\alpha+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}-(c_m-v\bar{y})Q_m-\eta \tag{6.12}$$

The following theorem characterizes the manufacturer's optimal production amount and product quality improvement investment in the decentralized system under wholesale price-only contract.

Theorem 6.3 *The expected profit function $\Pi_m(Q_m,\eta)$ is concave in both Q_m and η and the optimal input amount Q_m^d and improvement investment η^d satisfy the following equations*

$$(w-v)\int_{a}^{\frac{Q}{Q_m^d}}yg(y)dy-(w-v_r)\gamma\int_{a}^{\frac{Q}{Q_m^d}}\int_{yQ_m^d-(\alpha+k\sqrt{\eta})}^{u}yf(x)dxg(y)dy=(c_m-v\bar{y}) \tag{6.13}$$

$$\textit{and}\quad \frac{k}{2\sqrt{\eta^d}}(w-v_r)\gamma\Big\{\int_{a}^{\frac{Q}{Q_m}}\Big(\int_{l}^{yQ_m-(\alpha+k\sqrt{\eta^d})}f(x)dx\Big)g(y)dy$$
$$+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha+k\sqrt{\eta^d})}f(x)dx\Big)g(y)dy\Big\}=1 \tag{6.14}$$

Proof. Due to complexity, the concavity of $\Pi_m(Q_m,\eta)$ with respect to Q_m and η can not be proved analytically. The manufacturer's profit function $\Pi_{mc}(Q_1,Q_2)$ is jointly concave in Q_1 and Q_2, which can be checked graphically as shown in Fig. 6.1. To prove the theorem, we have

$$\frac{\partial\Pi_m(Q_m,\eta)}{\partial Q_m}=-(w-v_r)\gamma\int_{a}^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u}yf(x)dxg(y)dy$$
$$+(w-v)\int_{a}^{\frac{Q}{Q_m}}yg(y)dy-(c_m-v\bar{y})$$

$$\frac{\partial\Pi_m(Q_m,\eta)}{\partial\eta}=\frac{k}{2\sqrt{\eta}}(w-v_r)\gamma\Big\{\int_{a}^{\frac{Q}{Q_m}}\Big(\int_{l}^{yQ_m-(\alpha+k\sqrt{\eta})}f(x)dx\Big)g(y)dy$$
$$+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha+k\sqrt{\eta})}f(x)dx\Big)g(y)dy\Big\}+1$$

Solving the first order conditions for optimality of $\Pi_m(Q_m, \eta)$, we get the required optimal production quantity Q_m^d and product quality investment η^d as given in equations (6.13) and (6.14). ■

Theorem 6.3 demonstrates that, as the customer returns rate γ increases, the manufacturer decreases both the optimal production quantity (Q_m^d) and the optimal investment (η^d) under the wholesale price contract. As the rate of customer returns increases, both the manufacturer's and the retailer's revenues decrease. Since the manufacturer carries the risk of producing more than the order requires, he produces less and invests less on product development as he alone has to bear all costs related to this activities, although the entire system is benefited from this investment.

In the decentralized scenario, the manufacturer's self-interest brings the double marginalization effect by raising the wholesale price higher than the production cost. This is the only way he can make a positive expected profit. On the other hand, the retailer's ignorance to the fact that an additional unit ordered will make the manufacturer an additional profit leads to the double marginalization effect. This occurs because, although the firms are free to make their own decisions under the wholesale price-only contract, the manufacturer bears the risk of his own production uncertainty while the retailer bears the risk of his own demand uncertainty. This self-interested strategy has no flexibility to manage over-production and under-production difficulties associated with the manufacturer as well as the risk of demand uncertainty of the retailer.

Theorem 6.4 *Both the order and the production quantities in the decentralised model are strictly less than their counterparts in the centralized model. A lower order results in a lower expected supply chain profit in the decentralised setting.*

Proof. Since $v_r < v$ and $v < c_m < w$, we have $w > v_r$. Now, comparing equation (6.3) with (6.8) and using the fact that $w > v_r$, we find that, in the decentralized model, the retailer's optimal order quantity is strictly less than that in the centralized benchmark model. As the retailer orders less amount for the final product, the order quantity of the manufacturer for raw materials in the decentralized model is also less than that in the centralized benchmark model, which can be realized by comparing equations (6.13) and (6.14) with (6.4) and (6.5), respectively. Adding the supply chain members' expected profits, we get $\Pi_r + \Pi_m = \Pi_d$. It is easy to verify that $\Pi_c(Q^c, Q_m^c, \eta^c) > \Pi_d(Q^d, Q_m^d, \eta^d)$. Both the supplier's and the manufacturer's individual pricing policies are the reason behind the supply chain's inefficiency in the decentralized setting. ■

The above Theorem indicates that, in a decentralized environment, even if all the chain participants maximize their own profits, the total channel performance is not attained to its maximum level. Since the decision control is shared among the different chain members in a decentralized situation, there is a possibility of deviating from the optimal decisions made in the centralized model. Contract mechanisms are used to align each member's decisions with the centralized strategy to prevent double-marginalization by reducing competitiveness among the chain members without affecting the supply chain's structure or decision-making power.

6.5 Coordination mechanism for a decentralized supply chain

In the wholesale price contract, both the retailer and the manufacturer have to bear the risk of uncertainties on their own. That's why the 'double marginalization' problem occurs, which decreases profits of individual members as well as the whole supply chain. Therefore, a suitable contract mechanism needs to be developed so that both the retailer and the manufacturer are motivated to share the risk of other members. A decentralized supply chain is said to be coordinated under some contract if it generates the same profit potential as the centralized supply chain does. Both traditional manufacturer-led and growing manufacturer-retailer-led scenarios are taken into account in this article. To coordinate the supply chain in both circumstances, we consider the buy-back contract and the sharing contract. We use appropriate protocols to coordinate the supply chain under specific framework, the selection of which is based on existing business plan as well as context-specific characteristics.

6.5.1 Buy-back with revenue-sharing contract in the manufacturer-led scenario

Distortion in terms of double marginalization in the supply chain under decentralized model occurs in two ways. First, in case of uncertain demand, it is the retailer who deviates from the system's optimal action by ordering less product. Second, in case of random yield, it is the manufacturer who deviates from the system's optimal action by producing less product. A buy-back policy is a well-known strategy for correcting the retailer's under-ordering problem caused by demand uncertainty (Pasternack, 1985; Emmos and Gilbert, 1998). On the other hand, a revenue-sharing agreement may solve the problem of manufacturer's under-production by asking the retailer to share with the manufacturer a portion of his revenue generated from the expected sales. Considering consumer returns, Ruiz-Benitez and Muriel (2014) demonstrated that a buy-back policy can indeed encourage the retailer to raise his order quantity. Zhao and Yin (2018) showed that a supply chain in which the manufacturer only incurs the product quality improvement cost, is coordinated with revenue-sharing mechanism.

This section focuses on how to create a contract agreement by integrating these two contract mechanisms between the manufacturer and the retailer so that both the retailer and the manufacturer are motivated to share the risk of the other member. We consider a buy-back contract with revenue-sharing offered by the manufacturer to the retailer. The retailer has the option to accept or reject the offer. If the retailer accepts the offer, the manufacturer will purchase the remanning inventory of the retailer at a predetermined buy-back price b per unit at the end of the selling season. In addition, the retailer shares a proportion $(1-\phi)$ of his total revenue in exchange for the manufacturer's lower wholesale price, while keeping a fraction ϕ for himself. We will investigate whether the proposed contract is beneficial to each member and whether the retailer would be willing to accept it. Under this contract, for given production size and product quality improvement cost, the expected profit of the retailer can be obtained as

$$\Pi_{rbr}(Q) \;=\; pE[\min\{D,Q,yQ_m\}] + bE[(\min\{Q,yQ_m\}-D)^+] - g_rE[(D-\min\{Q,yQ_m\})^+]$$

$$
\begin{aligned}
& -(p-w)\gamma E[\min\{D,Q,yQ_m\}] - S - (w+c_r)E[\min\{Q,yQ_m\}] \\
= \; & (p+g_r-b-(p-w)\gamma)E[\min\{D,Q,yQ_m\}] - (w+c_r-b)E[\min\{Q,yQ_m\}] \\
& -g\bar{x} - S \qquad (6.15)
\end{aligned}
$$

We can rewrite the profit function (6.15) as follows:

$$
\begin{aligned}
&\Pi_{rbr}(Q) = (p+g_r-b-(p-w)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx \\
&+\int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx \\
&+\int_{Q-(\alpha+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}-(w+c_r-b)\Big\{\int_a^{\frac{Q}{Q_m}}yQ_m g(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\} \\
&-g\bar{x}-S \qquad (6.16)
\end{aligned}
$$

From the above, the optimal order quantity Q^{br} can be obtained as

$$
Q^{br} = (\alpha+k\sqrt{\eta})+F^{-1}\Big(\frac{(p-w)(1-\gamma)+g_r-c_r}{(p+g_r-b-(p-w)\gamma)}\Big) \qquad (6.17)
$$

This gives the global maximum since the second derivative of $\Pi_{rc}(Q)$ with respect to Q is non-positive. The manufacturer's expected profit function under this contract is given by

$$
\begin{aligned}
\Pi_{mbr}(Q_m,\eta) \; = \; & wE[\min\{Q,yQ_m\}]+vE[(yQ_m-Q)^+]-(b-v)E[(\min\{Q,yQ_m\}-D)^+]\Big\} \\
& -c_mQ_m-(w-v_r)\gamma E[\min\{D,Q,yQ_m\}]-\eta \\
= \; & (b-v)-(w-v_r)\gamma)E[\min\{D,Q,yQ_m\}]+(w-b+)E[\min\{Q,yQ_m\}] \\
& -(c_m-v\bar{y})Q_m-\eta \qquad (6.18)
\end{aligned}
$$

The above can be rewritten as

$$
\begin{aligned}
&\Pi_{mbr}(Q_m,\eta) = (b-v)-(w-v_r)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx \\
&+\int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx
\end{aligned}
$$

$$+\int_{Q-(\alpha+k\sqrt{\eta})}^{u} Qf(x)dx\Big)g(y)dy\Big\} - (w-b)\times\Big\{\int_{a}^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy\Big\}$$
$$-(c_m - v\bar{y})Q_m - \eta \tag{6.19}$$

Taking derivatives of $\Pi_{mbr}(Q_m, \eta)$ with respect to Q_m and η and setting them equal to zero, we obtain the optimal production amount Q_m^{br} and CSR investment η^{br} of the manufacturer satisfying the following equations:

$$(b-v)-(w-v_r)\gamma)\int_{a}^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy$$
$$+(w-b)\int_{a}^{\frac{Q}{Q_m}} yg(y)dy \quad = \quad (c_m - v\bar{y}) \tag{6.20}$$

$$\frac{k}{2\sqrt{\eta}}(w-b)\Big\{\int_{a}^{\frac{Q}{Q_m}}\Big(\int_{l}^{yQ_m-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} = 1 \tag{6.21}$$

If the supply chain is coordinated by a buy-back policy with revenue-sharing, the optimal choices for the integrated supply chain should maximise both the retailer's and the manufacturer's expected profits in (6.16) and (6.19) simultaneously. Comparing (6.3) with (6.17), (6.4) with (6.20) and (6.5) with (6.21), we find that there exist a wholesale price $w = b$ and buy-back price $b = (\phi - \gamma)p + g + w\gamma$ so that $Q^{br} = Q^c$, $Q_m^{br} = Q_m^c$ and $\eta^{br} = \eta^c$; i.e., the optimal decisions of both the retailer and manufacturer are aligned with the centralized system, when $c_r = 0$. We, therefore, conclude in general that the manufacturer cannot offer such contract to coordinate the supply chain. This leads to the following theorem.

Theorem 6.5 *A decentralized supply chain with customer returns that are proportional to sales cannot be coordinated by a buy-back agreement along with revenue-sharing contract.*■

Since the buy-back contract mechanism does not address returned products from customers, it is unable to fix the retailer's under-ordering problem in the case of customer returns. Similarly, while the revenue sharing arrangement incentives the manufacturer to enhance his production capacity, the manufacturer is still responsible for all product development costs. Therefore, we must modify the contract mechanism described above to coordinate the supply chain.

6.5.2 Differentiated buy-back with a modified revenue sharing contract

Differentiating between unsold and returned items is one feasible way to modify buy-back contract to align retailer's objective with the centralized one. We term this as differentiated buy-back contract in which the manufacturer trades with the retailer at the wholesale price w but offers to purchase all remaining products at the end of the selling season. For each unit of unsold (new) product, the retailer gets a credit of b. However, for returned items, the credit is enhanced by $b^{'}$ per unit to share the expenditure of returns processing with the retailer. In other sense, the manufacturer purchases unsold units at the rate b and return units at the rate $b_r = b + b^{'}$.

Although the revenue-sharing contract can encourage the manufacturer to raise the retailer's on-hand inventory by managing the manufacturer's under-production problem, some other barriers may not allow raising the expected sales due to lower customer demand. This implies that, in order to enhance the expected sales, the contract mechanism should attempt to increase the customer demand too by aligning the manufacturer's investment decision for product quality improvement with the centralized system. The retailer's cost-sharing contract is a technique that can correct the manufacturer's reduced investment decision by asking the retailer to share the manufacturer's investment costs in order to raise customer demand. In this approach, the retailer encourages the manufacturer to raise his investment in product development so that the rewarded customer demand boosts the market resulting in increased expected sales. As the revenue-sharing contract can't give right incentive to the manufacturer to align his production and investment decisions with the centralized system, we merge the above two contracts to define revenue-sharing-cost-sharing contract offered by the retailer under which the retailer not only shares with the manufacturer a portion $(1-\phi)$ of revenue made from the expected sales but also shares a portion (ϕ) of the manufacturer's investment on product quality development activities. It's worth noting that the manufacturer's profit now depends not only on the retailer's order but also on the market demand. Such a contract can only be arranged if the demand information is available to the manufacturer. In this case, the retailer's profit is given by

$$\begin{aligned}\Pi_{rc}(Q) &= \phi\Big\{pE[\min\{D,Q,yQ_m\}]-\eta\Big\}+bE[(\min\{Q,yQ_m\}-D)^+]-g_rE[(D-\min\{Q,yQ_m\})^+]\\&\quad-(p-b_r)\gamma E[\min\{D,Q,yQ_m\}]-S-(w+c_r)E[\min\{Q,yQ_m\}]\\&=(\phi p+g_r-b-(p-b_r)\gamma)E[\min\{D,Q,yQ_m\}]-(w+c_r-b)E[\min\{Q,yQ_m\}]\\&\quad-g\bar{x}-S-\phi\eta \qquad (6.22)\end{aligned}$$

The above can be rewritten as

$$\begin{aligned}\Pi_{rc}(Q)&=(\phi p+g_r-b-(p-b_r)\gamma)\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx\\&+\int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})}(\alpha+k\sqrt{\eta}+x)f(x)dx\\&+\int_{Q-(\alpha+k\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}-(w+c_r-b)\Big\{\int_a^{\frac{Q}{Q_m}}yQ_mg(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\}\\&-g\bar{x}-S-\phi\eta \qquad (6.23)\end{aligned}$$

Theorem 6.6 *For the decentralized model under differentiated buy-back with revenue-sharing-cost-sharing contract, the expected profit function* $\Pi_{rc}(Q)$ *is concave in* Q *and the optimal order quantity* Q^* *is obtained from the following equation:*

$$Q^* = (\alpha+k\sqrt{\eta})+F^{-1}\Big(\frac{\phi p+g_r-(p-b_r)\gamma-w-c_r}{\phi p+g_r-b-(p-b_r)\gamma}\Big) \qquad (6.24)$$

Proof. We have

$$\frac{\partial\Pi_{rc}(Q)}{\partial Q} = (\phi p+g_r-b-(p-b_r)\gamma)\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{Q-(\alpha+k\sqrt{\eta})}^{u}f(x)dx\Big)g(y)dy$$

$$-(w + c_r - b)\int_{\frac{Q}{Q_m}}^{b} g(y)dy$$

$$\frac{\partial^2 \Pi_{rc}(Q, Q_m, \eta)}{\partial Q^2} = -(\phi p + g_r - b - (p - b_r)\gamma)\Big\{\bar{F}\Big(Q - (\alpha + k\sqrt{\eta})\Big)\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big) + f\Big(Q - (\alpha + k\sqrt{\eta})\Big)\bar{G}\Big(\frac{Q}{Q_m}\Big)\Big\} + (w + c_r - b)\Big(\frac{1}{Q_m}\Big)g\Big(\frac{Q}{Q_m}\Big)$$

From the first order optimality condition $\frac{\partial \Pi_{rc}(Q)}{\partial Q} = 0$, we get the retailer's optimal order quantity Q^* as

$$Q^* = (\alpha + k\sqrt{\eta}) + \bar{F}^{-1}\Big(\frac{w + c_r - b}{\phi p + g_r - b - (p - b_r)\gamma}\Big) \tag{6.24}$$

Using Q^* as presented in (6.24), we get

$$\frac{\partial^2 \Pi_{rc}}{\partial Q^2} = -(\phi p + g_r - b - (p - b_r)\gamma)f(Q - (\alpha + k\sqrt{\eta}))\bar{G}\Big(\frac{Q}{Q_m}\Big)\Big) \leq 0.\blacksquare$$

Equation (6.24) shows that the retailer's optimal order quantity is an increasing function of the investment η and a decreasing function of its purchasing cost w and treating cost c_r, as expected. Taking into account the retailer's optimum responses, we now determine the manufacturer's optimal decisions. The manufacturer's expected profit function is given by

$$\begin{aligned}\Pi_{mc}(Q_m, \eta) &= (1-\phi)\{pE[\min\{D, Q, yQ_m\}] - \eta\} + wE[\min\{Q, yQ_m\}] + vE[(yQ_m - Q)^+] \\ &\quad - c_m Q_m - (b_r - v_r)\gamma E[\min\{D, Q, yQ_m\}] - (b - v)E[(\min\{Q, yQ_m\} - D)^+] \\ &= \{(1-\phi)p - (b_r - v_r)\gamma + (b - v)\}E[\min\{D, Q, yQ_m\}] - (w - b)E[\min\{Q, yQ_m\}] \\ &\quad - (c_m - v_m\bar{y})Q_m - (1-\phi)\eta \end{aligned} \tag{6.25}$$

An alternative representation of the manufacturer's profit function is given below:

$$\begin{aligned}\Pi_{mc}(Q_m, \eta) &= \{(1-\phi)p - (b_r - v_r)\gamma + (b - v)\}\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m - (\alpha + k\sqrt{\eta})}(\alpha + k\sqrt{\eta} + x)f(x)dx \\ &\quad + \int_{yQ_m - (\alpha + k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q - (\alpha + k\sqrt{\eta})}(\alpha + k\sqrt{\eta} + x)f(x)dx \\ &\quad + \int_{Q - (\alpha + k\sqrt{\eta})}^{u} Qf(x)dx\Big)g(y)dy\Big\} - (w - b) \times \Big\{\int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy\Big\} \\ &\quad - (c_m - v_m\bar{y})Q_m - (1-\phi)\eta \end{aligned} \tag{6.26}$$

Theorem 6.7 *The profit function $\Pi_{mc}(Q_m, \eta)$ is concave in both Q_m and η and the optimal input amount Q_m^* and product improvement expenditure η^* satisfy the following equations:*

$$\{(1-\phi)p - (b_r - v_r)\gamma + (b-v)\}\gamma) \int_a^{\frac{Q}{Q_m}} \int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy$$
$$+(w-b)\int_a^{\frac{Q}{Q_m}} yg(y)dy = (c_m - v\bar{y}) \qquad (6.27)$$

and

$$\frac{k}{2\sqrt{\eta}}\{(1-\phi)p - (b_r - v_r)\gamma + (b-v)\}\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy$$
$$+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} = 1 \qquad (6.28)$$

Proof. Due to complexity, the concavity of $\Pi_{mc}(Q_m, \eta)$ with respect to Q_m and η cannot be proved analytically. The manufacturer's profit function $\Pi_{mc}(Q_1, Q_2)$ is jointly concave in Q_m and η, which can be checked graphically as shown in Fig. 6.2. To prove the theorem, we have

$$\begin{aligned}
\frac{\partial \Pi_m(Q_m,\eta)}{\partial Q_m} &= \{(1-\phi)p - (b_r - v_r)\gamma + (b-v)\}\gamma \int_a^{\frac{Q}{Q_m}} \int_{yQ_m-(\alpha+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy \\
&\quad -(w-b)\int_a^{\frac{Q}{Q_m}} yg(y)dy - (c_m - v\bar{y}) \\
\frac{\partial \Pi_m(Q_m,\eta)}{\partial \eta} &= \frac{k}{2\sqrt{\eta}}\{(1-\phi)p - (b_r - v_r)\gamma + (b-v)\}\gamma\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+k\sqrt{\eta})} f(x)dx\Big) \\
&\quad \times g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+k\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} - 1
\end{aligned}$$

Solving the first order optimality conditions for the manufacturer's expected profit function $\Pi_m(Q_m, \eta)$, we get the required optimal production quantity Q_m^d and product quality investment η^d as given in equations (6.27) and (6.28). ■

Considering the optimal decisions of the centralized system and the corresponding ones of the manufacturer and the retailer in the decentralized system under the proposed differentiated buy-back with a modified revenue sharing contract, we see that when

$$b = (1-\phi) \times c_r + w \tag{6.29}$$

$$b_r = \frac{1}{\gamma}[(1-\phi)(\gamma p + c_r - g_r) - \phi(v - v_r\gamma) + w] \tag{6.30}$$

hold simultaneously, the proposed contract scheme achieves perfect coordination of the supply chain.

Theorem 6.8 *Under the voluntary compliance, in a decentralized supply chain where the retailer decides order quantity while facing uncertain demand and the manufacturer simultaneously decides the production quantity and product improvement investment in the presence the production uncertainty and the customer returns are proportional to sales, the proposed differentiated buy-back contract with revenue-sharing-cost-sharing contract that satisfy equations* (6.29) *and* (6.30)*, ensures the channel coordination. Furthermore, the supply chain's profit can be arbitrarily allocated between the two parties by varying* ϕ*.*

Proof. Using the supply chain members' optimal decisions of the decentralized model under differentiated buy-back with revenue-sharing-cost-sharing contract given in Theorems 6.6 and 6.7, we obtain the conditions such that the supply chain is coordinated. Comparing (6.3) with (6.24), (6.4) with (6.27) and (6.5) with (6.28) we get $b = (1-\phi) \times c_r + w$ and $b_r = \frac{1}{\gamma}[(1-\phi)(\gamma p + c_r - g_r) - \phi(v - v_r\gamma) + w]$. As a result, the retailer orders the amount of final product which is same as that of the centralized model and also, the manufacturer takes the production decision and investment decision for product improvement, which are same as in the centralized model. Further, using conditions in equations (6.29) and (6.30), the total expected profit of the decentralized system under differentiated buy-back with revenue-sharing-cost-sharing contract is $\Pi_{rc}(Q^*) + \Pi_{mc}(Q_m^*, \eta^*) = \Pi_c(Q^c, Q_m^c, \eta^c)$. ■

Equations (6.29) and (6.30) show that, for the growing consumer returns rate (γ), the manufacturer should increase the buy-back price (b) of unsold inventory under coordination contract at a given return premium (l). Since the manufacturer's loss increases as more customers return products, he is compelled to cut his production quantity. Thus, for efficient collaboration, the contract mechanism should allow the manufacturer to raise the buy-back price b in order to mitigate its profit loss and also encourage the manufacturer to produce more. Equation (6.29) reveals that $b = w + c_r - \phi r_r$ i.e., $b < w + c_r$, the retailer's total cost for each unit of unsold item, implying that our proposed contract arrangement prohibits the retailer to earn from over-ordering for return purpose.

The retailer earns an amount b for each unit of unsold items but, for returned items, the amount is raised by $b^{'}$ per unit as a return premium for maintaining charge paid by the manufacturer to the retailer for processing a customer return. Therefore, there persists an incentive for the retailer to cheat by declaring unsold product as customer returns leading to coordination failure. Thus, maintaining accurate data on both returns and unsold inventory is essential to the manufacturer for successful supply chain coordination.

We are concerned about whether our proposed differentiated buy-back with revenue-sharing-cost-sharing contract is capable to maintain authentic data on returns and unsold products. The contract mechanism is accomplished by requiring the retailer to fill out a return form with the customer's contact details, and the cause why the product was disappointing or didn't meet the customer's expectations when returning the product (as is the common practice at major retailers such as Wal-Mart). To receive the extra return premium ($b^{'}$), the retailer must provide the filled-out review documentation of the returning consumer to the manufacturer.

There are some other economic reasons why the manufacturer would benefit from collecting accurate customer returns data and why the retailer is incentivised to do so. In specific, the consumer returns data on why the product was disappointing or didn't fulfill customer's expectations, facilitates the manufacturer to learn about customer's tastes and preferences, which leads her to take investment decision for product improvement in a correct direction. The retailer, on the other hand, deals directly with the customer and handles all returns with no apparent interest to collect extensive information about the reason for the return. The additional return premium $b^{'}$ for returned products from consumers could be used by the manufacturer to encourage the retailer to submit more detailed documentation on the basis for customer returns and customer reactions to the product.

Since the manufacturer is concerned about the customer's better experience with the product, she prefers that the retailer should sell the new products first. This is because the quality of returned products is lesser than the new ones which may increase the probability of consumer disappointment. Now we'll see whether contract mechanism gives the retailer the right incentive to sell new products first. If a retailer has new products in stock and sells a return product at price p instead of a new unit, the retailer will lose the return premium $b^{'}$. In the developed scheme, the retailer has the right to claim $b + l$ for each returned item from customer but she may also be able to trade the returned item at $p_r < p$. Reselling is profitable for $p_r > b + b^{'}$ when the store has run out of new product. If the retailer had the option of selling a new unit for p or a returned unit for $p_r < p$, then the preference for the resale requires $p + b + b^{'} < pr + b$ i.e., $p_r - p = b^{'} > 0$, which is not feasible. Therefore, the contract mechanism can encourage the retailer to trade new products first.

Equation (6.30) demonstrates that the manufacturer would assign a higher return premium ($b^{'}$) for returned products from customers if the customer returns rate (γ) becomes smaller. Therefore, in order to obtain a higher return premium for returned products, the retailer can make an attempt to decrease consumer returns. The retailer is capable to do so because the rate of returns is influenced by the retailer's business practices. Thus the contract mechanism could indeed encourage the retailer to minimize customer returns. This encouragement is aligned with the manufacturer's desire for the retailer to improve system efficiency, as lower consumer returns rate leads to a higher system probability. We'll illustrate later how the contract mechanism also encourages the manufacturer to reduce consumer returns. Hence we reach at the conclusion that the coordinated framework can enhance system performance through decreasing expected return rate by the activities of both the retailer and the manufacturer.

6.5.3 Implementation of the contract

A differentiated buy-back with revenue-sharing-cost-sharing contract has three parameters $b, b^{'}$, and ϕ where b represents the buy-back price for products returned to the manufacturer, $b^{'}$ represents the handling fee paid by the manufacturer to the retailer for executing customer returns, and ϕ is defined as sharing parameter for both the revenue-sharing and cost-sharing contracts. For revenue-sharing, the fraction ϕ is described as the portion of the revenue the retailer keeps for himself and shares a fraction $(1-\phi)$ with the manufacturer, and for cost-sharing, ϕ is described as the ratio of investments on generating rewarded customer demand, that the retailer shares with the manufacturer. When bargaining and agreeing with the contract, the retailer and the manufacturer must be very careful about the sharing parameter ϕ. If the retailer wants to keep a large portion ϕ of the revenue generated from expected sales, he must also share a large portion ϕ of the manufacturer's product development costs. The same thing will happen on the manufacturer's side. Therefore, ϕ will be bargained in such a way that both parties are benefited from the deal.

Theorem 6.8 indicates that by adjusting the contract parameter ϕ, even for a fixed wholesale price w, buy-back price b and returns premium l, the supply chain profit can be distributed between the two parties in different ways. It also means that, according to contract design, the manufacturer will benefit from a higher share of the retailer's sales revenue. On the other hand, a lower buy-back price even hurts the manufacturer because the contract mechanism leads to a lower percentage of revenue-share for the manufacturer and a lower order quantity Q from the retailer. With a falling buy-back price, the retailer's cost-sharing percentage grows. Despite a bigger cost-sharing percentage from the retailer and a lower buy-back price (b) for unsold inventory, the revenue loss cannot be mitigated, and the manufacturer finds herself in a worse position. This is because the wholesale and retail prices are fixed; the manufacturer earns a lesser profit on both the regular delivery and shared revenue.

When the sharing parameter is negotiated and fixed, according to the coordination conditions in equations (6.29) and (6.30), the other two contract parameters b and l are also fixed. Since both the wholesale and retail prices are predetermined in our case, therefore, the profit allocation between the manufacturer and the retailer is also settled for a agreed value of ϕ. If more customers return products (i.e., γ rises), the retailer's profit declines. Equation (6.30) demonstrates that when γ rises, the manufacturer should increase the returns premium (l) for customer returns to minimize the retailer's profit loss. The greater return premium encourages the retailer to exert an effort to reduce consumer returns. It may be concluded that the differentiated buy-back with revenue-sharing-cost sharing contract is successful in incentivizing chain members to find ways to reduce consumer returns when it tends to increase.

6.5.4 A comparison of the designed contracts with other contracts

According to Cachon (2003), though a buy-back agreement with two components (such as wholesale price w and buy-back price b) is sufficient for coordinating a supply chain along with a single dimension (order quantity Q), extra free parameters are required if more actions are taken into consideration. In our model, the retailer has to think about consumer return policy in addition to his ordering decisions that introduce one additional dimension to the traditional supply chain conception. In this circumstance, we add an additional free parameter to the buy-back contract mechanism to tackle the returned products, and the new mechanism may reasonably include one buy-back price for unsold (new) items and an another buy-back price for returned items from consumers. Similarly, a revenue-sharing arrangement is sufficient for the manufacturer to deal with production uncertainty (Giri et al., 2021), but the manufacturer's investment in product development is a separate dimension for the traditional supply chain. To tackle the additional action, in this scenario, we introduce a cost-sharing contract along with revenue-sharing contract.

Hence, to address both the retailer and the manufacturer's additional actions (customer returns and product development investment), we introduce a composite contract with three parameters $(b, b^{'}, \phi)$ by combining the above described modified contracts. This mechanism is different from put option contract discussed in the literature (Wang et al., 2020) or buy-back agreement (Chen and Bell, 2011) in which they have to accompany a side arrangement to allocate the supply chain profit among its members. A contract with an excessive number of parameters may be difficult to implement in real business scenario. Since the supply chain profits can be arbitrarily allocated between the two parties by varying ϕ, the $(b, b^{'}, \phi)$ coordination policy does not require any additional side arrangement to share the channel profit among the members.

The differentiated buy-back contract is fundamentally similar to Pasternack (1985)'s buy-back contract for a stochastic demand scenario. In a buy-back agreement, the retailer has the option of returning unsold products to the manufacturer. While the differentiated buy-back contract allows the retailer to return both unsold and returned products to the manufacturer for two different buy-back prices. Analogous to the buy-back policy, there are various combinations of contract parameters $(b, b^{'}, \phi)$ which can coordinate the supply chain, and the combination of a triplet controls how the supply chain's wide profit could be distributed between the manufacturer and the retailer. This shows that, as long as we design our contract parameters to satisfy conditions in equations (6.29) and (6.30) in our proposed contract mechanism, the active incentive mechanisms of the buy-back contract will basically remain unchanged.

The revenue-sharing mechanism discussed in subsection 6.5.1 operates similarly to the one that has been extensively studied in the literature (Cachon and Lariviere, 2005). However, the sharing mechanism considered in subsection 6.5.2 is different, because the traditional revenue-sharing contract is offered by the upstream member (manufacturer) to correct the downstream member's (retailer) under-ordering problem. Here, our revenue-sharing mechanism is offered by the downstream member (retailer) to correct the upstream member's (manufacturer) under-producing problem. In fact, the traditional revenue-sharing agreement with stochastic demand, enables the manufacturer to swap some of the retailer's demand uncertainty by allowing the retailer to pay a smaller wholesale price at first and the compensate the manufacturer with a fraction of the retailer's sales revenue afterwards. However, in our model with stochastic demand and production uncertainties, the retailer's revenue-sharing mechanism is used to reallocate the manufacturer's production risk rather than the demand risk of the retailer, to incentivize the manufacturer to produce more.

6.6 Numerical analysis

In this section, we take a numerical example to explain the optimal decisions of different supply chain models developed in the chapter, investigate the impact of demand and production uncertainties on coordination, and the influence of customer returns on the profitability of the individual members as well as the entire system. Customer demand (X) is assumed to be uniformly distributed with a mean ($\bar{x}$) of 50 units and a standard deviation (σ_x) of $50/\sqrt{3}$ units. The production yield rate (Y) is also assumed to be uniformly distributed in the interval [a, b], $0 \leq a < b \leq 1$ with a mean ($\bar{y}$) of 0.8 unit and a standard deviation (σ_y) of 0.05 units. Other parameter-values are $\alpha = 500; k = 1.5; S = 200; c_m = 5; c_r = 2.5; g_r = 1.5; v = 5; v_r = 4.5; \gamma = 0.25;\ \phi = .37; w = 20; p = 32.58$ in their appropriate units. The parameter-values used in our analysis are reasonable match with the values from a few secondary sources such as Giri et al. (2016) and Chen and Bell (2011).

For the above set of values, the concavity of the manufacturer's objective function $\Pi_m(Q_m, \eta)$ under wholesale price contract, and $\Pi_{mc}(Q_m, \eta)$ under differentiated buy-back with revenue-sharing-cost-sharing contract with respect to Q_m and η are checked graphically as shown in Figs. 6.1 and 6.2, respectively.

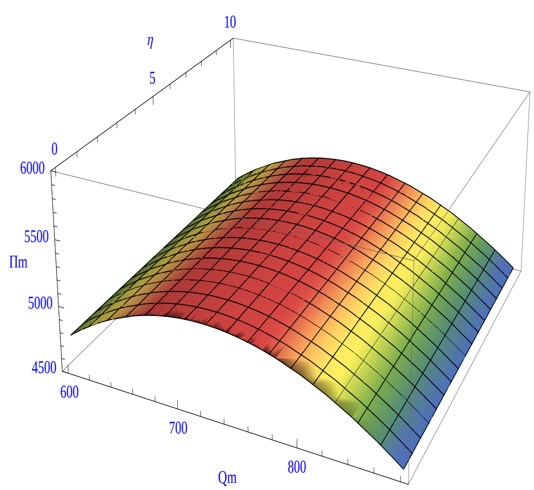


Fig. 6.1: Concavity of the profit function $\Pi_m(Q_m, \eta)$.

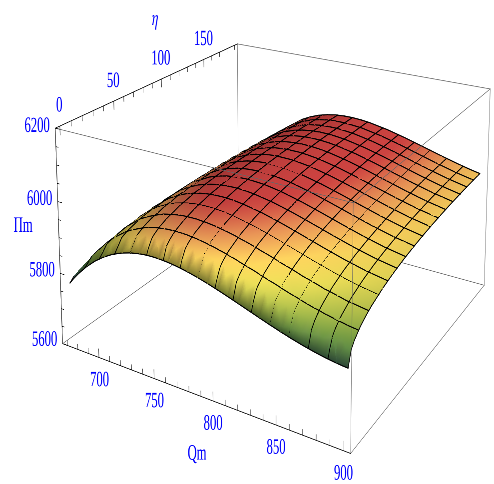


Fig. 6.2: Concavity of the profit function $\Pi_{mc}(Q_m, \eta)$.

Table 6.1 shows the optimal results for the centralized model and models under wholesale price contract and differentiated buy-back with revenue-sharing-cost-sharing contract. We find that the coordinating contract generates higher profits for both the manufacturer and the retailer than those under wholesale price contract. In addition, under the differentiated buy-back with revenue-sharing-cost-sharing contract, the manufacturer and the retailer together achieve the highest profit level. Under coordinating contract, the optimal order quantity,

Table 6.1: A comparison of results of different models

Model	Q	Q_m	η	Π_r	Π_m	Π
Centralized	609	795	186	-	-	9771
Decentralized	531	598	0	4755	3599	8355
Coordinated	609	795	186	5709	4061	9771

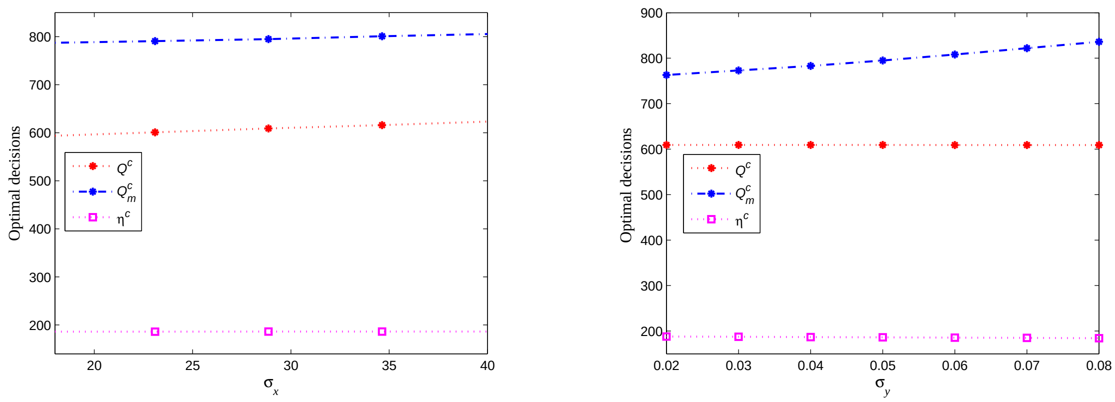
production amount and investment for product development rise compared to wholesale price contract resulting in larger expected sales. This is the reason for higher profitability for chain members under coordinating contract. It reflects the insights obtained from Theorems 6.4 and 6.8.

Table 6.2: Impact of customer returns on optimal decisions and profits for coordinated supply chain

γ	Q^*	Q_m^d	η^*	b^*	l^*	Π_{mc}	Π_{rc}	Π_c
0.1	612.152	802.194	226.805	21.575	208.365	4137.28	6710.62	10847.9
0.15	610.131	797.983	198.937	21.575	145.765	3866.89	6254.42	10121.3
0.2	608.041	793.557	172.897	21.575	114.465	3597.19	5799.62	9396.81
0.25	605.868	788.881	148.684	21.575	95.685	3328.18	5346.27	8674.45
0.3	603.595	783.911	126.299	21.575	83.165	3059.89	4894.41	7954.3
0.35	601.2	778.589	105.741	21.575	74.2221	2792.3	4444.12	7236.42

As shown in Table 6.2, when the customer returns rate (γ) grows up, the retailer should decrease order quantity to reduce the returned products from customers, resulting in lower production requirement for the manufacturer and thus provides a negative impact on both members' profits (as Chen and Bell (2009) predicted). Table 6.2 also shows that, as the customer return rate rises, the value of the coordinating parameter l decreases. It denotes that supply chain coordination is a proper loss sharing mechanism among the channel members. Whenever the customer return rises, its negative effect gets transmitted to the manufacturer and the retailer.

Fig. 6.3 shows that the manufacturer's investment decisions on product improvement are independent of both demand uncertainty (σ_x) and production uncertainty (σ_y). This finding is interesting and important as it means that the manufacturer's investment decision can ignore both demand and supply uncertainty information. However, as the demand uncertainty (σ_x) increases, the retailer orders more products and accordingly the manufacturer adjusts its scheduled production amount of raw material. For a given demand uncertainty



(a) Effect of σ_x on the optimal decisions. **(b)** Effect of σ_y on the optimal decisions.

Fig. 6.3: Performance of coordinated chain w.r.t. demand and supply uncertainties

(σ_x), as the yield uncertainty (σ_y) increases, the retailer's order remains unchanged while the manufacturer increases production lot size. This is because, as the demand uncertainty (σ_x) increases, the retailer increases the order quantity to avoid the stock-out situation. But, when the yield uncertainty (σ_y) increases, it cannot affect the market demand i.e., retailer's ordering decision, though the manufacturer increases its production size compared to the retailer's order.

6.6.1 Discussion

One of the primary findings of our research is coordinating a supply chain in the presence of consumer return policy. The majority of previous research works (Wang et al., 2020; Ruiz-Benitez and Muriel, 2014; Su, 2009) deal with the complicacy of consumer returns by assuming that the manufacturer can discriminate between unsold (new) and returned products. Our proposed coordination policy (b, b', ϕ) does not require any such specific skill of the manufacturer because the retailer must submit the documentation of unsold and returned products in order to collect the buy-back credits at the end of the selling season.

It is clear from relevant existing literature (Wang et al., 2020; Chen and Bell, 2011; Ruiz-Benitez and Muriel, 2014) that researchers are mainly concerned on how to respond when a customer returns a product. In our work, incorporating investment in product development depending on the customer preference and taste information helps to not only efficiently manage the reverse product flow, but also facilitate the discussion on the difficulties related to what move would be taken by channel members to reduce the customer returns.

Previous researches that provide analytical findings on coordinating contracts for a supply chain with consumer returns usually focus on the manufacturer-led scenario. The influence of a manufacturer's production uncertainty on the issue of supply chain coordination under customer returns has not been taken into account. Here, we exhibit the impact of both demand and production uncertainties on the coordination in the context of customer returns and product quality improvement. Sointu (2018) illustrated the influence of changes in a supply chain due to retailer-led structure on coordination. It inspires us to develop a contract mechanism for coordinating a supply chain in a manufacturer-retailer-led scenario with both demand and production uncertainties.

Finally, in the context of customer returns, previous researches restrict the retailer to deal with the selling product and handling returns from customers without gathering any specific information for the returns. Here, we introduce a holistic perspective of gathering and updating such documents at the time of return on the supply chain performance by exploring how the manufacturer can encourage the retailer to acquire and send information on the customers' product expectations and taste by providing different credits for unsold inventory and returned products to reduce customer returns rate.

6.7 Conclusion

In the business world today, a high variation of customer preferences is observed while purchasing a product. Product return policies allow customers to choose whether to keep or return a product once they have purchased it. Motivated by this dilemma, we study a two-level supply chain model with customer returns under simultaneous demand and supply uncertainties. We demonstrate how uncertainty influences ordering, production as well as investment decisions.

In the context of consumer returns, we present two contract mechanisms to coordinate the supply chain. For a manufacturer-led scenario, we combine a buy-back contract in which retailer credits only for unsold products with a revenue-sharing contract where manufacturer shares the retailer's revenue for her reduced wholesale price. For a manufacturer-retailer-led scenario, we combine a differentiated buy-back policy with two buy-back prices - one for unsold product and another for product returned by the customer with a revenue-sharing-cost-sharing scheme where the manufacturer is shared with both the retailer's revenue and the cost of investment for product improvement. We find that the buy-back with revenue sharing contract is unable to coordinate the supply chain, whereas the differentiated buy-back policy with revenue-sharing-cost-sharing scheme is able to do so. Apart from coordinating the supply chain, we demonstrate how the manufacturer might entice the retailer to gather data on the basis of return at the time when a customer returns a product and submit it to the manufacturer. We also look into how a manufacturer may learn about a customer's tastes and preferences by using customer review data, which leads to an investment decision for product improvement.

Our model is limited to one entity at each stage of the supply chain. Multiple entities at each level under competition or collaboration among them can be an exciting research topic. Moreover, the proposed model is based on the assumption of complete information symmetry throughout the supply chain. Incorporating information asymmetry at various levels of this supply chain can produce interesting results. Many retailers offer a wide range of products, and customers who return one may choose to purchase another. In reality, several stores only offer store credit when customers return a product. A multi-product model is needed to analyze consumer purchase and exchange decisions, which can lead to new insights.

Chapter 7

Coordinating a socially responsible closed-loop supply chain with product improvement and recycling under demand supply uncertainties*

7.1 Introduction

It is now universally noted that human activities have imposed a massive environmental burden on the planet in the form of industrial and residential wastes. Every year almost two billion tones of waste are produced around the world (United Nations Environment Programme, 2009). These wastes contain harmful compounds which have inflicted environment as well as human health. However, several of these waste products' elements (metals) are recyclable and thereby can reduce the need for natural resources (Huisman, 2003). In 2016, the World Bank calculated that 2.01 billion tones of municipal solid wastes were generated, and forecast that this figure will rise to 3.40 billion tones by 2050. As a result, several countries have enacted laws and regulations aimed at reducing wastes through reuse, recycle, and

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product recovery.

An important form of product recovery is remanufacturing which restores a used product to as-new condition, meaning that the standard of quality for a remanufactured product is the same as for a new product. Remanufacturing is common in several industries and for a wide range of products including ink cartridges (Kittell and Page, 2008), disposable cameras (Kodak, 2008), medical devices (Hosseini-Motlagh et al., 2020), motor vehicle components (Bosch, 2016a; Ferrer and Whybark, 2001), aerospace equipment (Treat, 2012), consumer products (Apple, 2016; Bosch, 2016b), and retreaded tires (Debo et al., 2006), to mention a few. Remanufacturing is an opportunity for the manufacturer because it incurs relatively low cost of remanufacturing used products compared to producing new ones. As a result, closed-loop supply chains (CLSCs) have steadily become the target of industries and academics in order to successfully study the utilisation of waste products and establish a resource-saving society while increasing profits via waste recycling. Waste product remanufacturing reduces the demand for natural resources as well as waste generation (Qiang, 2015). As a result, the CLSC's implementation has economical, social and environmental aspects (Chaudhary et al., 2017).

The implementation of a CLSC is mainly influenced by the amount of used items collected and delivered to the manufacturer's warehouse, which is mostly determined by consumers' willingness and environmental consciousness to return their waste products. A few forward-thinking consumers (Japanese consumers) are highly environmentally conscious and eager to return unwanted things on their own (Geyer and Blass, 2010). However, in less environmentally sensitive countries, some efforts are required to raise consumer awareness about the environment; however, this is a more common practice. CSR effort such as raising customer awareness about environmental issues can be effective incentive strategy to encourage customers to return unwanted items and increase the collection amount (Sarkis et al., 2010). CSR can be defined as a company's voluntary initiatives that demonstrate the linkages between day-to-day business with social and environmental concerns (Van Marrewijk, 2003). Furthermore, CSR boosts the company's reputation and it has a major impact on customer goodwill (Komodromos and Melanthiou, 2014), which increases the market demand (Hsueh, 2014). There are numerous examples of well-known companies who are using CSR to gain a competitive advantage, and produce sustainable products. For example, technology giant Hewlett-Packard has made a significant effort to encourage the recycling of its used laptops and printers.

Apart from CSR and recycling, a free-repair warranty for repairable products that malfunction prematurely is another competitive strategy to increase customer demand. Due to the competitive nature of the market, it is usual for manufacturers to include a free-repair warranty (FRW) with their sales activities in order to safeguard consumers from premature failures and signify product quality and reliability, especially for complicated and/or expensive items (Murthy and Djamaludin, 2002; Xie, Shen and Zhong, 2017). A free-repair warranty is a promise made by a manufacturer or other third party that their product will fulfil specific quality and performance standards for a reasonable period of time. This is commonly known as a manufacturer's warranty. For example, Apple provides one-year warranty against faults in materials and workmanship from the date of actual product purchase. If a problem occurs during the warranty coverage, Apple will repair the product at no cost using new or equivalent-to-new parts in reliability and performance.

Since the product's premature failures are intimately linked to the firm's processing technologies, design, and raw material procurement, so defects can be minimized by the manufacturer's appropriate practices. This necessitates additional quality improvement investments in the form of improved product design, updated equipment, higher-quality raw materials, and improved quality control processes (Xiao and Yang, 2009). Quality enhancement efforts, on the other hand, are another competitive instrument for increasing customer desire to acquire products. As a result, for the CLSC system, enterprise quality improvement efforts not only effect customer demand, but also have a beneficial impact on the reduction of products' premature failures in the reverse supply chain. According to an annual auto reliability report, Ford, as one of the most well-known automobile companies, slipped 10 places in 2011 due to poor quality of its own vehicles (Durbin and Krisher, 2011). After that, it managed to stay in the fourth place, beating Hyundai, BMW, and Toyota, due to a long-term investment in quality development (Rosevear, 2017).

In this chapter, we study a supply chain that considers remanufacturing, return policy, CSR initiative, and quality improvement effort to improve its performance under demand and supply uncertainties. The supply chain consists of two members - a manufacturer and a retailer. The retailer is responsible for collecting waste products from customers and increasing the return amount by participating in CSR activities. This is because retailers are at the end of the supply chain, directly addressing the market, and so have a unique perspective on the influence of CSR investment behaviour on customers. As a result, studying retailer CSR investment behaviour is extremely practical. The retailer also returns the collected waste products to the manufacturer for recycling. Because premature product failure is intimately tied to the manufacturer's production error, the manufacturer should employ a maintenance service to restore the product to functioning condition. Even though premature failures can be fixed through a repairing process, but for reducing the defect products in the reverse flow, the manufacturers may invest in new production technologies, design and procurement to improve the product quality. In this study, we attempt to incorporate product quality and quality-related returns into the design of a single-channel CLSC coordination mechanism. In the decentralized setting, a combination of buy-back contract and pay-back contract is considered. Then the contract is modified to a buy-back with pay-back-cost sharing contract in which the retailer promises not only to buy the manufacturer's excessive output above his order but also to share a portion of the quality improvement expenditure with the manufacturer. It is shown that the former contract cannot coordinate the supply chain but the later one can achieve coordination if an additional incentive is provided to the manufacturer in the form of cost-sharing in the context of demand supply uncertainties in the forward flow, and two types of returned products viz. defective products for repair and waste products for remanufacturing in the reverse flow.

The aim of this chapter is to address a research gap in the literature on supply chain coordination by examining the influence of CSR efforts on waste product returns and remanufacturing for waste recycling in a linear supply chain under supply and demand uncertainties. It is also aimed to explore the impact of defect products return for fixing and quality improvement to reduce defects on optimal decisions of SC as well as how channel partners participate on product quality improvement investment.

The conflicts of creating two types of uncertainty (demand and supply uncertainties) in the forward flow and the return of two types of product (defective and waste products) in the reverse flow dictate value of the research objective. The attempt to improve quality has a significant impact to reduce defect return rate and increase product demand but rising costs of the SC. As a result, selecting the finest quality improvement option can help SCs to solve their financial issues. CSR efforts, on the other hand, play a positive influence in raising waste product collection rates, which in turn promotes demand. Because product quality and CSR activities are distinguishing features in attracting client happiness, they are created and implemented at the same time in a SC. Finding their optimal settings and understanding their effects on one another can help SC to achieve its social and economical goals.

The remaining chapter is structured as follows: Section 7.2 provides model description, notations and assumptions for developing the proposed model. In Section 7.2.2, we derive the equilibrium solution of the integrated supply chain model. In Section 7.4, we develop the non-collaborative model under the well known wholesale price contract and investigate the individual decisions of the corresponding supply chain members. Section 7.5 illustrates the collaborative decentralized model, and discusses two contracts- buy-back-pay-back contract in subsection 7.5.1 and buy-back along with pay-back-cost-sharing contract in subsection 7.5.2. We conduct a sensitivity analysis in Section 7.6 to study the effect of a few key-parameters on the optimal decisions and profitability of the supply chain. In Section 7.7, the chapter is concluded.

7.2 Design of a socially responsible CLSC

7.2.1 Model description

We consider a single-manufacturer (she) single-retailer (he) closed-loop supply chain (CLSC) model where the manufacturer recycles the waste products and invests in product development to maintain the quality of the product. The retailer contributes to the environment by participating in CSR programmes that encourage customers to return their used products. Incorporating CSR activity, customer return, remanufacturing, and quality improvement, this study examines the CLSC under demand and supply uncertainties.

In forward logistics, the manufacturer produces a product at unit cost c_m and sells it to the retailer with wholesale price w_m. Consumers can purchase this item from the retailer at price p. In reverse logistics, we consider two types of product return from consumers. The first one is the return of defective products due to quality issues, which is negatively related to the product quality improvement level. The manufacturer is obligated to accept such products and perform maintenance services during the warranty period under the free repair warranty. After being repaired when the damaged products are returned to their owners, they will be in working order. For example, if your iPhone battery shrinks due to a non-artificial cause, you can request a free component replacement from Apple to ensure that the phone continues to function properly. The other type is the return waste products that have reached the end of their useful life (EOL), and consumers return their used products as part of their social responsibilities (Geyer and Blass, 2010). Because retailers are the nearest to customers, they could collect those products from customers and send them to the manufacturer to be utilised as raw materials for remanufacturing (Savaskan et al., 2004). In Korea, Samsung Electronics has formed a "take-back network" with significant local retailers to recycle unwanted home appliances and electronic goods.

The following assumptions are made for developing the proposed model:

(i) We deal with a production-sale-return-remanufacture/repair cycle, similar to prior research (Galbreth and Blackburn, 2006; Xie, Shen and Zhong, 2017; Inderfurth et al., 2005; Jamal et al., 2004). According to Xie, Liang, Liu and Ieromonachou (2017), we set unit remanufacturing cost c'_m to be lower than production cost c_m to define the effectiveness of remanufacturing on manufacturer profit in the same cycle.

(ii) The defective products and waste products are returned with rates r_1 and r_2, respectively.

(iii) When a product fails prematurely, customers can return it, and the manufacturer is supposed to provide free maintenance and cover all repair costs. The retailer is not responsible for any product quality issues. Furthermore, because the restored products are faultless, a product is only returned once.

(iv) The manufacturer, in contrast to defect product returns, has the advantage of knowing why a product is returned. As a result, the return rate of defective items can clearly represent the quality level, allowing the manufacturer to make necessary changes by investing in quality improvement in order to reduce defect returns.

(v) The unit repair cost of a defective product c_m^r is less than the unit manufacturing cost of a new product c_m (i.e., $c_m^r < c_m$); else, the manufacturer would always choose reproduction over repair.

(vi) Early test production or a second production run is not achievable due to production and assembly lead time and tight consumer response windows.

(vii) Both the manufacturer and the retailer are risk-neutral and they want to maximise their own expected profits with a zero reservation profit.

7.2.2 Mathematical model formulation

Similar to the study of Swami and Shah (2013), Dong et al. (2016), and Wang et al. (2016), the stochastic market demand is defined as: $D = \alpha + \beta e + \gamma\sqrt{\eta} + x$, where α represents the market demand's capability, which is independent of CSR investment (η) and quality improvement effort (e). β represents the positive coefficient value of the demand's quality effort, γ represents the positive coefficient value of the demand's CSR investment, and X is a random variable supposed to be distributed on $[l, u]$ with probability density function $f(\cdot)$, cumulative distribution function $F(\cdot)$, and mean $\bar{x}$. The manufacturer's supply uncertainty is treated as a production process with a stochastic proportional yield (see Yano and Lee, 1995). The output of finished product for input of raw material of size Q_m is yQ_m, where the random yield rate Y is distributed on $[a, b]$, $0 \leq a < b \leq 1$ and has continuous pdf $g(\cdot)$, cdf $G(\cdot)$, and mean $\bar{y}$, all are independent of lot size Q_m.

According to Atasu et al. (2008), the money saved by remanufacturing each unit of waste product is $c_m - c_m^{'} > 0$. The cost of investing in enhancing product quality by applying innovative technology to reduce faulty product return as well as CSR investment for collecting waste products may raise product price as well as demand. While it may be against the customer's affordability and may prevent them from purchasing, we assume that the supply chain specifies a fixed retail price for their items to avoid losing market share (similar to Wang et al., 2016; He et al., 2015). In this situation, the benefit of increased market demand can balance the cost of implementing the above strategies and the use of a fixed retail price. Furthermore, c_r represents the cost of value-added or handling activity conducted at the retail shop, v represents the salvage value for an unsold unit, and g_r represents the shortage penalty.

We consider the return rate r_1 of defect products as $r_1 = q_1 - \delta_1 e$ (identical to Li et al., 2013), where q_1 is the fixed amount of returned faulty products that are independent to the quality effort and δ_1 is the quality effort's negative impact coefficient on the return rate. The investment cost function for quality improvement to reduce the number of defect products is $\frac{1}{2}\mu e^2$, where μ is the investment coefficient of the quality improvement effort, which is same as the one used by Liu et al. (2012), Dong et al. (2016), Ji et al. (2017) and Chakraborty et al. (2019). We suppose that the retailer is responsible for collecting waste products and sending them to the manufacturer at a wholesale price w_r per unit, and the manufacturer is not involved in the return process directly. To obtain more remanufacturing resources, the retailer conducts a CSR effort to increase waste product return rate.

According to Hosseini-Motlagh et al. (2020), the amount of returned waste products is as follows: $r_2 = q_2 + \delta_2\sqrt{\eta}$, where δ_2 is the sensitivity of the return rate r_2 to the retailer's CSR action and q_2 is the minimum amount of wastes returned by environmentally concerned customers. Thus, the retailer can benefit by collecting and sending waste products to the manufacturer, while the manufacturer can reduce production cost and enhance revenue by remanufacturing waste products (Wei and Choi, 2010). Furthermore, the investment η in CSR activities to raise environmental awareness among consumers to return their used items improves customer interest and brand image related to the product, similar to the one used by Chao et al. (2009) and Li et al. (2013), which generates $\gamma\sqrt{\eta}$ rewarded demand at the CSR investment η, where γ is the coefficient of customer eagerness to purchase the product under CSR activities. Order quantity Q and CSR investment η are the decision variables on the retailer edge whereas quality improvement level e and production amount Q_m are the decision variables planned by the manufacturer. The notations used throughout the chapter are listed bellow:

x : stochastic part of customer demand with mean $\bar{x}$ and variance σ_x^2

y : random yield with mean $\bar{y}$ and variance σ_y^2

c_m : unit production cost for each new product of the manufacturer

$c_m^{'}$: unit remanufacturing cost for returned EOL product of the manufacturer

c_m^r : unit repair cost for returned defect products of the manufacturer

c_r : unit handling cost of the retailer

g_r : unit goodwill lost of the retailer for unmet customer demand

S : handling expenses relating to the waste return from customers

v : unit salvage price of leftover at the manufacturer or retailer

r_1 : return rate of defective products, $0 \leq r_1 \leq 1$

r_2 : return rate of waste products, $0 \leq r_2 \leq 1$

q_1 : fixed portion of the quantity of returned defect products

q_2 : fixed portion of the quantity of the returned waste products

δ_1 : sensitivity coefficient effect of the quality on the return rate of defect products

δ_2 : sensitivity coefficient of the CSR activities on the return rate of waste products

w_r : collection fee paid by the manufacturer to the retailer for each EOL returned item.

η : CSR investment level of the retailer

e : product improvement effort of the manufacturer

p : retail price of the unit final product at the retailer

Q : order amount of the retailer placed at the manufacturer

Q_m : aimed production lot size of the manufacturer

w_m : unit wholesale price charged by the manufacturer to the retailer.

7.3 Centralized supply chain

In this case, conceptually only one decision-maker is consulted in order to optimize the profit of the whole system. Profit distribution among supply chain members can be considered as an internal money transfer. The centralized supply chain's expected profit function can be written as

$$\begin{aligned}\Pi_c(Q,Q_m,e,\eta) &= pE[\min\{D,Q,yQ_m\}] + vE[(\min\{Q,yQ_m\}-D)^+] + vE[(yQ_m-Q)^+]\\ &+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta})E[\min\{D,Q,yQ_m\}] - c_rE[\min\{Q,yQ_m\}]\\ &-g_rE[(D-\min\{Q,yQ_m\})^+] - c_m^r(q_1-\delta_1 e)E[\min\{D,Q,yQ_m\}]\\ &-c_mQ_m - S - \eta - \frac{1}{2}\mu e^2 \qquad (7.1)\end{aligned}$$

The first four terms capture the full revenue of SC from the products that are sold, unsold goods at retail store and excessive stock for over-production which are salvaged after selling season and the revenue from remanufacturing, respectively. The supply chain's overall cost is covered by the last eight terms. Production costs associated with fresh raw material, product handling costs at the retailing store, goodwill lost when items are out of stock, the cost of exhibiting CSR activities, the investment of product improvement, defect products' repairing costs, and the cost of employees who handle returned products are all included in the total cost. A alternate presentation of the above expression is shown below.

$$\begin{aligned}\Pi_c(Q,Q_m,e,\eta) &= \Big\{p+g_r-v+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e)\Big\}\\ &\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx\\ &+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy\\ &+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx\\ &+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\} - c_r\Big\{\int_a^{\frac{Q}{Q_m}}yQ_mg(y)dy\\ &+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\} - (c_m-v\bar{y})Q_m - \eta - g\bar{x} - S - \frac{1}{2}\mu e^2 \qquad (7.2)\end{aligned}$$

According to Petruzzi and Dada (1999), it is typically difficult to show the joint concavity of the objective function in all the decision variables for a newsvendor problem with many decision variables in its objective function. The employment of a repeating method to demonstrate the objective function's concavity is a popular strategy in the literature (for example Wang, Wang, Li, Liu, Zhu and Wang, 2019). The objective function of the centralised system in this article is comprised of four decision variables, and exact methods cannot be used to get the optimal solution. As such we implement a repetitive method. Assume that for the centralised model, there exists a finite but not necessarily unique optimal decision set $(Q^c, Q^c_m, p^c, \eta^c)$. First, we consider that, for given e and η, the whole supply chain profit function Π_c in equation (7.2) is a function of Q and Q_m only. But due to complexity, the concavity of Π_c with respect to Q and Q_m can not be proved analytically. The following proposition explores the whole system's optimal order and production quantities assuming that Π_c is concave, which can be checked graphically as shown in Fig. 7.1.

Proposition 7.1 *For given quality improvement effort e and CSR investment η in the entire system's objective function $\Pi_c(Q, Q_m, e, \eta)$, the optimal order quantity Q^c and the optimal production decision Q^c_m satisfy the following equations:*

$$Q^c = (\alpha + \beta e + \gamma\sqrt{\eta}) + F^{-1}\Big(\frac{p + g_r - v + (c_m - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) - c^r_m(q_1 - \delta_1 e) - c_r}{p + g_r - v + (c_m - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) - c^r_m(q_1 - \delta_1 e)}\Big) \quad (7.3)$$

$$(p + g_r - v + (c_m - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) - c^r_m(q_1 - \delta_1 e)) \int_a^{\frac{Q}{Q^c_m}} \int_{yQ^c_m - (\alpha+\beta e+\gamma\sqrt{\eta})}^{u} y f(x)dx g(y)dy$$
$$-c_r \int_a^{\frac{Q}{Q^c_m}} y g(y)dy = (c_m - v\bar{y}) \quad (7.4)$$

Proof. We have

$$\frac{\partial \Pi_c(Q, Q_m, e, \eta)}{\partial Q} = \Big\{p + g_r - v + (c_m - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) - c^r_m(q_1 - \delta_1 e)\Big\} \times \int_{\frac{Q}{Q_m}}^{b} \Big(\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u} f(x)dx\Big) g(y)dy - c_r \int_{\frac{Q}{Q_m}}^{b} g(y)dy$$

$$\begin{aligned}\frac{\partial \Pi_c(Q,Q_m,e,\eta)}{\partial Q_m} &= \Big\{p+g_r-v+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e)\Big\}\\ &\quad\times\int_a^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u} yf(x)dxg(y)dy\\ &\quad-c_r\int_a^{\frac{Q}{Q_m}} yg(y)dy-(c_m-v\bar{y})\end{aligned}$$

Solving the first order conditions for optimality of $\Pi_c(Q,Q_m,e,\eta)$ *i.e.*, $\frac{\partial \Pi_c(Q,Q_m,e,\eta)}{\partial Q}=0$ and $\frac{\partial \Pi_c(Q,Q_m,e,\eta)}{\partial Q_m}=0$, we get the required optimal order quantity Q^c and production quantity Q_m^c as given in equations (7.3) and (7.4). ■

Again, for given Q and Q_m, the whole supply chain's profit function $\Pi_c(Q,Q_m,e,\eta)$ in equation (7.2) is a function of e and η only. But due to complexity, the concavity of $\Pi_c(Q,Q_m,e,\eta)$ with respect to e and η can not be proved analytically. The following proposition explores the whole system's optimal quality improvement effort e^c and CSR investment η^c assuming that $\Pi_c(Q,Q_m,e,\eta)$ is jointly concave with respect to e and η, which can be checked graphically as shown in Fig. 7.1b.

Proposition 7.2 *For given order quantity Q and production size Q_m in the entire system's objective function $\Pi_c(Q,Q_m,e,\eta)$, the optimal quality improvement effort e^c and CSR investment η^c, satisfy the following equations:*

$$\begin{aligned}&\beta(p+g_r-v+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e^c))\\ &\quad\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy\\ &\quad+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\}\\ &+\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})}(\alpha+\beta e^c+\gamma\sqrt{\eta}+x)f(x)dx\\ &\quad+\int_{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy\\ &\quad+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})}(\alpha+\beta e^c+\gamma\sqrt{\eta}+x)f(x)dx\\ &\quad+\int_{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})}^{u} Qf(x)dx\Big)g(y)dy\Big\}\delta_1 c_m^r=\mu e^c\end{aligned} \tag{7.5}$$

$$
\begin{aligned}
\textit{and}\quad & \frac{\gamma}{2\sqrt{\eta^c}}(p+g_r-v+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta^c})-c_m^r(q_1-\delta_1 e)) \\
& \times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^c})} f(x)dx\Big)g(y)dy \\
& +\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^c})} f(x)dx\Big)g(y)dy\Big\} \\
& +\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^c})}(\alpha+\beta e+\gamma\sqrt{\eta^c}+x)f(x)dx \\
& +\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^c})}^u (yQ_m)f(x)dx\Big)g(y)dy \\
& +\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^c})}(\alpha+\beta e+\gamma\sqrt{\eta^c}+x)f(x)dx \\
& +\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta^c})}^u Qf(x)dx\Big)g(y)dy\Big\}\frac{\delta_2}{2\sqrt{\eta^c}}(c_m-c_m^{'})=1
\end{aligned}
\tag{7.6}
$$

Proof. We have

$$
\begin{aligned}
& \frac{\partial\Pi_c(Q,Q_m,e,\eta)}{\partial e}=\beta(p+g_r-v+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e^c)) \\
& \times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} \\
& +\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})}(\alpha+\beta e^c+\gamma\sqrt{\eta}+x)f(x)dx \\
& +\int_{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})}^u (yQ_m)f(x)dx\Big)g(y)dy \\
& +\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})}(\alpha+\beta e^c+\gamma\sqrt{\eta}+x)f(x)dx \\
& +\int_{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})}^u Qf(x)dx\Big)g(y)dy\Big\}\delta_1 c_m^r-\mu e^c
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{\partial\Pi_c(Q,Q_m,e,\eta)}{\partial \eta}=\frac{\gamma}{2\sqrt{\eta^c}}(p+g_r-v+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta^c})-c_m^r(q_1-\delta_1 e)) \\
& \times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^c})} f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^c})} f(x)dx\Big)g(y)dy\Big\} \\
& +\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^c})}(\alpha+\beta e+\gamma\sqrt{\eta^c}+x)f(x)dx
\end{aligned}
$$

$$+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^c})}^{u}(yQ_m)f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^c})}(\alpha+\beta e+\gamma\sqrt{\eta^c}+x)f(x)dx$$
$$+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta^c})}^{u}Qf(x)dx\Big)g(y)dy\Big\}\frac{\delta_2}{2\sqrt{\eta^c}}(c_m-c_m^{'})-1$$

Solving the first order optimality conditions for the entire system's expected profit function $\Pi_c(Q, Q_m, e, \eta)$, we get the required optimal quality improvement effort e^c and CSR investment η^c, as given in equations (7.5) and (7.6).■

Using the optimal values of the decision variables in (7.2), the maximum channel profit is obtained. Due to the model's complexity, we are unable to derive a closed form solution. According to Propositions 7.1 and 7.2, the demand and supply uncertainties are both key elements in determining the system's optimal decisions. When there is a growth in production uncertainty, the ordering and production decisions must be affected. In order to avoid under-production, the system increases the anticipated raw material production quantity due to increased production uncertainty. In contrast, when the system's efficiency decreases in response to rising production uncertainty, the order quantity of the finished product decreases.

7.4 Decentralized supply chain with wholesale price contract

Here we'll look at the decentralised supply chain where the retailer and the manufacturer have a wholesale price-only contract between them. In this contract, the manufacturer provides its goods to the retailer at a unit wholesale price of w_m and wishes to invest a total of e in product quality enhancement activities in order to increase customer demand. To focus on examining the impacts of demand and supply uncertainties on supply chain coordination, we suppose that the unit wholesale price w_m is fixed (similar to Wang et al., 2020). Obtaining agreement approval, the sequence of events follows the approach outlined in Section 7.2.2. We look at a Nash sequence under which the manufacturer chooses the initial decision and the system is solved by backward substitution. As a result, the retailer must first determine his best course of action regarding order quantity Q and CSR investment η. The retailer's profit function $\Pi_r(Q,\eta)$ can be calculated for any given Q_m and e as:

$$\begin{aligned}\Pi_r(Q,\eta) &= pE[\min\{D,Q,yQ_m\}] + w_r(q_2+\delta_2\sqrt{\eta})E[\min\{D,Q,yQ_m\}] \\ &+vE[(\min\{Q,yQ_m\}-D)^+] - g_rE[(D-\min\{Q,yQ_m\})^+] \\ &-(w_m+c_r)E[\min\{Q,yQ_m\}] - \eta - S \end{aligned} \tag{7.7}$$

The first three terms represent the total revenue generated by the retailer from products sold to consumers, waste products sent to the manufacturer for remanufacturing, and unsold items sold at salvage price, respectively. The last four terms are, in order, the amount of goodwill lost due to unfulfilled demand, overall cost of acquiring and handling the order quantity, CSR investment to increase customer demand as well as waste product return rate, and cost of staff who handle returned items. The following is an alternative form of the above expression:

$$\begin{aligned}\Pi_r(Q,\eta) &= (p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta}))\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})} Df(x)dx \\ &+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\ &+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\ &+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\} - (w_m+c_r-v) \\ &\times\Big\{\int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy\Big\} - g\bar{x} - \eta - S \end{aligned} \tag{7.8}$$

The following theorem characterizes the optimal decisions of the retailer in decentralized model under wholesale price-only contract.

Theorem 7.1 *Under wholesale price-only contract, for given production decision and improvement investment of the manufacturer, the retailer's objective function is jointly concave in Q and η, the optimal order quantity $Q^d$and CSR investment η^d satisfy the following equations:*

$$Q^d = (\alpha+\beta e+\gamma\sqrt{\eta}) + F^{-1}\Big(\frac{p+g_r+w_r(q_2+\delta_2\sqrt{\eta})-w_m-c_r}{p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta})}\Big) \tag{7.9}$$

$$\begin{aligned}
\textit{and} \qquad & \frac{\gamma}{2\sqrt{\eta^d}}(p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta^d}))\\
&\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^d})}f(x)dx\Big)g(y)dy\\
&+\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^d})}f(x)dx\Big)g(y)dy\Big\}\\
+\Big\{\int_a^{\frac{Q}{Q_m}}\Big(&\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^d})}(\alpha+\beta e+\gamma\sqrt{\eta^d}+x)f(x)dx\\
&+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^d})}^u(yQ_m)f(x)dx\Big)g(y)dy\\
+\int_{\frac{Q}{Q_m}}^b\Big(&\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^d})}(\alpha+\beta e+\gamma\sqrt{\eta^d}+x)f(x)dx\\
&+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta^d})}^u Qf(x)dx\Big)g(y)dy\Big\}\frac{\delta_2}{2\sqrt{\eta^d}}w_r=1
\end{aligned}\tag{7.10}$$

Proof. Due to complexity, the concavity of $\Pi_r(Q,\eta)$ with respect to Q and η can not be proved analytically. The retailer's profit function $\Pi_r(Q,\eta)$ is jointly concave in Q and η, which can be checked graphically as shown in Fig. 7.2. To prove the theorem, we have

$$\begin{aligned}
\frac{\partial\Pi_r(Q,\eta)}{\partial Q} &= (p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta}))\int_{\frac{Q}{Q_m}}^b\Big(\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^u f(x)dx\Big)g(y)dy\\
&\quad -(w_m+c_r-v)\int_{\frac{Q}{Q_m}}^b g(y)dy
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\Pi_r(Q,\eta)}{\partial\eta} &= \frac{\gamma}{2\sqrt{\eta^d}}(p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta}))\\
&\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^d})}f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^d})}f(x)dx\Big)g(y)dy\Big\}\\
&+\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^d})}(\alpha+\beta e+\gamma\sqrt{\eta^d}+x)f(x)dx\\
&+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta^d})}^u(yQ_m)f(x)dx\Big)g(y)dy+\int_{\frac{Q}{Q_m}}^b\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta^d})}(\alpha+\beta e+\gamma\sqrt{\eta^d}+x)f(x)dx\\
&+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta^d})}^u Qf(x)dx\Big)g(y)dy\Big\}\frac{\delta_2}{2\sqrt{\eta^d}}w_r-1
\end{aligned}$$

Solving the first order conditions for optimality of $\Pi_r(Q,\eta)$, we get the required optimal production quantity Q^d and product quality investment η^d as given in equations (7.9) and (7.10). ■

It can be shown that the optimal order quantity Q^d is decreasing with respect to handling cost, while it is increasing with respect to customer satisfaction improvement investment. Also, $\frac{\partial Q}{\partial w_m} < 0$.

Theorem 7.1 shows that when the manufacturer raises its investment in product quality enhancement, the retailer increases his order quantity. This supports our hypothesis that a greater quality investment generates more demand, encourages the retailer to place more orders to the manufacturer. According to equation (7.9), an increased handling cost and a salvage value have bad and good impacts, respectively on the retailer's order quantity.

Comparing the order quantity in equation (7.9) with that in equation (7.3) of the centralised system, we see that the retailer orders less in the decentralised system, which results in a lower expected sales. As a result, we can observe that in a wholesale price contract, the supply chain is distorted by the retailer's action by ordering less than the optimal order quantity of the centralized system. After examining the retailer's issue and obtaining the optimal decisions $\left(Q^d, \eta^d\right)$, we now obtain the manufacturer's expected profit function $\Pi_m(Q_m, e)$ as follows:

$$\begin{aligned}\Pi_m(Q_m, e) \quad = \quad & w_m E[\min\{Q, yQ_m\}] + vE[(yQ_m - Q)^+] + (c_m - w_r - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) \\ & \times E[\min\{D, Q, yQ_m\}] - c_m Q_m - c_m^r(q_1 - \delta_1 e)E[\min\{D, Q, yQ_m\}] \\ & -\frac{1}{2}\mu e^2 \end{aligned} \tag{7.11}$$

The first three terms in (7.11) indicate that the manufacturer's total revenue includes revenue from wholesale price, salvage price for excess output and remanufacturing revenue, respectively; the last three terms are the production cost of finished product, loss due to repairing defect products returned by customers and investment on product quality improvement. The above equation can be written as

$$\Pi_m(Q_m, \eta) \quad = \quad (c_m - w_r - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) - c_m^r(q_1 - \delta_1 e)$$

$$
\begin{aligned}
&\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}+(w_m-v) \\
&\times\Big\{\int_a^{\frac{Q}{Q_m}}yQ_m g(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\}-(c_m-v\bar{y})Q_m-\frac{1}{2}\mu e^2 \qquad (7.12)
\end{aligned}
$$

The following theorem characterizes the manufacturer's optimal production amount and product quality improvement investment in the decentralized system under wholesale price-only contract.

Theorem 7.2 *The expected profit function* $\Pi_m(Q_m, e)$ *is concave in both* Q_m *and* e *and the optimal input amount* Q_m^d *and improvement investment* e^d *satisfy the following equations*

$$
\begin{aligned}
&\{(c_m-w_r-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e)\}\times\int_a^{\frac{Q}{Q_m^d}}\int_{yQ_m^d-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}yf(x)dxg(y)dy \\
&+(w_m-v)\int_a^{\frac{Q}{Q_m^d}}yg(y)dy=(c_m-v\bar{y}) \qquad (7.13)
\end{aligned}
$$

and

$$
\begin{aligned}
&\beta\{(c_m-w_r-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e^d)\} \\
&\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^d+\gamma\sqrt{\eta})}f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^d+\gamma\sqrt{\eta})}f(x)dx\Big)g(y)dy\Big\} \\
&+\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^d+\gamma\sqrt{\eta})}(\alpha+\beta e^d+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{yQ_m-(\alpha+\beta e^d+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^d+\gamma\sqrt{\eta})}(\alpha+\beta e^d+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{Q-(\alpha+\beta e^d+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}\delta_1 c_m^r=\mu e^d \qquad (7.14)
\end{aligned}
$$

Proof. Due to complexity, the concavity of $\Pi_m(Q_m, e)$ with respect to Q_m and e can not be proved analytically. The manufacturer's profit function $\Pi_m(Q_m, e)$ is jointly concave in Q_m and e, which can be checked graphically as shown in Fig. 7.3. We have

$$\begin{aligned}\frac{\partial \Pi_m(Q_m, e)}{\partial Q_m} &= ((c_m - w_r - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) - c_m^r(q_1 - \delta_1 e)) \\ &\quad \times \int_a^{\frac{Q}{Q_m}} \int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u} yf(x)dxg(y)dy - (w_m - v)\int_a^{\frac{Q}{Q_m}} yg(y)dy - (c_m - v\bar{y})\end{aligned}$$

$$\begin{aligned}&\frac{\partial \Pi_m(Q_m, e)}{\partial e} = \beta((c_m - w_r - c_m^{'})(q_2 + \delta_2\sqrt{\eta}) - c_m^r(q_1 - \delta_1 e)) \\ &\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} \\ &+\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})}(\alpha+\beta e^c+\gamma\sqrt{\eta}+x)f(x)dx \\ &+\int_{yQ_m-(\alpha+\beta e^c+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})}(\alpha+\beta e^c+\gamma\sqrt{\eta}+x)f(x)dx \\ &+\int_{Q-(\alpha+\beta e^c+\gamma\sqrt{\eta})}^{u} Qf(x)dx\Big)g(y)dy\Big\}\delta_1 c_m^r - \mu e^c\end{aligned}$$

Solving the first order optimality conditions for the manufacturer's expected profit function $\Pi_m(Q_m, \eta)$, we get the required optimal production quantity Q_m^d and product quality investment η^d as given in equations (7.13) and (7.14). ■

Theorem 7.2 shows that under a wholesale pricing contract, the manufacturer reduces both the optimal production quantity (Q_m^d) and the optimal investment (η^d) in comparison to the centralised system. Because the manufacturer bears the risk of producing more than the order requires, she produces less and invests less in product development because she bears the costs associated with these activities, despite the fact that the entire system benefits from this investment.

Even though the firms are able to make their own choices under the wholesale price-only contract in the decentralised scenario, the manufacturer is responsible for his own production uncertainty, while the retailer is responsible for his own demand uncertainty. This self-serving strategy misses the flexibility to deal with the manufacturer's over-production issue as well as the retailer's risk of demand unpredictability.

Theorem 7.3 *Both the order and the production quantities in the decentralised model are strictly less than their counterparts in the centralized model. A lower order results in a lower expected supply chain profit in the decentralised setting.*

Proof. Since $c_r > 0$ and $v < w_m$, we have $w + c_r - v > c_r$. Now, by comparing (7.3) with (7.9) and using the fact that $w + c_r - v > c_r$, we can see that the retailer's optimal order quantity in the decentralised model is strictly less than in the centralised benchmark model. Because the retailer orders a smaller quantity of the finished product, the manufacturer's raw material production size in the decentralised model is also smaller than in the centralised benchmark model, as can be seen by comparing equation (7.13) with equation (7.4). Adding the supply chain members' expected profits, we get $\Pi_r + \Pi_m = \Pi_d$. It is easy to verify that $\Pi_c(Q^c, Q^c_m, e^c, \eta^c) > \Pi_d(Q^d, Q^d_m, e^d, \eta^d)$. Individual pricing decisions of both retailers and manufacturers are to blame for the SC's inefficiencies in a decentralised scenario. ■

According to the above Theorem, even if all chain members optimize their own earnings, the total channel performance does not reach its maximum level in a decentralized system. In a decentralized environment, because decision control is shared among the various chain members, it is possible to deviate from the best decisions made in the centralized model. Contract strategies are used to match each member's decisions with the centralized objective in order to avoid double-marginalization effect by reducing competitiveness among chain members without changing the supply chain's design or decision-making power.

7.5 Coordination mechanism for the supply chain

Both the retailer and the manufacturer carry the risk of their own uncertainties in the wholesale price contract. As a result, the 'double marginalization' problem arises, reducing profitability for both the individual members and the entire supply chain. Hence, a proper contract mechanism need to be designed to encourage both the retailer and the manufacturer to bear the risk of the other member. If a decentralized supply chain under some contract generates the same profit potential as the centralized supply chain, it is said to be coordinated under that contract. In this chapter, we suggest a composite contract that constructively incorporates three well-known coordination mechanisms in order to coordinate the supply chain, encompassing both classic manufacturer-led and increasing retailer-led contract approaches. We coordinate the supply chain using appropriate protocols within a specific framework that is chosen based on the available business plan as well as context-specific characteristics.

7.5.1 Buy-back pay-back contract

In the decentralized supply chain model with wholesale price contract, distortion in the context of double marginalization occurs in two ways. First, in uncertain demand situation, it is the retailer who deviates from achieving the system's optimal performance by ordering less product. Second, in random yield situation, the manufacturer deviates from achieving the system's optimal performance by manufacturing less product. To coordinate the supply chain, the participants must be given the appropriate incentive levels via various contract mechanisms, which will help the supply chain to perform optimally. A buy-back contract is a well-known technique for rectifying the retailer's under-ordering issues faced by demand uncertainty (Pasternack, 1985; Emmos and Gilbert, 1998), in which the manufacturer buys back the retailer's excess stock at a discount price (a price below the wholesale price). A pay-back arrangement, on the other hand, solves the problem of the manufacturer's under-production in the presence of random yield by allowing the retailer to purchase the manufacturer's excess output beyond his order at a discount price (Tang and Kouvelis, 2014). We discuss how to build a composite contract by merging these two contract mechanisms so that both the retailer and the manufacturer are encouraged to share the risk of the other party.

We consider a buy-back contract offered by the manufacturer to the retailer as well as a pay back contract offered by the retailer to the manufacturer. If the retailer adopts the manufacturer's offer, the manufacturer will pay for the retailer's leftover inventory at the end of the selling season at a predetermined buy-back price $v^{'}$ per unit. On the other hand, if the manufacturer adopts the retailer's offer, the retailer will compensate for the manufacturer's excess output beyond his order at a predetermined pay-back price $v^{''}$ per unit at the end of the selling season. Under this contract, for given production size Q_m and quality improvement level e, the expected profit of the retailer can be obtained as

$$\begin{aligned}\Pi_{rbp}(Q,\eta) &= pE[\min\{D,Q,yQ_m\}] + w_r(q_2+\delta_2\sqrt{\eta})E[\min\{D,Q,yQ_m\}] \\ &\quad +vE[(\min\{Q,yQ_m\}-D)^+] - g_rE[(D-\min\{Q,yQ_m\})^+] \\ &\quad -(w_m+c_r)E[\min\{Q,yQ_m\}] + v^{'}E[(\min\{Q,yQ_m\}-D)^+] \\ &\quad -v^{''}E[(yQ_m-Q)^+] - \eta - S \end{aligned} \tag{7.15}$$

where the sixth term is the payment of the manufacturer for the retailer's unsold inventory and the seventh term is the payment of the retailer for the manufacturer's excess production output over his order. We can rewrite the above profit function as

$$\begin{aligned}\Pi_{rbp}(Q,\eta) &= (p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta})-v^{'})\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})} Df(x)dx \\ &\quad +\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\ &\quad +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\ &\quad +\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u} Qf(x)dx\Big)g(y)dy\Big\} - (w_m+c_r-v-v^{'}-v^{''}) \\ &\quad \times\Big\{\int_a^{\frac{Q}{Q_m}} yQ_mg(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy\Big\} - g\bar{x} - \eta - S - v^{''}\bar{y}Q_m \end{aligned} \tag{7.16}$$

Theorem 7.4 *For the decentralized model under buy-back pay-back contract, the expected profit function $\Pi_{rbp}(Q,\eta)$ is jointly concave in Q and η. The optimal order quantity Q^{bp} and CSR investment η^{bp} are obtained from the following equations:*

$$Q^{bp} = (\alpha+\beta e+\gamma\sqrt{\eta}) + F^{-1}\Big(\frac{p+g_r+w_r(q_2+\delta_2\sqrt{\eta})-w_m-c_r+v^{''}}{p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta})-v^{'}}\Big) \tag{7.17}$$

$$
\begin{aligned}
\textit{and} \qquad & \frac{\gamma}{2\sqrt{\eta}}(p+g_r-v+w_r(q_2+\delta_2\sqrt{\eta})-v^{'}) \\
& \times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy \\
& +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} \\
& +\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\
& +\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
& +\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\
& +\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}\frac{\delta_2}{2\sqrt{\eta}}w_r=1 \qquad (7.18)
\end{aligned}
$$

Proof. The proof is similar to the Theorem 7.1. ■

Eq.(7.17) shows that the retailer's optimal order quantity is an increasing function of the CSR investment η and a decreasing function of its purchasing cost w_m and treating cost c_r, as expected. Also we find that retailer's optimal order quantity increases with customer eagerness to purchase the product under CSR activities.

Taking into account the retailer's optimum responses, we now determine the manufacturer's optimal decisions. The expected profit function of the manufacturer is given by

$$
\begin{aligned}
\Pi_{mbp}(Q_m,e) \;=\; & w_mE[\min\{Q,yQ_m\}]+vE[(yQ_m-Q)^+]-c_m(Q_m-(q_2+\delta_2\sqrt{\eta}) \\
& \times E[\min\{D,Q,yQ_m\}])-(w_r+c_m^{'})(q_2+\delta_2\sqrt{\eta})E[\min\{D,Q,yQ_m\}] \\
& -c_m^r(q_1-\delta_1 e)E[\min\{D,Q,yQ_m\}]-v^{'}E[(\min\{Q,yQ_m\}-D)^+] \\
& +v^{''}E[(yQ_m-Q)^+]-\frac{1}{2}\mu e^2 \qquad (7.19)
\end{aligned}
$$

An alternative representation of the manufacturer's profit function as given below:

$$
\begin{aligned}
\Pi_{mbp}(Q_m,e) \;=\; & ((c_m-w_r-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e)+v^{'}) \\
& \times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_{l}^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}+(w_m-v-v^{'}-v^{''}) \\
&\times\Big\{\int_{a}^{\frac{Q}{Q_m}}yQ_m g(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\Big\} \\
&-(c_m-v\bar{y}-v^{''}\bar{y})Q_m-\frac{1}{2}\mu e^2
\end{aligned}
\tag{7.20}
$$

Theorem 7.5 *The profit function $\Pi_{mbp}(Q_m, e)$ is concave in both Q_m and e and the optimal input amount Q_m^{bp} and product improvement expenditure e^{bp} satisfy the following equations:*

$$
\begin{aligned}
&((c_m-w_r-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e)+v^{'})\int_a^{\frac{Q}{Q_m^{bp}}}\int_{yQ_m^{bp}-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}yf(x)dxg(y)dy \\
&\quad+(w_m-v-v^{'}-v^{''})\int_a^{\frac{Q}{Q_m^{bp}}}yg(y)dy=(c_m-v\bar{y}-v^{''}\bar{y})
\end{aligned}
\tag{7.21}
$$

$$
\begin{aligned}
\text{and}\quad &\beta((c_m-w_r-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e)+v^{'}) \\
&\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}f(x)dx\Big)g(y)dy\Big\} \\
&+\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{yQ_m-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}(\alpha+\beta e+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{Q-(\alpha+\beta e+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}\delta_1 c_m^r=\mu e
\end{aligned}
\tag{7.22}
$$

Proof. The proof is similar to the Theorem 7.2. ■

If a buy-back with pay-back contract is used to coordinate the supply chain, the optimum decisions for the integrated supply chain should maximise both the retailer's and the manufacturer's expected profits given in equations (7.16) and (7.20) at the same time. Comparing (7.22) with (7.5), we find that $e^{br} = e^c$, i.e., the optimal quality effort of the manufacturer is aligned with the centralized system, when $p + g_r - v + w_r(q_2 + \delta_2\sqrt{\eta}) - v^{'} = 0$. But under this condition, the order quantity of the retailer Q^{bp} will be undefined. So we can conclude that neither the manufacturer nor the retailer could offer such a contract to coordinate the supply chain.

7.5.2 Buy-back pay-back cost-sharing contract

It can be shown that even when simply a pay-back mechanism is employed to rectify the manufacturer's under-production problem, her production quantity remains below the optimum level because the retailer doesn't really place a large enough order, and that's why the manufacturer does not spend sufficiently in quality improvement. This occurs because, under the buy-back pay-back contract, the manufacturer is only responsible for product quality improvement efforts, although both the retailer and the manufacturer might benefit from these efforts. However, because of the increased quality investment costs, the manufacturer struggles to implement such initiative from the system's standpoint. This implies that the manufacturer should be given incentives to improve the quality of her products.

The retailer's cost-sharing contract is a method of correcting the manufacturer's low-investment decision by requiring the retailer to share the manufacturer's investment expenditures in order to improve product quality. In this strategy, the retailer motivates the manufacturer to enhance his quality improvement investment so that rewarding customer demand raises the market, resulting in higher expected sales. Because the pay-back contract alone cannot provide the necessary incentive to the manufacturer to align his production and investment decisions with the centralised system, we combine the two contracts to establish a pay-back cost-sharing contract offered by the retailer, in which the retailer not only shares the manufacturer's production uncertainty by purchasing the excess output of the manufacturer at a predetermined pay-back price $v^{''}$ per unit, but also shares a proportion (ϕ) of the manufacturer's investment on quality improvement. It's worth emphasising that the manufacturer's profit now depends on market demand as well as the retailer's order. Such a contract can only be executed if the manufacturer has access to demand information. More significantly, we want to know whether the cost-sharing agreement benefits the retailer and why the retailer would be encouraged to split the cost of improving product quality with the manufacturers.

We propose a composite contract that combines two mechanisms to align the decentralised model with centralised system: a pay back-cost sharing mechanism to align manufacturer's production and investment decisions with the centralised system, and a buy-back mechanism to align retailer's ordering and CSR decisions with the centralised system. The profit functions of the retailer and the manufacturer are given in this situation by

$$\begin{aligned}\Pi_{rc}(Q,\eta) &= pE[\min\{D,Q,yQ_m\}] + w_r(q_2+\delta_2\sqrt{\eta})E[\min\{D,Q,yQ_m\}] \\ &\quad + vE[(\min\{Q,yQ_m\}-D)^+] - g_rE[(D-\min\{Q,yQ_m\})^+] \\ &\quad -(w_m+c_r)E[\min\{Q,yQ_m\}] + v^{'}E[(\min\{Q,yQ_m\}-D)^+] \\ &\quad -v^{''}E[(yQ_m-Q)^+] - \eta - S - \phi\frac{1}{2}\mu e^2 \end{aligned} \tag{7.23}$$

$$\begin{aligned}\Pi_{mc}(Q_m,e) &= w_mE[\min\{Q,yQ_m\}] + vE[(yQ_m-Q)^+] - c_m(Q_m-(q_2+\delta_2\sqrt{\eta}) \\ &\quad E[\min\{D,Q,yQ_m\}]) - (w_r+c_m^{'})(q_2+\delta_2\sqrt{\eta})E[\min\{D,Q,yQ_m\}] \\ &\quad -c_m^r(q_1-\delta_1 e)E[\min\{D,Q,yQ_m\}] - v^{'}E[(\min\{Q,yQ_m\}-D)^+] \\ &\quad +v^{''}E[(yQ_m-Q)^+] - (1-\phi)\frac{1}{2}\mu e^2 \end{aligned} \tag{7.24}$$

The profit function $\Pi_{rc}(Q,\eta)$ and $\Pi_{mc}(Q_m,e)$ can be shown concave in their respective decisions variables in a way similar to that used for the previous contract mechanism scenario and the optimal order quantity Q^*, production size Q_m^* and CSR expenditure η^* satisfy the equations (7.17), (7.18) and (7.21), the optimal solutions for the previous contract mechanism. The manufacturer's optimal product improvement effort e^* satisfies

$$\beta((c_m - w_r - c_m^{'})(q_2+\delta_2\sqrt{\eta}) - c_m^r(q_1-\delta_1 e^*) + v^{'})$$

$$
\begin{aligned}
&\times\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^*+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy \\
&\quad+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^*+\gamma\sqrt{\eta})} f(x)dx\Big)g(y)dy\Big\} \\
&+\Big\{\int_a^{\frac{Q}{Q_m}}\Big(\int_l^{yQ_m-(\alpha+\beta e^*+\gamma\sqrt{\eta})}(\alpha+\beta e^*+\gamma\sqrt{\eta}+x)f(x)dx \\
&\quad+\int_{yQ_m-(\alpha+\beta e^*+\gamma\sqrt{\eta})}^{u}(yQ_m)f(x)dx\Big)g(y)dy \\
&+\int_{\frac{Q}{Q_m}}^{b}\Big(\int_l^{Q-(\alpha+\beta e^*+\gamma\sqrt{\eta})}(\alpha+\beta e^*+\gamma\sqrt{\eta}+x)f(x)dx \\
&+\int_{Q-(\alpha+\beta e^*+\gamma\sqrt{\eta})}^{u}Qf(x)dx\Big)g(y)dy\Big\}\delta_1 c_m^r = (1-\phi)\mu e^* \qquad (7.25)
\end{aligned}
$$

By comparing the manufacturer's quality level e^* with e^{bp} under pay-back contract in Equation (7.22), we can see that the manufacturer is willing to set a relatively high quality level under the pay-back-cost-sharing contract. Besides the retailer's contribution to the quality improvement expenditure, since the manufacturer expects potential revenue from additional demand due to product quality improvement and lower repairing cost for defect product, she is willing to set product quality level as the centralised system does.

Under the proposed contract, we must align the optimal decisions of the centralised system with the equivalent decisions of the manufacturer and the retailer in the decentralised system in order to coordinate the supply chain. Comparing the optimal decisions for the centralised system in equations (7.3) (7.4), (7.5), and (7.6) with the corresponding ones for the retailer in equations (7.17) and (7.18) and manufacture in equations ,(7.21) and (7.25), respectively, we see that when

$$
\begin{aligned}
w_r &= p+g_r-c_r\phi(p+g_r-v+(\frac{c_m}{\bar{y}}-v)-c_m^r(q_1-\delta_1 e)) \\
&\quad+c_r\phi\times\Big(\frac{p+g_r-v+(c_m-c_m^{'})-c_m^r(q_1-\delta_1 e)}{p+g_r-v+(c_m-c_m^{'})(q_2+\delta_2\sqrt{\eta})-c_m^r(q_1-\delta_1 e)}\Big) \qquad (7.26)\\
v^{'} &= (1-\phi)(p+g-v)+\phi c_r^m(q_1-\delta_1 e) \qquad (7.27)\\
w_m &= v+v^{'}+v^{''} \qquad (7.28)\\
\textit{and}\quad v^{''} &= (\frac{c_m}{\bar{y}}-v)\phi \qquad (7.29)
\end{aligned}
$$

hold simultaneously, then the optimal decisions of the manufacturer and the retailer under this contract maximizes the centralized model's objective function *i.e.,* the proposed contract scheme achieves perfect coordination of the supply chain.

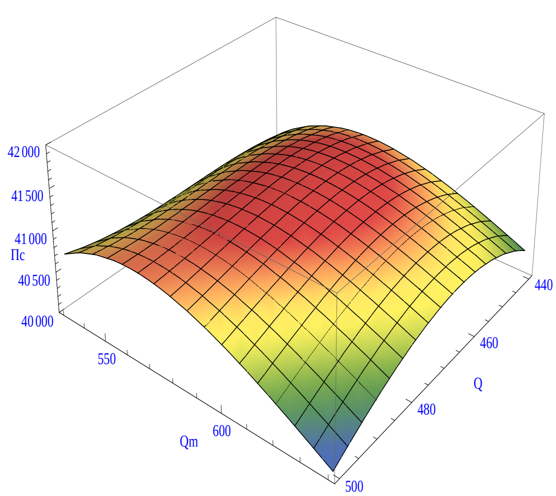
Theorem 7.6 *Under the voluntary compliance, in a decentralized supply chain where the retailer simultaneously decides order quantity and CSR investment while facing uncertain demand and the manufacturer simultaneously decides the production quantity and quality improvement in the presence the production uncertainty; additionally the customer return rate of used product is influenced by the CSR activities of the retailer and the return rate of defected product from customer is influenced by the manufacturer's quality improvement effort, a buy-back contract with pay-back-cost-sharing contract that satisfy equations* (7.26) *-* (7.29)*, ensures the channel coordination. Furthermore, the chain profit can be arbitrarily split between the two parties.*

Proof. We obtain the criteria for supply chain coordination by using the optimal decisions of supply chain members in the decentralised model under the buy-back with pay-back-cost-sharing contract. We find the conditions stated in equations (7.26)- (7.29) by comparing (7.3) with (7.17) and (7.4) with (7.21), (7.5) with (7.25) and (7.6) with (7.18). As a result, the retailer makes the same final product ordering decisions and CSR investments as the centralised model, and the manufacturer makes the same manufacturing decisions and product enhancement investments as the centralised model. Using criteria (7.26) - (7.29), the decentralised system's total expected profit under a buy-back with pay-back-cost-sharing contract is $\Pi_{rc}(Q^*, e^*) + \Pi_{mc}(Q_m^*, \eta^*) = \Pi_c(Q^c, Q_m^c, e^c, \eta^c)$. ■

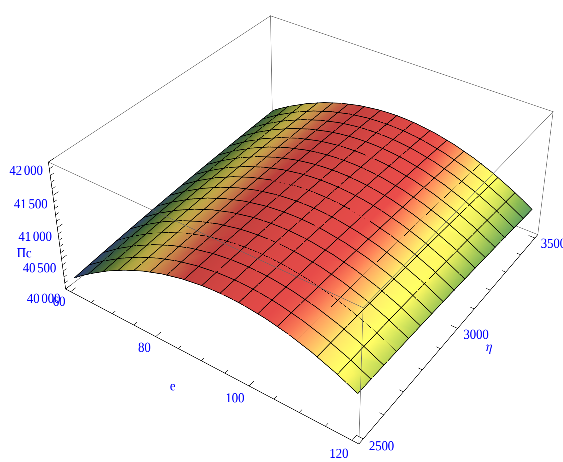
7.6 Numerical analysis

In this part, we conduct computational investigations to discuss the performance of the above mentioned coordinating contract on equilibrium solution as well as the impact of waste products remanufacturing and defect products repairing on quality improvement levels and CSR activities. We also perform a sensitivity analysis to see how demand and supply uncertainties as well as quality and CSR investment sensitive coefficient parameters influence the equilibrium solution. We compare coordinating contracts with simple wholesale price contract to measure the efficiency gain of coordination. The basic parameter-values are taken as $\alpha = 250; \beta = 0.7; \gamma = 0.9; q_1 = 0.1; q_2 = 0.5; \delta_1 = 0.0001; \delta_2 = 0.001; S = 200; c_m = 20; c_m^{'} = 10; c_m^r = 8; c_r = 2; g_r = 1.5; v = 10; w = 100; p = 140;$ $\phi = 0.4;$ and $\mu = 0.9$ in there appropriate units. The demand follows a uniform distribution with mean $\bar{x} = 50$ and standard deviation $\sigma_x = 50/\sqrt{3}$, and supply follows a uniform distribution having mean $\bar{y} = 0.8$ and standard deviation $\sigma_y = 0.05$. The parameter-values considered in our analysis are closely matched with those found in a few secondary sources including Giri et al. (2016), Chen and Bell (2011), Taleizadeh et al. (2019), and Zhang et al. (2020).

For the above set of values, the concavity of the intreageted system's objective function Π_c for given e and η with respect to Q and Q_m and Π_c for given Q and Q_m with respect to e and η are checked graphically as shown in Figs. 7.1a and 7.1b, respectively. Also, for the same data set, the concavity of the retailer's expected profit function $\Pi_r(Q, \eta)$ with respect to Q and η and the manufacturer's expected profit function $\Pi_m(Q_m, e)$ with respect to Q_m and e under the wholesale price contract are checked graphically as shown in Figs. 7.2 and 7.3, respectively.



(a) Concavity of Π_c w.r.t. Q and Q_m.



(b) Concavity of Π_c w.r.t. e and η.

Fig. 7.1: Concavity of the profit function $\Pi_c(Q, Q_m, e, \eta)$ w.r.t. its decision variables

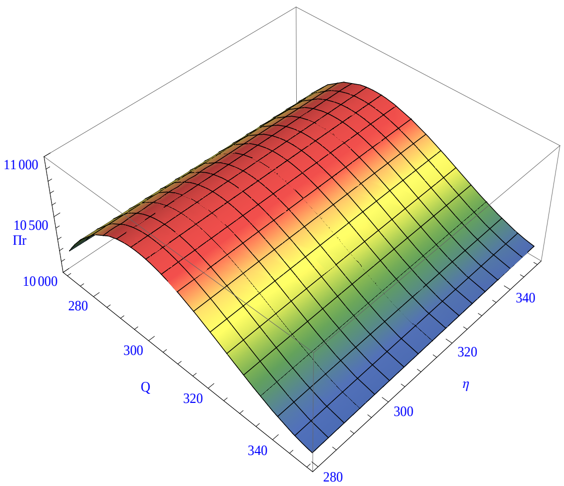


Fig. 7.2: Concavity of the function $\Pi_r(Q, \eta)$.

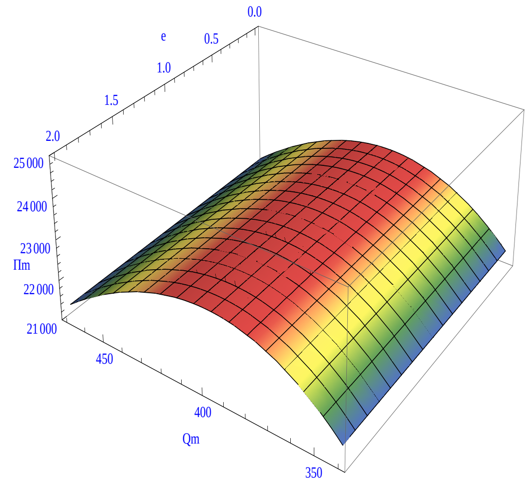


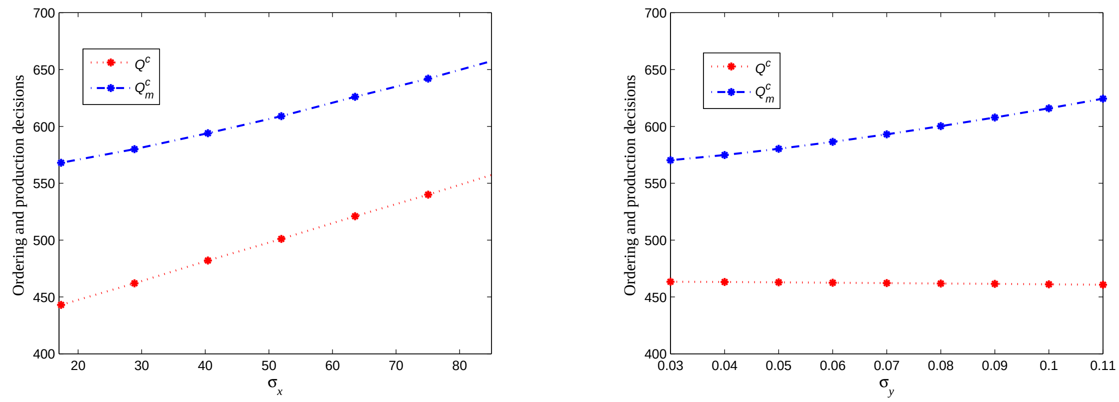
Fig. 7.3: Concavity of the function $\Pi_{mc}(Q_m, e)$.

Table 7.1: A comparison of results of different models

Model	Q	Q_m	η	e	Π_r	Π_m	Π
Centralized	462	580	3049	92.25	-	-	41168
Decentralized	296	398	306	1.13	10685	22956	8355
Coordinated	462	580	3049	92.25	14460	26707	41168

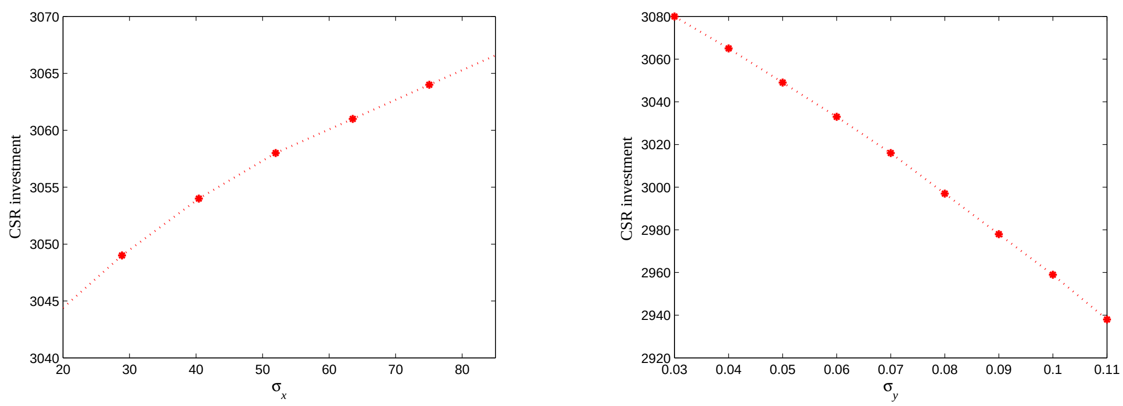
Table 7.1 illustrates the optimized results for the centralized model and also decentralized models with wholesale price contract and buy-back with pay-back cost-sharing contract. We find that the coordinated contract provides higher profit for both the manufacturer and the retailer over wholesale price contract. Furthermore, under the buy-back with pay-back-cost-sharing contract, the manufacturer and the retailer obtain the highest profit level together. Under coordinating contract, the optimal order quantity, production amount, quality improvement level and investment for CSR activities increase compared to wholesale price contract, leading to larger expected sales. As a result, supply chain members gain higher profit under coordinating contract. It also supports our theoretical results given in Theorems 7.3 and 7.6.

Fig. 7.4 depicts the impact of supply and demand uncertainties on supply chain members' optimal ordering and production decisions. An increased uncertainty, whether it is from the the manufacturer's production or from the retailer's customer demand, forces to an increased



(a) Effect of σ_x on the Q and Q_m. **(b)** Effect of σ_y on the Q and Q_m.

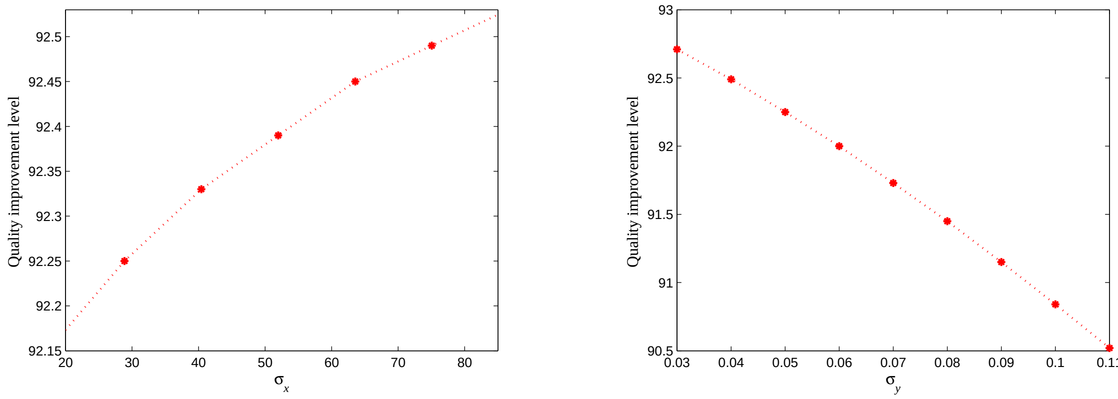
Fig. 7.4: Effect on ordering and production decisions w.r.t. demand and supply uncertainties



(a) Effect of σ_x on the optimum η. **(b)** Effect of σ_y on the optimum η.

Fig. 7.5: Performance of CSR investment w.r.t. demand and supply uncertainties

backlog in manufacturer's production. Although the retailer's stock level rises in response to customer demand uncertainty, it falls in response to the manufacturer's production risk. This is because, as demand becomes more unpredictable, more inventory must be kept on-hand to hedge against demand risk and, as a result, the production amount must be increased. On the other hand, when production becomes pretty fluctuating, the manufacturer prefers to design bigger production size in order to achieve large output, which compensates for the shortage in the event of lower yield realization. Although the supply of the manufacturer is equal to the retailer's order in the context of large yield realization, the supply usually experiences shortage due to the uncertainties, resulting in low yield realization. As a result, when supply yield fluctuation worsens, the manufacturer decides to increase production. Thus we find



(a) Effect of σ_x on the optimum e.

(b) Effect of σ_y on the optimum e.

Fig. 7.6: Performance of quality improvement w.r.t. demand and supply uncertainties

that, with a rising demand uncertainty, more inventories are preserved at the retailer's hand. However, a high demand is needed to escape from salvage. As seen in Figs. 7.5a and 7.6a, a higher demand uncertainty leads to an increased effort level in both the retailer's CSR investment and the manufacturer's quality improvement, leading to increased demand. Also Figs. 7.5b and 7.6b indicate that, a higher production uncertainty leads to an decreased effort level in both the retailer's CSR investment and the manufacturer's quality improvement.

Table 7.2: Effects of consumers' sensitivity to manufacturer's quality investment level e on the optimal decisions and profits of the coordinated supply chain.

β	Q^*	Q_m^*	η^*	e^*	Π_{rc}	Π_{mc}	Π_c
0.5	431.079	538.347	3032.32	65.9727	13724.4	25574.9	39299.3
0.6	445.626	557.553	3040.33	79.1128	14062	26094.4	40156.4
0.7	462.813	580.269	3049.76	92.2544	14460.8	26707.8	41168.7
0.8	482.641	606.507	3060.61	105.397	14920.8	27415.4	42336.2
0.9	505.111	636.278	3072.9	118.542	15442	28217	43659

The effects of the parameter β on the chain members' decision variables as well as individual profits are shown in Table 7.2. As seen in the table, an increasing β in the interval [.5, .9] improves the product's quality level (e). The improvement in quality persuades consumers to buy more products, i.e. increases market demand. Although the cost of improving quality must be paid by the chain members, the higher demand generates enough money to cover the quality investment expenditure. As a result, the retailer's and the manufacturer's profits increase with β [see the trends of Π_{rc} & Π_{mc}]. Furthermore, because of the product's high quality (e), the return rate of defect products is reduced, which is favourable to the manufacturer, although the increment of the retailer's CSR investment is lower than the one of the manufacturer's quality investment. These findings show that the quality coefficient β has a beneficial impact on the SC's performance.

Table 7.3: Effects of consumers' sensitivity to retailer's CSR activity level η on the optimal decisions and profits of the coordinated supply chain.

γ	Q^*	Q_m^*	η^*	e^*	Π_{rc}	Π_{mc}	Π_c
0.7	443.334	554.502	1872.84	92.1551	14701.9	25303.6	40005.5
0.8	452.471	566.586	2425.06	92.2044	14589	25962.7	40551.7
0.9	462.813	580.269	3049.76	92.2544	14460.8	26707.8	41168.7
1	474.362	595.561	3747.31	92.3051	14317.4	27539.3	41856.6
1.1	487.123	612.467	4518.14	92.3566	14158.4	28457.4	42615.8

From Table 7.3, we see that as CSR coefficient γ increases, the retailer tends to set higher CSR investment, which in turn increases the quantity of returned waste products from customers. A higher CSR coefficient indicates higher remanufacturing, which is good for the environment. Remanufacturing reduces manufacturing cost and increases income for the manufacturer. This cost savings may inspire the manufacturer to spend more in quality development in order to attain a higher market demand [see the trend of e]. More specifically, the manufacturer uses the money obtained from remanufacturing to improve the product's quality, which benefits the SC. Table 7.3 shows that the profits of the manufacturer and the entire system are increased, but the profit of the retailer is declined due to the large investment in CSR. This finding demonstrates the effects of CSR on the decision variables and profits; a larger CSR investment means a higher return amount of waste products, which leads to rise in remanufacturing profit and improvement level of the product.

7.6.1 Discussion

The insights gained from our study are given in the following. Existing scholarly studies such as Taleizadeh et al. (2019), Hosseini-Motlagh et al. (2020), and Ghosh et al. (2021)

mostly concentrated on waste products return and remanufacturing in CLSC. In our study, the inclusion of defect products return due to premature failure is also considered. Product quality improvement can reduce the number of defect products returned but raises the cost of production. To compensate the manufacturer's expenditure due to quality improvement, the suggested cost-sharing contract might be adjusted in the coordinated model.

Secondly, although remanufacturing raises both enterprises' profits but it is constricted by the retailer's recycling efforts. As a result, the retailer should better fulfil his social responsibilities by participating in CSR initiatives to collect more waste products and pursue corporate profit. Also, when the retailer executes its social responsibilities, it could win huge attention from the society and, thereby result in more customer demand and achieve more profit.

Third, we consider that there is no secondary resource for the manufacturer at low yield realization, i.e., the manufacturer hardly provides up to the quantity ordered by the retailer with yield uncertainty, putting the retailer in a more uncertain position than a supply chain with stochastic demand because the retailer's order decision now depends on both yield and demand uncertainties. This makes the study more innovative, which assists in gaining insights into several supply uncertainty problems such as production, transportation, and assembly systems.

Finally, from consumers' stand point, the community must raise environmental consciousness, study and learn scientific methods and ways of dealing with waste home appliances, and expand recycling channels, all of which may be accomplished successfully through retailers' improved CSR efforts.

7.7 Conclusion

The most popular waste recycling policy is product take-back and remanufacturing, which has spread fast over the world in the last decade (Atasu and Subramanian, 2012). In our proposed model, we have included two types of return - defect products return and waste products return. Most of the existing studies have given importance to the later one even though the former one is directly connected to the firm's production technologies, and consumers are generally willing to continue utilizing these products after the manufacturer provides maintenance services. Today's growing environmental and social concerns have forced firms to consider sustainability initiatives in their business decisions (Safarzadeh et al., 2020). In this article, we have supposed that the retailer performs CSR to increase customer trust in the product and raise environmental awareness, which influences the market demand and the amount of waste products returned to the manufacturer at the same time. Furthermore, the manufacturer invests in quality enhancement operations in order to reduce the defect products return rate, which also increases the market demand.

Due to coordination challenge and different uncertainties, the globe has recently seen a lot of fluctuations in supply chain performance. Motivated by this issue, we have modeled a two-tiered close-loop supply chain with a retailer and a manufacturer under demand and supply uncertainties. Although considering two sources of uncertainty brings the problem closer to the real-world business environment, interaction between the two sources of uncertainty complicates the challenge. Although earlier researches (e.g., Govindan and Popiuc, 2014; Heydari and Ghasemi, 2018; Heydari, Govindan and Jafari, 2017) provide many important insights into maximising the returned waste amount in RSC, they do not provide adequate guidance to decision makers where demand and supply are both uncertain. We have demonstrated how uncertainty affects ordering, manufacturing, quality improvement, and CSR investment decisions. Because of the interrelation and uncertainties in the decisions of the members, the cooperation of the members' decisions must be coordinated in order to establish an efficient system.

We have shown that, under the wholesale price contract, the double marginalisation" effect is induced in the decisions of the the firms which deviate its response from the system's optimal one, while the retailer orders less from the manufacturer than the centralised system does. The manufacturer sets a production quantity lower than that of the centralised system. A coordinating contract mechanism aims at creating larger production and order quantities. We have first proposed a buy-back pay-back contract in which buy-back contract provides the retailer incentives to order more while the pay-back contract allows the retailer to pay a discount for the manufacturer's excess output and thus providing the necessary incentives to the manufacturer. The buy-back pay-back contract alone cannot coordinate in this configuration, since the manufacturer still tends to reduce the product quality and, as a result, the demand drops down. In this case, we have proposed a buy-back pay-back cost-sharing arrangement, in which the retailer, in addition to the prior incentive, also contributes a fraction of the manufacturer's cost of quality improvement investment. We have demonstrated that this contract can accomplish coordination and allocate supply chain profit to the manufacturer and retailer in a number of different ways.

We have found that the CLSC's sustainability performance improves as a result of the coordination plan, which allows CLSC members to invest more in their sustainability efforts such as CSR investment level and product quality improvement effort, while also increasing the recycling rate of waste products, reducing pollution and resource wastes. On the one hand, investing in CSR and improving product quality enhance consumer faith in products, resulting in increased market demand and business profitability. As a result, CSR investment behaviour and product quality improvement effort in CLSC are always economically beneficial to members and the entire CLSC system.

Chapter 8

Conclusion and future span

The supply chain is the most significant organ of all the businesses engaged in the design, manufacture, and delivery of product to customers. In today's globalised business, with continuous advancements in technology, fluctuating demand, unpredictable supply, and unstable customer behaviour, companies are currently experiencing high complexity and competition. As a result, supply chain managers must consider all parameters that are relevant to the decisions they will undertake. The complicated business environment, competitive market scenario, and growing uncertainties in a supply chain force strategy makers to collaborate among chain members in order to survive in the most unfavorable scenarios. Thus, supply chain coordination is the most critical operation for increasing operational efficiency, responding to customer requests, and decreasing inventory costs. This is the process of coordinating production, inventory, distribution, and transportation among supply chain actors to obtain the highest mix of efficiency and responsiveness for the market addressed.

A supply chain often consists of numerous participants who are primarily concerned with optimizing their individual objectives instead of thinking about the entire chain. The optimal decisions of specific individuals may not meet with that of the supply chain resulting in poor system efficiency. To improve channel efficiency and responsiveness, the supply chain system must be flexible enough to response to unexpected changes in demand and supply, must communicate information vertically and horizontally among the chain members to simplify necessary functions, and align the interests of all channel members to distribute risks, expenditures, and rewards effectively across the network. This requires the evolution of supply chain coordination pushed by the need of business in world markets.

Supply chain coordination is a regular phenomenon in global marketplaces using various contract mechanisms, and it brings economic benefits to both the supplier and the buyer. It reduces the system's operating costs and increases its efficiency. To survive in global marketplaces, it is critical to adjust to market changes, and in this regard, the importance of supply chain coordination can no longer be neglected. As a result, in order to receive the potential advantages of supply chain coordination through various contract mechanisms, in this thesis, we aim to model some of the more complex market situations by considering various demand patterns, and collaborate the chain through various contracts, as well as to investigate their role and impact in real business practice. In Chapters 3-7, supply chain models under various uncertainties and influences are built, solved, and evaluated, and significant managerial implications of the decisions achieved in each model are discussed. In each scenario, coordinated contracts are provided to assure the maximum profit from a specific marketplace while also demonstrating the contract's applicability.

Let us now look at some of the limitations or restrictions of the models mentioned in this thesis. To mention a few of these limitations, the first is that the various models presented here have all been explored strictly for demand types that are continuous in nature, and no discrete demand forms have been considered. So, in future research work, it is of priority to be executed discrete types of demand in several aspects. Next, in most situations, we have only considered a single objective at a time; nevertheless, these works may be extended to include multiple objects at once. We've simply discussed regarding single items so far in our model, but we can also use them for multi items. Furthermore, the majority of the cost or profit variables in our models have been treated as constants. Extending the same models to include variable cost or variable profit is achievable. Finally, information asymmetry may be considered at many levels of the supply chain, and utilizing individual and composite contracts with the goal of exchanging information will provide novel situations for future research.

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List of Publications

1. **Coordinating a Socially Responsible Supply Chain with Random Yield under CSR and Price Dependent Stochastic Demand**, Joyanta Kumar Majhi, Bibhas C. Giri and K. S. Chaudhuri, *International Journal of Supply and Operations Management*, (2021), Vol. 8, No. 2, pp. 194-221.

2. **Coordinating a three-level supply chain with effort and price dependent stochastic demand under random yield**, Bibhas C. Giri, Joyanta Kumar Majhi, Sudarshan Bardhan and K. S. Chaudhuri, *Annals of Operations Research*, (2021), Vol. 307, No. 1, pp. 175-206.

3. **Coordination mechanisms of a three-layer supply chain under demand and supply risk uncertainties**, Bibhas C. Giri, Joyanta Kumar Majhi, and K. S. Chaudhuri, *RAIRO-Operations Research*, (2021) , Vol. 55, pp. 2593-2617.

4. **Coordination of a supply chain with customer returns and quality improvement through customer feedbacks under demand and supply uncertainties**, Joyanta Kumar Majhi and Bibhas C. Giri, *European Journal of Operational Research*, (2021) (Submitted).

5. **Coordinating a socially responsible closed-loop supply chain with product improvement and recycling under demand supply uncertainties**, Joyanta Kumar Majhi, Bibhas C. Giri and K. S. Chaudhuri, *International Journal of Production Research*, (2022) (Submitted).

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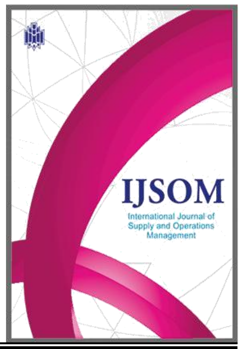

Coordinating a Socially Responsible Supply Chain with Random Yield under CSR and Price Dependent Stochastic Demand

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Abstract

Corporate social responsibility plays an important role in associating customers with socially responsible firms. Faithful consumers are willing to give extra money for commodities or services that incentive the firms to take corporate social responsibility (CSR). This article studies the coordination issue in a two-stage supply chain which is composed of a manufacturer and a retailer who sells a short shelf-life product in a single period. The manufacturer exhibits CSR and simultaneously determines its CSR investment and production quantity, as his production process is subject to random yield. On the other hand, the retailer decides the selling price and order quantity simultaneously while facing price and CSR sensitive stochastic demand. We construct an agreement between the retailer and the manufacturer which comprises a revenue-sharing along with cost-sharing contract. We show that the supply chain can perfectly coordinate under this composite contract and allow arbitrary allocation of total channel profit to ensure that both the retailer and the manufacturer are benefited. We further analyze the impact of randomness in production as well as the effect of CSR investment on the performance of the entire supply chain. A numerical example is provided to explain the developed model and gain more insights.

Keywords: Random yield; Demand uncertainty; Corporate social responsibility; Channel coordination; Pricing.

1. Introduction

One of the most essential concerns of today's supply chain management is to prevent the 'double marginalization' phenomenon (Spengler, 1950) because all the players want to take advantage of both competitive and cooperative relationships. Therefore, they individually seek to optimize their profits that usually lead to a situation where the players have different and sometimes conflicting objectives. For this reason, a supply chain needs collaboration of the members to remove the conflicting objectives among them. One of the interesting collaboration instruments to remove the conflictive objectives is a contract mechanism among the channel members. Contract mechanism induces the members in a decentralized supply chain to work as a centralized supply chain to improve the whole supply chain-wide profit. A contract with this efficiency has been called a 'perfect coordination contract' (Bernstein and Federgruen, 2005). A great amount of literature has discussed contract-based coordination with the help of popular contracts such as quantity discounts (Jeuland and Shugan, 1983; Mandal and Giri, 2019), quantity flexibility (Tsay, 1999; Xiong et al., 2011), buy-back policy (Pasternack, 1985; Ding and Chen, 2008), and so on. For a detailed survey on the contract mechanism, we refer readers to Cachon (2003) and Tsay et al. (1999).

A revenue sharing contract is commonly used in the video renting industries such as Hollywood Entertainment and Blockbuster Inc. (Giannoccaro and Pontrandolfo, 2009). It offers a buyer the right to buy a certain quantity of products at a comparatively lower wholesale price before the information on demand is settled, and gives a certain portion of his revenue to the supplier after selling season is over (Nezhad et al., 2015). Thus, by offering

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ORIGINAL RESEARCH

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Coordinating a three-level supply chain with effort and price dependent stochastic demand under random yield

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Abstract

One of the major objectives of modern supply chain management is dealing with the negative impact of decentralization among the involved entities and minimizing double marginalization effect within the chain, especially when the end-customers' demand is not deterministic. This paper investigates coordination issue in a three-level supply chain with one raw-material supplier, one manufacturer, and one retailer. The retailer determines the retail price, sales effort, and order quantity simultaneously before the selling season starts. Both the supplier and the manufacturer face random yield in production. A composite contract having two components—a contingent buyback with target sales rebate and penalty between the retailer and the manufacturer, and a revenue sharing contract between the manufacturer and the supplier is proposed. The proposed composite contract is shown to achieve supply chain coordination and allows arbitrary allocation of total channel profit among all the chain members. The impact of randomness in both demand and production, and the impact of non-existence of emergency resource for the final product on the performance of the entire supply chain are analyzed. A numerical example is provided to illustrate the developed model and draw some important managerial insights.

Keywords Random yield · Demand uncertainty · Three-echelon supply chain · Channel coordination · Secondary market · Sales effort and pricing

Notations

c_s: Unit production cost at the raw material supplier
c'_s: Unit procurement cost of raw material from the secondary market
c_m: Unit manufacturing cost at the manufacturer
v: Unit salvage value of the final product

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COORDINATION MECHANISMS OF A THREE-LAYER SUPPLY CHAIN UNDER DEMAND AND SUPPLY RISK UNCERTAINTIES

Bibhas C. Giri*, Joyanta Kumar Majhi and Kripasindhu Chaudhuri

Abstract. This paper considers a newsvendor model for a single product to focus on the importance of coordination under demand and supply uncertainties where the raw materials are procured from two unreliable suppliers without any emergency resource; the main supplier (which is cheaper but more unreliable) is prone to random supply disruption and, therefore, it can satisfy all or nothing of the buyer's order, while the backup supplier (which is expensive but less unreliable) is prone to random yield and, therefore, can satisfy only a random fraction of the buyer's order. From the numerical results, we observe that it would be optimal to over-utilize the backup supplier and under-utilize the main supplier if the maximum growth in supply risk results from supply disruption. On the other hand, when the growth in supply risk occurs mainly due to increase in yield risk, the optimal risk mitigation strategy would be to increase the use of the backup supplier and decrease the use of the main supplier. We propose the price only contract and a new revenue sharing contract to mitigate demand and supply uncertainties in the decentralized model, and observe that the revenue sharing contract can fully coordinate the supply chain with win–win outcome for all entities involved in the supply chain.

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1. Introduction

From the World Trade Center terrorist attack on 11 September, 2001 and blackout on 14 August, 2003 in the U.S. to recent political instability, natural disasters and destructive competitive acts increase the complexity, uncertainty and ambiguity of globalized supply chain. There are mainly two kinds of risk of uncertainty that affect supply chain management and network design. The first risk of uncertainty grows from the matter of demand and supply coordination and the second one grows from supply uncertainty which is emblematically modelled as complete supply disruption where supply halts completely, or yield uncertainty where the supplied quantity can fulfil a random fraction of the placed order size. We incorporate such supply uncertainties with normal demand-supply coordination risks. This paper builds on the literature regarding the management of the risk of uncertainty, and on the framework of supply chain coordination.

Keywords. Demand uncertainty, random yield, supply disruption, dual sourcing, three-echelon supply chain, channel coordination.

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