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Analysis of some lead-time reduction strategies in two-echelon supply chain systems

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CERTIFICATE FROM THE SUPERVISOR(S)

This is to certify that the thesis entitled "**Analysis of some lead-time reduction strategies in two-echelon supply chain systems**" submitted by Sri **Sumon Sarkar** who got his name registered on **13/02/2018 (Index No. 45/18/Maths./25)** for the award of Ph. D. (Science) degree of Jadavpur University, is absolutely based upon his own work under the supervision of **Prof. Ashis Kumar Sarkar** and **Prof. Bibhas Chandra Giri** that neither this thesis nor any part of it has been submitted for either any degree / diploma or any other academic award anywhere before.

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Abstract

Analysis of some lead-time reduction strategies in two-echelon supply chain systems

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The overall goal of the supply chain (SC) is to deliver the product or service to the customer on time and to make a profit and increase the value of the shareholders. To ensure that a SC system is profitable, SC managers typically evaluate the assets based on both financial and operational performance to analyze whether the assets are contributing to the financial return. Inventory and lead-time management are the most critical resources and competitive advantages that a SC have. Lead-time variation can have a huge impact on the reorder point and safety stock for any SC system, so it is very important to control lead-time by the SC manages. There are very few practical tools available to support managers' decisions when it comes to combining lead-time with inventory and financial performance for a SC system. The purpose of the research was, therefore, to study different lead-time reduction strategies, as well as inventory decision for two-echelon supply chain with the implications of the strategies on SC's financial performance. To meet the research objectives, a combination of analytical research and numerical research were used. In general, the study concludes that the reduction of the lead-time, as defined in the study, has a high impact on the financial performance of the SC and the length of the strategic lead-times affect the safety stocks of the system. The outcomes from this study contribute to the literature focused on inventory and lead-time management by providing worthwhile information for the SC managers, which permits them for better understanding the impact of lead-time on the financial performance of a SC.

Keywords: Supply chain; pricing; effort; lead-time; investment; backordering; service level constraint; distribution-free approach.

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Dedicated to my parents

Chapter 1

Introduction

Supply chains have been around since ancient times, starting with the first product or service created and sold. With the advent of industrialization, the supply chain became more sophisticated, allowing companies to do a more efficient job of producing and delivering goods and services. For example, Henry Ford's standardization of auto parts was a turning point that allowed mass production of goods to meet the demands of an expanding client. Over time, incremental changes (such as the invention of computers) have brought additional levels of sophistication to supply chain systems. However, for generations, the supply chain remained essentially an isolated linear function handled by supply chain specialists.

The Internet, technological innovation, and the explosion of the global economy driven by demand have changed all that. Today's supply chain is no longer a linear entity. Rather, it is a complex collection of disparate networks that can be accessed 24 hours a day. At the center of these networks there are consumers who expect their orders to be fulfilled, when they want them, and the way they want them.

We now live in an age of unprecedented global commerce and business, not to mention continuous technological innovation and rapidly changing customer expectations. Today's best supply chain strategies call for a demand-driven operating model that can successfully bring people, processes, and technology together around integrated capabilities to deliver goods and services with extraordinary speed and precision.

Today's supply chain is broad, deep, and constantly evolving, which means it must be agile to be effective. In the past, supply chains met the needs of companies and customers through an end-to-end model that was not greatly affected by the change. Now, consumers have multiple options in how to buy products in stores, online, etc. They have also come to expect increased levels of customization. An agile supply chain can meet those expectations.

1.1 Supply chain

The supply chain is a set of activities that ensure that a product reaches the customer. This set of activities is very broad and ranges from obtaining raw materials and their subsequent transformation to transportation and distribution in each of the phases from the beginning of the process until the final product reaches the consumer.

Within an organization, the supply chain includes all the activities such as marketing, delivery, operations, funding, product improvement, customer service, etc. that are required for receiving and fulfilling customer needs. A supply chain consists of a series of activities involving many organizations through which the materials move from supplier to the end customers. There may be a different supply chain for each product. In addition to manufacturer and supplier, a supply chain involves warehouses, transporters, retailers, and even customers who are also a part of the supply chain. Everything that involves the delivery of raw materials from the supplier to the manufacturer and delivering them to the end customers in the form of final products is done through an SC. The supply chain involves various resources such as materials, people, information, money, etc., and the flow of these resources has to be efficiently managed to create profitable business models. Various researchers and economists from time to time provide some similar expressions to define the supply chain. We can mention a few in the following:

> *A supply chain is a network of facilities and distribution options that perform the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers-* Ganeshan, 1995.

> *A supply chain consists of all stages involved, directly or indirectly in fulfilling a customer request. The supply chain not only includes the manufacturers and suppliers, but also transporters, warehouses, retailers, and customers themselves-* Chopra and Meindl, 2001.

> *The flow and management of resources across the enterprise for the purpose of maintaining the business operations profitably-* Sehgal, 2009.

Figure 1.1 depicts how each stage of a supply chain is connected. The product flows from upstream to downstream and the information flows from downstream to upstream. These flows sometimes occur in both directions and may be managed by one of the stages or an intermediary. Each stage as presented here may not be present in every supply chain. End customers' needs and the roles played by the

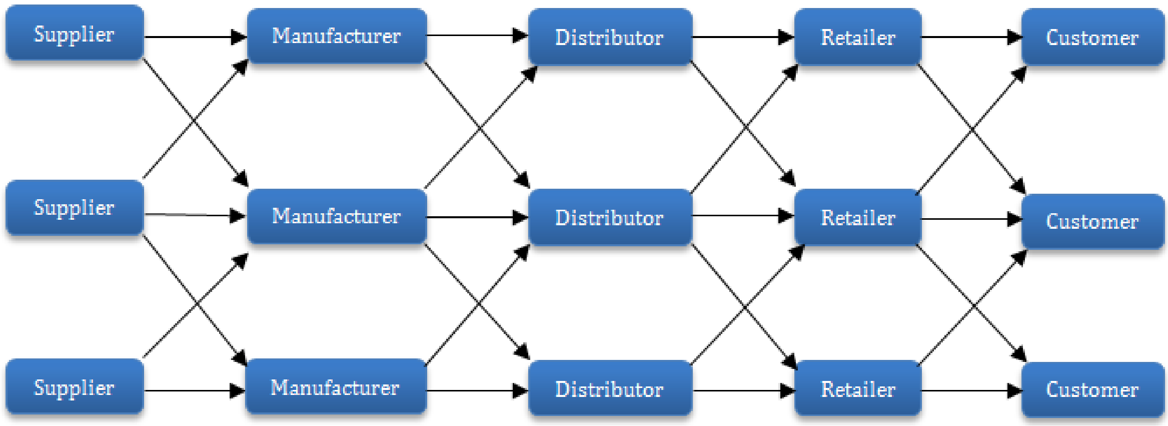

Figure 1.1: A network of supply chains

stages involved decide the appropriate design of the supply chain. In a direct channel, the manufacturer sells directly to the customers. In this case, there is no need for the retailers.

1.1.1 Supply chain example

Assuming that a customer entered the Spencer store to buy beauty soap. The supply chain starts with the customer and his or her need for a beauty soap. The next step for this supply chain is the Spencer Retail Store where customers go for the specific product. Spencer keeps his shelves using a list of items that may have been supplied from a finished goods store owned by Wal-Mart or obtained from a third party (vendor). The vendor, in turn, was supplied by the manufacturer [e.g, Hindustan Uni Liver (HUL)]. The HUL manufacturing industry acquires raw materials from various suppliers who are further supplied by various lower tire suppliers. For example, packaging materials may be received from India Foils limited (a foil company) while India Foils limited receives raw materials for making packaging materials from other suppliers. This forms a typical supply chain (see 1.2).

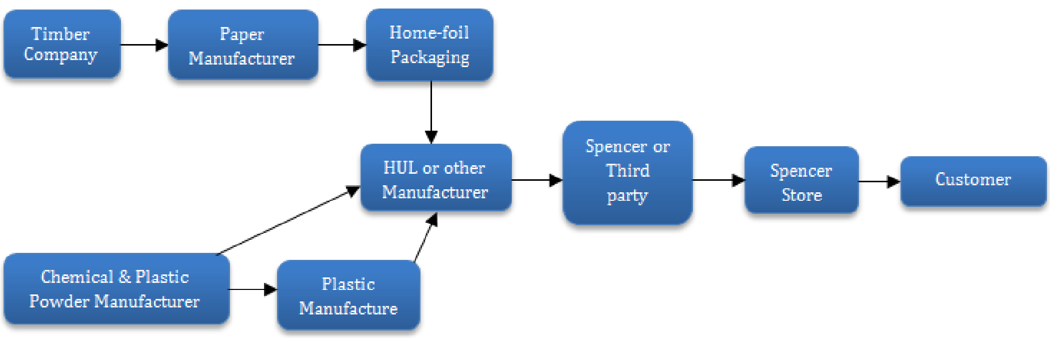

Figure 1.2: Stages of a beauty soap supply chain

1.1.2 Supply chain management (SCM)

Supply Chain Management is the backbone of any business. This particular branch of management works from the beginning of the business until the service reaches the customer. The details include many things. At present, the expansion of trade and commerce extends beyond the borders of the country to the rest of the world. This has increased the level of competition. And delivering products or services to customers at a relatively low cost while maintaining quality in a competitive market has become a major challenge for all organizations. To address this challenge, there is a growing demand for supply chain management professionals in businesses.

Generally, supply chain management refers to the supply of goods. Supply chain management is the process of purchasing an organization's raw materials, planning, source, storage, preparation, and marketing activities to be completed on time and at a low cost. The definition of supply chain management is the delivery of a product from the primary source to the people through various processes. Items are usually outsourced from anywhere, and those are supplier, manufacturer, seller, distributors, and retailers. Items can undergo various hands before reaching customers. The SCM is then tasked with coordinating and integrating inside and outside the organization and behind it. Its main purpose is to increase customer service without increasing the cost of the organization, which at the same time helps to reduce the total cost of the chain. However, in modern times, it is not necessary to just supply products. Maintaining product quality and promised time are important conditions of product delivery. Even after-sales services are included in supply chain management. Therefore, we must treat supply chain management, not as a functional or operational process of the company, but as a management and planning model within the organization that can lead to competitive advantages and make a difference in the market.

1.1.3 Goal of supply chain management

The primary goal of supply chain management is to meet customer demand through the most efficient use of resources, including distribution capacity, inventory, and manpower. The basic idea behind supply chain management for companies and corporations is to get involved in a supply chain by exchanging information about market fluctuations and production capacities. If all relevant information for any company is accessible, each company in the supply chain will have the ability to help optimize the entire chain, rather than sub-optimize it based on a local interest. This will lead to better planning in global production and distribution, which can reduce costs and offer a more attractive end product, generating better sales and better overall results for the companies involved. This is a form of vertical integration. Supply chain management must be executed to ensure customer satisfaction and business success. Due to the critical role of supply chain management within organizations, employers are always looking for employees with a wealth of skills and knowledge to enable them to conduct business operations efficiently. Due to new trends, supply chain management is the most important business discipline globally, regardless of whether it is a small company or a multinational.

The impact of supply chain management on business is quite noticeable and two clear examples of the main aspects where this impact can be seen include:

- ***Increase in customer service***: Supply chain management ensures delivery of the right product and quantity at the right time. In addition, these products have to be available in the location that customers have specified; customers should also receive quality after-sales support.
- ***Increase in cash flow***: Supply chain management increases cash flow, and if product delivery can be accelerated, benefits are also received quickly. Companies value it highly since the use of large fixed assets such as plants, warehouses as well as transport vehicles is reduced throughout the operation.

Importantly, it also helps simplify just about everything from day-to-day products to unexpected natural disaster flows. By using the right management tools and techniques, companies gain the ability to correctly diagnose problems, while avoiding interruptions and in turn determining the best way in which products can be efficiently moved if necessary of a crisis. All these are related to the logistics of the company and therefore it should never be lost sight of.

1.1.4 Supply chain systems

1.1.4.1 Centralized

Under a centralized structure, all members place their orders in a coordinated manner, transmitting information in real-time about their inventory levels, products in transit, and consumer sales data. The supplier makes production decisions based on market demand and centralizing all the inventories of the members of the chain. In this way, all members of the chain benefit synergistically from the chain's performance. Orders flow to the point that each consumer receives their order exactly at the time and in the required quantity. This timing strategy eliminates the "bullwhip effect" and lowers inventory levels and relative costs by up to 50% without compromising customer service. With shorter storage times, the risk of obsolescence and investment in working capital is significantly reduced. We can see the uses of a centralized system in the hotel or restaurant business where they make their own products, market them, sell them, provide service by themselves.

1.1.4.2 Decentralized

In decentralized structure, each member makes his/her decisions independently from the decisions of his/her partners. Organizations place orders depending solely on their own inventory levels without considering the situation of other members. There is no direct interaction between the supplier and the final consumer; consequently, the former does not know the actual sales data, limiting itself to forecasting the market trend only based on the orders it receives from the retailer. As there is no synergistic coordination between the actors involved in the value creation process for the end customer, the ordering process and the delivery of products between retailer and supplier suffers continuous delays caused by production and transportation times, and by delays in the flow of information. As a consequence, there is a global inefficiency in the production distribution network, which is shown in the "bullwhip effect".

1.1.4.3 Coordinated

In order to coordinate the supply chain, the exchange of information at each step of the supply chain as well as the impact of its activities at other stages need to be considered. Supply chain coordination improves if all stages of the chain undertake actions that together increase the total profit of the supply chain. There are various types of coordination policy mentioned in the literature such as revenue sharing, price discount, profit sharing, buy-back policy, trade-credit policy, etc.

1.2 Different types of inventory in supply chain

A business can run efficiently as a result of different variables. Probably, among these variables, the most important variable is customer satisfaction. After all, no business can exist without a customer. It is very important for a business owner and manager to have an idea about the different types of inventory in order to meet the needs of the customers. This concept of inventory will help the business owner to better manage, plan and budget. Moreover, it helps to maintain its success by meeting the demand of the market.

- ***Raw material:*** Raw materials are essential to keep any business alive. Raw materials include all those items that take the form of a finished product through processing. For example, if we want to start a lemonade business, we must first procure lemons, water, and sugar as raw materials. The supply of raw materials in inventory exists only in the production industry. There is no need for raw materials in the trade industry as it does not deal with processing or manufacturing.

- ***Work-In-Progress:*** The term work-in-progress (WIP) inventory refers to a list of inventories that are partially completed and awaiting completion. Usually, WIP includes raw materials that have been delivered for initial processing. It also covers the whole process of production. For example, in the auto industry, brake pads would be part of WIP.

- ***Finished goods:*** Finished goods inventory refers to any complete product that is ready for sale in the market. These goods have gone through all stages of production and quality testing. For the pre-packaged ice cream business, the packed and boxed ice cream cones are ready to deliver would be finished goods inventory.

- ***Buffer inventory (or safety stock)*:** This is to meet the risks of unplanned production stoppage or unexpected increase in customer demand. There is no need of safety inventories if everything is safe. But, in reality, there is a variation in demand. This is normal and so we need to resort to safety inventories if we want to meet the service objectives.

1.2.1 Why keep inventories?

The main objectives of keeping inventory are:

- Mitigation of fluctuations in demand by offering an assurance against market uncertainties.

- It facilitates a proactive role in the face of anticipated changes in supply and demand.

- It allows a continuous flow of the manufacturing and assembly processes, giving flexibility to the programming processes.

- It improves the process of buying and selling supplies and materials, having the possibility of taking advantage of volume discounts.

For this and more, it can be concluded that obviously, the process by which the organization seeks to maintain a certain level of inventory is a "necessary evil" and that the search for the minimization of costs associated with it generates the need to apply multiple tools.

1.2.2 Inventory management in supply chain

Among the processes and activities associated with supply chains, inventory management is prominent, greatly conditioning the performance of other activities related to the supply chain, such as the acquisition of raw materials, production, storage of goods, and logistics (from dispatch to delivery of the good or service to the customer). A supply chain of a company, as we know, is the set of agents, departments, and units of the company that participate in the production-dispatch-delivery sequence of goods and /or services to an end customer. Proper inventory management is key to determining delivery modes and times, especially in companies dedicated to the production of goods; however, inventory management is also a key strategic piece for companies focused on providing services, as we will see. Inventory management is an activity inherent to the field of cost management of a company and refers, in a clear and obvious way, to the management of inventories, reducing their levels to the maximum without compromising the capacity to respond to the demand of goods and services.

Therefore, what is expected is to keep inventories to a minimum. The just-in-time philosophy is based on the concept of zero inventory. When there are high levels of inflation, the concept of zero inventory loses validity, because in this case, the best way to protect against inflation is to maintain high levels of inventory, especially of those items whose inflation rate is higher than the average. Another negative factor in inventories is the uncertainty of demand, which makes it difficult to maintain an inventory that can satisfy all requirements. There are conditions where shortages of inventories cannot be covered as quickly as they are depleted, causing costs due to shortages. On other occasions, there are products that deteriorate due to existing as excess.

The problem with inventory is that it's level should not be so high that it represents an extreme expense. Similarly, the level should not be so low that it causes a stock-out. Therefore, the organization must determine the appropriate level of inventory that balances these two extremes.

1.2.3 Objectives of inventory management

The objectives of inventory management within a company are defined as follows:

- One of the main goals is to ensure that customers have access to the products they need whenever they want. The system must include constant stock replenishment. Time and effort must be taken into account in promoting products, which attracts customer interest.

- Efficient control avoids excess inventory, which can be achieved with technological tools such as logistics control software and thus satisfy demand. This type of technology is usually applied to large companies due to the cost involved.

- It also allows for effectively managed inventory control that complies with profit margins.

Managing inventory for both products and service-based businesses is a crucial question since the existence of human beings is a very important economic entity. Interest in optimal inventory management problems at a scientific level goes back to the early 20th century. But the most important persuasion came after World War II when caliber scientists from Jacob, Marshak, Kenneth Arrow, Samuel Carlin faced the problem of optimal stocking according to stochastic demand. It was a feature of this discipline that such problem-solving methods were developed before arranging the commercial electronic data processing required for their ready-made application.

1.2.4 Costs involved in supply chain inventory

In general, inventory-related costs include item costs, order placement costs (process organization), maintenance costs, and out-of-stock (shortage) costs.

- ***Purchase cost*:** It refers to the purchase price of an item that the company acquires or produces. For purchased items, the total price list includes price, transportation and shipping costs, taxes, and duties. In the case of manufactured items, the costs of raw materials, labor, and distribution expenses are to be included. They can be constant, or they can be offered at a discount depending on the volume of the order.

- ***Ordering cost*:** This cost occurs as a result of the transport of items ordered. It covers activities such as specification and preparation of documents, purchase orders, follow-up with suppliers, and inspection of orders when delivered.

- ***Maintenance costs***: These costs denote the expenses incurred in maintaining inventories, e.g., rent, electricity, taxes, losses, obsolescence, insurance premiums, and labor costs.
- ***Shortage costs***: They are caused when the company cannot fully satisfy a customer's order. The company loses the contribution margin from that sale and may lose it in future sales. Sometimes a penalty must be paid.
- ***Holding costs***: These costs are associated with storing goods for a period of time and are proportional to the average number of goods available. It includes the cost of space, capital costs, cost of insurance and taxes, inventory risk costs, etc.

1.3 Lead-time in supply chain

The lead-time refers to the time that elapses from the moment an order is generated to a supplier until the merchandise is delivered from that supplier to the customer (it can be a private individual or a shop). Managing this concept is essential for the organization of all processes throughout the entire supply chain. Lead-time, also known as "replenishment time" could be defined as the time that elapses from the start of the production process until it is fully completed, or what is easier, the time that passes since it originates. This includes the distribution time, that is, the time that passes in the process of delivering the product to the end customer. Well, as expected, the delivery time or lead-time should be reduced as much as possible, since by minimizing the time it takes to restock merchandise.

1.3.1 Lead-time components

- ***Pre-processing time***: Time taken for receiving the request, understanding the request and creating a purchase order
- ***Processing Time***: Time taken to produce or procure the item
- ***Waiting Time***: Amount of time the item is in queue waiting for production
- ***Transportation Time***: Time the item is in transit to reach the customer
- ***Storage time***: Time the item is waiting at warehouse or factory
- ***Inspection time***: Time taken for checking the product for any non-conformity

1.3.2 Benefits of reduced lead-time

Nowadays, all companies focus on reducing their lead-time as much as possible, since this will be a directly proportional indicator of the productive efficiency of the business. Good knowledge of the replenishment time or lead-time is essential to carry out correct planning of the processes of the supply chain. Likewise, the main advantages of having a reduced delivery time are:

- ***Inventory reduction*:** Through the optimization of the resupply times there will be more capacity to respond to new orders, so there will be a large amount of merchandise in stock. Supply times have a linear effect on inventory policy: the longer the lead-time, the greater the stock stored, and vice versa.
- ***Competitive advantages*:** Having a short lead-time allows speeding up delivery times and meeting customer commitments. This is so since when the order is delivered to the consumer quickly, the company is positioned above its competitors in a simple way in the aspects related to the production, distribution, and delivery of the products.
- ***More accurate demand planning***: Performing short-term demand forecasting tasks allows gaining accuracy. One of the main rules of demand management is that the more in the future it is organized, the more uncertainty there will be. With shorter delivery times it is possible to reduce this uncertainty, as this facilitates more reliable and accurate forecasts.

1.4 Factors involved in supply chain

1.4.1 Demand

The meaning of demand encompasses a wide range of goods and services that can be purchased at market prices, either by a specific consumer or by the total set of consumers in a given place, to satisfy their needs and wants. Although it may seem like an obvious question, knowing the type of demand for a product is not always easy, especially when a business is young and does not have significant historical data to analyze patterns or fluctuations in demand. The demand pattern of a commodity may be either ***deterministic*** or ***probabilistic***.

- ***Deterministic*** demand is the one that we know with certainty. This applies to both inventories with dependent and independent demand. It can be static if it does not vary over time or dynamic if it varies in each period.

- In case of ***probabilistic*** demand, demand for a product is not easily predictable and as a consequence of this, the demand assumes a probabilistic distribution on which the control models can be determined.

1.4.2 Safety stock

Safety stock is the extra product that is stored to be able to cope if a stock break occurs. It is part of the correct management of the logistics department to calculate this data so as not to fall into breakage and, therefore, lose sales. The amount of safety stocks can also influence the development of a company. When the figure is high, it can lead to high inventory maintenance costs. In addition, items that are stored for a long time can deteriorate, break or expire. For its part, few safety stocks can mean a loss in sales and therefore a higher percentage of losses. The main idea of the safety stock and through which success will be achieved is to find a balance. To calculate the safety stock, we must assess the following aspects:

- Delivery term
- Demand for safety stock.
- Normal deviation of the delivery time of the orders.
- Standard deviation of demand.
- Desired service level

This allows companies to satisfy consumer demand, even if any of the following situations arise:

- Excessive and unforeseen growth in the demand for a certain product.
- Faults in the production phase.
- Delays of the suppliers or suppliers of the materials.
- Workers' strike.

1.4.3 Service level

The level of service is the percentage that results from the orders that the company can fulfill in a certain period or period of time. This means that the level of service is directly related to the level of customer satisfaction.

1.4.4 Stock replenishment systems

- ***Continuous review system*:** It is also known as a reorder point system, fixed quantity system, or (Q, r) model. The models classified in this system are characterized by the fact that an order is placed when the inventory reaches a certain level. This means that the remaining inventory is checked frequently and in many cases, each time an item is withdrawn to determine whether a new order should be generated. If it is considered too low, the system automatically prepares a new order.

- ***Periodic review system*:** It is also often called a fixed-interval reorder system, a fixed-period system, a periodic reorder system, or the P model. Inventory is reviewed periodically (every week, every 10 days, every month, etc.) and not continuously, therefore the issuance of orders is carried out at the end of each period or at its beginning.

1.4.5 Reorder point

The order point or reorder point (ROP) refers to the moment when the firm must order a new purchase of stock from its suppliers to avoid falling into a stock out of stock. This is the minimum quantity of a product that the firm keeps in the store and is re-ordered when the inventory level falls at this stage. The reorder point for stock occurs when the inventory level drops to zero. After replenishment of stock, the level of inventory moves from zero to the original level. In determining the reorder point, three factors need to be at hand: demand, lead-time, and safety stocks.

1.4.6 Trade-credit

Trade-credit consists of the offer of credit by a provider of products and services to its client, allowing him to pay for them later, that is, in the future. Both agree on a payment date that the client must respect and comply with because otherwise, he may take some legal action against him for not respecting the stipulated terms. With a trade-credit agreement, the buyer can pay at another time. Trade-credit provides several benefits. It can help a business with cash flow problems obtain needed goods and services. Trade-credit can also help finance a short-term project that would not be feasible if the business had to pay upfront.

1.4.7 Discount cash flow

Discounted cash flow (DCF) is an investment appraisal technique that, unlike the payback technique or the accounting rate of return, takes the value of money over time into consideration. One of the most common methods of valuing companies is discounting cash flows. Discounting cash flows consists of valuing a company for its ability to generate free cash flows (FCF) in the future. To carry out the valuation correctly, these future cash flows must be updated to the present. Valuing a company with discounted cash flows consists of updating the FCFs.

1.4.8 Inflation

Inflation, in economics, is a generalized and sustained increase in the prices of goods and services in the market over a period of time, generally one year. In other words, inflation reflects the decrease in the purchasing power of the currency: a loss in the real value of the internal medium of exchange and the unit of measure of an economy. A common measure of inflation is the price index, which corresponds to the annualized percentage of the general change in prices over time (the most common is the consumer price index). The effects of inflation in an economy are diverse and can be both positive and negative. The negative effects of inflation include a decline in the real value of the currency over time, discouragement from saving and investing due to uncertainty about the future value of money, and a shortage of goods. Positive effects include the possibility for state central banks to adjust nominal interest rates to mitigate a recession and to encourage investment in non-cash capital projects.

1.5 Problem statement

The importance of SCM has created significant interest in building a kind of partnership between supply chain companies. Integration of management and marketing, as well as other business activities, has become a preliminary research domain of SCM. To improve the profitability of the overall channels, supply chain coordination (SCC) among the activities of the members is considered an essential principle of modern SCM (El Ouardighi, Jørgensen, and Pasin, 2008). In the supply chain, the decision of one member is greatly influenced by the decision of the other, and so coordination among SC members helps to maximize the profit of the entire SC (Cachon and Lariviere, 2005). Traditionally, the buyer and vendor used a policy according to Economic Order Quantity (EOQ) or Economic Production Quantity (EPQ) classical methods. The concept of joint economic lot size (JELS) has been introduced to refine the classical methods, to satisfy the needs of today's market. The concept of JELS is a kind of integration between two different business entities for competitive advantage.

Inventory management, a critical element of the supply chain, is tracking inventory from the time of manufacture to warehouses, and from these facilities to the point of sale. The goal of inventory management is to have the right products in the right place at the right time. This requires inventory visibility-knowing when to order, how much to order, and where to store stock. Inventory visibility is knowing what inventory you have and where it is located. Businesses need an accurate view of inventory to ensure customer order fulfillment, reduce shipment lead-times, and minimize stockouts, overselling, and price reductions. Too little inventory of when and where it is needed can create unhappy customers. But a large inventory also has its own downsides: the cost to store and insure it, and the risk of spoilage, theft, and damage. Companies with complex supply chains and manufacturing processes must find the right balance between having too much inventory on hand or not having enough.

The lead-time for an order is a characteristic factor that must be taken into account within a logistics network for a production-delivery supply chain, since it is the period of time that passes between the issuance of an order and the item is received. Godinho Filho and Saes, 2013 stated that reducing lead-time plays a vital role in today's logistics management. Companies that fail to make their supply chain systems more efficient, flexible, and customer-oriented will have a harder time surviving in the future. According to Glock, 2012a, if companies neglect the importance of efficiency and flexibility in the supply chain, they may face various problems. Further Jamshidi, Ghomi, and Karimi, 2015 stated that the effect of lead-time is even more significant when demand is uncertain, which can be decreased with efficient and flexible lead-time management. This means that if a company has a long replenishment lead-time, there is a greater risk that the product will run out of stock in response to fluctuating demand. From a logical point of view, Jamshidi, Ghomi, and Karimi, 2015 argued that reducing lead-time can reduce safety stocks and the likelihood of stock running out, as uncertainty in replacement lead-time is an important factor in carrying safety stocks. Another important factor behind carrying safety stock is according to Van Kampen, Van Donk, and Van Der Zee, 2010 the uncertainty in demand. According to the authors, it is possible to reduce the safety stock if the reliability of the demand information can be increased or the variability of lead-time can be decreased. In addition, it can lead to increased service level and competitiveness.

1.6 Scope of this thesis

Despite rapid progress over the past decade, there is still a large undiscovered research area in the SCM industry that cannot be covered by this thesis. The long-term aim of this research is to develop a practical (e.g. method or approach) way for SC managers to evaluate optimal ordering and lead-time decision based on financial performance. We develop supply chain models addressing the issues like ordering, lead-time, investment, and coordination. The issues covered in this thesis and the contributions of this work are presented below. This thesis explores some important issues in the two-echelon supply chain management (SCM) to fill the gap in the literature work, covering production, replenishment, coordination planning under lead-time demand uncertainty.

- Supply chain model for defective items with variable demand under zero lead-time is addressed. The model is developed under decentralized and centralized decision-making for price and green sensitive demand. Moreover sampling inspection is considered to remove the defective items. A coordination approach based on credit period policy is also proposed to increase the supply chain (SC) member's individual profit.

- Then, we develop a supply chain model where replenishment lead-time is a function of production time, setup time, and transportation time. This study considers that the lead-time for the first shipment is different from the rest of the shipments. Also there are two different reorder points. An investment was considered to reduce transportation lead-time.

- Further, the concept of variable lead-time is considered into integrated supply chain model with time value of money under net present value (NPV) method and variable backorder. The replenishment lead-time is assumed to be a function of order quantity and production rate. It is also assumed that the lead-time could be reduced by changing the regular production rate of the vendor at the risk of paying additional cost.

- Then, the concept of deterministic lead-time is considered into integrated supply chain model where the replenishment lead-time is decomposed into various components and investment is made to reduce the components. Backorder rate is considered as a function of lead time.
- Finally, we consider variable lead-time in a production-delivery supply chain model under a service level constraint and defective production. Compared to several models, the proposed models and approaches show a significant computational advantage.

1.7 Outline of the thesis

The thesis includes seven chapters and five appended papers. The content of each chapter is summarized below to give the reader an idea of the structure.

The thesis starts with **Chapter 1: Introduction**. Being an introductory chapter, it provides a brief overview of SCM. Various terminologies and basic concepts of SCM relevant to the thesis are also provided. The chapter ends by providing the problem statement and outline of the thesis.

In **Chapter 2: Literature review**. a brief background of the literature used in this study is presented.

Chapter 3: Two-echelon supply chain model considering product quality assessment and green retailing. In this chapter, a joint economic lot size (JELS) model is developed to enhance the greening efforts of a product that flows along a two-level supply chain (supplier-retailer). The impact of both selling price and greening effort level on demand function has been considered. It is assumed that every individual lot shipped to the retailer carries some random defective items. Hence, each lot goes through an error-free sub-lot sampled inspection process to remove the defective items. The profit function is developed under three decision-making scenarios: centralized, decentralized, and coordinated. Coordination is made based on a trade-credit scheme. The coordinated model suggests that more emphasis should be given to the greening effort level for higher profit. It is observed that, in many cases, sub-lot inspection gives better results compared to full lot inspection.

Chapter 4: Lead-time reduction in an integrated supply chain model with stochastic demand. In this chapter, lead-time is considered as a function of production, setup, and transportation time. The buyer receives normally distributed stochastic lead-time demands from its customers. Here we consider different lead-times for the first shipment and the rest of the shipments. An investment is considered to reduce the transportation time.

Chapter 5: Lead-time reduction in a two-echelon integrated supply chain model with variable backorder. This chapter studies an integrated vendor-buyer model with shortages under order size and production rate dependent lead-time. Shortages are partially backlogged and the backlogging rate depends on the length of the lead-time. It is assumed that the replenishment lead-time can be reduced by changing the regular production rate of the vendor at the risk of paying an additional cost. The proposed model is formulated to obtain the net present value (NPV) of the expected total cost of the integrated system.

Chapter 6: Integrated supply chain model with controllable lead-time and trade-credit financing. This chapter investigates a lead-time reduction strategy for a single-manufacturer single-retailer integrated SC system with controllable backorder rate and trade-credit financing. The corresponding problems are formulated and solved for both cases when lead-time demand distribution is known/unknown. Min-max distribution-free approach is adopted for unknown lead-time demand distribution.

Chapter 7: Coordinated joint economic lot size model with variable demand and lead-time under service level constraint. This chapter addresses a joint economic lot size (JELS) model that focuses on ordering, pricing, effort, and lead-time decisions under a service level constraint. The buyer is faced with price and effort-dependent stochastic lead-time demand. Here we consider lead-time as an added control parameter that can be reduced through some additional cost which is a negative exponential function of the lead-time. We propose both centralized and decentralized approaches considering that the distribution of the lead-time demand is unknown, and adopt a distribution-free approach to solve those models. Coordination is made based on the price discount contract.

Chapter 2

Literature review

Supply chain is a worldwide network that covers all the activities related to moving goods from the raw-materials supplier to the end-users. It is considered one of the most effective management for sustainability and competitiveness of industries. SC sustainability plays an important role in ensuring business continuity and managing operational costs (Gold, Seuring, and Beske, 2010). The SC literature model has improved over the years since the pioneer attempt of Goyal, 1977, who formulated an integrated inventory model composed of a single buyer and a single vendor with infinite production rate, which is known as the joint economic lot size (JELS) model. Banerjee, 1986 generalized Goyal, 1977 considering the vendor as a manufacturer with finite production rate. Following Goyal's paper, many researchers worked on joint economic lot size (JELS) mode with a variety of realistic assumptions, such as imperfect quality (Dey and Giri, 2019; Sarkar and Giri, 2020; Tiwari et al., 2020), permissible delay in payments (Aljazzar, Jaber, and Moussawi-Haidar, 2017), controllable lead-time (Heydari, 2014a; Tiwari, Sana, and Sarkar, 2018) vendor-managed inventory (Rad, Khoshalhan, and Glock, 2014; Taleizadeh et al., 2020), environmental issues (Kazemi et al., 2018; Tiwari, Daryanto, and Wee, 2018). For literature reviews on JELS problem, the reader is referred to Glock, 2012b.

2.1 Demand

The selling price of any product is a key factor that greatly influences market demand. Selling price is considered as an important vehicle for revenue growth of any SC. Whitin, 1955, was one of the first to consider price-sensitive demand in the inventory model. The author developed an EOQ model by considering linearly price-dependent demand. Lau and Lau, 1988 developed a Newsboy model and showed that market demand could be increased by reducing sales prices. There is a large number of literature dedicated to the price-dependent demand with newsboy/EOQ/JELS model (e.g., Yang, Ouyang, and Wu, 2009; Johari et al., 2018; Giri, Mondal, and Maiti, 2019; Maihami, Govindan, and Fattahi, 2019; Modak and Kelle, 2019; Mishra, Wu, and Tseng, 2019). Sajadieh and Jokar, 2009 developed a coordinating SC model with price-dependent demand under centralized and decentralized decision-making policies. Zanoni et al., 2014 investigated coordinated inventory replenishment decisions considering the price and environmentally sensitive demand where investment was used to improve the production process resulting in the improved environmental performance of the product. Giri and Roy, 2016 formulated a single-manufacturer multi-buyer stochastic SC model with price-dependent demand and controllable lead-time. Modak et al., 2018 examined pricing policy for a SC model considering greenhouse gas emissions from the production system. Feng, Zhang, and Tang, 2018 treated price as one of the decision variables in a joint dynamic pricing and production problem for perishable products with quantity-dependent deterioration rate and price-dependent and time-varying demand. Tiwari et al., 2018 studied joint control for optimal ordering and pricing policies in a SC with limited storage capacity. Qiu, Qiao, and Pardalos, 2019 integrated the inventory replenishment and pricing policy for perishable products with routing problem. Chen et al., 2019 examined the inventory management for a short life cycle product with a finite horizon multi-period setting where demand is deterministic, stock- level dependent, time-varying, and price-dependent. Canyakmaz, Özekici, and Karaesmen, 2019 investigated optimal inventory control policy where consumer demand is strongly affected by fluctuating prices. Agrawal and Yadav, 2020 proposed a pricing and profit allocation policy to coordinate a two-stage production-inventory supply chain with one manufacturer and multiple suppliers. They proposed four different schemes to allocate profit between the SC members.

In today's marketing environment, in addition to the retail price, promotional efforts are also an important vehicle for increasing market demand. Tsao and Sheen, 2008 examined a SC problem of dynamic pricing and promotion policy for a deteriorating item. Li et al., 2016 considered e-commerce in the dual-channel supply chain by analyzing pricing and the greening effort in both centralized and decentralized scenarios and achieved a positive correlation between green degree and corresponding green investment. Results showed that a two-part tariff coordination contract can increase the degree of green level which can create a win-win-win for the retailer, manufacturer, and the environment. He et al., 2009 investigated coordination policy for a system with price and effort sensitive stochastic demand. Huang, Nie, and

Zhang, 2018 considered a SC model consisting of a manufacturer and two competing retailers and studied the effect of promotional strategy on demand. They have shown that the retailer's promotional efforts have a positive impact on demand, but also have a negative impact on the manufacturer's brand image. Malekian and Rasti-Barzoki, 2019 developed a game-theoretic approach to investigate the price promotion and advertising in a manufacturer-retailer supply chain. They found that the advertising effort improved the channel's effectiveness. Ghosh et al., 2021 discussed in detail how consumers' green preferences of product and environmental protection awareness affect the decision-making of the supply chain. Due to the existence of some uncertain conditions, Jamali, Gorji, and Iranpoor, 2021 modeled a pricing and greening policy in a supply chain under fuzzy parameters. They employed a game theory approach to obtain equilibrium solutions for cooperative and non-cooperative scenarios and highlighted the importance of vertical cooperation for the improvement of entire supply chain profit, product greenness, and the environment.

2.2 Lead-time

In inventory management, lead-time has always been an important factor to consider (Naddor, 1966; Das, 1975; Magson, 1979; Foote, Kebriaei, and Kumin, 1988). Lead-time is defined as the duration of time between placing an order and receiving it. A general assumption of lead-time refers to it as a fixed time (Ravichandran, 1995; Rabinowitz et al., 1995). Although constant or deterministic lead-time assumption follows JIT (just-in-time) philosophy, but it is not fitted in most modern complex setups where overseas, containerized, and air-freight transportation are involved. According to Tersine, 1993, lead-time involves order preparation time, order shipment/delivery time, set-up time, etc. Recognizing that manufacturing lead-time is highly dependent on lot-size, Kim and Benton, 1995 questioned on the assumption of fixed lead time and established a relationship between lot-size and lead-time. They showed that significant savings can be occurred by considering the interrelationships between lot size and safety stock decisions. Hariga, 1999 revisited Kim and Benton, 1995's model to rectify the expression of the annual backorder cost, and proposed another relation for the revised lot-size. However, the above two models were considered only from buyer's/manufacturer's perspective. Ben-Daya and Hariga, 2004 were the first researchers to consider lot-size dependent lead-time in a vendor–buyer integrated supply chain model with stochastic demand. However, they assumed that the reorder points for all replenishment cycles are the same. Hsiao, 2008a improved this model by assuming that there are two different reorder points and service levels. Hsiao, 2008b investigated an equal-sized batch shipment model with variable lead-time by controlling the reorder and shipping points with information sharing. He showed that significant cost savings can be archived through controlling reorder and the shipping points. Glock, 2012a used the similar formulation for the lead-time as of Hsiao, 2008a and developed different reduction strategies for lead-time. Glock and Ries, 2013 considered dual sourcing SCM model under stochastic demand and lot-size dependent variable lead-time. Rad, Khoshalhan, and Glock, 2014 proposed a two-echelon integrated inventory system with a single vendor and two buyers where lead-time varies with lot-size and delay time where they adopt integrated vendor managed inventory (VMI) policy and traditional retail managed inventory (RMI) policy to solve the model. Mou, Cheng, and Liao, 2017 modified/corrected Glock, 2012a by incorporating the lead-time crashing cost function and by taking transportation time as a decision variable under two distinct safety stocks. Yang et al., 2017 considered a news-vendor model for perishable products by considering delivery lead-time. Heydari, Zaabi-Ahmadi, and Choi, 2018 adopted the concept of lead-time crashing as a coordination mechanism between seller and buyer by using various modes of shipment to deliver items. Ponte et al., 2018 studied the effect of mean and variability of delivery lead-times and production for a multi-tier supply chain. They pointed out that the reduction strategy for lead-time can increase SC profit as well as satisfactory level of the consumers. Glock, Rekik, and Ries, 2020 extended a news-vendor problem by considering the delivery lead-time as a function of order quantity and transportation delay where both the retailer and the manufacturer have the option to shorten the lead-time.

One of the first papers dealing with a controllable lead-time in an inventory model is due to Liao and Shyu, 1991. The authors assumed that lead-time can be decomposed into several components, each having a different piece-wise linear crashing cost function for lead-time reduction, and that each component may be reduced to a given minimum duration. Later, several researchers (Ben-Daya and Raouf, 1994; Ouyang, Yeh, and Wu, 1996; Ouyang and Wu, 1997; Wu, 2000) studied lead-time reduction under various assumptions. Pan and Yang, 2002 considered lead-time as a decision variable to generalize Goyal, 1988 model and obtained lower total cost and shorter lead-time than Banerjee, 1986 and Goyal, 1988. Ouyang, Wu, and Ho, 2004 extended Pan and Yang, 2002 model by considering shortages and taking reorder point as a decision variable. They obtained a lower total cost than Pan and Yang, 2002 model. Ben-Daya and Hariga, 2004 relaxed the assumption of deterministic demand and developed an integrated inventory model assuming lot size dependent lead-time and shortages. Yang and Pan, 2004 addressed an integrated vendor-buyer model with lead-time reduction taking into account quality-related cost. Ouyang, Wu, and Ho, 2007 extended Yang and Pan, 2004 model by allowing shortages and considering reorder point as one of the decision variables. Li, Xu, and Ye, 2011 developed a supply chain coordination model with controllable lead-time and service level constraint. Arkan and Hejazi, 2012 designed a supply chain model for the coordination between a single buyer and a single supplier considering the credit period and controllable lead-time. Jha and Shanker, 2013 considered a two-echelon single-vendor multi-buyer supply chain model with controllable lead-time and service level constraint. Yi and Sarker, 2013 considered a single-vendor and single-buyer production inventory system where the lead-time is controllable with an extra investment under a long-term agreement between the two trading partners. Mandal and Giri, 2015 considered controllable lead-time in integrated supply chain model to maximize the benefits for all the participating players. Jamshidi, Ghomi, and Karimi, 2015 formulated a mixed-integer non-linear model for a five-tier supply chain with controllable lead-time and multiple transportation options, and develop a novel meta-heuristic method. Jindal and Solanki, 2016 considered the effects of inflation rate on total SC cost for a two single-vendor single-buyer integrated system under controllable lead-time. Tiwari, Sana, and Sarkar, 2018 developed a JELS model with stochastic demand and controllable lead-time under ordering and setup cost reduction. Sarkar and Giri, 2020 considered an integrated production-delivery supply chain model to consider the quality-related issue by assuming an investment option in process quality improvement.

2.3 Imperfect production

Over the past several decades, research has been conducted to increase the applicability of the inventory model in real-world manufacturing systems (Hax and Candea, 1984; Silver and Peterson, 1985). The classical inventory model was developed based on the assumption that the production facility is failure-free, i.e., all the items produced are of perfect quality, which is rarely satisfied in reality. In practice, failure-free production is not possible due to long-run processes, human mistakes, or incomplete process controls, the machine may move to an out-of-control state from an in-control state thereby resulting in the production of defective/imperfect quality items. In the direction of production models with unreliable machines, numerous studies have been conducted. Rosenblatt and Lee, 1986 were among the pioneers who looked at a deteriorating production system that is subject to random shipment from an in-control to an out-of-control state. Groenevelt, Pintelon, and Seidmann, 1992a addressed an economic lot-size model to study the effects of machine breakdowns and corrective maintenance. Groenevelt, Pintelon, and Seidmann, 1992b reconsidered this work assuming machine failure rate as exponentially distributed and repair time with a general probability distribution. Kim and Hong, 1999 further considered the model of Groenevelt, Pintelon, and Seidmann, 1992a where the time to shift from in-control to the out-of-control state follows a general distribution and they obtained exact optimal production run length. Thereafter, several studies have been conducted to incorporate the imperfect quality (e.g., Salameh and Jaber, 2000; Goyal and Cárdenas-Barrón, 2002; Chang, 2004; Papachristos and Konstantaras, 2006; Wee, Yu, and Chen, 2007). Most of the research, however, focused on determining the optimal policy from the buyer's or the vendor's point of view. Chen and Kang, 2010 developed an integrated inventory model to coordinate between the vendor and the buyer considering trade-credit and imperfect quality. Dey and Giri, 2014 proposed an integrated inventory model with an imperfect production process to study the effects of reducing defective rate. Fajrianto, Jauhari, and Rosyidi, 2019 proposed an integrated inventory model for deteriorating items where the production process is imperfect, thus producing a proportion of defective items. Jauhari, Purnasari, and Rosyidi, 2021 considered pricing and inventory decisions in a two-level supply chain including learning process and errors in inspection with full backordering.

2.4 Inspection policy

Product inspection plays an important role especially when the production system is imperfect. Salameh and Jaber, 2000 considered imperfect items, which could be characterized by a screening process and accumulated in a single batch for sale at the end of the production cycle. In a note, Goyal and Cárdenas-Barrón, 2002 presented Salameh and Jaber, 2000 model through a simple approximated way. Further, Goyal, Huang, and Chen, 2003 considered the model of Goyal and Cárdenas-Barrón, 2002 to include the vendor and to extend it to the integrated SC model. Khan, Jaber, and Ahmad, 2014 developed a simple integrated supply chain model to determine the optimal vendor-buyer inventory policy with the consideration of inspection errors at the buyer's end and learning in production at the vendor's end. Kang et al., 2018 considered offline inspection policy to determine the optimal number of inspectors for different products in a multi-objective optimization model. Lopes, 2018 considered imperfect production where the inspection process goes through two types of inspection errors under preventive maintenance action. Dey and Giri, 2019 developed a new approach to deal with learning in a batch-wise inspection policy in an integrated vendor-buyer model with an imperfect production process. Zhao et al., 2020 presented a joint inspection policy for a system with two levels of defective states, namely severe and minor defectives. They examined that a minor defective triggers the normal order, which shortens the inspection interval to check system status more frequently.

While most of the literature addressing defective items adopts full inspection, there is some literature where sampling inspection policy is adopted. Shih, 1980 is one of them who first came up with the idea of sampling inspection in an EOQ model. Pulak and Al-Sultan, 1996 incorporated sampling inspection in Economic Manufacturing Quantity (EMQ) model. While in most of the cases, sampling inspection is considered for deterministic demand in EOQ/EPQ models, Wu and Ouyang, 2000, Wu, Lee, and Tsai, 2004 studied sampling inspection for stochastic demand cases in continuous review models. Moreover, the authors considered penalty costs for uninspected items. Wu, Ouyang, and Ho, 2007 considered sampling inspection in an integrated supply chain model with stochastic demand and controllable lead-time. Rezaei, 2016 extended the model of Salameh and Jaber, 2000 to incorporate sampling inspection in the EOQ model and numerically they showed that sampling inspection is more profitable for the buyer compared to full-inspection. Al-Salamah, 2016 investigated an inventory model for both perfect and imperfect quality items under a sampling inspection plan with inspection errors and destructive testing. According to his model, if a sample fails to meet certain criteria, the whole lot is rejected and sold at a lower price in the secondary market. Tuan, Yang, and Hung, 2020 considered an EOQ model with controllable lead-time under imperfect items and sub-lot inspection policy. Öztürk, 2020 developed an EOQ model for the shipment with defective items considering inspection errors and a sub-lot inspection policy.

2.5 Variable backorder

Due to variable lead-time, sometimes the vendor may fail to deliver a lot within the desired lead-time. As a result, the buyer may face a stock-out situation, in which case, customers' demand is not fulfilled resulting in a financial loss. Moreover, the unsatisfied customers may not turn up next time to meet their demands from the same source. This indicates that, in reality, the backorder rate should not be constant.

Ouyang and Chuang, 2001 were the first authors to consider a shortage quantity dependent backorder rate with normally distributed lead-time demand. However, for the case when the lead-time demands of different customers are not identical, the model of Ouyang and Chuang, 2001 cannot be used. Therefore, Lee, 2005 modified Ouyang and Chuang's model by considering the mixture of normal distribution. Lee, Wu, and Hsu, 2006 developed a computational algorithm to solve an inventory model where the backorder rate is dependent on the length of lead-time through the number of shortages. Lee, Wu, and Lei, 2007 investigated an inventory model with a mixture of normally distributed lead-time demand where backorder rate varies with lead-time and price discount. Soni and Patel, 2014 investigated the vendor-buyer supply chain model with lead-time dependent backlogging rate and service level constraint.

A stock-out situation not only disappoints customers but also makes doubt in customers' minds about the storage capacity of the buyer. Unsatisfied customers may not turn up next time to meet their demand from the same buyer. Therefore, the buyer in this case loses the opportunity to earn some more profit. However, for fashionable goods such as certain brand gum shoes, hi-fi equipment, cosmetics, and clothes, some customers may wait up to a certain period for backorder and some may not wait at all (Montgomery, Bazaraa, and Keswani, 1973; Rosenberg, 1979; Park, 1982). Therefore, motivating the customers for backorder becomes a challenging problem for the buyers. Price discount policy on backordered items is a well-known factor that can motivate the customers for backorder as well as increase the rate of backorder. In this direction, Pan and Hsiao, 2001 extended Ouyang, Yeh, and Wu, 1996 model by assuming backorder price discount as a decision variable. Later, Ouyang, Chuang, and Lin, 2003 considered a periodic review inventory model with review period and backorder discounts as decision variables, but the lead-time is treated as a fixed constant. Pan, Lo, and Hsiao, 2004 and Pan and Hsiao, 2005 discussed two inventory models with backorder price-discount. For both models, they considered the lead-time crashing cost as a function of reduced lead-time. Lin, 2008 analyzed a continuous review inventory model with backorder price discount in which the lead-time and ordering cost reductions are inter-related. Kurdhi, Prasetyo, and Handajani, 2016 developed a continuous review inventory model to investigate the effect of backorder price discounts when the amount received quantity is uncertain. Tiwari et al., 2020 examined a vendor-buyer integrated supply chain model under defective items and backorder price discount.

2.6 Service level constraint

When demand is stochastic and replenishment lead-time is variable, it is very natural that the system may face a shortage of items. Moreover, according to Aardal, Jonsson, and Jönsson, 1989, it is very difficult to determine the cost of shortage in practice. To overcome this situation, Aardal, Jonsson, and Jönsson, 1989 suggested the use of a service-level constraint rather than explicitly defining shortages costs in the objective function. They introduced a service level as a constraint form in a continuous review inventory model and developed a relation between shortages and service level. There are many types of service level but typically we can observe the widespread use of two types of service level, namely Type 1 service level and Type 2 (or fill rate) service level. The first one denotes the probability of not stocking out over a planning horizon and the other denotes the proportion of demand satisfied with the existing inventory. Moon and Choi, 1994 solved a distribution-free inventory model with a service level constraint and developed an iterative procedure to find the optimal quantity and reorder point. Jha and Shanker, 2009 developed a JELS model for single-vendor single-buyer for decaying items with controllable lead-time and service-level constraint. Moon, Shin, and Sarkar, 2014 developed a continuous-review inventory system under a service constraint on fill rate where a negative exponential crashing cost function was used to reduce lead-time. Jauhari and Saga, 2017 discussed a periodic review integrated SC model for a vendor-buyer system with stochastic demand and service-level constraint. Bhuiya and Chakraborty, 2020 studied a continuous-review production-inventory model assembling lost sales and backorders with service level constraint.

2.7 Distribution-free (DF) approach

It is worth noting that the deterministic JELS models often ignore the situation of shortages and service level as it is assumed that shortages should not be followed when demand is known in advance. In practice, however, it is often understandable to consider shortages when demand is stochastic or lead-time is variable. In order to make the final formulation of the shortages, a decision needs to be made on the distribution of demand during the lead-time. There is a tendency to use normal distribution to count the shortages quantity (e.g., Priyan and Uthayakumar, 2014; Castellano et al., 2019; Tiwari et al., 2020). However, using the normal distribution for a larger variety of demand can lead to additional orders and large financial losses, as observed by Gallego, Katircioglu, and Ramachandran, 2007. Moreover, in many cases, decision-makers do not get the idea of specific distribution as they have very little information about demand distribution. To overcome this situation, Gallego and Moon, 1993 introduced the concept of a distribution-free approach under which decision- makers can get an idea of measuring the shortages amount without knowing the standard pattern distribution function. Castellano, Gallo, and Santillo, 2021 applied the distribution-free approach to a single-vendor multi-buyer integrated SC model where lead-time is considered as controllable under a back-order lost sales mixture. Malik and Kim, 2020 investigated a flexible production system under three constraints, budget, space, and service level and solved the issue of unknown distribution of lead-time demand using distribution-free approach.

2.8 Coordination policy

A supply chain usually has several members with conflicting objectives. Coordination problems occur if a supply chain member does not work toward an optimal solution to the overall supply chain. If an agreement can be found so that each member reasonably acts in accordance with the optimal solution of the supply chain, this agreement is said to coordinate the supply chain (Toktaş-Palut and Ülengin, 2011). The above mentioned papers are based on determining the best management decisions in a decentralized or centralized SC. However, without efficient coordination among channel members, it is very difficult to run SC with maximum profit. Yang and Chang, 2013 noted that offering delay-in-payment to the buyer could effectively increase the sales volume. Moussawi-Haidar et al., 2014 considered delay-in-payment as a coordination mechanism in a three-echelon SC model and showed that their proposed coordination mechanism manages to reduce a significant amount of expenditure compared to decentralized models. Heydari, 2014b developed a coordination policy between the buyer and the seller using time-based temporary price-discount policy. He revealed that a SC profit is highly related to joint decision- making on safety stock volume. Aljazzar, Jaber, and Moussawi-Haidar, 2017 dealt with a three-echelon SC model by coupling permissible delay in payment and price discount as a coordination mechanism and they obtained more profit using those mechanisms separately. Taleizadeh, Rabiei, and Noori-Daryan, 2019 developed a coordinated SC model for retailer and manufacturer under three distinct coordination policies and addressed higher profits in every situation than decentralized condition.

Chapter 3

Two-echelon supply chain model considering product quality assessment and green retailing

3.1 Introduction

The last two chapters conferred about the basic concepts of SC management and presented a brief overview of the existing models in the literature of SC. In this chapter*, a SC model for items with price and green sensitive demand is investigated under decentralized, centralized, and coordinated decision-making policy. While formulating the SC models for variable demand, usually it is assumed that the items supplied to the retailers are defect-free, which is not true in practice. Here we assume that every individual lot shipped to the retailer carries some random defective items, hence each lot goes through an error-free sub-lot sampled inspection process to remove the defective items. We develop the profit function under three decision-making scenarios: centralized, decentralized, and coordinated. Coordination is made based on a trade-credit scheme under which the retailer changes his/her optimal decisions according to a centralized policy. We obtain the minimum and maximum credit periods which encourage both the retailer and the supplier to follow coordinated decision-making policy. The coordinated model suggests that more emphasis should be given to the greening effort level for higher profit. It is observed that, in many cases, sub-lot inspection gives better results compared to full lot inspection.

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3.2 Preliminary aspects

In this chapter, we consider a two-echelon SC that consists of a single supplier supplying a single product type to a single buyer. The retailer encounters variable demand from his/her customers. Each batch received by the retailer undergoes an inspection process carried out checking not all units of the batch, but a sample. Discovered defectives are sold in the secondary market at reduced price, while the fraction of undiscovered ones enter in retailer's inventory to meet demand. The retailer incurs some extra penalty for each defective items. The problem is to establish the inventory replenishment decision that maximizes the expected total profit per time unit.

In the next section, we highlight the notation and assumptions that will be used during model development.

3.2.1 Notation and assumptions

We adopt the following notation to develop the proposed model.

- **Decision variables**

Q	Order quantity of the retailer [quantity unit]
B	Backorder quantity of the retailer [quantity unit]
s	Unit selling price of the retailer [$/unit quantity]
g	Greening effort level
m	Number of shipments from supplier to retailer

- **Parameters**

$D(s,g)$	Annual demand of the product [quantity unit/time unit]
c	Unit purchasing price for the retailer [$/quantity unit]
p	Unit purchasing price for the supplier [$/quantity unit]
v	Salvage price for the retailer [$/quantity unit]
K_r	Ordering cost for the retailer [$/order]
K_s	Ordering cost for the supplier [$/order]
H_r	Holding cost for the retailer [$/quantity unit/time unit]
H_s	Holding cost for the supplier [$/quantity unit/time unit]
π_r	Unit backordering cost for the retailer [$/quantity unit]
r	Disposal cost for the retailer [$/quantity unit]
i_c	Inspection cost for the retailer [$/quantity unit]
w	Penalty cost for the retailer for uninspected defective items [$/quantity unit]

•**Parameters**

F	Green innovation investment efficiency coefficient
f	Fraction of items inspected per shipment
Y	Number of defective units among the inspected fQ units, a random variable [quantity unit]
α	Percentage of defective items in every batch (units) (random variable)
δ	Fraction of defective items that will be disposed
$1-\delta$	Fraction of defective items that will be sold at salvage price

Additional notation will be introduced as needed.

3.2.2 Assumptions

1. A single retailer deals with a single supplier for a single type of product.

2. Inventory replenishments are instantaneous.

3. The retailer's economic order quantity (EOQ) is Q units and the supplier's lot size $mQ, (m = 1, 2, ...)$ is an integer multiple of the retailer's order size (Lee and Rosenblatt, 1986).

4. The demand rate for the buyer is a decreasing function of selling price (Qin, Tang, and Guo, 2007, Sajadieh and Jokar, 2009). The greening effort by the buyer also affects the demand via a multiplying effect i.e., when the buyer takes part in a greening policy, the demand is modified by a multiplier g. Therefore, we have the demand function $D(s, g) = (a - bs)g^{\mu}$, where $a > 0$ is a scaling factor, $b > 0$ is the price elasticity of demand, and $\mu > 0$ is the green elasticity coefficient.

5. The retailer incurs a green innovation investment cost $C(g) = F(g-1)^2$ which is convex, increasing, and continuously differentiable with respect to the greening level (g) for any $g > 1$ (Krishnan, Kapuscinski, and Butz, 2004; Pal, Sana, and Chaudhuri, 2015).

6. The goal of an organization while producing a product is always to make the product as high-quality as possible. However, defective products may be produced during the manufacturing or assembly process for a variety of reasons. Therefore, we assume that each lot received by the retailer carries some defective items. Also, we assume that an error-free and non-destructive sample inspection is performed at the retailer's end to identify those defective items.

The inspection is processed so quickly that the length of inspection is negligible (Wu, Ouyang, and Ho, 2007).

7. Most of the papers assume that all defective products are discarded or returned. In reality, for some products (e.g., clay products) or fruits, it is common in practice that there are two categories of defective items. Some are partially damaged/spoiled that can be sold in the secondary market but some products that are completely damaged/spoiled and are completely unusable and so those are disposed of. Keeping this in mind, we assume that a portion of the defective items are considered usable and sold at a salvage price before receiving the next batch. The remainder of the defective units is disposed of at additional disposable cost. To save the storage costs, all processes are completed after the inspection (Su, 2012).

8. In reality, it is often seen that many companies store less amount of inventory to optimize inventory holding costs, and as a result, the system faces shortages of stock. Moreover, the system may face shortages due to the elimination of defective items. Keeping this in mind, we consider shortages that are fully backlogged (Wee, Yu, and Chen, 2007).

3.3 Model development

Once the retailer receives a lot, an error-free and non-destructive inspection is performed to eliminate the defective items on fQ items with a fixed cost i_c per unit. The retailer incurs some extra penalty for each uninspected item that is found defective. The remaining $(Q - Y)$ items are stored in inventory to meet the demand which contains $(\alpha Q - Y)$ defective items. Among the defective items Y, δ percentage are classified as useless and are disposed of through fixed unit disposal cost r. On the other hand, the remaining $(1 - \delta)$ percentage of Y are usable and are sold in a lot at a salvage value to the contracted wholesalers before receiving the next batch. The proposed model is formulated under three decision-making scenarios: decentralized, centralized, and coordinated. The logistic diagram is given in 3.1.

To formulate the model, we first discuss the structure of the demand function. Generally, there are different demand functions of which the widely used demand function is the linear demand function. Normally, the linear demand function is formed as $D(s) = a - bs$. To reflect current market competition behavior, we adopt the nonlinear multiplicative form of greening level to the basic linear price-dependent demand to highlight its interacted effects on the market demand. Therefore, when the retailer deploys green innovation effort, the demand is modified by a multiplier g^{μ}, where $\mu > 0$ is the sensitivity of greening level and there is no greening innovation when $\mu = 0$. Then the demand function with the joint effect of price and greening level is defined as $D(s, g) = (a - bs)g^{\mu}$. The retailer incurs a green innovation effort cost $C(g) = F(g-1)^2$ which is convex, increasing, and continuously differentiable with respect to the greening level (g) for any $g > 1$.

In the next section, we derive the expected total system profit per time unit for decentralized, centralized, and coordinated models.

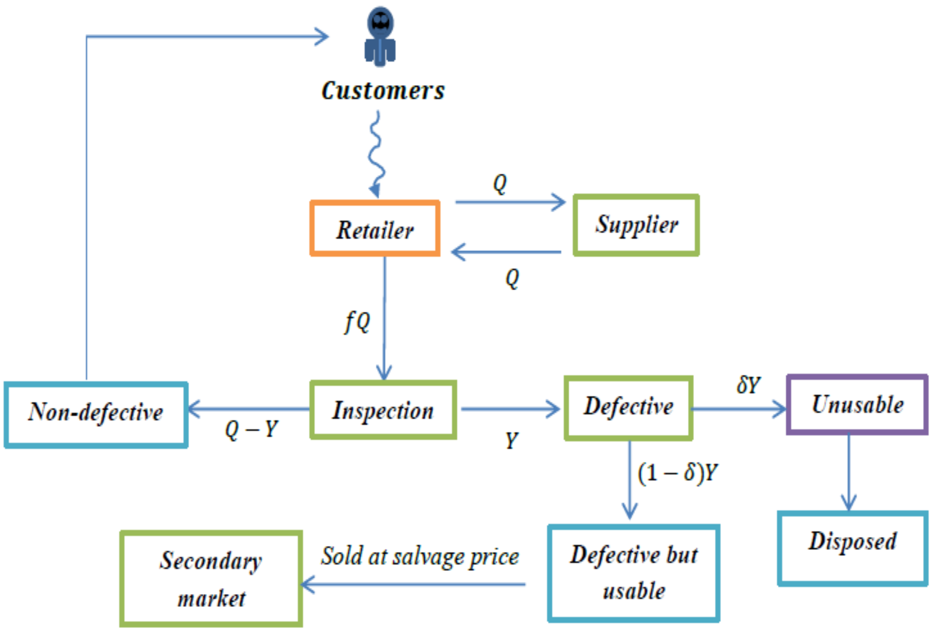

Figure 3.1: The logistic diagram of supplier-retailer supply chain system

3.3.1 Decentralized model

In the decentralized decision-making policy, the retailer and the supplier maximize their profits with a self-interested motive. The retailer earns profit from good quality items as well as from defective items. Hence, the retailer's total revenue per cycle is

$$\begin{aligned} R &= \text{Revenue from non-defective items+revenue from defective items} \\ &= s(Q-Y) + v(1-\delta)Y. \end{aligned}$$

Now, the retailer's expected total revenue per cycle is

$$E(R) = sE(Q-Y) + v(1-\delta)E(Y).$$

On the other hand, the retailer's total cost per cycle can be obtained as follows:

$$\begin{aligned} C &= \text{Ordering cost+purchasing cost+holding cost+backorder cost} \\ &+ \text{inspection cost+penalty cost+disposal cost+green innovation cost} \\ &= K_r + cQ + \frac{H_r(Q-Y-B)^2}{2D(s,g)} + \frac{\pi_r B^2}{2D(s,g)} + i_c f Q + w(\alpha Q - Y) \\ &+ r\delta Y + F(g-1)^2 T \end{aligned}$$

Now, the retailer's expected total cost is

$$\begin{aligned} E[C] &= K_r + cQ + \frac{H_r}{2D(s,g)}[E(Q-Y)^2 - 2E(Q-Y)B + B^2] + \frac{\pi_r B^2}{2D(s,g)} \\ &+ i_c f Q + w[QE(\alpha) - E(Y)] + r\delta E(Y) + F(g-1)^2 E(T) \end{aligned}$$

With defective rate α in a lot, the number of defects found in the sub-lot sampled is a random variable Y, which has a hypergeometric distribution with parameters Q, f, and α. That is, Y has a hypergeometric probability mass function (p.m.f.)

$$Pr(Y|\alpha) = \frac{C_Y^{\alpha Q} C_{fQ-Y}^{Q-\alpha Q}}{C_{fQ}^{Q}},$$

where $0 \le Y \le \min\{fQ, \alpha Q\}$.
In this case,

$$E(Y|\alpha) = f\alpha Q$$

and

$$Var(Y|\alpha) = \frac{f(1-f)\alpha(1-\alpha)Q^2}{Q-1}.$$

Hence, unconditioning on α, we have

$$E(Y) = \int_0^1 E(Y|\alpha) g(\alpha) d\alpha = fQE(\alpha)$$

and

$$
\begin{aligned}
E(Y^2) &= \int_0^1 E(Y^2|\alpha)g(\alpha)d\alpha \\
&= \int_0^1 \left\{[E(Y|\alpha)]^2 + Var(Y|\alpha)\right\} g(\alpha)d\alpha
\end{aligned}
$$

$$
= f^2Q^2E(\alpha^2) + \frac{f(1-f)E[\alpha(1-\alpha)]Q^2}{Q-1}.
$$

The expected cycle length is

$$
E(T) = \frac{E(Q-Y)}{D(s,g)} = \frac{Q-E(Y)}{D(s,g)} = \frac{Q[1-fE(\alpha)]}{D(s,g)}
$$

Therefore, the retailer's expected total profit per unit time in decentralized decision-making policy is

$$
\begin{aligned}
\Pi_r^d(Q,B,s,g) &= \frac{E(R)-E(C)}{E(T)} \\
&= \left[s + \frac{fE(\alpha)}{1-fE(\alpha)}(v+w) - \frac{fE(\alpha)}{1-fE(\alpha)}\left(\delta(v+r)+\frac{w}{f}\right)\right] D(s,g) \\
&\quad - \frac{[K_r + Q(c+fi_c)]D(s,g)}{(1-fE(\alpha))Q} - \frac{(\pi_r+H_r)B^2}{2Q[1-fE(\alpha)]} + BH_r - F(g-1)^2 \\
&\quad - \frac{H_r}{2}\left[Q\left(\frac{1-2fE(\alpha)+f^2E(\alpha^2)}{1-fE(\alpha)}\right) + \frac{f(1-f)E[\alpha(1-\alpha)]Q}{[1-fE(\alpha)](Q-1)}\right] \quad (3.1)
\end{aligned}
$$

For large value of Q, it is noticed that $\frac{Q}{Q-1} \approx 1$.
Hence, (3.1) can be approximated to

$$
\begin{aligned}
\Pi_r^d(Q,B,s,g) &\approx \left[s + \frac{fE(\alpha)}{1-fE(\alpha)}(v+w) - \frac{fE(\alpha)}{1-fE(\alpha)}\left(\delta(v+r)+\frac{w}{f}\right)\right] D(s,g) \\
&\quad - \frac{[K_r + Q(c+fi_c)]D(s,g)}{(1-fE(\alpha))Q} - \frac{(\pi_r+H_r)B^2}{2Q[1-fE(\alpha)]} + BH_r - F(g-1)^2 \\
&\quad - \frac{H_r}{2}\left[Q\left(\frac{1-2fE(\alpha)+f^2E(\alpha^2)}{1-fE(\alpha)}\right) + \frac{f(1-f)E[\alpha(1-\alpha)]}{[1-fE(\alpha)]}\right] \quad (3.2)
\end{aligned}
$$

Letting, $Z_1 = \frac{fE(\alpha)}{1-fE(\alpha)}, Z_2 = 1 - fE(\alpha), Z_3 = \frac{1-2fE(\alpha)+f^2E(\alpha^2)}{1-fE(\alpha)}, Z_4 = \frac{f(1-f)E[\alpha(1-\alpha)]}{[1-fE(\alpha)]}$, we have (3.2) as follows:

$$\begin{aligned}\Pi_r^d(Q,B,s,g) &= \left[s + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{c+fi_c}{Z_2}\right] D(s,g) \\ &- \frac{K_r D(s,g)}{Z_2 Q} - \frac{H_r}{2}(QZ_3 + Z_4) - \frac{(\pi_r + H_r)B^2}{2QZ_2} + BH_r - F(g-1)^2 \quad (3.3)\end{aligned}$$

Now, the problem is to find the optimal order quantity, backorder quantity, selling price, and greening level that maximize the expected total profit per time unit of the retailer under decentralized decision-making policy.

Before proceeding with the optimization procedure, we need to introduce the following conjecture:

Conjecture 1. *The inequality*

$$\omega = (H_r + \pi_r)\frac{1-2fE(\alpha) + f^2E(\alpha^2)}{1-fE(\alpha)} - H_r(1 - fE(\alpha)) >$$

holds for any values adopted in practice.

Under Conjecture 1, the following property is satisfied:

Proposition 3.1. *For fixed (s,g), the retailer's decentralized profit function Π_r^d in (3.3) is concave in (Q,B). Moreover, the retailer's optimal backorder quantity B_r^d and order quantity Q_r^d can be derived from $\frac{\partial \Pi_r^d}{\partial B} = 0$ and $\frac{\partial \Pi_r^d}{\partial Q} = 0$, namely*

$$B = B_r^d(s,g) = \frac{H_r Z_2}{H_r + \pi_r} Q \tag{3.4}$$

$$Q = Q_r^d(s,g) = \sqrt{\frac{2K_r(H_r+\pi_r)(a-bs)g^{\mu}}{H_r Z_2\left[Z_3(H_r+\pi_r) - H_r Z_2\right]}} \tag{3.5}$$

Proof. To prove the concavity, we evaluate the Hessian matrix, H of the profit function (3.3); then the minors are examined. The associated Hessian matrix is given by

$$H = \begin{pmatrix} \frac{\partial^2 \Pi_r^d(Q,B)}{\partial Q^2} & \frac{\partial^2 \Pi_r^d(Q,B)}{\partial Q \partial B} \\ \frac{\partial^2 \Pi_r^d(Q,B)}{\partial B \partial Q} & \frac{\partial^2 \Pi_r^d(Q,B)}{\partial B^2} \end{pmatrix}$$

The principal minors are

$$|H_1| = -\frac{B^2(\pi_r + H_r) + 2K_r D(s,g)}{Q^3 Z_2} < 0$$

and

$$|H_2| = \frac{2K_r D(s,g)(H_r + \pi_r)}{Q^4 Z_2^2} > 0$$

which indicate Π_r^d is concave in Q and B.

Hence, the values of Q_r^d and B_r^d can be obtained explicitly by solving the following equations simultaneously:

$$\begin{aligned}\frac{\partial \Pi_r^d}{\partial B} &= H_r - \frac{B(H_r + \pi_r)}{QZ_2} = 0 \\ \frac{\partial \Pi_r^d}{\partial Q} &= 2K_r D(s,g) - H_r Q^2 Z_2 Z_3 + B^2(H_r + \pi_r) = 0\end{aligned}$$

This concludes the proof. □

Now, substituting the values of B and Q from (3.4) and (3.5) in (3.3), we have

$$\begin{aligned}\Pi_r^d(s,g) &= \left[s + Z_1\left\{v(1-\delta) + w\left(1 - \frac{1}{f}\right) - \delta r\right\} - \frac{c + fi_c}{Z_2}\right] \\ &\times \ (a - bs)g^{\mu} - F(g-1)^2 - \frac{H_r Z_4}{2} \\ &- \left(\sqrt{\frac{2K_r H_r\left[Z_3(\pi_r + H_r) - H_r Z_2\right](a - bs)g^{\mu}}{Z_2(H_r + \pi_r)}}\right)\end{aligned} \tag{3.6}$$

Letting $U = \frac{K_r H_r[Z_3(\pi_r + H_r) - H_r Z_2]}{Z_2(H_r + \pi_r)}$, (3.6) can be simplified as follows:

$$\begin{aligned}\Pi_r^d(s,g) &= \left[s + Z_1\left\{v(1-\delta) + w\left(1 - \frac{1}{f}\right) - \delta r\right\} - \frac{c + fi_c}{Z_2}\right](a - bs)g^{\mu} \\ &- \ F(g-1)^2 \frac{H_r Z_4}{2} - \sqrt{2Uag^{\mu}[1 - (b/a)s]}\end{aligned} \tag{3.7}$$

Since $0 < b/a < 1$ and $0 < s < a/b$ for $s \in [\frac{1}{2}a/b, a/b]$, $\sqrt{1-(b/a)s}$ can be approximated as $\eta_1 s^2 + \eta_2 s + \eta_3$, where $\eta_1 = (-8 + 4\sqrt{2})\left(\frac{b}{a}\right)^2, \eta_2 = (12 - 7\sqrt{2})\left(\frac{b}{a}\right), \eta_3 = 3\sqrt{2} - 4$ (Qin, Tang, and Guo, 2007).

Hence, using the approximation $\sqrt{1-(b/a)s} \approx \eta_1 s^2 + \eta_2 s + \eta_3$,(3.7) can be written as follows:

$$\begin{aligned}\Pi_r^d(s,g) &= \left[s + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{c+fi_c}{Z_2}\right](a-bs)g^{\mu} \\ &- F(g-1)^2 - \frac{H_r Z_4}{2} - \sqrt{2Uag^{\mu}}\left(\eta_1 s^2 + \eta_2 s + \eta_3\right) \end{aligned} \tag{3.8}$$

Proposition 3.2. *For fixed value of g, the retailer's decentralized profit function* Π_r^d *in (3.8) is concave in s if* $a^3 > 64b^2(3-2\sqrt{2})U/g^{\mu}$. *Moreover, the retailer's optimal decentralized selling price* s_r^d *can be derived from* $\frac{d\Pi_r^d}{dQ} = 0$, *namely*

$$s = s_r^d(g) = \frac{\left[Z_1\left\{w\left(\frac{1}{f}-1\right)+(r+v)\delta - v\right\}+\frac{c+fi_c}{Z_2}\right]bg^{\mu}+ag^{\mu}-\eta_2\sqrt{2Uag^{\mu}}}{2\left(bg^{\mu}+\eta_1\sqrt{2Uag^{\mu}}\right)} \tag{3.9}$$

Proof. We have

$$\frac{d^2\Pi_b^d}{ds^2} = -(2bg^{\mu} + 2\sqrt{2Uag^{\mu}}\eta_1)$$

For Π_r^d to be concave, it is required that the value of $\frac{d^2\Pi_r^d}{ds^2}$ be negative. This condition is achieved if

$$a^3 > 64b^2(3-2\sqrt{2})U/g^{\mu}$$

In practice, demand parameter a is usually very large. Hence, $a^3 > 64b^2(3-2\sqrt{2})U/g^{\mu}$ would be normally satisfied.

Hence, Π_r^d is concave in s, for fixed g with $Q = Q_r^d$ and $B = B_r^d$. The value s_r^d is obtained solving the equation in s:

$$\begin{aligned}\frac{\partial \Pi_r^d}{\partial s} &= (a-bs)g^{\mu} - bg^{\mu}\left[s + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{c+fi_c}{Z_2}\right] \\ &- \sqrt{2Uag^{\mu}}(2s\eta_1 + \eta_2) = 0.\end{aligned}$$

This concludes the proof. □

Substituting the value of s from (3.9), Eq. (3.8) becomes:

$$\begin{aligned}\Pi_r^d(g) &= \left[Z_1\left(v(1-\delta)+w\left(1-\frac{1}{f}\right)-r\delta\right)-\frac{c+fi_c}{Z_2}\right]ag^{\mu} \\ &- F(g-1)^2-\frac{H_rZ_4}{2}-\eta_3\sqrt{2ag^{\mu}U} \\ &+ \frac{\left[\left\{a+bZ_1(\frac{w}{f}+(r+v)\delta-v-w)+b\frac{(c+fi_c)}{Z_2}\right\}g^{\mu}-\eta_2\sqrt{2ag^{\mu}U}\right]^2}{4\left[bg^{\mu}+\eta_1\sqrt{2ag^{\mu}U}\right]} \end{aligned} \quad (3.10)$$

In Propositions (3.1-3.2), the existence of retailer's decentralized optimal decisions Q, B, and s that maximize the retailer's profit function is proved. Note that (3.10) contains a single decision variable g. Due to the mathematical complexity, the closed-form solution for g cannot be obtained. To find the unique solution for g, we must check the concavity of (3.10). Due to appearance of highly non-linear terms in (3.10), theoretically, it is not possible to prove. However, the concavity behavior of the objective function can be examined numerically. The NSolve method of MATHEMATICA can be applied to find the solution, as is done in this case.

Now, the supplier's expected total profit per time unit is

$$\Pi_s^d = \text{Revenue} - \text{setup cost} - \text{holding cost}$$

The first shipment of Q units is delivered to the retailer as soon as the supplier receives it. Thereafter, supplier delivers every shipment after interval $E[T] = \frac{E(Q-Y)}{D(s,g)}$. Therefore, the supplier's total inventory held up per ordering cycle is

$$I_s = E[T](Q+2Q+.........+(m-1)Q) = E[T]\frac{m(m-1)Q}{2}.$$

Now, the supplier's total profit per ordering cycle is

$$TP_s = (c-p)mQ - K_s - H_sI_s.$$

Therefore, the supplier's expected total profit per unit time in decentralized decision-making policy can be obtained by dividing TP_s by ordering cycle $mE[T]$ as

$$\Pi_s^d(Q,s,g,m) = (c-p)\frac{(a-bs)g^{\mu}}{Z_2} - \frac{K_s(a-bs)g^{\mu}}{mQZ_2} - \frac{H_s(m-1)Q}{2}. \quad (3.11)$$

It is important to note that the information about Q, B, s, and g are required for the supplier to decide the value of m. Here it is assumed that the retailer acts as a leader and determines optimal (Q, B, s, g) that maximize $\Pi_r^d(Q, B, s, g)$. Then based on this, the supplier determines m which maximizes $\Pi_s^d(m)$.

Proposition 3.3. *The supplier's profit function Π_s^d in (3.11) is concave in m with $Q = Q_r^d, s = s_r^d$, and $g = g_r^d$. Moreover, the supplier's optimal number of shipments m_s^d can be derived from $\frac{d\Pi_s^d}{dm} = 0$, namely*

$$m = m_s^d = \frac{1}{Q}\sqrt{\frac{2K_s(a-bs)g^{\mu}}{H_s Z_2}} \tag{3.12}$$

Proof. We now prove the concavity of Π_s^d in m for fixed (Q, B, s, g). We have

$$\frac{d^2\Pi_s^d}{dm^2} = -\frac{2K_s D(s,g)}{m^3 Q Z_2} < 0$$

Hence Π_s^d is concave in m. The value m_s^d is obtained solving in m the equation

$$\frac{d\Pi_s^d}{dm} = -\frac{K_s D(s,g)}{m^2 Q Z_2} - \frac{H_s}{2} = 0.$$

This concludes the proof. □

The value of m must be an integer; however, (3.12) may give a decimal value. Therefore, we propose the following which will give the integer value of m:

$$m^* = \begin{cases} \lfloor m^*_{Decimal} \rfloor & \text{if } \Pi_s^d(\lfloor m^*_{Decimal} \rfloor) > \Pi_s^d(\lfloor m^*_{Decimal} \rfloor + 1) \\ \lfloor m^*_{Decimal} \rfloor + 1 & \text{if } \Pi_s^d(\lfloor m^*_{Decimal} \rfloor) \le \Pi_s^d(\lfloor m^*_{Decimal} \rfloor + 1) \end{cases}$$

The expected joint total profit of the SC under the decentralized optimization scheme Π_{chain}^d is obtained by summing the retailer's and the supplier's individual expected total profits *i.e.*, $\Pi_{chain}^d(g, m) = \Pi_r^d(g) + \Pi_s^d(m)$.

3.3.2 Centralized model

In the centralized framework, rather than focusing on individual member's profits, supply chain members jointly optimize the profit of the whole supply chain system as a single entity. Although the perfect coordination is very difficult to achieve in real life, centralized decision-making can produce more profit compared to decentralized decision-making system. Some instances of centralized decision-making system can be realized in restaurant, electronics business, and online businesses where the core important decisions are taken by those at a higher level of authority and after the decision has been taken, it is reported to lower level employees who are expected to follow the order. Here, in our model, both the channel members jointly take decision in the centralized supply chain to maximize the total supply chain's profit. The SC's expected total profit per unit time under centralized decision-making policy is

$$\begin{aligned}
\Pi^c_{chain}(Q,B,s,g,m) &= \text{Retailer's profit} + \text{Supplier's profit} \\
&= \left[s - \frac{p}{Z_2} + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{fi_c}{Z_2}\right] D(s,g) \\
&- \frac{D(s,g)}{QZ_2}\left(K_r + \frac{K_s}{m}\right) - \frac{H_r}{2}(QZ_3 + Z_4) - \frac{(\pi_r + H_r)B^2}{2QZ_2} \\
&+ BH_r - F(g-1)^2 - \frac{H_s(m-1)Q}{2} \qquad (3.13)
\end{aligned}$$

Now, the problem is to find the optimal order quantity, backorder quantity, selling price, greening level, and number of shipments that maximize the expected total profit per time unit of the SC under centralized decision-making policy.

Under Conjecture 1, the following properties are satisfied:

Proposition 3.4. *For fixed (s,g,m), the centralized profit function Π^c_{chain} in (3.13) is concave in (B,Q). Moreover, the retailer's optimal centralized backorder quantity B^c_r and order quantity Q^c_r can be derived from $\frac{\partial \Pi^c_{chain}}{\partial B} = 0$ and $\frac{\partial \Pi^c_{chain}}{\partial Q} = 0$, namely*

$$B = B^c_r(Q) = \frac{H_r Z_2}{H_r + \pi_r} Q \qquad (3.14)$$

$$Q = Q^c_r(s,g,m) = \sqrt{\frac{2D(s,g)\,(K_r + K_s/m)}{Z_2\left(\frac{H_r}{H_r+\pi_r}[Z_3(H_r+\pi_r) - H_r Z_2] + H_s\,(m-1)\right)}} \qquad (3.15)$$

Proof. To prove the concavity, we evaluate the Hessian matrix, H of the profit function (3.13); then the minors are examined. The associated Hessian matrix is given by

$$H = \begin{pmatrix} \frac{\partial^2 \Pi^c_{chain}(Q,B)}{\partial Q^2} & \frac{\partial^2 \Pi^c_{chain}(Q,B)}{\partial Q \partial B} \\ \frac{\partial^2 \Pi^c_{chain}(Q,B)}{\partial B \partial Q} & \frac{\partial^2 \Pi^c_{chain}(Q,B)}{\partial B^2} \end{pmatrix}$$

The principal minors of the Hessian matrix are

$$|H_1| = -\frac{2D(s,g)(K_r+K_s/m)+B^2(H_r+\pi_r)}{Q^3Z_2} < 0, |H_2| = \frac{2D(s,g)(K_s+mK_r)(H_r+\pi_r)}{mQ^4Z_2^2} > 0,$$

which indicate Π^c_{chain} is strictly concave in Q and B.

Hence, the values of Q^c_r and B^c_r can be obtained explicitly by solving the following equations simultaneously:

$$\begin{aligned}
\frac{\partial \Pi^c_{chain}}{\partial Q} &= -\frac{1}{2}H_s(m-1) + \frac{D(s,g)\left(K_r + \frac{K_s}{m}\right)}{Q^2Z_2} - \frac{H_rZ_3}{2} + \frac{B^2(H_r+\pi_r)}{2Q^2Z_2} = 0 \\
\frac{\partial \Pi^c_{chain}}{\partial B} &= H_r - \frac{B(H_r+\pi_r)}{QZ_2} = 0
\end{aligned}$$

This concludes the proof. □

Substituting the values of B and Q from (3.14) and (3.15) in (3.13), we have

$$\begin{aligned}
\Pi^c_{chain}(s,g) &= \left[s - \frac{p}{Z_2} + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - r\delta\right\} - \frac{fi_c}{Z_2}\right] D(s,g) \\
&- \sqrt{2\left(H_rZ_3 - \frac{H_r^2Z_2}{H_r+\pi_r} + H_s(m-1)\right)\left(\frac{K_r+K_s/m}{Z_2}\right)D(s,g)} \\
&- F(g-1)^2 - \frac{H_rZ_4}{2} \qquad (3.16)
\end{aligned}$$

Proposition 3.5. *For fixed s and g with* $B = B^c_r$ *and* $Q = Q^c_r$, Π^c_{chain} *in (3.16) is concave in m. Moreover, the supplier's optimal number of shipments* m^c_s *can be derived from* $\frac{d\Pi^c_{chain}}{dm} = 0$, *namely*

$$m = m^c_s = \sqrt{\frac{K_s}{K_rH_s}\left[H_r\left(\frac{Z_3(H_r+\pi_r) - H_rZ_2}{H_r+\pi_r}\right) - H_s\right]} \qquad (3.17)$$

Proof. For given values of s and g, maximizing (3.16) with respect to m is equivalent to minimizing the following expression:

$$\Pi'(m) = 2\left(H_rZ_3 - \frac{H_r^2Z_2}{H_r+\pi_r} + H_s(m-1)\right)\left(\frac{K_r+K_s/m}{Z_2}\right)D(s,g)$$

Now we will show that $\Pi'(m)$ is a convex function in m. We have

$$\frac{d^2\Pi'}{dm^2} = \frac{4K_sD(s,g)}{m^3Z_2}\left(\frac{H_r}{H_r+\pi_r}[Z_3(H_r+\pi_r) - H_rZ_2] - H_s\right)$$

In order that Π' to be convex, it is required that the value of $\frac{d^2\Pi'}{dm^2}$ be positive. In practice $H_r > H_s$, hence $\frac{d^2\Pi'}{dm^2} > 0$ is likely to hold for real world situation. Hence, we can conclude that Π^c_{chain} is concave in m, for fixed (s, g) with $Q = Q^c_r$ and $B = B^c_r$.

The value m^c_s is obtained solving in m the equation

$$\frac{d\Pi'}{dm} = H_s\left(K_r + \frac{K_s}{m^2}\right) + H_r K_s\left(\frac{H_r Z_2}{H_r + \pi_r} - Z_3\right) = 0$$

and taking the positive root. □

The integer value of m can be found from the following relation:

$$m = \begin{cases} \lfloor m_{Decimal} \rfloor & \text{if } \Pi^c_{chain}(\lfloor m_{Decimal} \rfloor) > \Pi^c_{chain}(\lfloor m_{Decimal} \rfloor + 1) \\ \lfloor m_{Decimal} \rfloor + 1 & \text{if } \Pi^c_{chain}(\lfloor m_{Decimal} \rfloor) \leq \Pi^c_{chain}(\lfloor m_{Decimal} \rfloor + 1) \end{cases}$$

Now, using the approximation $\sqrt{1 - (b/a)s} \approx \eta_1 s^2 + \eta_2 s + \eta_3$, (3.16) reduces to

$$\begin{aligned} \Pi^c_{chain}(s, g|_{m^c_s}) &= \left[s + Z_1\left\{v(1-\delta) + w\left(1 - \frac{1}{f}\right) - r\delta\right\} - \frac{p + fi_c}{Z_2}\right] D(s, g) \\ &- F(g-1)^2 - \frac{H_r Z_4}{2} - \sqrt{2L(m^c_s) a g^\mu}(\eta_1 s^2 + \eta_2 s + \eta_3) \end{aligned} \tag{3.18}$$

where $L(m^c_s) = \left[H_r\left(\frac{Z_3(H_r+\pi_r) - H_r Z_2}{H_r + \pi_r}\right) + H_s(m^c_s - 1)\right]\left(\frac{K_r + \frac{K_s}{m^c_s}}{Z_2}\right) > 0$.

Proposition 3.6. *For fixed g with $B = B^c_r$, $Q = Q^c_r$, and $m = m^c_s$, Π^c_{chain} in (3.18) is concave in s if $a^3 > 64b^2(3 - 2\sqrt{2})L(m^c_s)/g^\mu$. Moreover, the supplier's optimal selling price s^c_r can be derived from $\frac{d\Pi^c_{chain}}{ds} = 0$, namely*

$$s = s^c_r(g) = \frac{\left[a + bZ_1\left\{\frac{w}{f} + (r+v)\delta - v - w\right\} + \frac{b}{Z_2}(fi_c + p)\right]g^\mu - \eta_2\sqrt{2ag^\mu L(m^c_s)}}{2\left(bg^\mu + \eta_1\sqrt{2ag^\mu L(m^c_s)}\right)} \tag{3.19}$$

Proof. We have

$$\frac{d^2\Pi^c_{chain}}{ds^2} = -2bg^\mu - 2\sqrt{2L(m^c_s)ag^\mu}\eta_1$$

In order that Π^c_{chain} to be concave, it is required that the value of $\frac{d^2\Pi^c_{chain}}{ds^2}$ be negative. This condition is achieved if $a^3 > 64b^2(3 - 2\sqrt{2})L(m^c_s)/g^\mu$.

In practice, the demand parameter a is usually very large. Hence, $a^3 > 64b^2(3 - 2\sqrt{2})L(m^c_s)/g^\mu$ would be normally satisfied.

Hence, Π_c^{chain} is concave in s, for fixed g with $Q = Q_r^c, B = B_r^c$, and $m = m_s^c$. The value s_r^c is obtained solving in s the equation

$$\begin{aligned}\frac{\partial \Pi^c_{chain}}{\partial s} &= (a-bs)g^{\mu} - bg^{\mu}\left[s + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{p+fi_c}{Z_2}\right] \\ &- \sqrt{2L(m_s^c)ag^{\mu}}(2s\eta_1 + \eta_2) = 0.\end{aligned}$$

This concludes the proof. □

Substituting (3.19) into (3.18), the latter becomes:

$$\begin{aligned}\Pi^c_{chain}(g) &= \left[Z_1\left(v + w - (r+v)\delta - \frac{w}{f}\right) - \frac{p+fi_c}{Z_2}\right]ag^{\mu} - F(g-1)^2 \\ &+ \frac{\left\{\left[a+bZ_1(\frac{w}{f}+(r+v)\delta-v-w)+\frac{b}{Z_2}(fi_c+p)\right]g^{\mu}-\eta_2\sqrt{2ag^{\mu}L(m_s^c)}\right\}^2}{4\left[\eta_1\sqrt{2ag^{\mu}L(m_s^c)}+bg^{\mu}\right]} \\ &- \frac{H_rZ_4}{2} - \eta_3\sqrt{2ag^{\mu}L(m_s^c)} \qquad (3.20)\end{aligned}$$

In Propositions (3.4-3.6), the existence of centralized optimal decisions Q, B, s, and m that maximize the centralized profit function is proved. Note that (3.20) contains a single decision variable g. Due to the mathematical complexity, the closed-form solution for g cannot be obtained. To find the unique solution for g, we must check the concavity of (3.20). Due to appearance of highly non-linear terms in (3.20), theoretically, it is not possible to prove. However, the convexity behavior of the objective function can be examined numerically. The NSolve method of MATHEMATICA can be applied to find the solution, as is done in this case.

3.3.3 Coordination model

It is well known that the centralized decision-making policy gives better results than decentralized decision-making policy. However, in reality it is very difficult to establish a centralized policy due to various reasons (e.g., lack of information sharing, long distance etc.,). Therefore, in this section, we try to establish a coordinate between the SC parties (supplier and retailer) through a trade credit contract. Under this policy, the supplier and the retailer sign a contract with the agreement that the supplier offers a certain credit period to the retailer, and in turn the retailer guarantees to change his optimal decisions according to globally optimal decisions.

Under this policy, the parameters need to be tuned in such a way that both upstream and downstream have sufficient motivation to participate in this scheme. In this coordination, the upstream party tries to change the decision variables (Q, B, s, g) of the downstream party from their locally optimal values $(Q_r^d, B_r^d, s_r^d, g_r^d)$ to globally optimal values $(Q_r^c, B_r^c, s_r^c, g_r^c)$ by providing trade-credit period T_c to the downstream party.

Let us recall the decentralized profit function of the retailer and supplier:

$$\begin{aligned}\Pi_r^d(Q_r^d, B_r^d, s_r^d, g_r^d) &= \left[s_r^d + Z_1\left\{v(1-\delta) + w\left(1 - \frac{1}{f}\right) - \delta r\right\} - \frac{c + fi_c}{Z_2}\right] D(s_r^d, g_r^d) \\ &- \frac{K_r D(s_r^d, g_r^d)}{Z_2 Q_r^d} - \frac{H_r}{2}\left(Q_r^d Z_3 + \frac{Z_4 Q_r^d}{Q_r^d - 1}\right) - \frac{(\pi_r + H_r) B_r^{d^2}}{2 Q_r^d Z_2} \\ &+ B_r^d H_r - F(g_r^d - 1)^2 \end{aligned} \tag{3.21}$$

and

$$\Pi_s^d(Q_r^d, s_r^d, g_r^d, m_s^d) = (c - p)\frac{D(s_r^d, g_r^d)}{Z_2} - \frac{K_s D(s_r^d, g_r^d)}{m_s^d Q_r^d Z_2} - \frac{H_s(m_s^d - 1) Q_r^d}{2} \tag{3.22}$$

According to the coordination policy, the retailer has to change his/her optimal decision according to the centralized policy and instead the supplier will give a credit period of time T_c. Therefore, after accepting the credit period and the centralized optimal decisions, the retailer's newly generated profit function becomes

$$\begin{aligned}\Pi_r^{co}(Q_r^c, B_r^c, s_r^c, g_r^c, T_c) &= \left[s_r^c + Z_1\left\{v(1-\delta) + w\left(1 - \tfrac{1}{f}\right) - \delta r\right\} - \tfrac{c + fi_c}{Z_2}\right] D(s_r^c, g_r^c) \\ &- \frac{K_r D(s_r^c, g_r^c)}{Z_2 Q_r^c} - \frac{H_r}{2}\left(Q_r^c Z_3 + \frac{Z_4 Q_r^c}{Q_r^c - 1}\right) - \frac{(\pi_r + H_r) B_r^{c^2}}{2 Q_r^c Z_2} \\ &+ B_r^c H_r - F(g_r^c - 1)^2 + cI_e D(s_r^c, g_r^c) T_c \end{aligned} \tag{3.23}$$

where the last term i.e., $cI_e D(s_r^c, g_r^c) T_c$ is the earning from delay in payment.

On the other hand, after providing the credit period to the retailer and accepting the centralized optimal decisions, the supplier's expected profit function changes to

$$\begin{aligned}\Pi_s^{co}(Q_r^c, s_r^c, g_r^c, m_s^c, T_c) &= (c - p)\frac{D(s_r^c, g_r^c)}{Z_2} - \frac{K_s D(s_r^c, g_r^c)}{m_s^c Q_r^c Z_2} - \frac{H_s(m_s^c - 1) Q_r^c}{2} \\ &- cI_e D(s_r^c, g_r^c) T_c \end{aligned} \tag{3.24}$$

• *Conditions for the retailer to participate*

Given (3.21) and (3.23), the retailer's goal is to know the minimum duration of the credit period so that the profit after execution of the contract is more or equal to the decentralized profit without the contract. Therefore, to determine the minimum credit period, we have the following condition:

$$\Pi_r^{co}(Q_r^c, B_r^c, s_r^c, g_r^c, T_c) \geq \Pi_r^d(Q_r^d, B_r^d, s_r^d, g_r^d) \tag{3.25}$$

Based on the above inequality, the retailer's profit under trade-credit agreement should be more than or equal to the retailer's decentralized profit. Otherwise, the retailer does not accept supplier's proposed credit period.

After substituting (3.21) and (3.23) in the inequality (3.25), we have

$$\begin{aligned}
&\left[s_r^c + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{c+fi_c}{Z_2}\right] D(s_r^c, g_r^c) - \frac{K_r D(s_r^c, g_r^c)}{Z_2 Q_r^c} \\
-\ & \frac{H_r}{2}\left(Q_r^c Z_3 + \frac{Z_4 Q_r^c}{Q_r^c - 1}\right) - \frac{(\pi_r + H_r) B_r^{c^2}}{2 Q_r^c Z_2} + B_r^c H_r - F(g_r^c - 1)^2 + c I_e D(s_r^c, g_r^c) T_c \geq \\
&\left[s_r^d + Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{c+fi_c}{Z_2}\right] D(s_r^d, g_r^d) - \frac{K_r D(s_r^d, g_r^d)}{Z_2 Q_r^d} \\
-\ & \frac{H_r}{2}\left(Q_r^d Z_3 + \frac{Z_4 Q_r^d}{Q_r^d - 1}\right) - \frac{(\pi_r + H_r) B_r^{d^2}}{2 Q_r^d Z_2} + B_r^d H_r - F(g_r^d - 1)^2
\end{aligned}$$

which gives

$$T_c \geq \frac{\Gamma}{c I_e D(s_r^c, g_r^c)} \tag{3.26}$$

where

$$\begin{aligned}
\Gamma\ =\ & s_r^d D(s_r^d, g_r^d) - s_r^c D(s_r^c, g_r^c) + \left[Z_1\left\{v(1-\delta) + w\left(1-\frac{1}{f}\right) - \delta r\right\} - \frac{c+fi_c}{Z_2}\right] \\
& (D(s_r^d, g_r^d) - D(s_r^c, g_r^c)) + \frac{H_r Z_3}{2}(Q_r^c - Q_r^d) + \frac{H_r Z_4}{2}\left(\frac{Q_r^c}{Q_r^c - 1} - \frac{Q_r^d}{Q_r^d - 1}\right) \\
& + \frac{K_r}{Z_2}\left(\frac{D(s_r^c, g_r^c)}{Q_r^c} - \frac{D(s_r^d, g_r^d)}{Q_r^d}\right) + \frac{(\pi_r + H_r)}{2Z_2}\left(\frac{B_r^{c^2}}{Q_r^c} - \frac{B_r^{d^2}}{Q_r^d}\right) \\
& + H_r(B_r^d - B_r^c) + F(g_r^c - 1)^2 - F(g_r^d - 1)^2
\end{aligned}$$

Inequality (3.26) indicates the minimum value (T_{cmin}) of the proposed credit period to achieve win-win outcomes for the retailer and channel coordination, where

$$T_{cmin} = \frac{\Gamma}{c I_e D(s_r^c, g_r^c)} \tag{3.27}$$

• *Conditions for the supplier to participate*

Given (3.22) and (3.24), the target of the supplier is to mark the maximum credit period level so that the profit after implementation of the contract is greater or equal to the decentralized profit without contract. Therefore, to determine the maximum discount level, we have the following condition:

$$\Pi_s^{co}(Q_r^c, s_r^c, g_r^c, m_s^c, T_c) \geq \Pi_s^d(Q_r^d, s_r^d, g_r^d, m_s^d) \tag{3.28}$$

Using (3.22) and (3.24) in (3.28), we have

$$\begin{aligned} &(c-p)\frac{D(s_r^c, g_r^c)}{Z_2} - \frac{K_s D(s_r^c, g_r^c)}{m_s^c Q_r^c Z_2} - \frac{H_s(m_s^c - 1)Q_r^c}{2} - cI_e D(s_r^c, g_r^c)T_c \geq \\ &(c-p)\frac{D(s_r^d, g_r^d)}{Z_2} - \frac{K_s D(s_r^d, g_r^d)}{m_s^d Q_r^d Z_2} - \frac{H_s(m_s^d - 1)Q_r^d}{2} \end{aligned} \tag{3.29}$$

which gives

$$T_c \leq \frac{\Omega}{cI_e D(s_r^c, g_r^c)} \tag{3.30}$$

where

$$\begin{aligned} \Omega &= \frac{(c-p)}{Z_2}\left[D(s_r^c, g_r^c) - D(s_r^d, g_r^d)\right] + \frac{K_s}{Z_2}\left(\frac{D(s_r^d, g_r^d)}{m_s^d Q_r^d} - \frac{D(s_r^c, g_r^c)}{m_s^c Q_r^c}\right) \\ &+ \frac{H_s}{2}\left[(m_s^d - 1)Q_r^d - (m_s^c - 1)Q_r^c\right] \end{aligned}$$

Inequality (3.30) indicates the maximum value (T_{cmax}) of the proposed credit period that the supplier can offer to achieve win-win outcome and channel coordination, where

$$T_{cmax} = \frac{\Omega}{cI_e D(s_r^c, g_r^c)} \tag{3.31}$$

Equations (3.27) and (3.31) provide the range of the credit period to execute the proposed coordination policy. $[T_{cmin}, T_{cmax}]$ is the interval within which each value of T_c represents how both the SC members will share the profit. As T_c moves closer to T_{cmax}, the retailer will be more benefited. On the other hand, as T_c approaches closer toward T_{cmin}, the supplier will be more benefited.

3.4 Numerical illustrations

This section investigates numerical experiments and the sensitivity of the developed models. The aim is to draw insights into how optimal inventory decisions should be modified according to decentralized, centralized, and coordinated decision-making policies. We consider an inventory system with the following parameter-values $a = 210; b = 0.6; K_r = \$100/\text{order}; K_s = \$500/\text{setup}; p = \$70/\text{item}; c = \$130/\text{item}, v = \$45/\text{item}; H_r = \$4/\text{unit}/\text{unit time}; H_s = \$1/\text{unit}/\text{unit time}; \pi_r = \$7.5/\text{unit item}; w = \$100/\text{item}; i_c = \$1.2/\text{item}; I_e = \$0.12/\text{year}; F = 500; r = 1.5; \mu = 0.5; \delta = 0.3; f = 0.15$.

The defective rate, α follows the Beta distribution with $s = 1$, and $t = 9$ (Wu, Ouyang, and Ho, 2007). That is, the pdf of α is given by

$$g(\alpha) = 9(1-\alpha)^8, \quad 0 < \alpha < 1$$

Hence $E(\alpha) = s/(s+t) = 1/10$ and $E(\alpha^2) = s(s+1)/[(s+t)(s+t+1)] = 1/55$.

We analyze optimal ordering, pricing, backordering, greening level, and shipment decisions under three decision-making policies: (1) decentralized, (2) centralized, and (3) coordinated. The optimal outcomes using exact and approximated formulations for decentralized, centralized, and coordinated scenarios are presented in Table 3.1. As it can be seen that the deviation of the SC profit is acceptable with the deviation of 0.12% for the decentralized model and 0.16% for the centralized model. Note that, the expected total annual profit of the centralized supply chain has \$1457 more than the total expected annual profit in the decentralized scenario. The centralized channel has 31.86% more demand for the product than the decentralized channel. This happens as the demand depends on the selling price and greening level of

	Decentralized			Centralized			Coordinated
Parameters	Exact	Approx	Parameters	Exact	Approx	Parameters	Exact
Q_r^d	91.8428	91.7657	Q_r^c	136.26	136.261	Q_r^{co}	136.26
B_r^d	31.4661	31.4397	B_r^c	46.684	46.6841	B_r^{co}	46.684
s_r^d	245.712	245.879	s_r^c	215.696	215.672	s_r^{co}	215.696
g_r^d	2.91202	2.91154	g_r^c	3.78238	3.78103	g_r^{co}	3.78238
m_s^d	4	4	m_s^c	3	3	m_s^{co}	3
	–	–	–	–	–	T_{cmin}	0.629641
	–	–	–	–	–	T_{cmax}	1.22567
	–	–	–	–	–	T_c	0.927656
Π_r^d	9189.68	9182.54	Π_r^c	–	–	Π_r^{co}	9918.27
Π_s^d	6218.94	6208.27	Π_s^c	–	–	Π_s^{co}	6947.53
Π_{chain}^d	**15408.6**	**15390.8**	Π_{chain}^c	**16865.8**	**16837.9**	Π_{chain}^{co}	**16865.8**

Table 3.1: Optimal solutions under the decentralized, centralized and coordinated scenarios

the product. Note that, customers get more benefits (lower product price and better customer service) in the centralized channel compared to the decentralized one. However, a centralized policy does not necessarily increase the economic gain of all SC members. In the centralized policy, the retailer loses some of its profits compared to the decentralized policy due to the larger order quantity. As a result, the retailer may not participate in the centralized policy. So it is not always possible to bring two different entities under the centralized policy. As shown in Table 3.1, the proposed trade-credit coordination policy can increase the total SC profit as well as the individual SC member's profit as compared to the decentralized policy. Thus, this collaboration policy guarantees the participation of both SC members in practice.

Now, it is necessary to find out when the coordination policy will give a win-win situation. Under the proposed credit-period coordination policy, the minimum and maximum credit periods, i.e., T_{cmin} and T_{cmax} obtained as $[0.629, 1.226]$. Using the credit-period, the retailer's and the supplier's profits change within $[9189, 10647]$ and $[7676, 6219]$, respectively. Figure 3.2 shows changes in the retailer's and the supplier's profits over the credit period. Significant changes in retailer's and supplier's profits compared to the decentralized profit have been observed with changes in the credit period T_c. Looking at Figure 3.2, it can be seen that while the credit period is minimum, i.e., $T_{cmin} = 0.629$, the retailer's coordinated profit is the same as its decentralized profit and thereafter gradually increases with increasing credit period. Therefore, in the worst possible case, even if the supplier provides a minimum credit-period, the buyer will still be willing to accept the centralized decision. On the other hand, from the supplier's point of view, the situation goes in the opposite direction and gives the maximum profit at $T_{cmin} = 0.629$ whereas the minimum profit (which is the same as decentralized profit) at $T_{cmax} = 1.226$. So it is clear that the coordination model can increase the profits of the SC members as compared to

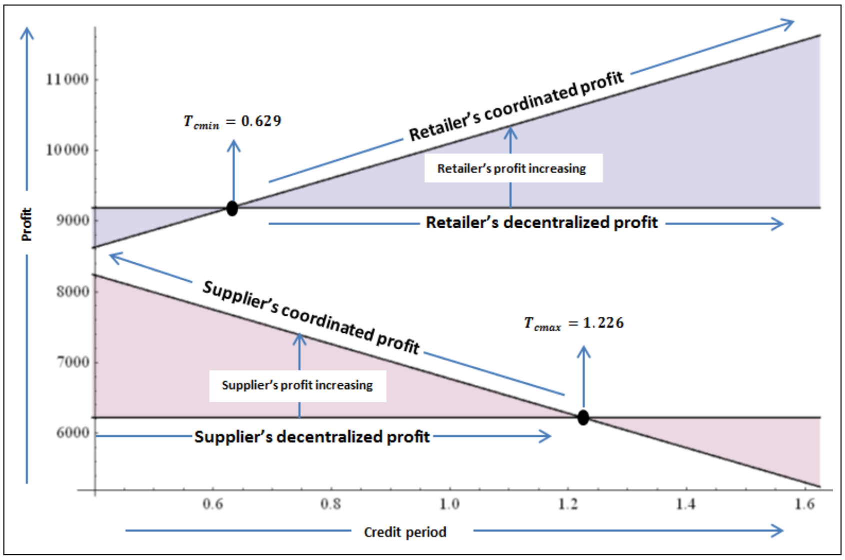

Figure 3.2: Credit period versus expected profit

the decentralized model. The above observation reveals that both parties will agree to coordinate if $T_c \in [T_{cmin}, T_{cmax}]$; T_c will be determined based on the bargaining power of the members. Here, we consider T_c as the average of T_{cmin} and T_{cmax}.

3.4.1 Some special cases

In this section, some special cases are presented to investigate how defective items and inspection policy change optimal decisions.

3.4.1.1 Case I: If the received batch contains all perfect quality items, i.e., $E[Y] = var[Y] = 0$.

To highlight this case, we consider $E[Y] = var[Y] = 0$ in the base model, and the results are given in Table 3.2.

	Decentralized		Centralized		Coordinated
Q_r^d	93.7671	Q_r^c	138.241	Q_r^{co}	138.241
B_r^d	32.6147	B_r^c	48.0837	B_r^{co}	48.0837
s_r^d	240.623	s_r^c	211.055	s_r^{co}	211.055
g_r^d	3.05378	g_r^c	3.92384	g_r^{co}	3.92384
m_s^d	4	m_s^c	3	m_s^{co}	3
	—	—	—	T_{cmin}	0.593906
	—	—	—	T_{cmax}	1.15756
	—	—	—	T_c	0.875735
Π_r^d	10312.2	—	-	Π_r^{co}	11038.6
Π_s^d	6587.38	—	—	Π_s^{co}	7313.79
Π_{chain}^d	**16899.6**	Π_{chain}^c	**18352.4**	Π_{chain}^{co}	**18352.4**

Table 3.2: Optimal solutions under the decentralized, centralized and coordinated scenarios when $E[Y] = var[Y] = 0$

3.4.1.2 Case II: If the items are sold without inspection, i.e., $f = 0$

This is another special case of the proposed model where the retailer receives each batch with a known percentage of defective items, and without any inspection, all items are sold in the market at the regular selling price. To highlight this case, we consider $f = 0$ in the base model, and the results are given in Table 3.3.

	Decentralized		Centralized		Coordinated
Q_r^d	90.5821	Q_r^c	134.442	Q_r^{co}	134.442
B_r^d	31.5068	B_r^c	46.7625	B_r^{co}	46.7625
s_r^d	245.552	s_r^c	215.992	s_r^{co}	215.992
g_r^d	2.91644	g_r^c	3.77343	g_r^{co}	3.77343
m_s^d	4	m_s^c	3	m_s^{co}	3
	—	—	—	T_{cmin}	0.612439
	—	—	—	T_{cmax}	1.19244
	—	—	—	T_c	0.902441
Π_r^d	9223.84	—	-	Π_r^{co}	9930.44
Π_s^d	6137.83	—	—	Π_s^{co}	6844.43
Π_{chain}^d	**15361.7**	Π_{chain}^c	**16774.9**	Π_{chain}^{co}	**16774.9**

Table 3.3: Optimal solutions under the decentralized, centralized and coordinated scenarios when $f = 0$

3.4.1.3 Case III: If the items are sold with full inspection, i.e., $f = 1$

In this case, we consider that every batch is inspected before selling it to the market, i.e., $f = 1$ and the results are shown in Table 3.4. This case is generally observed where the consequences of giving up a defective item can be quite fatal e.g. for avionic systems.

	Decentralized		Centralized		Coordinated
Q_r^d	99.0376	Q_r^c	147.091	Q_r^{co}	147.091
B_r^d	31.0031	B_r^c	46.0459	B_r^{co}	46.7625
s_r^d	246.725	s_r^c	213.838	s_r^{co}	213.838
g_r^d	2.88411	g_r^c	3.83881	g_r^{co}	3.77343
m_s^d	4	m_s^c	3	m_s^{co}	3
	—	—	—	T_{cmin}	0.742535
	—	—	—	T_{cmax}	1.44297
	—	—	—	T_c	1.09275
Π_r^d	8974.98	—	-	Π_r^{co}	9849.51
Π_s^d	6719.42	—	—	Π_s^{co}	7593.94
Π_{chain}^d	**15694.4**	Π_{chain}^c	**17443.5**	Π_{chain}^{co}	**17443.5**

Table 3.4: Optimal solutions under the decentralized, centralized and coordinated scenarios when $f = 1$

3.4.1.4 Comparative analysis of the above cases

From Table 3.2, we can see that the values of order quantity Q and backorder quantity B obtained in the model with defective items (i.e., proposed model) are smaller than those of the model with perfect items (i.e., $E(\alpha) = 0$).

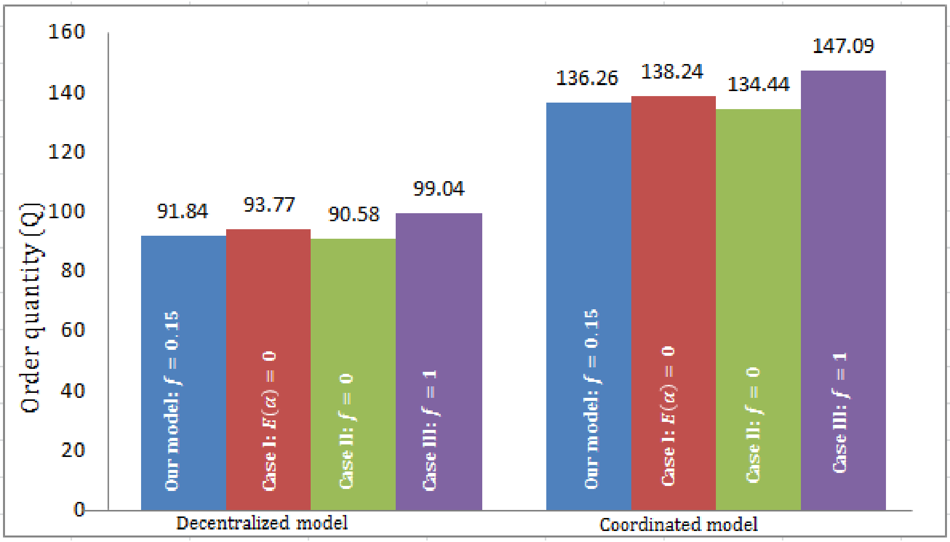

Figure 3.3: Order quantity for different cases

These findings are in stark contrast to previous results (Salameh and Jaber, 2000; Goyal, Huang, and Chen, 2003). It is seen that if a batch contains no defective items, then the supply chain's profit is increased by 8.82% in the decentralized model and by 8.10% in the coordinated model compared to the proposed model (i.e., sampling inspection model). Thus, it can be concluded that the reduction of defective items can be beneficial to the supply chain system. Referring to Figure 3.3, it is evident that, in the case with no inspection (i.e., $f = 0$), the order quantity Q is minimum and in the case with full inspection (i.e., $f = 1$), the order quantity Q is maximum. This is because when the whole lot is inspected, all the defective products in the lot are identified which are scraped or returned resulting in increased order quantity. It can be seen that the model with no defective items (i.e., $E(\alpha) = 0$) leads to the maximum profit while the no inspection model (i.e., $f = 0$) leads to the minimum profit. As can be seen in Figure 3.4, the model that inspects a small portion of the batch (i.e., $f = 0.15$) leads to better results than models that do not inspect the batch. Increasing the value of f, the relative advantage of the model with sample inspection can be increased to the model with full inspection (i.e., $f = 1$).

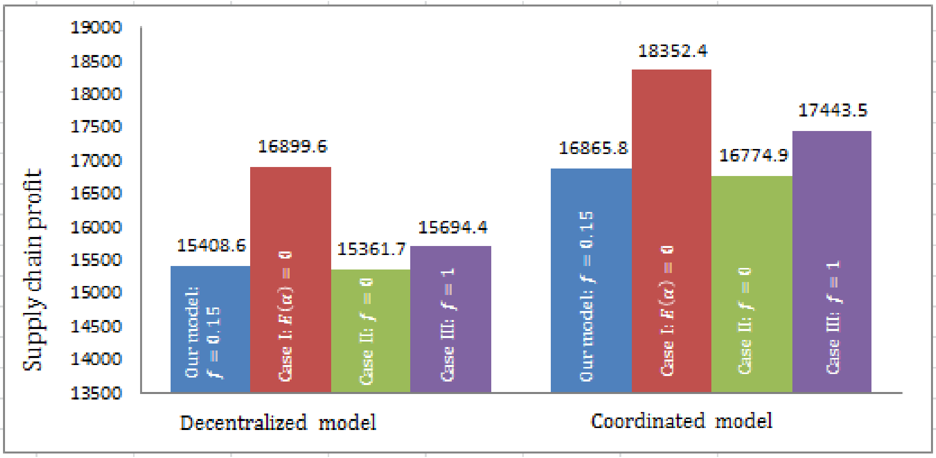

Figure 3.4: Supply chain profit for different cases

3.4.2 Sensitivity analysis

This subsection is dedicated to analyze the sensitivity of the parameters on the decision variables. To check the effect of the parameters, we will change the value of one parameter at a time. That is, all other parameters remain the same when we check the sensitivity of a parameter.

3.4.2.1 Effect of price elasticity coefficient b

Figure 3.5 shows the changes in order quantity Q, selling price s, and greening level g with changes in price elasticity coefficient b. From Figure 3.5 it is seen that increasing value of b leads to a decrease in the greening level and selling price. This indicates that if the demand for a product becomes more price-sensitive, the emphasis should be on reducing the selling price rather than increasing the amount of investment for green innovation. In short, when the demand is more price-sensitive, the manager should attract the customer by decreasing the selling price.

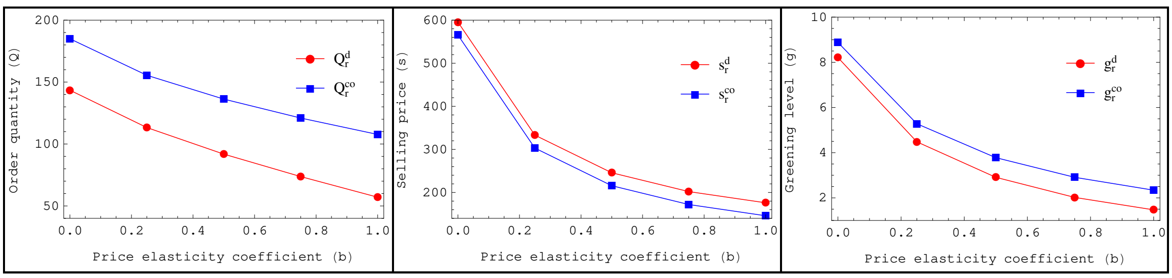

Figure 3.5: Optimal decisions for different price elasticity coefficient b

On the other hand, when the demand is less sensitive to the price, the manager should attract the customer by increasing the greening level. Such a situation could occur, when a new company emerges in the high-tech market, the number of technologically different products in the market decreases, resulting in increased product rejection based on selling price among customers. This is where coordination becomes more important because the coordination policy can attract customers by reducing the selling price of the product as compared to the decentralization policy which can be seen from Figure 3.5. From Figure 3.5, an adverse effect of b on the order quantity is noticed. As the value of b increases, the optimal order quantity decreases.

In general, increasing customers' sensitivity to product price greatly increases the risk of losing customers for any company which reduces the amount of profit earned. Although it is possible to hold the market share by lowering the price of the product, it reduces the sales revenue. So in this case, ordering less would be a wise thing to do, which would help to reduce holding costs, and hence the company will be protected from huge profit losses.

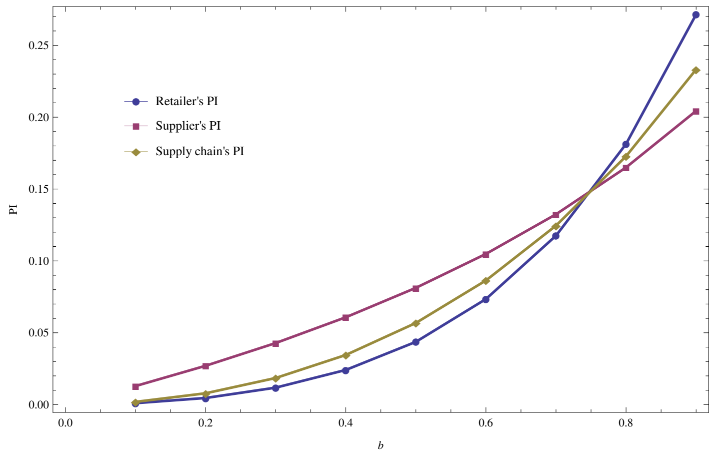

Figure 3.6: PI with respect to price elasticity coefficient b

To describe the benefits of trade-credit coordination, when market demand is more price-sensitive, we calculated PI for the retailer, supplier, and chain using the formula $PI = \frac{Profit\ in\ coordination\ model - Profit\ in\ decentralized\ model}{Profit\ in\ coordination\ model} \times 100\%$ and depicted in Figure 3.6. Figure 3.6 shows that PIs for the retailer, the supplier, and the supply chain increase strongly with few increase in the price elasticity parameter (b). After this observation, we can conclude that the more price-sensitive the demand is, the more profitable the coordination is.

3.4.2.2 Effects of greening elasticity coefficient μ

Figure 3.7 shows the path of greening level (g) with changes in its elasticity coefficient (μ) in demand function. It can be seen that greening level increases as the

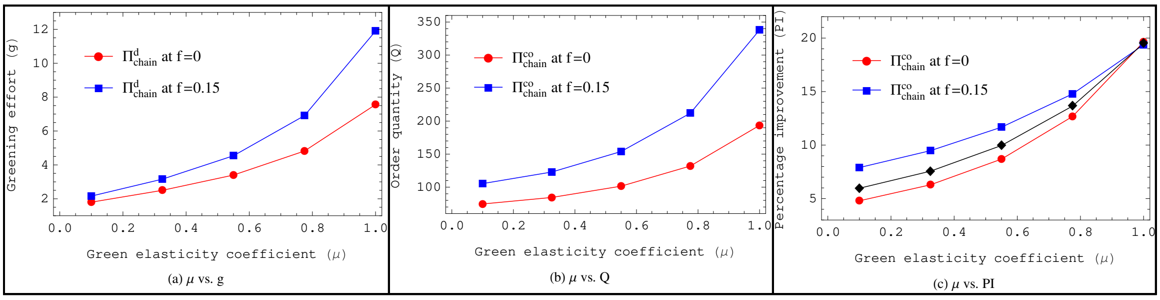

Figure 3.7: Optimal decisions for different green elasticity coefficient μ

sensitivity of demand in the greening level increases. The level of greening in the coordinated model is always higher than in the decentralized model, which indicates that for the same effect of greening level on demand function, the retailer applies more green innovation effort under the coordinated model than the decentralized model. From Figure 3.7, a favorable effect of μ on order quantity (Q) is noticed. As the value of μ increases, the optimal order quantity increases. In general, the higher the customer perception of greening, the higher the demand for green products resulting in increased order quantity. PI increases with increasing sensitivity of greening elasticity coefficient.

3.4.2.3 Effects of defective percentage $E[\alpha]$

To understand how the profit changes as a result of the inclusion of defective items in the batch received by the buyer, different defective rates have been considered. The results are shown in Figure 3.8.

It is seen that profits of both the supply chain and its members have been declined as the defective rate has increased, which is similar to what was obtained by Chang and Ho, 2010 for their EOQ-type model. However, it can be seen that the profits of the retailer and the supplier in the coordinated model are always higher than that of the decentralized model. Importantly, the bargaining space between the retailer and the supplier increases when $E[\alpha]$ increases. This happens as the rate of increment of T_{cmax} is more than that of T_{cmin}. This indicates that if a batch contains a large percentage of defective items, the credit period given to the supplier should be extended. Implementing this credit period, both the retailer and the supplier can

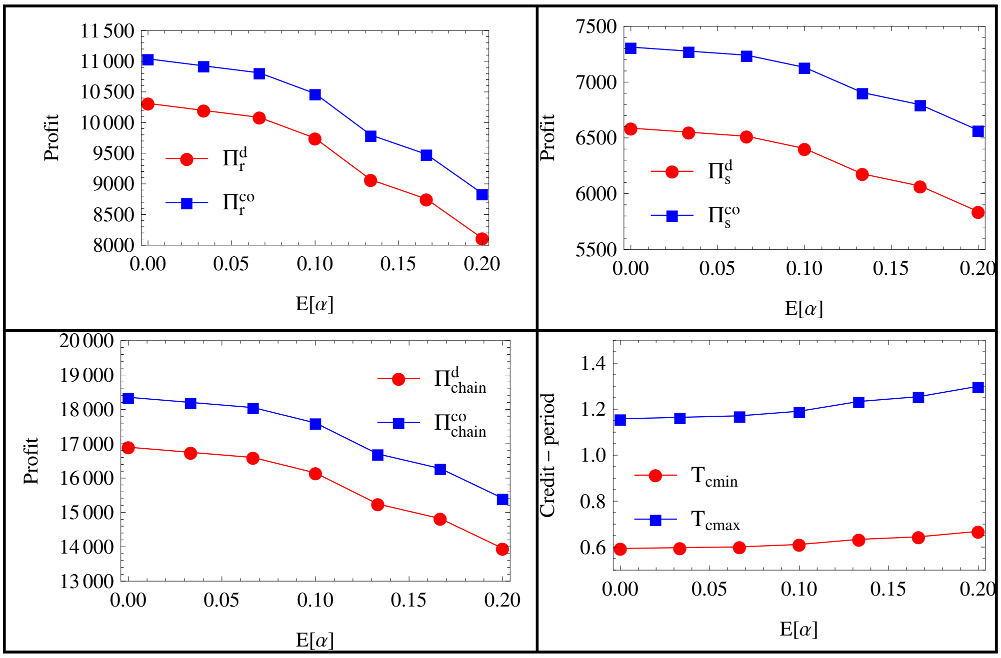

Figure 3.8: Changes in profit and credit-period when $E[\alpha]$ changes

enhance their profits when this kind of situation occurs. Therefore, it can be concluded that the coordinated model certainly is of great benefit for both the retailer and the supplier when defective items appear in a batch.

3.4.2.4 Effect of penalty cost (w) on inspection scenarios

It is interesting to show the importance of inspecting the lot when defective units are considered in an inventory model. We now investigate inspection scenarios depend-

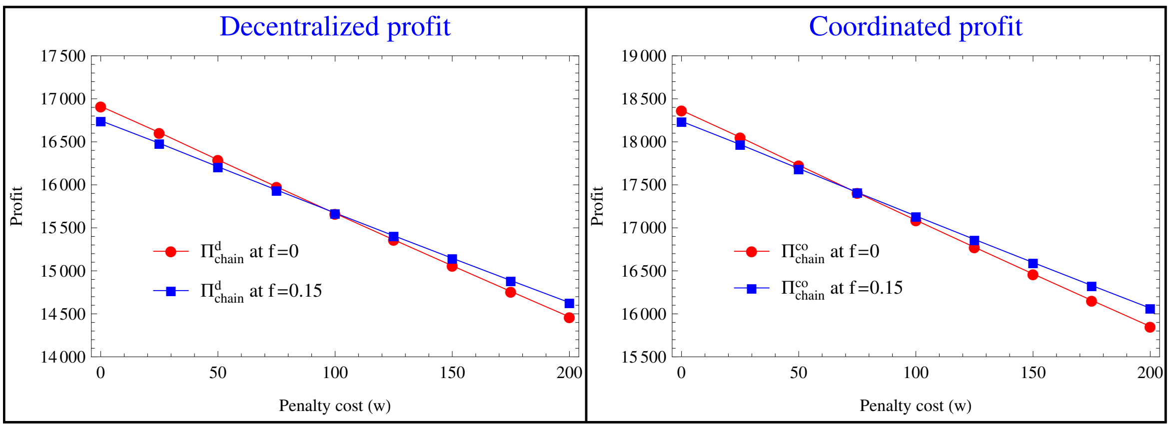

Figure 3.9: Changes in profit when w changes

ing on unit penalty cost of unidentified defective items (w). To this aim, we set the range of w from 1 to 160 and carried out similar experiments for $f = 0$, i.e., the case without inspection, $f = 0.15$, i.e., the case with inspection. The results are depicted in Figure 3.9. As shown in Figure 3.9, the expected total profits of the supply chain in the decentralized and the coordinated scenarios decrease with an increase in w. Figure 3.9 shows that for the decentralized model when $w < 124$ (approximately), the optimum profit is being obtained by adopting without inspection strategy. In other words, when $w < 124$, inspection of items would not be a viable option. While on the other hand, when w exceeds a critical threshold value of 124, it is profitable to inspect a portion of the lot. For the coordinated model, the critical threshold of w is 58, i.e., when $w < 58$ it will not be profitable to inspect the lot but for $w > 58$ inspecting a lot will be profitable. This implies that under the coordinated model, the retailer may decide to inspect a batch for a smaller value penalty cost than the value in the decentralized model. The profit of the coordinated supply chain for different values of f with increasing w is depicted in Figure 3.10. From Figure 3.10 it is seen that after a certain value of w, inspecting the whole lot will be profitable for the supply chain. So, when the penalty cost is relatively low, the manager should not invest money in inspections but rather it is profitable to sell the items in the market without any inspection. It can be concluded that the coordinated supply chain performs better over the decentralized supply chain in the case of green product.

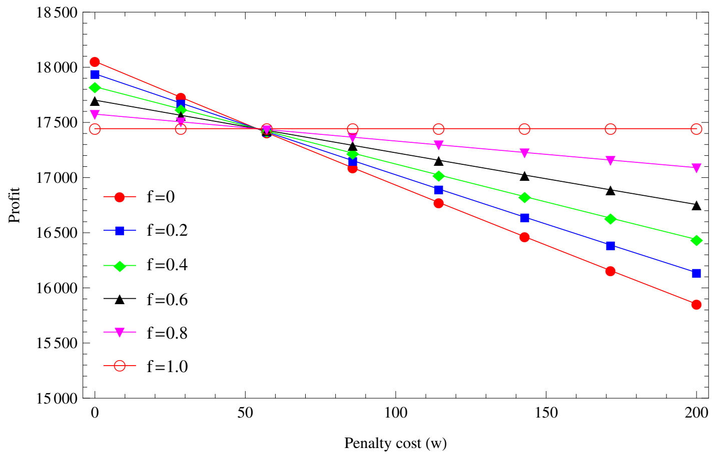

Figure 3.10: Changes in profit for different f and w

3.4.2.5 Effect of inspection fraction f

Figure 3.11 illustrates the effects of inspection portion (f) on optimal Q and PI for different values of penalty cost w. From Figure 3.11, it is worthwhile to notice that the order quantity for both the decentralized and the coordinated models increases as the inspection portion increases. The result is reasonable because the more items we inspect, the more likely we can remove defective items from each batch, which

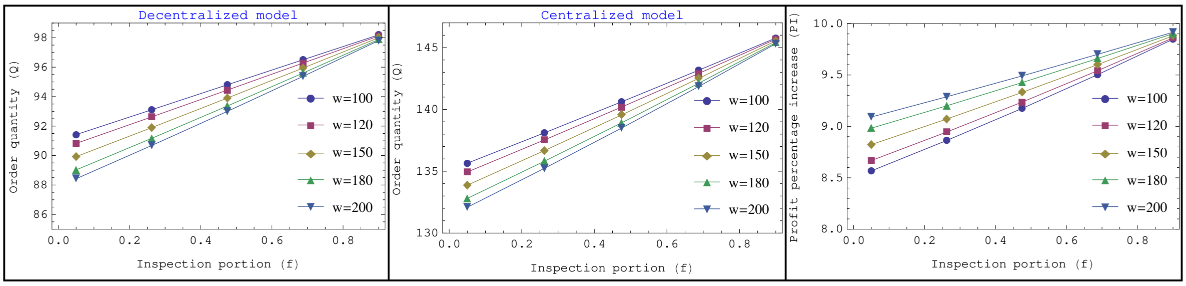

Figure 3.11: Variation of the optimal order quantity and PI when f changes

increases the order quantity. The percentage improvement, PI, increases as the inspection fraction (f) increases. This indicates that coordination between SC members is very important for deciding how many items of should be inspected when there are defective items in a batch. To describe this, we can consider industries such as automotive, aerospace, and nuclear where the presence of defective items in a lot can be fatal. In these cases, product inspection is very crucial, and sometimes a full batch inspection has to be done so that no defective item is delivered. So the managers have to be very careful about the inspection strategy depending on where the product is being shipped.

3.4.3 Managerial insights

Green marketing is a term prevalent in recent times. Today, various studies have shown that more and more people are interested in conscious use and willing to pay a little more for products or services that contribute to the environment. This is where ecological marketing comes into play, a tool that allows companies to communicate sustainable production strategies. The result illustrates the strategic and core insights for the managers in managing the green supply chain under sample inspection. From the numerical analysis, it is observed that the greening level of the product increases as the product becomes more green sensitive. Although the growth rate has been observed to be higher in the coordinated model than in the decentralized model. So it can be concluded that the supply chain manager should put a lot of emphasis on establishing coordination among the supply chain members to produce environmentally friendly products. Our observations revealed that increasing the greening level of the product is perhaps the most efficient approach to increase SC profit when market demand is relatively less price sensitive. Thus, to increase SC profit managers should increase the effort to increase the greening level.

In today's competitive market, the manager has to constantly think of new strategies to attract customers to the product. One strategy is an effective pricing strategy which is very much essential for continued sales success. From the consumer's point of view, relatively low priced products have a lot of demand in the market. For that, within companies, it is important to have good logistics and great coordination. Our numerical analysis has shown that due to lack of coordination between the supplier and the retailer in the decentralized policy, the selling price has increased drastically resulting in a significant reduction in SC's profit. On the other hand, if the coordination between the two parties is completed, it is possible to increase the service level of the company by delivering the products to the customers at a much lower price. Thus, the manager should pay special attention to establish coordination among the members of the supplier chain. While the decline in product prices can improve market share, on another side it also reduces the sales revenue. So in this case, managers should order fewer items which would help to reduce holding costs, and hence the company will be protected from huge profit losses.

It is very important to check the quality and quality assurance of the products before selling any product in the market and there are two ways to perform the quality check operations. One of the most common conflicts encountered by an industry manager is to perform the quality check with either sampling inspection or 100% inspection (full inspection). In this chapter, we briefly explained the possible causes of quality inspection that an industry manager could encounter in practice. Any industry manager should choose the inspection method based on its own conditions. It is observed that, if the product is not inspected, the supply chain profit is minimal but maximum profit can be achieved with a full inspection. This means if the products are of high quality and if there is sufficient budget and time, then 100% quality inspection method should be chosen. On the other hand, if the volume of the product is high, the quality of the product is low, and the budget for inspection is a bit tight, or can be relied on probability based methods then sampling inspection would be the best choice for product quality inspection. Therefore, the SC managers have to cleverly emphasis on the inspection policy to obtain maximum profit. With the increase in the number of items inspected, the number of orders has increased significantly, indicating that all industry managers must be careful when dealing with the suppliers regarding defective items. It is obvious that the downstream channel member has to face some operating and financial obligations initially when it implements coordinated decisions. The retailer has to increase its order quantity, back-order quantity, and the level of green activity in the coordinated model. These will increase its operational costs. But the retailer willingly accepts coordinated decisions as it gets enough financial supports from the supplier through the trade-credit contract.

All of these have opened up a new opportunity for managers to explore the maximum total profit solution for a SC model, synchronizing coordinated policy with price and greening effort dependent demand under shortages and sampled inspection. Numerical and sensitivity analysis provides the insights needed to create the best decision policies for SC management that can achieve the best results based on economic and environmental performance.

3.5 Conclusions

Coordination among members has become an integral part of any SC system, and there are numerous examples to support the effectiveness of this policy in terms of overall profit performance. This chapter examined a single supplier-single retailer JELS model where credit-period as an incentive scheme was proposed for coordination. Each batch received by the supplier is supposed to contain a defective percentage, leading to the implementation of an inspection policy. We proposed a sample inspection policy where a portion of each batch is inspected instead of the full batch resulting in the retailer incurring penalty cost for the unidentified defective items. We formulated the SC model under three decision-making scenarios, namely decentralized, centralized, and coordinated. In all cases, we derived the expected total SC profit as well as the individual profit of the SC members, and an optimization problem was formulated. We then found the minimum and maximum satisfactory trade credit periods which enabled us to establish a win-win situation among the retailer and supplier. Several experiments were finally carried out to draw insights into how SC decisions modify in presence of defective items. These experiments also permitted to analyse the model sensitivity to changes in some parameter values.

From the results, it is clear that the coordination among the supply chain members has resulted in an improvement in the level of the greening of the product and profits of the supply chain. In general, the higher the customer perception of greening, the higher the demand for green products resulting in increased order quantity. According to our proposed trade-credit policy, retailers must be interested in participating in a joint decision-making policy using a lower trade-credit period, which is more acceptable in a real-world case. The proposed coordination mechanism could influence the retailer for altering its ordering policy. Under the proposed decision-making policy, it is observed that the consumers are likely to buy large quantities of products at a low selling price. Generally, in a low income economy, customers are more price sensitive than a developed economy. Hence, business firms have more opportunities to make a better profit in developed economic countries. A growing economy has high market potential and moderate price sensitive customers. As a result, firms have high chances to make business in growing countries. From the consumer's perspective, the consumer can enjoy a lower purchase price when supply chain decisions are made through a coordinated decision-making policy. A firm manager can earn more profit using full inspection policy if time and other circumstances for inspection permit. Following the proposed trade-credit coordination policy, a firm manager may increase its profitability by providing lower sales prices.

Chapter 4

Lead-time reduction in an integrated supply chain model with stochastic demand

4.1 Introduction

The last chapter discussed a two-echelon supply chain model for imperfect items where the replenishment lead-time was zero. However, in reality, the lead-time duration may primarily be affected by the number of items ordered and additional time for transportation, set-up, packaging, etc., (for example, the case of the serial production process). Hence, variable lead-time conceptualization in SC makes the model more practical and applicable to the industry. With this motivation, the present chapter* develops a continuous-review vendor-buyer integrated (SC) model wherein the lead-time (taken as replenished) is considered as a factor affected by the time stamp required for setup and production followed by transportation. The buyer receives normally distributed stochastic lead-time demand from its customers. Due to the stochastic nature of lead-time demand, shortages may arise on the buyer's side which is fully backlogged. We presume imperfect production at the vendor's end, which leads to the generation of a certain ratio/percentage of defective products, which results in additional warranty costs for the vendor. This chapter intends to uncover the best policy that minimizes the system's total expected cost by optimizing the order quantity, safety factor, investment amount, and the number of shipments.

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*This chapter is based on the work published in *Annals of Operations Research*, 2021; doi.org/10.1007/s10479-021-04105-0.

4.2 Preliminary aspects

In this section, we discuss in detail the problem statement of the inventory system under investigation. We consider that a single item is procured from one vendor. The buyer manages the inventory as per continuous-review policy and considering stochastic lead-time demand. Due to high setup costs, the vendor follows a single setup multiple delivery (SSMD) policy to manage his/her inventory system. The buyer places an order when the stock level touches the reorder point and thereafter the vendor starts to produce the order quantity. The buyer receives the batches after a certain lead-time that consists of three main complements: setup time, production time, and transportation time. The buyer has the option of investing some money to reduce the transportation time, which will help to reduce the replenishment lead-time. Besides, the buyer can reduce the replenishment lead-time by managing the order quantity. Each batch received by the buyer undergoes a human inspection process to ensure that there are no defective items in the batch. Defective items found during the inspection are sold with a warranty. The warranty cost is to be borne by the vendor as the vendor is responsible for the production.

4.2.1 Notation and assumptions

For mathematical model formulation, the following notation are used:

•Decision variables

Q	buyer's order quantity (units)
r_1	reorder point for the first batch (units)
r_2	reorder point for the batches $2, 3, ..., m$
k_1	safety factor for the first batch, equivalent to r_1
k_2	safety factor for the batches $2, 3, ..., m$, equivalent to r_2
W	capital expenditure to reduce transportation time
m	number of shipments

•Parameters

D	annual demand of buyer (units/year)
P	rate of production of vendor (units/year)
S_V	setup cost of vendor (\$/setup)
S_B	ordering cost of buyer (\$/order)

κ_{t0}	original transportation time per shipment before any investment (time unit)
κ_t	transportation time per shipment after investment (time unit)
s_t	setup time per setup at the vendor (time unit)
α_1	lead-time demand for the first batch, (random variable)
α_2	lead-time demand for batches 2, .., m, (random variable)
Y	percentage of defective items (random variable)
$f(y)$	probability density function of Y
$f(\alpha_1)$	probability density function of α_1
$f(\alpha_2)$	probability density function of α_2
x	screening rate
d	unit screening cost ($/unit item)
v	vendor's unit warranty cost of defective item ($/defective item)
σ	standard deviation of lead-time demand
•Functions	
$\kappa_t(W)$	reduced transportation time, a function of capital expenditure W
L_1	buyer's replenishment lead-time for the first shipment
L_1	buyer's replenishment lead-time for shipments 2, .., m,
Π_b	buyer's expected annual cost
Π_v	vendor's expected annual cost
Π_{sc}	joint expected annual cost of the system

We use the following assumptions while developing the model:

1. A single-buyer deals with a single-vendor for one type of product.

2. The buyer monitors the inventory level continuously. When the inventory level drops to the reorder point r, the buyer places an order. The reorder point = safety stock + expected demand during the lead-time (Glock, 2012a).

3. The buyer orders a quantity mQ and the vendor produces mQ units in one setup at a fixed production rate P, where $P > D$. After that, the vendor delivers Q quantity over m times (Ben-Daya and Hariga, 2004, Mou, Cheng, and Liao, 2017).

4. For the first shipment, the replenishment lead-time depends on production time, transportation time, and setup time, i.e., $L_1 = p_t + \kappa_t + s_t$; however the lead-time of the remaining batches depends only on transportation time, i.e.,

$L_2 = \kappa_t$ (see Hsiao, 2008a). Additional investment is made to reduce the lead-time through reducing transportation time.

5. At screening time, the number of perfect units is at least equal to the demand of the product.

6. To avoid shortages at the vendor's end, the production rate is considered more than the demand rate; however, due to stochastic lead-time demand, the buyer faces shortages that are fully backlogged (Glock, 2012a).

7. If the buyer wants to reduce the lead-time, then the additional cost has to be borne by the buyer.

4.3 Model formulation

Here we consider a vendor-buyer system where both of them try to cooperatively investigate the optimal lot-size, which minimizes the total cost of the system. The buyer follows a continuous-review inventory policy and places an order of mQ units when its inventory level drops to the reorder point. In order to reduce the production cost, the vendor produces mQ items at one go and transfers m batches of size Q each. Each lot is inspected at the buyer's end from which Y percentage of items are found to be defective which are separated and the rest $(Q - Y)$ non-defective items meet the customer's demand. Hence, $Q(1 - Y)/D$ is the ordering cycle length and $mQ(1 - Y)/D$ is the complete production cycle length (See 4.1).

The buyer's replenishment lead-time is a function of three main components.

1. Set-up time-time needed to prepare the machine for it to be ready for the production run such as changing molds tools, fixtures, etc.

2. Production time- time needed to produce a lot for delivery, which is a function of lot-size and production rate.

3. Transportation time- time needed to reach the lot to the buyer.

Hence, the lead-time takes the following form:

$$L = p_t + s_t + \kappa_t, \tag{4.1}$$

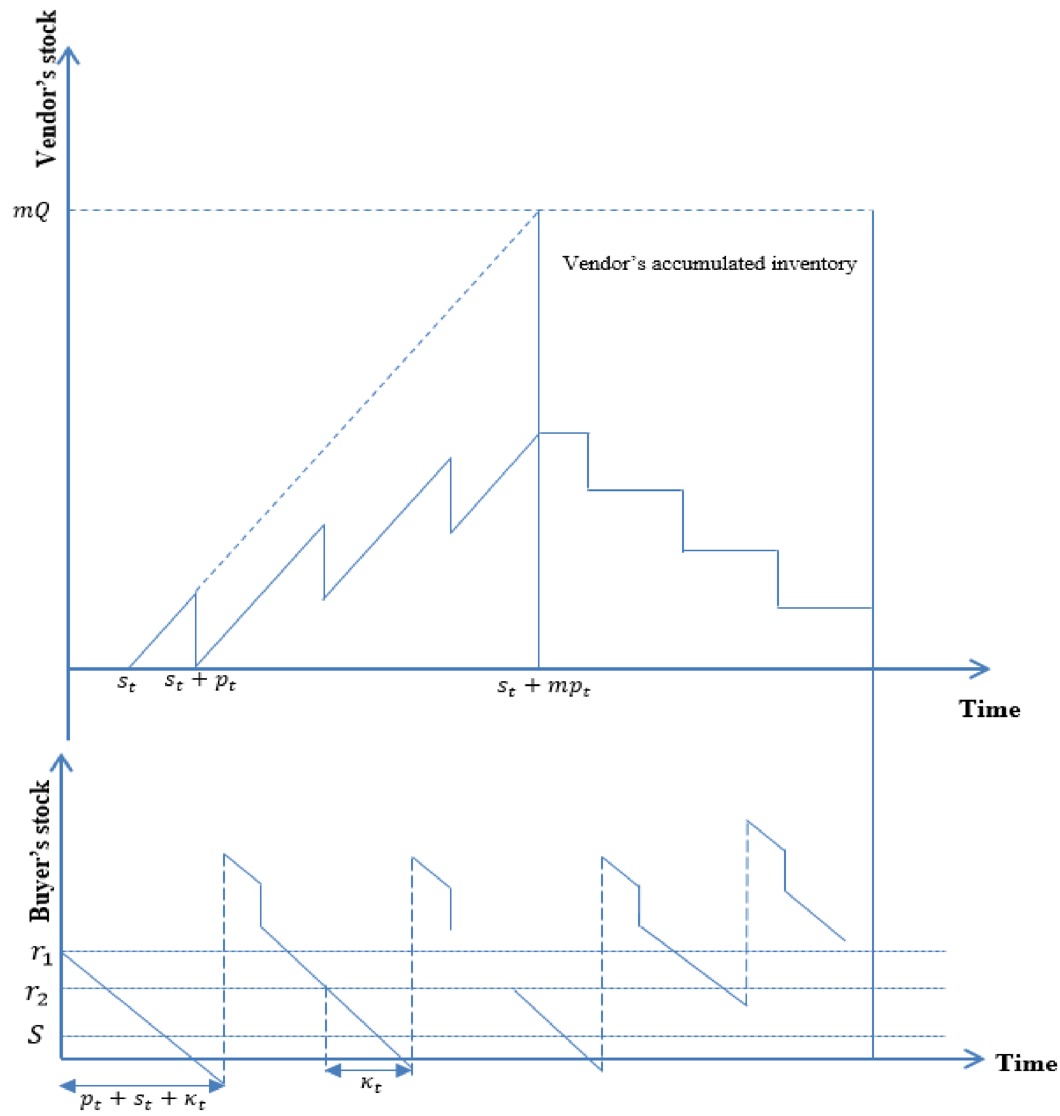

Figure 4.1: Vendor-Buyer's Inventory level

where p_t is the vendor's production lead-time required to produce Q units, s_t is the fixed setup time needed to setup the machine for a production run, and κ_t represents transportation lead-time.

Since lead-time is a function of transportation time, the vendor can control the transportation time using fast or slow shipping mode. Here we assume that transportation lead-time is a strictly decreasing function of capital expenditure W:

$$\kappa_t(W) = \kappa_{t0}e^{-aW} \tag{4.2}$$

where $W = W_i, i = 0, ..., n$; κ_{t0} is the original transportation time per shipment, and a is the fixed parameter that can be estimated using previous data.

Therefore, (4.1) becomes

$$L(Q, W) = \frac{Q}{P} + \kappa_{t0}e^{-aW} + s_t \tag{4.3}$$

According to Assumption 4, we have two different replenishment lead-times: lead-time for the first batch, which is $L_1 = \frac{Q}{P} + \kappa_{t0}e^{-aW} + s_t$, and lead-time for the rest of the each batch is $L_2 = \kappa_{t0}e^{-aW}$.

Following Hadley and Whitin, 1963 model, the expected average inventory level for the first shipment is

$$\frac{Q - Y}{2} + r_1 - DL_1$$

While, for shipments $2, .., m$, it is

$$\frac{Q - Y}{2} + r_2 - DL_2$$

Moreover, Hsiao, 2008a stated that the safety stock is identical for all cycles. Hence, we can write

$$r_1 - DL_1 = r_2 - DL_2 \tag{4.4}$$

Taking into reference of r_1, we have

$$r_2 = r_1 - D(L_1 - L_2) \tag{4.5}$$

Moreover, we can write safety stock as a function of safety factor, i.e., $r_1 - DL_1 = k_1\sigma\sqrt{L_1}$ and $r_2 - DL_2 = k_2\sigma\sqrt{L_2}$.

Now, from the relation (4.4), we have $k_1\sigma\sqrt{L_1} = k_2\sigma\sqrt{L_2}$ which implies

$$k_2 = k_1\sqrt{\frac{L_1}{L_2}} \tag{4.6}$$

Therefore, the average inventory level per replenishment cycle of non-defective items for the buyer can be obtained as

$$\frac{Q(1 - Y)}{2} + k_1\sigma\sqrt{L_1} \tag{4.7}$$

Hence, the expected holding cost (for defective and non-defective items) per replenishment cycle is

$$\begin{aligned} & h_B T\left[\frac{Q(1-Y)}{2}+k_1\sigma\sqrt{L_1}\right]+\frac{h_B Y Q^2}{x} \\ = \; & h_B\frac{Q(1-Y)}{D}\left[\frac{Q(1-Y)}{2}+k_1\sigma\sqrt{L_1}\right]+\frac{h_B Y Q^2}{x} \end{aligned} \tag{4.8}$$

where $\frac{h_B Y Q^2}{x}$ is the buyer's total holding cost for defective items.

The expected shortage quantity for the first shipment is B_1 while for shipments $2, ..., m$ is B_2. Therefore, the expected shortage quantity for m replenishment cycles is

$$B_1+(m-1)B_2.$$

Hence, the expected shortage quantity per replenishment cycle is

$$\frac{1}{m}\left[B_1+(m-1)B_2\right].$$

Therefore, the expected shortage cost is

$$\frac{\pi}{m}\left[B_1+(m-1)B_2\right].$$

Therefore, the buyer's total cost per replenishment cycle can be obtained as follows:

$$\begin{aligned} TC_B &= \text{Ordering cost} + \text{transportation cost} + \text{inspection cost} + \text{holding cost} \\ &+ \text{backorder cost} + \text{lead-time crashing cost} \\ &= S_B+F+Qd+h_B\frac{Q(1-Y)}{D}\left[\frac{Q(1-Y)}{2}+k_1\sigma\sqrt{L_1}\right]+\frac{h_B Y Q^2}{x} \\ &+ \frac{\pi}{m}[B_1+(m-1)B_2]+TW \end{aligned} \tag{4.9}$$

Now the buyer's expected total cost per replenishment cycle can be calculated as

$$\begin{aligned} E[TC_B] &= S_B+F+Qd+\frac{h_B Q^2 E(1-Y)^2}{2D}+h_B\frac{Q(1-E[Y])}{D}k_1\sigma\sqrt{L_1} \\ &+ \frac{m h_B E[Y] Q^2}{x}+\frac{\pi}{m}[B_1+(m-1)B_2]+WE[T] \end{aligned} \tag{4.10}$$

and the expected duration of a replenishment cycle is $E[T] = \frac{(1-E[Y])Q}{D}$. Then the buyer's expected total cost per time unit is

$$\begin{aligned}\Pi_b(Q,k_1,W) &= \frac{E[TC_B]}{E[T]} \\ &= \frac{S_B D}{m(1-E[Y])Q} + \frac{FD}{(1-E[Y])Q} + h_B\left[\frac{QE(1-Y)^2}{2(1-E[Y])} + k_1\sigma\sqrt{L_1}\right] \\ &+ \frac{dD}{1-E[Y]} + \frac{\pi D}{mQ(1-E[Y])}\{B_1(r_1,L_1)+(m-1)B_2(r_2,L_2)\} \\ &+ \frac{h_B E[Y]DQ}{x(1-E[Y])} + W \qquad (4.11)\end{aligned}$$

On the other side, the vendor's average inventory is (Goyal, 1977):

$$\begin{aligned}&\frac{1}{mE[T]}\left\{\left[mQ\left(\frac{Q}{P}+(m-1)E[T]\right) - \frac{mQ(mQ/P)}{2}\right] - E[T][Q+2Q+......+(m-1)Q]\right\} \\ =& \frac{D}{mQ(1-E[Y])}\left[mQ\left(\frac{Q}{P}+(m-1)\frac{Q(1-E[Y])}{D}\right) - \frac{m^2Q^2}{2P}\right] - \frac{Q(1-E[Y])m(m-1)Q}{2D} \\ =& \frac{Q}{2} + \frac{(m-2)Q}{2}\left(1-\frac{D}{(1-E[Y])P}\right) \qquad (4.12)\end{aligned}$$

Thus, the vendor's expected holding cost per unit time is

$$h_V\left\{\frac{Q}{2}+\frac{(m-2)Q}{2}\left(1-\frac{D}{(1-E[Y])P}\right)\right\} \qquad (4.13)$$

The set-up cost per unit time for the vendor is $S_V D/mQ(1-Y)$.
Warranty cost per item is v. Thus, the vendor's warranty cost per unit time is

$$v\frac{mYQ}{mE[T]} = \frac{vDE[Y]}{(1-E[Y])}.$$

Therefore, the vendor's expected total cost per time unit can be obtained as follows (Huang, 2004):

$$\Pi_v(m) = \frac{S_V D}{m(1-E[Y])Q} + \frac{vDE[Y]}{(1-E[Y])} + \left\{\frac{Q}{2}+\frac{(m-2)Q}{2}\left(1-\frac{D}{(1-E[Y])P}\right)\right\}h_V \qquad (4.14)$$

Thus, the joint expected total system cost per time unit of the SC is

$$\begin{aligned}\Pi_{sc}(Q,k_1,W,m) &= \Pi_b(Q,k_1,W)+\Pi_v(m)\\ &= \left\{\left(\frac{S_B+S_V}{mQ}\right)D+\frac{FD}{Q}+(d+vE[Y])D-\frac{(m-2)QDh_V}{2P}\right\}\\ &\quad\times\left(\frac{1}{1-E[Y]}\right)+\frac{h_BE[Y]DQ}{x(1-E[Y])}+h_B\left[\frac{QE(1-Y)^2}{2(1-E[Y])}+k_1\sigma\sqrt{L_1}\right]\\ &+\frac{(m-1)Q}{2}h_V+\frac{\pi D}{mQ(1-E[Y])}\{B_1+(m-1)B_2\}+W \qquad (4.15)\end{aligned}$$

The aim is to identify the safety factor (k_1), order quantity (Q), transportation time crashing cost (W), and the number of shipments (m) that minimize the expected total system cost per time unit.

Shortages occur when the lead-time demand is larger than the reorder point. So, the expected shortage quantity at first batch is

$$B_1=\int_{r_1}^{\infty}(\alpha_1-r_1)g(\alpha_1)d\alpha_1, \qquad (4.16)$$

where $g(\alpha_1)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\alpha_1-\mu)^2}{2\sigma^2}}$, μ and σ being the mean and standard deviation, respectively.

For mean lead-time demand (DL_1) and standard deviation $(\sigma\sqrt{L_1})$, the first batch's expected shortage quantity is

$$B_1=\int_{r_1}^{\infty}\frac{(\alpha_1-r_1)}{\sqrt{2\pi}\sigma\sqrt{L_1}}e^{-\frac{1}{2}\left(\frac{\alpha_1-DL_1}{\sigma\sqrt{L_1}}\right)^2}d\alpha_1, \qquad (4.17)$$

Assuming $z=\frac{\alpha_1-DL_1}{\sigma\sqrt{L_1}}$ and $k_1=\frac{r_1-DL_1}{\sigma\sqrt{L_1}}$, (4.17) becomes

$$B_1=\sigma\sqrt{L_1}\int_{k_1}^{\infty}(z-k_1)\phi(z)dz \qquad (4.18)$$

where $\phi(z)$ is a standard normal pdf. Presuming $\Psi(k_1)=\int_{k_1}^{\infty}(z-k_1)\phi(z)dz$, (4.18) becomes

$$B_1=\sigma\sqrt{L_1}\Psi(k_1) \qquad (4.19)$$

Similarly, for each of the remaining $(m-1)$ batch, the expected shortage quantity is

$$B_2 = \int_{r_1}^{\infty} (\alpha_2 - r_2) g(\alpha_2) d\alpha_2 = \sigma\sqrt{L_2}\Psi(k_2), \tag{4.20}$$

where $\Psi(k_2) = \int_{k_2}^{\infty}(z-k_2)\phi(z)dz$.

Using (4.19) and (4.20), equation (4.15) becomes

$$\begin{aligned} \Pi_{sc}(Q,k_1,W,m) &= \left\{\left(\frac{S_B+S_V}{mQ}\right)D + \frac{FD}{Q} + (d+vE[Y])D - \frac{(m-2)QDh_V}{2P}\right\}\left(\frac{1}{1-E[Y]}\right) \\ &+ \frac{h_B E[Y]DQ}{x(1-E[Y])} + \frac{(m-1)Q}{2}h_V + h_B\left[\frac{QE(1-Y)^2}{2(1-E[Y])} + k_1\sigma\sqrt{L_1}\right] \\ &+ \frac{\pi D}{mQ(1-E[Y])}\left\{\sigma\sqrt{L_1}\Psi(k_1) + (m-1)\sigma\sqrt{L_2}\Psi(k_2)\right\} + W \end{aligned} \tag{4.21}$$

Substituting $L_1 = \frac{Q}{P} + \kappa_{t0}e^{-aW} + s_t$ and $L_2 = \kappa_{t0}e^{-aW}$, (4.21) becomes

$$\begin{aligned} \Pi_{sc}(Q,k_1,W,m) &= \left\{\left(\frac{S_B+S_V}{mQ}\right)D + \frac{FD}{Q} + (d+vE[Y])D - \frac{(m-2)QDh_V}{2P}\right\}\left(\frac{1}{1-E[Y]}\right) \\ &+ \frac{h_B E[Y]DQ}{x(1-E[Y])} + \frac{(m-1)Q}{2}h_V + h_B\left[\frac{QE(1-Y)^2}{2(1-E[Y])} + k_1\sigma\sqrt{\frac{Q}{P} + \kappa_{t0}e^{-aW} + s_t}\right] \\ &+ \left\{\sigma\sqrt{\left(\frac{Q}{P} + \kappa_{t0}e^{-aW} + s_t\right)}\Psi(k_1) + (m-1)\sigma\sqrt{(\kappa_{t0}e^{-aW})}\Psi(k_2)\right\} \\ &\times \frac{\pi D}{mQ(1-E[Y])} + W \end{aligned} \tag{4.22}$$

Due to the presence of highly complicated terms in (4.22), analytical proof of convexity by considering all decision variables at a time is not possible. Therefore, a sequential search algorithm will be used to obtain the optimal values of the decision variables. Here we use the similar approach as of Moon and Cha, 2005.

Lemma 4.1. *For given values of (k_1, W, m), Π_{sc} is not convex in Q but the near-optimal solution can be determined uniquely and have either the possible minimum lot size Q_{min} or Q that satisfies $\frac{\partial \Pi_{sc}}{\partial Q} = 0$.*

Proof. We have the second order partial derivative derivative of Π_{sc} with respect to Q as

$$\frac{\partial^2 \Pi_{sc}}{\partial Q^2} = L_1^{-3/2}\left[L_1\left\{\frac{E_5}{Q^2} + \frac{E2}{Q^2} + \frac{2(E_4+\sqrt{L_1}E_3)}{Q^3}\right\} + \frac{1}{2P}\left(\frac{E5}{Q} + \frac{E2}{Q} + \frac{E4}{Q^2}\right) - \frac{E1}{2P}\right] \tag{4.23}$$

where $E_1 = \frac{\sigma k_1 h_B}{2P} > 0$,
$E_2 = \frac{\sigma\pi D\Psi(k_1)}{2mP(1-E[Y])} > 0$,
$E_3 = \frac{D}{1-E[Y]}\left\{F + \frac{S_B+S_V}{m} + \left(1-\frac{1}{m}\right)\pi\sigma\sqrt{\kappa_{t0}e^{-aW}}\Psi(k_2)\right\} > 0$,

$E_4 = \frac{D\sigma\pi}{m(1-E[Y])}\left(\kappa_{t0}e^{-aW} + s_t\right)\Psi(k_1) > 0,$
$E_5 = \frac{(m-1)\sigma\pi D k_1[1-\Phi(k_2)]}{2mP(1-E[Y])} > 0.$

As the first and second terms of (4.23) within the bracket converge to zero for large Q, and therefore it is obvious that $\frac{\partial^2\Pi_{sc}}{\partial Q^2} < 0$ for large value of Q, and hence clearly $\left|\frac{\partial^2\Pi_{sc}}{\partial Q^2}\right|_{Q=\infty} = 0$. Therefore, $\Pi_{sc}(Q, k_1, W, m)$ is not convex in Q.

The first-order partial derivative of $\Pi_{sc}(Q, k_1, W, m)$ with respect to Q for given $k_1, W,$ and m is given by

$$\begin{aligned}\frac{\partial\Pi_{sc}}{\partial Q} &= \left\{-\left(\frac{S_B+S_V}{mQ^2}\right)D - \frac{FD}{Q^2} - \frac{(m-2)Dh_V}{2P}\right\}\left(\frac{1}{1-E[Y]}\right) + \frac{(m-1)}{2}h_V + \frac{h_B k_1\sigma}{2P\sqrt{L_1}} \\ &+ \frac{h_B}{(1-E[Y])}\left(\frac{E[Y]D}{x} + \frac{E(1-Y)^2}{2}\right) - \frac{\pi D}{mQ^2(1-E[Y])}\Bigg\{\sigma\sqrt{L_1}\Psi(k_1) + (m-1)\sigma\sqrt{L_2} \\ &\Psi\left(k_1\sqrt{\frac{L_1}{L_2}}\right)\Bigg\} + \frac{\pi D}{mQ(1-E[Y])}\left\{\frac{\sigma\Psi(k_1)}{2P\sqrt{L_1}} - \frac{(m-1)k_1\sigma\left[1-\Phi\left(k_1\sqrt{\frac{L_1}{L_2}}\right)\right]}{2P\sqrt{L_1}}\right\}\end{aligned}$$

which can be rewritten as

$$\frac{\partial\Pi_{sc}}{\partial Q} = H(m) + L_1^{-1/2}\left(E_1 - \frac{E_2}{Q} - \frac{\sqrt{E_3 L_1}}{Q^2} - \frac{E_4}{Q^2} - \frac{E_5}{Q}\right) \tag{4.24}$$

where $H(m) = \frac{h_V}{2}\left(m - 1 - \frac{D(m-2)}{P(1-E[Y])}\right) + \frac{h_B}{1-E[Y]}\left(\frac{E[(1-Y)^2]}{2} + \frac{DE[Y]}{x}\right) > 0.$

In (4.24), the last four terms within the bracket converge to zero for large Q and it is certain that $\left|\frac{\partial\Pi_{sc}}{\partial Q}\right|_{Q=\infty} = H(m) > 0.$

$$\begin{aligned}\text{If } \left|\frac{\partial\Pi_{sc}}{\partial Q}\right|_{Q=1} &= H(m) + \left(E_1 - E_2 - \sqrt{E_3\left(\frac{1}{P} + \kappa_{t0}e^{-aW} + s_t\right)} - E_4 - E_5\right) \\ &\times \left(\frac{1}{P} + \kappa_{t0}e^{-aW} + s_t\right)^{-1/2} \geq 0\end{aligned}$$

then $\frac{\partial \Pi_{sc}}{\partial Q} \geq 0$ for all Q which implies that Π_{sc} is a strictly increasing function of Q and the optimal solution is the possible minimum lot size Q_{min}.

$$\text{If } \left|\frac{\partial \Pi_{sc}}{\partial Q}\right|_{Q=1} = H(m) + \left(E_1 - E_2 - \sqrt{E_3\left(\frac{1}{P} + \kappa_{t0}e^{-aW} + s_t\right)} - E_4 - E_5\right) \times \left(\frac{1}{P} + \kappa_{t0}e^{-aW} + s_t\right)^{-1/2} < 0$$

then it is clear that the sign of the first-order partial derivative changes from negative to positive only once for $1 \leq Q < \infty$. This means that $\frac{\partial \Pi_{sc}}{\partial Q} = 0$ has a unique solution for fixed k_1, W, and m. The sign of the first-order derivative indicates that Π_{sc} gradually increases after a point of inflection. Thus, for fixed k_1, W, and m, there exists an optimal solution that can be determined uniquely in Q.

Hence, Lemma 4.1 is proved. □

Equating $\frac{\partial \Pi_{sc}}{\partial Q}$ equal to 0 and solving it we have the optimal order quantity as

$$Q = \sqrt{\frac{\left(\frac{D}{1-E[Y]}\right)\left[F+\frac{S_B+S_V}{m}+\frac{\pi_0\sigma}{m}\left\{\sqrt{L_1}\Psi(k_1)+(m-1)\sqrt{L_2}\Psi\left(k_1\sqrt{\frac{L_1}{L_2}}\right)\right\}\right]}{H(m)+\frac{\sigma}{2P\sqrt{L_1}}\left[h_B k_1+\frac{\pi_0 D}{mQ(1-E[Y])}\left\{\Psi(k_1)-k_1(m-1)\left(1-\Phi\left(k_1\sqrt{\frac{L_1}{L_2}}\right)\right)\right\}\right]}} \tag{4.25}$$

Lemma 4.2. *For fixed (Q, W, m), Π_{sc} is convex w.r.t. k_1 and the optimal k_1 must satisfy* $\frac{\partial \Pi_{sc}}{\partial k_1} = 0$.

Proof. Now, evaluating first and second order partial derivatives of Π_{sc} w.r.t. k_1:

$$\frac{\partial \Pi_{sc}}{\partial k_1} = h_B\sigma\sqrt{L_1} - \frac{\pi D\sigma}{mQ}\left\{1 - \Phi(k_1) + (m-1)\left[1 - \Phi\left(k_1\sqrt{\frac{L_1}{L_2}}\right)\right]\right\}\sqrt{L_1}$$

and

$$\frac{\partial^2 \Pi_{sc}}{\partial k_1^2} = \frac{\pi D\sigma}{mQ}\left[\phi(k_1) + (m-1)\sqrt{\frac{L_1}{L_2}}\phi\left(k_1\sqrt{\frac{L_1}{L_2}}\right)\right]\sqrt{L_1}$$

Since $\frac{\partial^2 \Pi_{sc}}{\partial k_1^2} > 0$, we can conclude that the cost function Π_{sc} is convex w.r.t. k_1. Hence, Lemma 4.2 is proved. □

Setting $\frac{\partial \Pi_{sc}}{\partial k_1} = 0$ and solving for k_1, we have

$$1 - \Phi(k_1) + (m-1)\left[1 - \Phi\left(k_1\sqrt{\frac{L_1}{L_2}}\right)\right] = \frac{mQh_B}{\pi D} \tag{4.26}$$

One can see that the expressions in (4.25) and (4.26) are not independent of each other, i.e., for example, Q is required to calculate k_1 which in turn is a precondition for calculating the value of Q. Following the methodology as of Ben-Daya and Hariga, 2004 to calculate the value of Q, we can then find the value of k_1 from (4.26). The process will continue until a sufficiently stable solution is found. Note that we will run the algorithm for each value of W_i, $i = 0, 1, 2, ..., n$.

4.3.1 Solution algorithm

Step 1: Set $m = 1$.

Step 2: For each W_i, $i = 0, 1, 2, ..., n$, perform 2a to 2c.

2a: Set $Q_i = \left\lceil \sqrt{\frac{\frac{D}{1-E[Y]}\left(F + \frac{S_B + S_V}{m}\right)}{H(m)}} \right\rceil$. If $Q_i < Q_{min}, Q_i = Q_{min}$.
Compute k_{1i} from (4.26).

2b: Check the sign of $\left|\frac{\partial \Pi_{sc}}{\partial Q_i}\right|_{Q_i=1}$.
If $\left|\frac{\partial \Pi_{sc}}{\partial Q_i}\right|_{Q_i=1} \geq 0, Q_i^{'} = Q_{min}$.
Otherwise, compute $Q_i^{'}$ from (4.25).
If $Q_i^{'} < Q_{min}, Q_i^{'} = Q_{min}$.
Otherwise, set $Q_i^{'} = \left\lceil Q_i^{'} \right\rceil$.
Compute $k_{1i}^{'}$ from (4.26).

2c: If $Q_i^{'} = Q_i$, stop iteration and calculate $\Pi_{sc}(Q_i, k_{1i}, W_i, m)$ and $\Pi_{sc}(Q_i^{'}, k_{1i}^{'}, W_i, m)$.
If $\Pi_{sc}(Q, k_1, W_i, m) < \Pi_{sc}(Q_i^{'}, k_{1i}^{'}, W_i, m)$, then near-optimal solution is (Q_i, k_{1i}, W_i, m). Otherwise, the near-optimal solution is $(Q_i^{'}, k_{1i}^{'}, W_i, m)$.
Else if $Q_i^{'} \neq Q_i$, set $Q_i \leftarrow Q_i^{'}$,$k_{1i} \leftarrow k_{1i}^{'}$. Go to Step 2b.

Step 3: For each set of values (Q_i, k_{1i}, W_i, m), compute $\Pi_{sc}(Q_i, k_{1i}, W_i, m)$; $i = 1, 2, .., n$.

Step 4: Find $Min_{i=0,1,2,..,n}\Pi_{sc}(Q_i, k_{1i}, W_i, m)$.
If $\Pi_{sc}(Q_m^*, k_{1m}^*, W_m^*, m) = Min_{i=0,1,2,...,n}\Pi_{sc}(Q_i, k_i, W_i, m)$, then $(Q_m^*, k_{1m}^*, W_m^*, m)$ is the near-optimal solution for fixed m.

Step 5: Set $m = m + 1$, repeat steps (2), (3), and (4) to get $\Pi_{sc}(Q_m^*, k_{1m}^*, W_m^*, m)$.

Step 6: If $\Pi_{sc}(Q_m^*, k_{1m}^*, W_m^*, m) \leq \Pi_{sc}(Q_{(m-1)}^*, k_{1(m-1)}^*, W_{(m-1)}^*, (m-1))$, then go to Step 5; otherwise, go to Step 7.

Step 7: Set $\Pi_{sc}(Q_m^*, k_{1m}^*, W_m^*, m) = \Pi_{sc}(Q_{(m-1)}^*, k_{1(m-1)}^*, W_{(m-1)}^*, (m-1))$. Then (Q^*, k_1^*, W^*, m) is the near-optimal solution.

4.4 Numerical analysis

This section presents a numerical analysis to validate the developed model. We execute the results and discuss the findings from an extensive numerical experiment. The purpose of this analysis is to come-up with optimal decisions for a decision-maker or manager.

4.4.1 Numerical examples

In this subsection, we consider three numerical examples which are discussed as follows:

Example 1: We use the following data set for this example: $D = 1000$ units, $P = 3000$, $S_V = \$400/$ setup, $S_B = \$150/$ order, $h_B = \$5$/unit/unit time, $h_V = \$3$/unit/unit time, $F = \$25$/delivery, $d = \$0.5$/unit, $v = \$30/$ unit, $\pi_0 = \$150$/unit short, $\sigma = 85$ units/year, $x = 175200$ units/year, $a = 0.008$, $\kappa_{t0} = 0.12$, $s_t = 0.025$. The transportation lead-time has four components with data shown in Table 4.1.

Project i	Investment W_i	Transportation time
0	0	0.1200
1	80	0.0633
2	200	0.0242
3	300	0.0109

Table 4.1: Relationship between investment and transportation time reduction

i	W_i	m	Q	k_1	S	L_1	L_2	Π_{sc}
0	0	7	110.706	2.29152	83.0733	0.181902	0.1200	3529.33
1*	**80***	**7***	**109.518***	**2.12851***	63.9099	0.124781	0.0632751	**3507.7***
2	200	7	108.175	1.9755	49.0383	0.0852859	0.0242276	3553.26
3	300	7	107.644	1.96662	44.7821	0.0717676	0.0108862	3629.11

Table 4.2: Results of Example 1

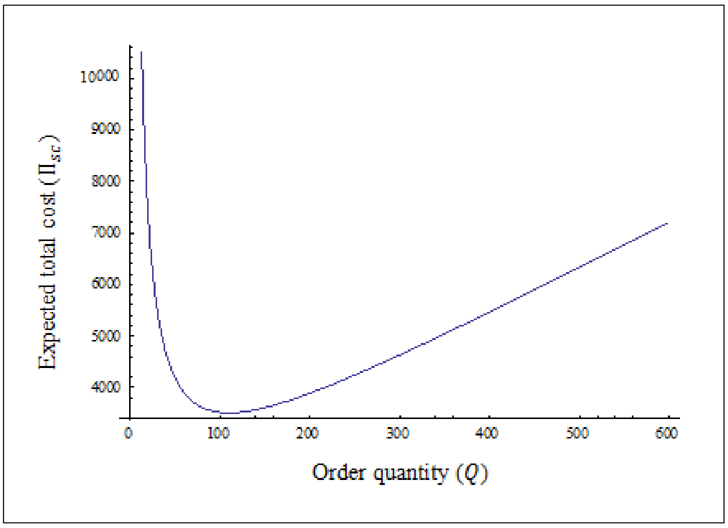

Figure 4.2: Convexity of the cost function

The lead-time demand follows normal distribution and percentage of defects in a lot (Y) is uniformly distributed on (e, f), i.e., $Y \sim U[e, f]$ where $E[Y] = (e+f)/2$, var $[Y] = (f-e)^2/12$ and $E[(1-Y)^2] = \frac{1}{f-e}\int_e^f (1-y)^2 dy = \frac{e^2+ef+f^2}{3} + 1 - e - f$ where $e = 0$ and $f = 0.04$.

The optimal results for Example 1 are shown in Table 4.2 as follows: Ordered quantity from the buyer to vendor per replenishment cycle is $Q^* = 109.52$, safety factor $k_1^* = 2.1285$, total number of delivered shipments $m^* = 7$, safety stock $S = 63.91$, replenishment lead-time for the first and rest of each shipment are $L_1^* = 0.1248$ and $L_2 = 0.0633$, the joint expected annual total cost of the SC is $\Pi_{sc}^* = 3507.7$.

To see the nature of the total cost function for this problem, a curve is drawn in Figure 4.2 by putting the values of Q along the horizontal axis versus the associated total cost along the vertical axis. The curve shows the convex nature of the total cost function (4.22) with respect to Q for the fixed value of (k_1, W, m). The nature of the total cost function (4.22) has been checked for various parameter values and it has been found convex in all cases.

Example 2: For this example, we consider $\sigma = 150$ and other parameters have the same values as in Example 1. The optimal results are shown in Table 4.3 as

i	W_i	m	Q	k_1	S	L_1	L_2	Π_{sc}
0	0	7	112.037	2.28609	146.431	0.182346	0.1200	3886.28
1	80	7	109.93	2.12676	112.752	0.124918	0.0632751	3787.23
2*	**200***	**8***	**97.4809***	**1.96974***	84.4631	0.0817212	0.0242276	**3768.81***
3	300	9	88.6066	1.94243	74.5241	0.0654217	0.0108862	3823.59

Table 4.3: Results of Example 2

follows: Order quantity from the buyer to the vendor per replenishment cycle is $Q^* = 97.48$ units, safety factor $k_1^* = 1.9697$, total number of delivered shipments are $m^* = 8$, safety stock $S = 84.46$, replenishment lead-time for the first and rest of each shipment are $L_1 = 0.0817$ and $L_2 = 0.0242$, the joint expected annual total cost of the SC is $\Pi^*_{sc} = 3768.81$.

Example 3: For this example, we consider $\sigma = 350$ and other parameters have the same values as in Example 1.

i	W_i	m	Q	k_1	S	L_1	L_2	Π_{sc}
0	0	7	116.161	2.26956	340.478	0.18372	0.1200	4982.86
1	80	8	101.096	2.14161	261.783	0.121974	0.0632751	4639.67
2	200	11	77.0268	1.9621	187.949	0.0749032	0.0242276	4415.791
3*	**300***	**11***	**75.5882***	**1.92466***	166.487	0.0610822	0.0108862	**4404.44***

Table 4.4: Results of Example 3

The optimal results are shown in Table 4.4 as follows: Ordered quantity from the buyer to vendor per replenishment cycle is $Q^* = 75.59$ units, safety factor $k_1^* = 1.9247$, total number of delivered shipments $m^* = 11$, safety stock $S = 166.49$, replenishment lead-time for the first and rest of the each shipment are $L_1 = 0.0611$ and $L_2 = 0.0109$, the joint expected annual total cost of the SC is $\Pi^*_{sc} = 4404.44$.

The optimal results for Examples (1-3) are summarized in Table 4.5.

	W_i^*	m^*	Q^*	k_1^*	S	L_1	L_2	Π^*_{sc}
Example 1	80	7	109.52	2.1285	63.91	0.1248	0.0633	3507.7
Example 2	200	8	97.48	1.9697	84.46	0.0817	0.0242	3768.81
Example 3	300	11	75.59	1.9247	166.49	0.0611	0.0109	4404.44

Table 4.5: Summary of the optimal results

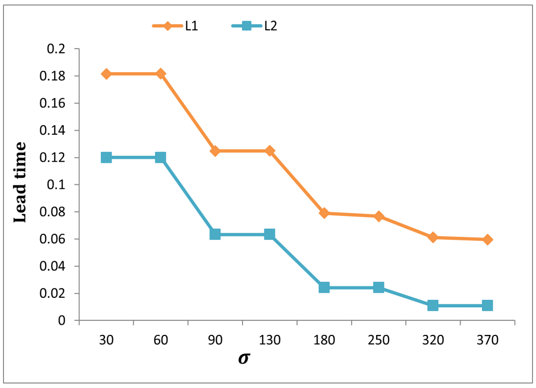

Figure 4.3: Impact of standard deviation (σ) on lead-time L_1 and L_2

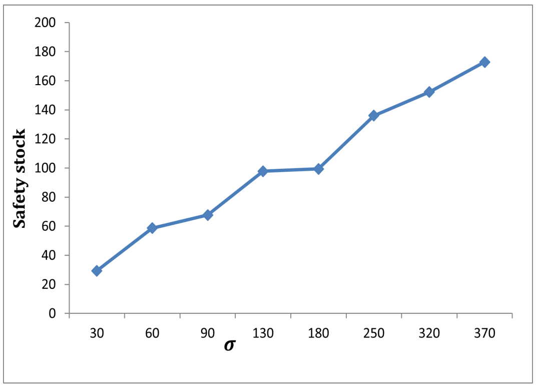

Figure 4.4: Impact of standard deviation (σ) on safety stocks S

4.4.2 Sensitivity analysis

Here, we explore the impact of key parameters on optimal decisions. We consider the parameter-values of Example 2 for the analysis.

4.4.2.1 Impact of standard deviation (σ) on optimal solution

Table 4.6 demonstrates the impact of lead-time demand uncertainty (σ) on optimal decisions. One can observe that with an increase in deviation of lead-time demand (σ), the expected annual total cost also increases. It is obvious that if the system faces a high demand deviation then it makes sense to invest money to reduce the replenishment lead-time, thereby driving up the total cost (Figure 4.3). A significant deviation in demand increases the safety stocks which helps the SC system to absorb the variability of customer demand (Figure 4.4). Too many safety stocks increase the holding cost but it also reduces the risk of running out of stock. Cost saving due to lead-time reduction has been depicted in Figure 4.5. From Table 4.6, it is seen that the shipment size (Q) and optimal number of shipments (m) are more sensitive for a higher value of (σ).

σ	W_i	m^*	Q^*	k_1^*	S	L_1	L_2	Π_{sc}^*	Savings ($)
30	0	7	109.583	2.29614	29.3489	0.181528	0.12	3227.08	0
60	0	7	110.195	2.29362	58.6662	0.181732	0.12	3391.97	0
90	80	7	109.549	2.12837	67.6679	0.124792	0.0632751	3529.21	27.59
130	80	7	109.804	2.1273	97.7264	0.124876	0.0632751	3701.22	75.26
180	200	9	89.266	1.96523	99.4153	0.0789829	0.0242276	3866.87	184.06
250	200	10	82.5316	1.9631	135.953	0.0767381	0.0242276	4095.12	339.77
320	300	11	75.7161	1.92393	152.212	0.0611249	0.0108862	4318.94	499.59
370	300	12	71.0603	1.91394	172.844	0.0595729	0.0108862	4460.79	631.58

Table 4.6: Effect of lead-time demand deviation (σ) on optimal decisions

4.4.2.2 Impact of transportation cost (F) on optimal solution

Table 4.7 shows the impact of transportation cost on optimal decisions. With the increase in the transportation cost (e.g., 10 to 200), the shipment size gradually increases while the number of shipments decreases (Figure 4.6), which is an expected outcome since, in real-life, with the increase in transportation cost, the buyer always tries to increase the shipment size to decrease the number of shipments. One can observe that lead-time changes in an increasing manner for increasing value of F (Figure 4.7). This is since transportation cost increases the order quantity which in turn increases the production time as well as the replenishment lead-time. Increased transportation cost increases the total supply chain cost (Figure 4.8).

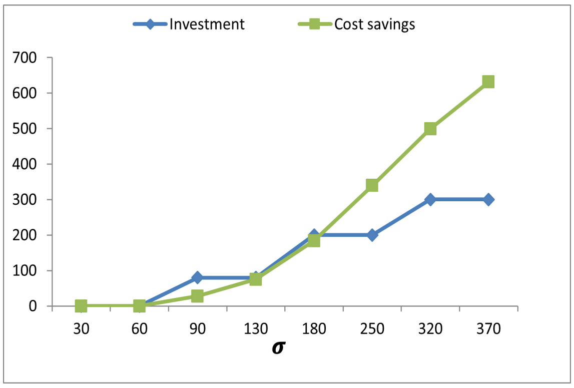

Figure 4.5: Impact of standard deviation (σ) on cost savings and investment

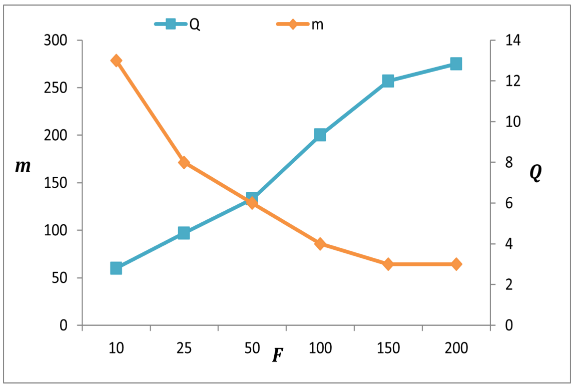

Figure 4.6: Effect of transportation cost (F) on m and Q

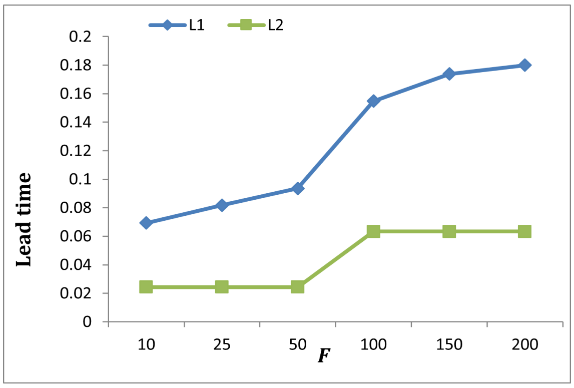

Figure 4.7: Effect of transportation cost (F) on lead-times L_1 and L_2

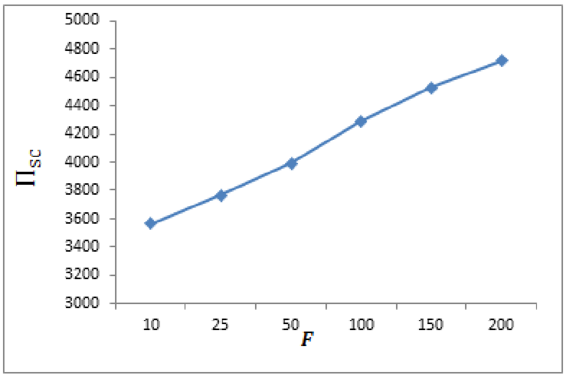

Figure 4.8: Impact of transportation cost (F) on annual total cost (Π_{sc})

F	W_i	m^*	Q^*	k_1^*	S	L_1	L_2	Π_{sc}^*
10	200	13	60.2673	2.01868	79.722	0.0693167	0.0242276	3567.77
25	200	8	97.4809	1.96974	84.4631	0.0817212	0.0242276	3768.81
50	200	6	132.771	1.94815	89.3477	0.0934847	0.0242276	3994.41
100	80	4	199.659	1.9898	117.442	0.154828	0.0632751	4291.79
150	80	3	256.519	1.97594	123.557	0.173781	0.0632751	4528.3
200	80	3	275.063	1.94453	123.736	0.179963	0.0632751	4720.26

Table 4.7: Impact of transportation cost (F) on optimal decisions

4.4.2.3 Impact of setup time (s_t) on optimal solution

Table 4.8 reflects the impact of setup time (s_t) on the optimal decision. Specifically, a linear increment in the safety stock as well as lead-time is observed as setup delay increases (Figures 4.9 & 4.10).

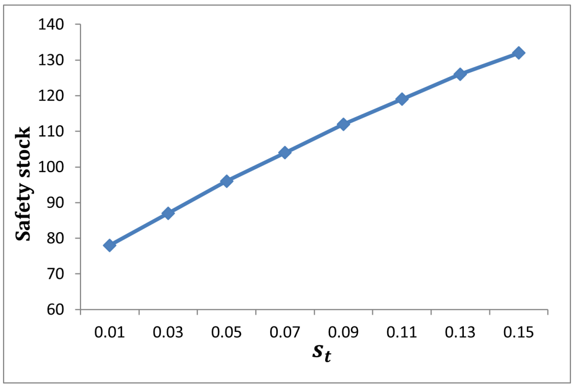

Figure 4.9: Impact of setup time (s_t) on safety stocks S

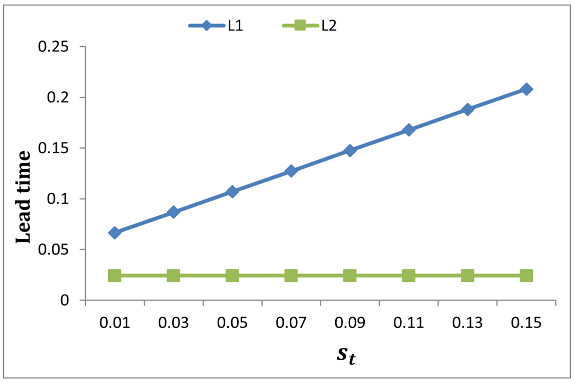

Figure 4.10: Impact of setup time (s_t) on lead-times L_1 and L_2

Since the setup time applies to the first shipment, no effect in lead-time is noticed for subsequent shipments. It is to be noted that the lead-time variability is directly related to delay in setup time and this reflects the importance of decreasing lead-time. Extra safety stocks increase stock holding cost and as a result, we see an increase in total supply chain cost. This behavior is observed in Figure 4.11.

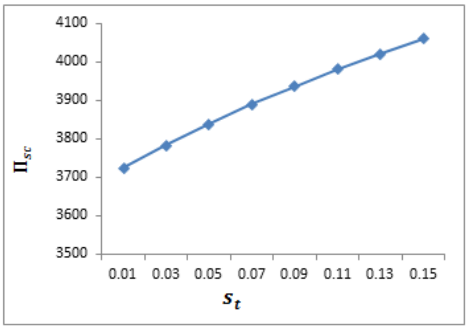

Figure 4.11: Impact of setup time (s_t) on joint expected annual total cost (Π_{sc})

s_t	W_i	m^*	Q^*	k_1^*	S	L_1	L_2	Π_{sc}^*
0.01	200	8	96.7821	2.00951	77.7239	0.0664883	0.0242276	3723.62
0.03	200	8	97.7233	1.96291	86.7474	0.086802	0.0242276	3783.43
0.05	200	8	98.6274	1.94931	95.6917	0.107103	0.0242276	3838.92
0.07	200	8	99.4053	1.94398	104.065	0.127363	0.0242276	3889.76
0.09	200	8	100.082	1.94067	111.833	0.147588	0.0242276	3936.74
0.11	200	8	100.686	1.938	119.077	0.16779	0.0242276	3980.5
0.13	200	8	101.233	1.93565	125.882	0.187972	0.0242276	4021.79
0.15	200	8	101.736	1.9335	132.316	0.20814	0.0242276	4060.83

Table 4.8: Effect of setup time (s_t) on optimal decisions

4.4.2.4 Effect of holding cost $(h_B \& h_V)$ on optimal decisions

Table 4.9 demonstrates the impact of holding costs on optimal decisions. One can observe that when the buyer's holder cost is higher than the vendor then in that case, small-sized shipment is good for him/her.

h_B	h_V	W_i	m^*	Q^*	k_1^*	S	L_1	L_2	Π_{sc}^*
	3	80	6	130.72	2.24484	122.268	0.131849	0.0632751	3437.37
3	6	80	3	179.091	2.32097	133.921	0.147972	0.0632751	4004.03
	9	80	2	222.191	2.38064	143.879	0.162339	0.0632751	4398.66
	3	200	9	87.612	1.90397	79.983	0.0784316	0.0242276	3893.45
6	6	200	6	91.9005	2.03244	86.1543	0.0798611	0.0242276	4500.89
	9	200	5	90.9765	2.10692	89.1391	0.0795531	0.0242276	4949.7
	3	200	12	68.5162	1.78653	71.9397	0.0720663	0.0242276	4232.41
9	6	200	8	70.7385	1.88759	76.3987	0.0728071	0.0242276	4858.1
	9	200	6	75.6024	1.95714	80.0908	0.0744284	0.0242276	5324.08

Table 4.9: Effect holding cost on optimal decisions

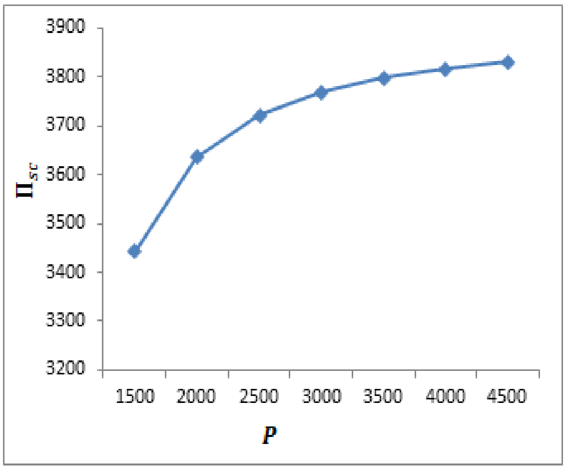

Figure 4.12: Impact of production rate (P) on annual total cost (Π_{sc})

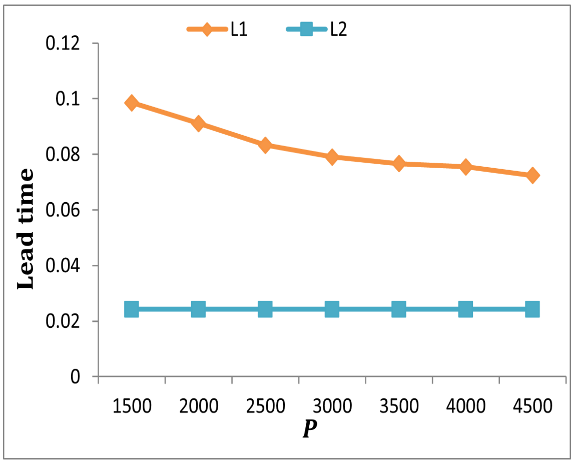

Figure 4.13: Impact of production rate (P) on lead-times L_1 and L_2

When the vendor's holding cost is greater or equal to the buyer's one, the shipment size increases but the number of shipments decreases. This is because, in this case, by delivering large-sized shipments, the vendor tries to reduce its holding cost, which yields a reduction in the number of shipments.

4.4.2.5 Impact of production rate (P) on optimal decisions

As the production rate (P) of the vendor varies, the total supply chain cost varies (see Table 4.10). This is obvious because if there is an increase in production rate, then more items will be produced, and thus it will increase the supply chain total cost (Figure 4.12).

P	W_i	m^*	Q^*	k_1^*	S	L_1	L_2	Π_{sc}^*
1500	200	16	73.9011	1.79415	84.4611	0.098495	0.0242276	3442.69
2000	200	11	83.7479	1.89691	85.8818	0.0911015	0.0242276	3635.25
2500	200	10	85.1453	1.93754	83.874	0.0832857	0.0242276	3721.18
3000	200	9	89.3646	1.96475	82.8428	0.0790158	0.0242276	3768.72
3500	200	8	95.8862	1.98451	82.3997	0.0766236	0.0242276	3797.36
4000	200	7	104.943	2.00002	82.4125	0.0754633	0.0242276	3816.9
4500	200	7	104.128	2.00911	81.0711	0.072367	0.0242276	3830.16

Table 4.10: Impact of production rate (P) on optimal decisions

It has been observed that lead-time decreases with an increase in production rate (Figure 4.13). This happens due to the inverse relation between order quantity and production rate in the lead-time function. This reduction in lead-time results in a decrease in lead-time demand which in turn increases the order quantity and reduces the amount of safety stock (Figure 4.14). As the production rate increases, the number of shipments decreases (Figure 4.15).

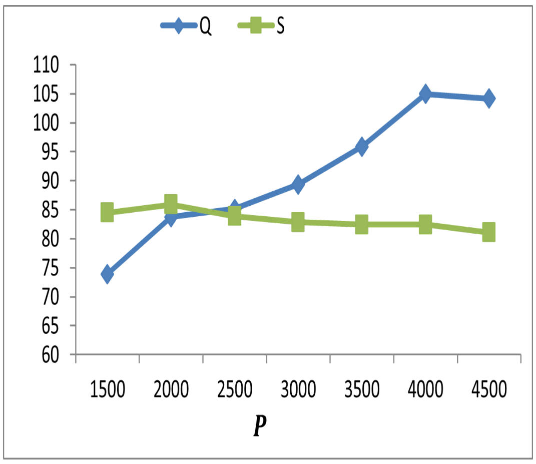

Figure 4.14: Impact of production rate (P) on S and Q

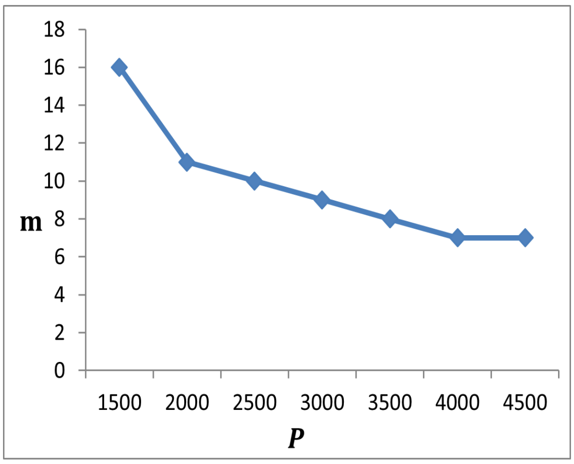

Figure 4.15: Impact of production rate (P) on number of shipments (m)

4.4.2.6 Impact of backorder cost (π) on optimal solution

Table 4.11 shows the impact of unit backorder cost (π) on optimal solutions. When the unit backorder cost is relatively low i.e., $\pi = 5$, there is no need to do more investment for lead-time reduction. The rise in lead-time reduction investment has

π	W_i	m^*	Q^*	k_1^*	S	L_1	L_2	Π_{sc}^*
5	80	6	128.548	0.846674	45.9885	0.131125	0.0632751	3482.42
20	200	7	110.841	1.19245	52.5076	0.0861746	0.0242276	3606.59
60	200	8	98.7751	1.60867	69.1622	0.0821526	0.0242276	3694.99
100	200	8	98.0444	1.80771	77.6043	0.081909	0.0242276	3736.48
180	200	9	89.1558	2.03569	85.7963	0.0789462	0.0242276	3782.6
210	200	9	88.9847	2.09551	88.2856	0.0788891	0.0242276	3794.19
250	200	9	88.7979	2.16273	91.0817	0.0788269	0.0242276	3807.13

Table 4.11: Effect of backorder cost (π) on optimal decisions

been noticed along with the rise in unit backorder cost. This is a reasonable result because when the backorder cost is too high, the buyer wants to reduce the amount of shortage, which is possible by reducing the time (Figure 4.16). Also, the buyer will increase the amount of safety stock so that shortages can be backordered as much as possible (Figure 4.17).

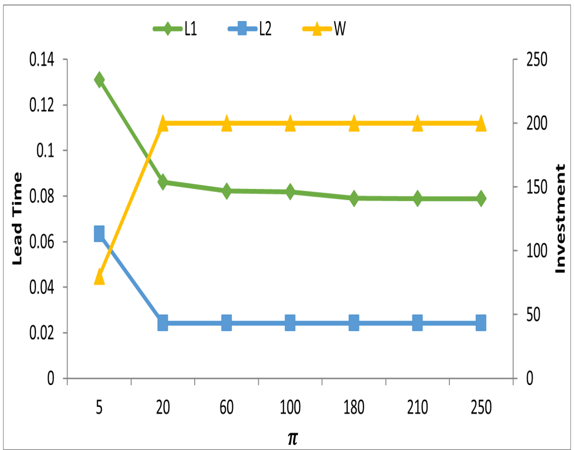

Figure 4.16: Impact of backorder cost (π) on L_1, L_2 and W

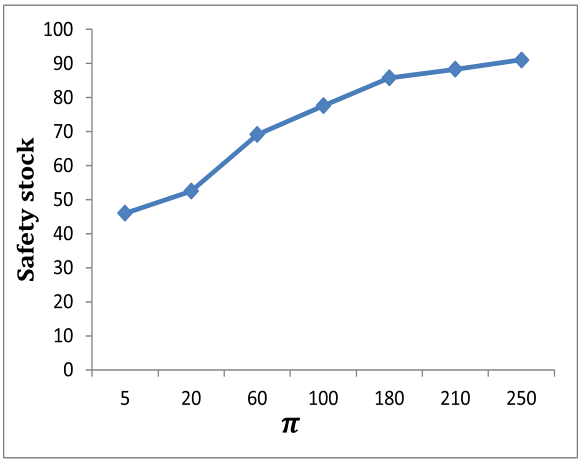

Figure 4.17: Impact of backorder cost (π) on safety stock S

4.4.2.7 Impact of inspection cost (d) on optimal solution

Figure 4.18 illustrates the behavior of the change in inspection cost to the total system cost for different values of the defective percentage. From Figure 4.18, it is observed that as the inspection cost (d) increases, the joint expected annual total cost of the system increases regardless of different values of defective rate $(E[Y])$. It is also observed that the percentage of defective products in a lot greatly increases the impact of inspection costs on total costs.

4.4.2.8 Impact of warranty cost (v) on optimal solution

Figure 4.19 illustrates the behavior of the change in warranty cost to the total system cost for different values of the defective percentage. From Figure 4.19, it is observed that, as the warranty cost (v) increases, the joint expected annual total cost of the system increases. This shows that both the vendor and the buyer are adversely affected by the increase in the warranty cost during the warranty period. In this

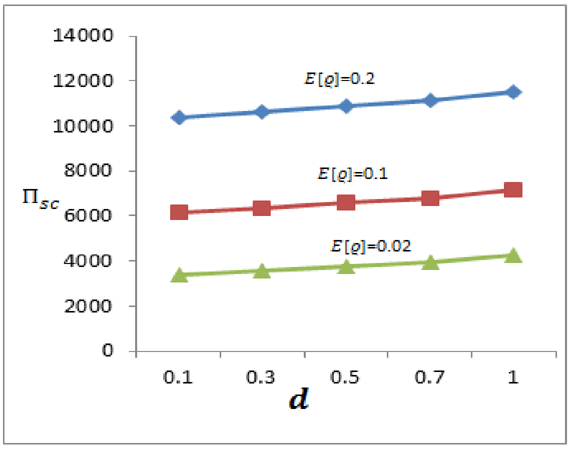

Figure 4.18: Impact of inspection cost (d) on Π_{sc}

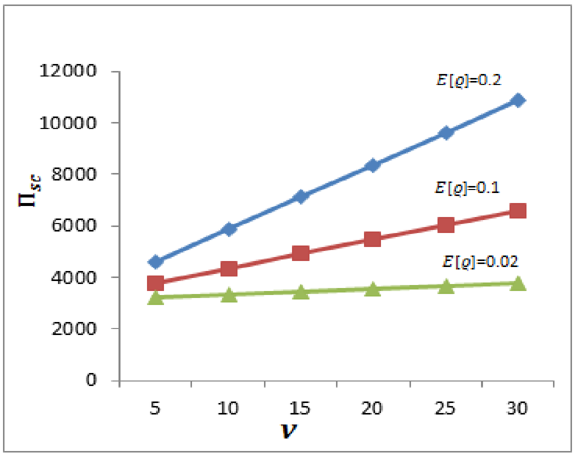

Figure 4.19: Impact of warranty cost (v) on Π_{sc}

case, the buyer and the vendor have to pay special attention to the quality of the product and the health of the production system so that fewer defective products are produced.

4.4.3 Managerial Insights

Due to uncertainty on economic grounds, alternative competitive strategies are being incorporated by the companies. One of those alternatives is customer satisfaction. To please the customer, the importance of satisfying the customers' needs has become a priority. Technological advancement is yet insufficient to predict reliable accuracy in customers' demand, quantity to be received, etc., in advance. Henceforth, the presumption of unknown demand is perhaps apt across industries around the world. In a situation of uncertain demand, lead-time serves as a crucial factor and enables a series of gains. As a result of short lead-time, the safety stock reduces. The business' competitive advantage increases due to an improvised customer-service level which is an outcome of losses due to stock-out. For lean supply conditions, the lead-time is minimised, helping in minimizing the safety stock level that further results in moderating the likelihood and stockouts' impact. Yet a well-maintained safety stock doesn't serve as a guarantee for a system to stay stock-out free. In an attempt to establish multiple backorders while a stock-out, the brand proposes a discount on the stock-out items that might result in the buyer staying longer for a desired stocked out item. Elevated customer loyalty can thus be provoked by a price-discount control. Considering this, several brands including Procter & Gamble, Nordstrom, Nike, Disney, South west Airlines, Marriott Hotels, Wal-Mart, McDonald's, Sony, Canon, Nokia, Toyota, Electrolux, IKEA, Bang & Olufsen, Club Med, Lego, and Tesco keenly emphasize upon customer behavior, hereby, respond efficiently to the dynamics of the customers' needs. These companies rely on price-discount to elevate the customers' loyalty and justify their choice of staying longer for backorders. The lead-time reduction model coupled with the price discount as well as lead-time dependent backlogging rate at the buyer's end proves to be a practically feasible approach towards real-world inventory systems.

- Figure 4.5 indicates that the investment amount increases as the deviation in lead-time demand (σ) increases. Also, the cost saving increases with an increasing value of σ. So, the managers may refrain from investing money for lead-time reduction when σ is low. However, it is always preferred to invest money to reduce the replenishment lead-time when σ is comparatively high.

- Figure 4.4 indicates that a higher value of σ increases the safety stock. Therefore, when σ is high, supply chain managers are advised to store more safety stocks which will protect their systems from the risk of stock-out.

- When the deviation in lead-time demand (σ) is high, a substantial amount of lead-time reduction is essential to reduce the risk of a stock-out situation. However, when σ is high, no matter how much transportation cost is, the buyer should invest money (optimal amount) to reduce the replenishment lead-time.

4.5 Concluding remarks

The present chapter optimizes a vendor-buyer model with variable lead-time and uncertain market demand. In order to represent a quite realistic scenario, we presumed an imperfect production system, which produces a certain percentage of faulty items. Unlike the traditional integrated supply chain model, we assume lead-time as a function of production time, setup time, and transportation time. Furthermore, the replenishment lead-time can be shortened by reducing transportation time. From the numerical analysis, it is found that the reorder point and safety stock can be reduced by reducing the replenishment lead-time. This reduction in safety stock reduces the holding cost of the buyer as well as the total supply chain cost. We also found that high demand uncertainty influences the supply chain members to reduce the replenishment lead-time to lessen the stock-out probability. A higher transportation cost would increase the shipment size but decreases the shipment numbers to reduce the total transportation cost.

Chapter 5

Lead-time reduction in a two-echelon integrated supply chain model with variable backorder

5.1 Introduction

The previous chapter has dealt with a two-echelon SC model under variable lead-time and fully backlogged shortages. But in reality, when a shortage occurs, some customers express a desire to wait for a certain period of time for backorder, while some customers meet their demands from other options without waiting. However, if the waiting time is too long, the customers may refuse to wait for the backorder. This phenomenon indicates that the longer the lead-time is, the higher the shortages are, and the smaller the backorder rate would be. Keeping in mind such a scenario, this chapter* studies an integrated vendor-buyer SC model with lead-time dependent backorder rate. Here the replenishment lead-time is considered as a function of the buyer's order size and the vendor's production rate. Further, the replenishment lead-time is assumed to be reduced by changing the regular production rate of the vendor at the risk of paying an additional cost. The proposed model is formulated to obtain the net present value (NPV) of the expected total cost of the integrated system through optimization of (i) the buyer's order quantity, (2) the buyer's safety factor, and (3) the vendor's production rate. Theoretical results are derived to demonstrate the existence and uniqueness of the optimal solution.

*This chapter is based on the work published in *RAIRO-Operations Research*, 54 (2020) 961-979.

5.2 Preliminary aspects

A two-echelon integrated vendor-buyer SC model is considered for a single type of item. The buyer faces stochastic lead-time demand from the customers and manages its inventory as per continuous-review policy. The buyer places an order when the stock level touches the reorder point. The vendor delivers the entire order quantity in a single shipment. The buyer receives the batch after variable lead-time dependent on order quantity and production rate. The vendor has the option to reduce the production lead-time by setting the production rate to the maximum level before starting production. Besides, the buyer can reduce the replenishment lead-time by managing the order quantity. The problem is to establish the production-inventory replenishment decision that minimizes the expected total cost per time unit under net present value (NPV). In the next two subsections, we highlight the notation and assumptions that will be used during model development.

5.3 Notation and assumptions

We use the following notation to develop the proposed model.

- **Decision variables**

Q	size of a shipment (units)
u	safety factor
P	production rate of the vendor (units/year)

- **Parameters**

D	annual demand at the buyer (units/year)
P_0	regular production rate of the vendor (units/year)
P_{max}	maximum production rate of the vendor (units/year)
C_s	vendor's setup cost per unit time ($/unit time)
C_o	buyer's ordering cost per order ($/order)
H_b	unit holding cost at the buyer ($/unit/year)
H_v	unit holding cost at the vendor ($/unit/year)
$l(Q, P)$	buyer's replenishment lead-time (time unit)
σ_l	standard deviation of lead-time demand (units)
r	reorder point (units)
$\delta(l)$	backorder rate during shortage period
b	penalty cost at the buyer for unit short ($/unit/year)
b_0	buyer's marginal profit ($/unit)
X	lead-time demand
j	yearly interest rate ($)
$E(X-r)^+$	expected shortage quantity

We consider the following assumptions to develop the model:

(i) We considers a supply chain consisting of a single-vendor and a single-buyer to deal with a single type of item.

(ii) The buyer faces stochastic demand during replenishment lead-time from his/her customers and the demand is normally distributed with a finite mean and standard deviation (Liao and Shyu, 1991).

(iii) Following continuous review (Q, r) inventory policy, the buyer places an order of size Q whenever the inventory level falls to the reorder point and the vendor manufactures the items with a finite production rate $P(> D)$ in a single setup and transfers the entire quantity to the buyer over a single shipment.

(iv) The buyer's reorder point is defined as the sum of the expected demand during lead-time and safety stock.

(v) Replenishment lead-time between the buyer and the vendor is variable, which is directly proportional to the buyer's order size and inversely proportional to the vendor's production rate (Moon and Cha, 2005).This is logical as a larger order size will take a longer production time than a smaller one.

(vi) Replenishment lead-time is controllable which can be controlled by monitoring the vendor's production rate through some additional investment. The extra costs incurred by the vendor will be fully transferred to the buyer if shortened lead-time is requested (Moon and Cha, 2005).

(vii) Shortages are allowed in the buyer's inventory. Unsatisfied demand is backlogged, and the fraction of shortages backordered is (Abad, 1996, Abad, 2001) $\delta(l) = e^{-\alpha l}, 0 \leq \delta(l) \leq 1$ with $\delta(0) = 1$ where l is the lead-time up to the next replenishment and α is a positive constant. Note that if $\delta(l) = 1$(or 0) for all l, then shortages are completely backlogged (or lost).

(viii) Time value of money is considered.

5.4 Mathematical model

We suppose that the buyer follows the (Q, r) inventory policy and places an order of size Q when the inventory level drops to the reorder point r. The vendor manufactures the entire order with a finite production rate $P(> D)$ in a single setup and transfers the entire quantity to the buyer over a single lot (see Figure 5.1 for the inventory pattern). The lead-time $l(Q,P)$ is directly proportional to the buyer's order quantity Q and inversely proportional to the vendor's production rate P, i.e, $l(Q,P) = \frac{Q}{P}, 0 < P_0 \leq P \leq P_{max}$, where P_0 is the vendor's regular production rate and P_{max} is the maximum production rate. During unpredictable replenishment lead-time, the buyer may face stock-out situation. Consequently, it is necessary to calculate the safety stock level to prevent stock-outs. Therefore, the safety stock level is calculated by multiplying the safety stock risk factor (u) with the standard deviation (σ_l) and the square root of the lead-time $(\sqrt{\frac{Q}{P}})$. Hence, the safety stock is $u\sigma_l\sqrt{\frac{Q}{P}}$. Therefore, we have the reorder point (r) as the sum of the expected demand during lead-time and safety stock, i.e., $r = D\frac{Q}{P} + u\sigma_l\sqrt{\frac{Q}{P}}$. The buyer's expected shortage quantity at the end of the replenishment cycle is

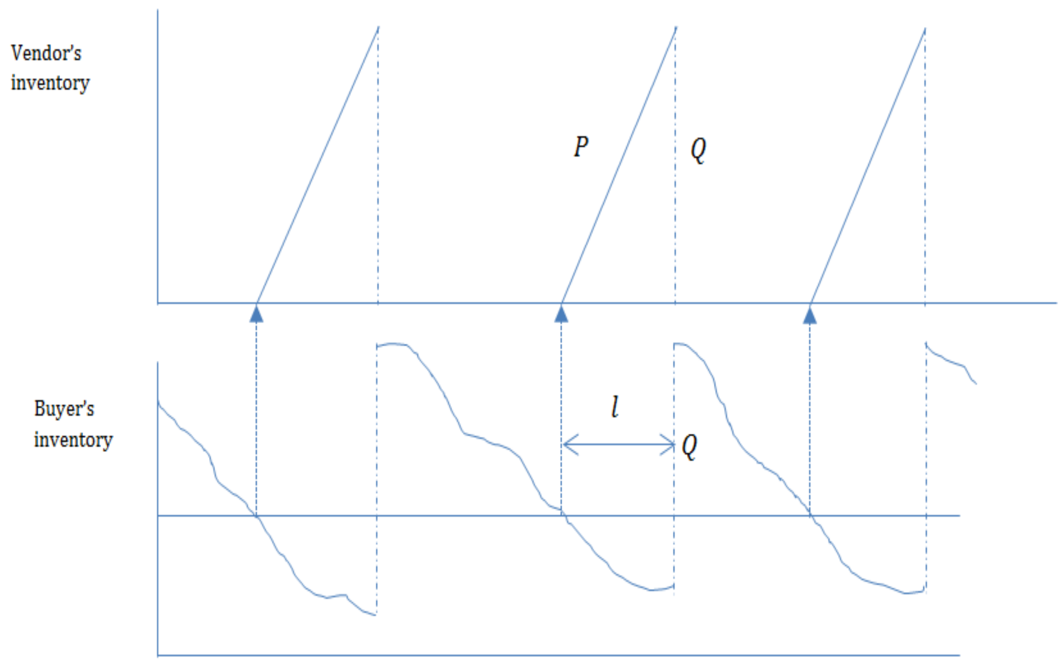

Figure 5.1: Inventory pattern for the vendor and buyer

$$B = E(X-r)^+ = \int_r^\infty (x-r)df(x) \tag{5.1}$$

where $f(x) = \frac{1}{\sigma_l\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma_l^2}}$ for mean μ and standard deviation σ_l.

The expected shortage quantity for a demand with mean $D\frac{Q}{P}$ and standard deviation $\sigma_l\sqrt{\frac{Q}{P}}$ during the lead-time is given by

$$B = \int_r^\infty \frac{(x-r)}{\sqrt{2\pi}\sigma_l\sqrt{\frac{Q}{P}}} e^{-\frac{1}{2}\left(\frac{x-D\frac{Q}{P}}{\sigma_l\sqrt{\frac{Q}{P}}}\right)^2} dx, \tag{5.2}$$

Assuming $z = \frac{x-D\frac{Q}{P}}{\sigma_l\sqrt{\frac{Q}{P}}}$ and $u = \frac{r-D\frac{Q}{P}}{\sigma_l\sqrt{\frac{Q}{P}}}$, (5.2) becomes

$$B = \sigma_l\sqrt{\frac{Q}{P}}\int_{z=u}^\infty (z-u)f(z)dz, \tag{5.3}$$

where $f(z)$ is the standard normal probability density function.
Assuming

$$G(u) = \int_{z=u}^\infty (z-u)f(z)dz, \tag{5.4}$$

(5.3) becomes

$$B = \sigma_l\sqrt{\frac{Q}{P}}G(u), \tag{5.5}$$

We assume that the backorder rate δ is a function of lead-time $(l(Q,P))$ i.e.,

$$\delta(l) = e^{-\alpha l(Q,P)} = e^{-\frac{\alpha Q}{P}} \tag{5.6}$$

Therefore, the expected backorder quantity is

$$\delta(l)E(X-r)^+ = e^{-\frac{\alpha Q}{P}}\sqrt{\frac{Q}{P}}G(u)\sigma_l \tag{5.7}$$

and hence the expected loss in sales per replenishment cycle is

$$\{1-\delta(l)\}E(X-r)^+ = \left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u). \tag{5.8}$$

When the replenishment lead-time is too long, i.e., $l(Q,P) \to \infty$ then total backordered quantity is unsold whereas all backordered quantities are sold when $\alpha \to 0$, i.e., the mean time of patience to wait $\left(\frac{1}{\alpha}\right)$ tends to infinity.

Further, at the beginning of each replenishment cycle, the retailer's expected net

inventory is the safety stock $u\sigma_l\sqrt{\frac{Q}{P}}$ plus the previous replenishment cycle's lost sales $\left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)$, and the expected net inventory level immediately after a replenishment is $Q+u\sigma_l\sqrt{\frac{Q}{P}}+\left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)$.
Therefore, the expected average inventory over a replenishment cycle is

$$Q+u\sigma_l\sqrt{\frac{Q}{P}}+\left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)-Dt \text{ for } t\in\left[0,\frac{Q}{D}\right]. \tag{5.9}$$

Therefore, the expected inventory holding cost for the buyer under the time value of money is

$$\begin{aligned} I_c &= \int_{t=0}^{Q/D} H_b\left[Q+u\sigma_l\sqrt{\frac{Q}{P}}+\left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)-Dt\right]e^{-jt}dt \\ &= \frac{H_b}{j}\left[\left\{Q+u\sigma_l\sqrt{\frac{Q}{P}}+\left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)\right\}\left(1-e^{-\frac{Qj}{D}}\right)\right. \\ &\quad \left.+Qe^{-\frac{Qj}{D}}+\left(e^{-\frac{Qj}{D}}-1\right)\frac{D}{j}\right] \end{aligned} \tag{5.10}$$

The backorder cost is

$$S_c=\left[b+\left(1-e^{-\frac{\alpha Q}{P}}\right)b_0\right]\sigma_l\sqrt{\frac{Q}{P}}G(u) \tag{5.11}$$

The buyer's expected total cost is

$$\begin{aligned} ETC_b &= \text{ordering cost } + \text{ holding cost } + \text{ backorder cost} \\ &= C_o+\frac{H_b}{j}\left[\left\{Q+u\sigma_l\sqrt{\frac{Q}{P}}+\left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)\right\}\left(1-e^{-\frac{Qj}{D}}\right)\right. \\ &+ \left. Qe^{-\frac{Qj}{D}}+\left(e^{-\frac{Qj}{D}}-1\right)\frac{D}{j}\right]+\left[b+\left(1-e^{-\frac{\alpha Q}{P}}\right)b_0\right]\sigma_l\sqrt{\frac{Q}{P}}G(u) \end{aligned} \tag{5.12}$$

The vendor's expected total cost is

$$ETC_v = \text{setup cost } + \text{ holding cost} \tag{5.13}$$

$$\begin{aligned} \text{where the vendor's holding cost} &= H_v\frac{QD}{2P}\int_{t=0}^{Q/D}e^{-jt}dt \\ &= \frac{H_v}{j}\left(1-e^{-\frac{Qj}{D}}\right)\frac{QD}{2P} \end{aligned} \tag{5.14}$$

Therefore, the vendor's expected total cost is

$$ETC_v = C_s + \frac{H_v}{j}\left(1 - e^{-\frac{Qj}{D}}\right)\frac{QD}{2P} \tag{5.15}$$

We now use discounted cash flow approach (Moon and Yun, 1993). There are cash outflows for the ordering cost, lead-time crashing cost, and stockout cost at the beginning of each cycle. Therefore, the expected total relevant cost of the supply chain is

$$\begin{aligned} ETC &= C_o + \frac{H_b}{j}\left[\left\{Q + u\sigma_l\sqrt{\frac{Q}{P}} + \left(1 - e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)\right\}\left(1 - e^{-\frac{Qj}{D}}\right)\right. \\ &+ \left. Qe^{-\frac{Qj}{D}} + \left(e^{-\frac{Qj}{D}} - 1\right)\frac{D}{j}\right] + \left[b + \left(1 - e^{-\frac{\alpha Q}{P}}\right)b_0\right]\sigma_l\sqrt{\frac{Q}{P}}G(u) + C_s \\ &+ \frac{H_v}{j}\left(1 - e^{-\frac{Qj}{D}}\right)\frac{QD}{2P} \end{aligned} \tag{5.16}$$

Our goal is to reduce the replenishment lead-time by increasing the production rate of the vendor. If the buyer requests the vendor to increase the production rate, the buyer will be asked by the vendor for added cost to achieve this. In this case, the extra cost that is induced by the difference between the desired production rate and the regular production rate is given by (see Moon and Cha, 2005)

$$(P - P_0)l(Q,P)S = \left(1 - \frac{P_0}{P}\right)QS \tag{5.17}$$

Hence, incorporating productivity improvement cost, (5.16) becomes

$$\begin{aligned} ETC &= C_o + \frac{H_b}{j}\left[\left\{Q + u\sigma_l\sqrt{\frac{Q}{P}} + \left(1 - e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)\right\}\left(1 - e^{-\frac{Qj}{D}}\right)\right. \\ &+ \left. Qe^{-\frac{Qj}{D}} + \left(e^{-\frac{Qj}{D}} - 1\right)\frac{D}{j}\right] + \left[b + \left(1 - e^{-\frac{\alpha Q}{P}}\right)b_0\right]\sigma_l\sqrt{\frac{Q}{P}}G(u) + C_s \\ &+ \frac{H_v}{j}\left(1 - e^{-\frac{Qj}{D}}\right)\frac{QD}{2P} + \left(1 - \frac{P_0}{P}\right)QS \end{aligned} \tag{5.18}$$

Then the net present value of the expected total cost of the supply chain is (Silver, Pyke, Peterson, et al., 1998),

$$\begin{aligned} PVETC(Q,u,P) &= \frac{1}{1-e^{-\frac{Qj}{D}}}\left[C_o + \frac{H_b}{j}\left\{\left(Q + u\sigma_l\sqrt{\frac{Q}{P}} + \left(1-e^{-\frac{\alpha Q}{P}}\right)\sigma_l\sqrt{\frac{Q}{P}}G(u)\right)\right.\right. \\ &\quad \left.(1-e^{-\frac{Qj}{D}}) + Qe^{-\frac{Qj}{D}} + (e^{-\frac{Qj}{D}}-1)\frac{D}{j}\right\} + \frac{H_v}{j}\left(1-e^{-\frac{Qj}{D}}\right)\frac{QD}{2P} \\ &+ \left.\left[b + \left(1-e^{-\frac{\alpha Q}{P}}\right)b_0\right]\sigma_l\sqrt{\frac{Q}{P}}G(u) + \left(1-\frac{P_0}{P}\right)QS + C_s\right] \end{aligned} \tag{5.19}$$

5.4.1 Solution methodology

As the cost function given by (5.19) is highly nonlinear, it is not possible to prove the convexity of the cost function with respect to all the decision variables jointly. Therefore, an iterative algorithm is developed to find the optimal solution of the developed model. Some convexity and concavity properties with respect to the control parameters are also derived, that will help to show that the solution obtained from the algorithm is a global minimum.

Now, for a fixed value of P, the first order partial derivatives of $PVETC$ in (5.19) with respect to u and Q yield

$$\frac{\partial PVETC}{\partial u} = \frac{H_b}{j}\sigma_l\sqrt{\frac{Q}{P}} - \sigma_l\sqrt{\frac{Q}{P}}\left[\frac{H_b}{j}\left(1-e^{-\frac{\alpha Q}{P}}\right) + \frac{b+\left(1-e^{-\frac{\alpha Q}{P}}\right)b_0}{1-e^{-\frac{Qj}{D}}}\right][1-F(u)] \tag{5.20}$$

and

$$\begin{aligned} \frac{\partial PVETC}{\partial Q} &= \frac{(1-e^{-\frac{Qj}{D}} - \frac{Qj}{D}e^{-\frac{Qj}{D}})\left[\frac{H_b}{j} + S\left(1-\frac{P_0}{P}\right)\right]}{\left(1-e^{-\frac{Qj}{D}}\right)^2} - \frac{je^{-\frac{Qj}{D}}\left(\frac{C_o+C_s}{D}\right)}{\left(1-e^{-\frac{Qj}{D}}\right)^2} + \frac{u\sigma_l\sqrt{\frac{Q}{P}}H_b}{2Qj} \\ &+ \frac{\sqrt{\frac{Q}{P}}\left(1-e^{-\frac{Qj}{D}} - 2\frac{Qj}{D}e^{-\frac{Qj}{D}}\right)\sigma_l G(u)\left(b+\left(1-e^{-\frac{\alpha Q}{P}}\right)b_0\right)}{2Q\left(1-e^{-\frac{Qj}{D}}\right)^2} + \frac{DH_v}{2Pj} \\ &+ \left[\frac{1-e^{-\frac{\alpha Q}{P}}}{2P\sqrt{\frac{Q}{P}}} + \frac{\alpha e^{-\frac{\alpha Q}{P}}\sqrt{\frac{Q}{P}}}{P}\right]\frac{H_b}{j}\sigma_l G(u) + \frac{b_0\alpha e^{-\frac{\alpha Q}{P}}}{P(1-e^{-\frac{Qj}{D}})}\sigma_l\sqrt{\frac{Q}{P}}G(u) \end{aligned} \tag{5.21}$$

Now, for fixed P and Q, $PVETC$ is convex in u, since

$$\frac{\partial^2 PVETC}{\partial u^2} = \sigma_l \sqrt{\frac{Q}{P}} \left[\frac{H_b}{j} \left(1 - e^{-\frac{\alpha Q}{P}}\right) + \frac{b + \left(1 - e^{-\frac{\alpha Q}{P}}\right) b_0}{1 - e^{-\frac{Qj}{D}}} \right] f(u) > 0 \tag{5.22}$$

for all $u > 0$. Solving $\frac{\partial PVETC}{\partial u} = 0$ for u, we get the optimal safety factor for a given lot-size (Q) and production rate (P) as

$$u = \overline{F}^{-1} \left(\frac{H_b \left(1 - e^{-\frac{Qj}{D}}\right)}{\left(1 - e^{-\frac{\alpha Q}{P}}\right) \left[H_b \left(1 - e^{-\frac{Qj}{D}}\right) + jb_0 \right] + jb} \right) \tag{5.23}$$

Next, differentiating (5.21) with respect to Q we get

$$\begin{aligned}
\frac{\partial^2 PVETC}{\partial Q^2} &= \left(\frac{H_b}{D} + \frac{Sj}{D}\left(1 - \frac{P_0}{P}\right)\right) f_1 + \frac{j^2(C_0 + C_s)}{D^2} f_2 + \frac{2\sigma_l \overline{\pi} j \sqrt{\frac{Q}{P}} G(u)}{DQ f_3^3} f_4 \\
&+ \frac{j^2 e^{-\frac{Qj}{D}} \sigma_l \sqrt{\frac{Q}{P}} G(u) \overline{\pi}}{D^2 f_3^2} + \frac{\alpha \sigma e^{-\frac{\alpha Q}{P}} G(u) \sqrt{\frac{Q}{P}}}{PQ} \left(\frac{H_b(P - Q\alpha)}{Pj} + \frac{b_0 f_5}{f_3^2} \right) \\
+ &\frac{\sigma_l \sqrt{\frac{Q}{P}} \frac{Qj}{D}^{\frac{Qj}{D}} \overline{\pi} G(u)}{Q^2 f_3^2} - \frac{\sigma_l \sqrt{\frac{Q}{P}} \overline{\pi} G(u)}{4 f_3 Q^2} - \frac{e^{-\frac{\alpha Q}{P}} e^{-\frac{Qj}{D}} (D f_4 \alpha + Pj) \sigma_l \sqrt{\frac{Q}{P}} G(u) b_0 \alpha}{P^2 D f_3^2} \\
- &\frac{\sigma_l \sqrt{\frac{Q}{P}}}{4Q^2} \left(u + (1 - e^{-Q\alpha/P}) G(u) \right) \frac{H_b}{j}
\end{aligned} \tag{5.24}$$

where $\overline{\pi} = b + \left(1 - e^{-\frac{\alpha Q}{P}}\right) b_0 > 0$, $f_1 = e^{-\frac{Qj}{D}} \frac{(2 + \frac{Qj}{D}) e^{-\frac{Qj}{D}} - 2 + \frac{Qj}{D}}{(1 - e^{-\frac{Qj}{D}})^3} > 0$, $f_2 = \frac{e^{-\frac{Qj}{D}}(1 + e^{-\frac{Qj}{D}})}{(1 - e^{-\frac{Qj}{D}})^3} > 0$, $f_3 = 1 - e^{-\frac{Qj}{D}} > 0$, $f_4 = 1 - e^{\frac{Qj}{D}} + \frac{Qj}{D} e^{\frac{Qj}{D}} > 0$, and $f_5 = 1 - e^{-\frac{Qj}{D}} - \frac{Qj}{D} e^{-\frac{Qj}{D}} > 0$.

From (5.24) it is difficult to check the sign of $\frac{\partial^2 PVETC}{\partial Q^2}$ analytically. We will check the sign of the second order derivative in the numerical section later.

Further, for given Q and u, $PVETC$ can not be shown to be convex in P. We develop the following lemma to obtain the optimal value of production rate P.

Lemma 5.1. *For fixed values of Q and u, the cost function $PVETC$ is a decreasing or increasing or concave function in P when $P_0 \leq P \leq P_{max}$. Therefore, the optimal value of P that minimizes $PVETC$ is either P_0 or P_{max}.*

Proof. The first order partial derivative of $PVETC$ with respect to P is

$$\begin{aligned}
\frac{\partial PVETC}{\partial P} &= \frac{1}{1-e^{-\frac{Qj}{D}}}\Bigg[-\frac{Q\sigma_l G(u)}{2P^2}\left\{b_0\left(1-e^{-\frac{\alpha Q}{P}}\right)+b_1\right\}\left(\frac{Q}{P}\right)^{-1/2} \\
&-\frac{b_0\alpha\sigma_l Qe^{-\frac{\alpha Q}{P}}\sqrt{\frac{Q}{P}}G(u)}{P^2}-\frac{H_b}{j}(1-e^{-\frac{Qj}{D}})\Bigg\{\frac{uQ\sigma_l}{2P^2\sqrt{\frac{Q}{P}}}+\frac{\alpha\sigma_l Qe^{-\frac{\alpha Q}{P}}\sqrt{\frac{Q}{P}}G(u)}{P^2} \\
&+\frac{Q\sigma_l\left(1-e^{-\frac{\alpha Q}{P}}\right)G(u)}{2P^2\sqrt{\frac{Q}{P}}}\Bigg\}-\frac{DQH_v(1-e^{-\frac{Qj}{D}})}{2P^2j}+\frac{QSP_0}{P^2}\Bigg] \\
&= \frac{1}{2P^2}\left(a_1-a_2\sqrt{P}+a3\frac{(P-2\alpha Q)e^{-\frac{\alpha Q}{P}}}{\sqrt{P}}\right), \qquad (5.25)
\end{aligned}$$

where $a_1(Q)=\frac{2QSP_0}{1-e^{-\frac{Qj}{D}}}-\frac{DQH_v}{j}, a_2(Q)=\sqrt{Q}\left(\frac{\sigma_l(b_1+b_0)G(u)}{1-e^{-\frac{Qj}{D}}}+\frac{\sigma_l H_b}{j}[u+G(u)]\right),$
$a_3(Q)=\sqrt{Q}\left(\frac{\sigma_l b_0 G(u)}{1-e^{-\frac{Qj}{D}}}+\frac{\sigma_l H_b}{j}G(u)\right).$

case (i) $a_1-a_2\sqrt{P_0}+a3\frac{(P_0-2\alpha Q)e^{-\frac{\alpha Q}{P_0}}}{\sqrt{P_0}}<0.$
In this case, $PVETC$ is a strictly decreasing function of P. Therefore, the minimum total cost will occur at the maximum point P_{max}. Hence P_{max} is the optimal solution minimizing $PVETC$.

case (ii) $a_1-a_2\sqrt{P_{max}}+a3\frac{(P_{max}-2\alpha Q)e^{-\frac{\alpha Q}{P_{max}}}}{\sqrt{P_{max}}}>0.$
In this case, $PVETC$ is a strictly increasing function of P. Therefore, the minimum cost will occur at the minimum point P_0. Hence P_0 is the optimal solution minimizing $PVETC$.

case (iii) $a_1-a_2\sqrt{P_0}+a3\frac{(P_0-2\alpha Q)e^{-\frac{\alpha Q}{P_0}}}{\sqrt{P_0}}>0$ and $a_1-a_2\sqrt{P_{max}}+a3\frac{(P_{max}-2\alpha Q)e^{-\frac{\alpha Q}{P_{max}}}}{\sqrt{P_{max}}}<0.$
In this case, $PVETC$ is a concave function of P for all $P\in[P_0,P_{max}]$. Therefore, the optimal production rate that minimizes $PVETC$ for fixed Q and u can be selected as either P_0 or P_{max} by comparing $PVETC(Q,u,P_0)$ and $PVETC(Q,u,P_{max})$.

□

5.5 Numerical experiments

In this section, we provide four numerical examples using different data sets to investigate how the optimal decision variables change with the model parameters.

Example 1: $D = 200$ units/year, $P_0 = 300$ units/year, $P_{max} = 400$ units/year $C_o = \$300/$ order, $C_s = \$500/$ order, $H_b = \$6$/unit/unit time, $H_v = \$4$/unit/unit time, $\sigma_l = 15$ units, $S = \$1.5$, $b_0 = \$150$, $b = \$100$, $j = \$0.12$, $\alpha = 0.85$. Using the

$P = P_0(= 300)$							$P = P_{max}(= 400)$							
Q	u	r	SS	l	$\delta(l)$	$PVETC$	Q	u	r	SS	l	$\delta(l)$	$PVETC$	LT reduction %
183	1.85	144	22	0.6097	0.5956	15700	190	1.80	114	19	0.4758	0.6673	15648	21.96

Table 5.1: Numerical results of Example 1

solution algorithm, the optimal results are found for the case when the lead-time demand follows normal distribution. We have

$$a_1 - a_2\sqrt{P_0} + a3\frac{(P_0 - 2\alpha Q)e^{-\frac{\alpha Q}{P_0}}}{\sqrt{P_0}} = -70127 < 0$$

Therefore, following lemma 1(i), we can say that $PVETC$ is strictly decreasing function of P and the minimum cost will occur when the production rate is maximum i.e., $P = P_{max}$ (see Figure 5.2). The detailed results are given in Table 5.1. From Table 5.1, we have the optimal results as follows: production rate $P(= P_{max}) = 400$, order quantity $Q = 190$ units, safety factor $u = 1.8045$, reorder point $r = 114$ units, safety stock $SS = 19$ units, lead-time $l = 0.4758$ time units, backorder rate $\delta(l) = 0.6673$ and the net present value of the expected total cost $PVETC = \$15648$. It is observed that reductions in lead-time by 21.96% and NPV of the expected total cost by \$52 are possible by running the production system at it's maximum level.

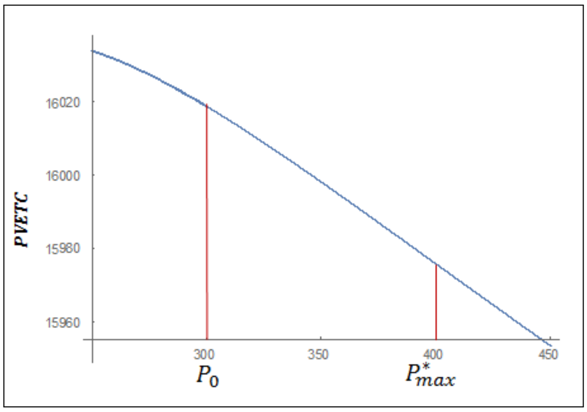

Figure 5.2: Graph of $PVETC$ in Example 1 (optimal solution P^*_{max})

Example 2: $D = 150$ units/year, $P_0 = 300$ units/year, $P_{max} = 400$ units/year $C_o = \$60/$ order, $C_s = \$300/$ order, $H_b = \$12$/unit/unit time, $H_v = \$9$/unit/unit time, $\sigma_l = 5$ units, $S = \$1.5$, $b_0 = \$100$, $b = \$20$, $j = \$0.11$, $\alpha = 0.85$.

We have

$$a_1 - a_2\sqrt{P_{max}} + a3\frac{(P_{max} - 2\alpha Q)e^{-\frac{\alpha Q}{P_{max}}}}{\sqrt{P_{max}}} = 171234 > 0.$$

Therefore, from lemma 1(ii) we can say that $PVETC$ is strictly increasing function of P and the minimum cost will occur when the production rate is minimum i.e., $P = P_{min}$ (see Figure 5.3). The detailed results are given in Table 5.2. From Table 5.2, we have the optimal results as follows: production rate $P(= P_{min}) = 300$, order quantity $Q = 79$ units, safety factor $u = 1.04$, reorder point $r = 42$ units, safety stock $SS = 3$ units, lead-time $l = 0.2645$ time units, backorder rate $\delta(l) = 0.7986$ and $PVETC = \$12799$.

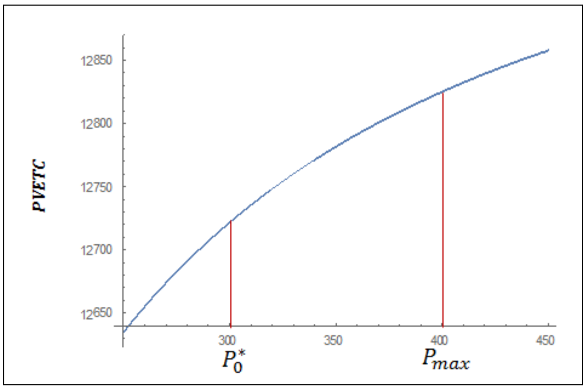

Figure 5.3: Graph of $PVETC$ in Example 2 (optimal solution P_0^*)

$P = P_0(= 300)$							$P = P_{max}(= 400)$							
Q	u	r	SS	l	$\delta(l)$	$PVETC$	Q	u	r	SS	l	$\delta(l)$	$PVETC$	LT reduction %
79	1.04	42	3	0.2645	0.7986	12799	82	0.94	33	2	0.2056	0.8396	12837	–

Table 5.2: Numerical results of Example 2

Example 3: $D = 100$ units/year, $R_0 = 300$ units/year, $R_{max} = 400$ units/year $C_o = \$100/$ order, $C_s = \$200/$ order, $H_b = \$4$/unit/unit time, $H_v = \$1$/unit/unit time, $\sigma_l = 45$ units, $S = \$1.5$, $b_0 = \$150$, $b = \$100$, $j = \$0.1$, $\alpha = 0.85$.

$P = P_0(= 300)$							$P = P_{max}(= 400)$							
Q	u	r	SS	l	$\delta(l)$	$PVETC$	Q	u	r	SS	l	$\delta(l)$	$PVETC$	LT reduction %
79	1.04	42	3	0.2645	0.7986	12799	100	1.89	68	43	0.2509	0.8079	7753	22.70

Table 5.3: Numerical results of Example 3

We have

$$a_1 - a_2\sqrt{P_0} + a3\frac{(P_0 - 2\alpha Q)e^{-\frac{\alpha Q}{P_0}}}{\sqrt{P_0}} = 82034 > 0$$

and

$$a_1 - a_2\sqrt{P_{max}} + a3\frac{(P_{max} - 2\alpha Q)e^{-\frac{\alpha Q}{P_{max}}}}{\sqrt{P_{max}}} = -31848 < 0.$$

Therefore, from lemma 1(iii), we can say that $PVETC$ is a concave function of P and the minimum present value of the joint expected total cost will occur either at the minimum production rate or maximum production rate. Now, we have $PVETC|_{P=P_0} = 7741 < PVETC|_{P=P_{max}} = 7753$. Therefore, the minimum present value of the joint expected total cost will occur when the production rate is minimum (see Figure 5.4). Detailed results are given in Table 5.3. From Table 5.3, we have the optimal results as follows: production rate $P(= P_{min}) = 300$, order quantity $Q = 97$ units, safety factor $u = 1.92$, reorder point $r = 82$ units, safety stock $SS = 49$ units, lead-time $l = 0.3246$ time units, backorder rate $\delta(l) = 0.7589$, and $PVETC = \$7741$.

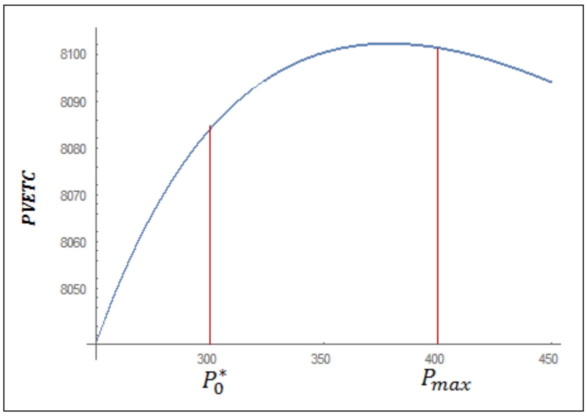

Figure 5.4: Graph of $PVETC$ in Example 3(optimal solution P_0^*)

$P = P_0(= 300)$							$P = P_{max}(= 400)$							
Q	u	r	SS	l	$\delta(l)$	$PVETC$	Q	u	r	SS	l	$\delta(l)$	$PVETC$	LT reduction %
142	2.09	175	89	0.4744	0.6682	12768	148	2.04	144	77	0.3701	0.7301	12745	21.98

Table 5.4: Numerical results of Example 4

Example 4: $D = 180$ units/year, $P_0 = 300$ units/year, $P_{max} = 400$ units/year $C_o = \$150/$ order, $C_s = \$250/$ order, $H_b = \$4$/unit/unit time, $H_v = \$1$/unit/unit time, $\sigma_l = 62$ units, $S = \$1.5$, $b_0 = \$200$, $b = \$100$, $j = \$0.1$, $\alpha = 0.85$. We have $a_1 - a_2\sqrt{P_0} + a3\frac{(P_0-2\alpha Q)e^{-\frac{\alpha Q}{P_0}}}{\sqrt{P_0}} = 48642 > 0$ and $a_1 - a_2\sqrt{P_{max}} + a3\frac{(P_{max}-2\alpha Q)e^{-\frac{\alpha Q}{P_{max}}}}{\sqrt{P_{max}}} = -169350 < 0$.

Therefore, from lemma 1(iii), we can say that $PVETC$ is a concave function of P and the minimum cost will occur either at the minimum production rate or maximum production rate. Now, we have $PVETC|_{P=P_0} = 12768 > PVETC|_{P=P_{max}} =$

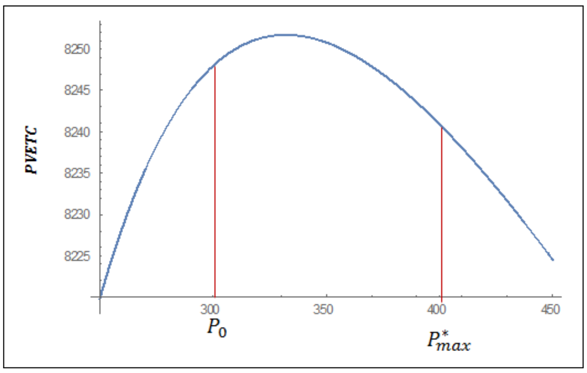

Figure 5.5: Graph of $PVETC$ in Example 5 (optimal solution P_{max}^*)

12745. Therefore, the minimum present value of joint expected total cost will occur when the production rate is maximum (see Figure 5.5). Detailed results are given in Table 5.4. From Table 5.4, we find the optimal results as follows: production rate $P(= P_{max}) = 400$, order quantity $Q = 148$ units, safety factor $u = 2.04$, reorder point $r = 144$ units, safety stock $SS = 77$ units, lead-time $l = 0.3701$ time units, backorder rate $\delta(l) = 0.7301$ and $PVETC = \$12745$.

In Figure 5.6, the convexity of $PVETC$ with respect to Q is shown.

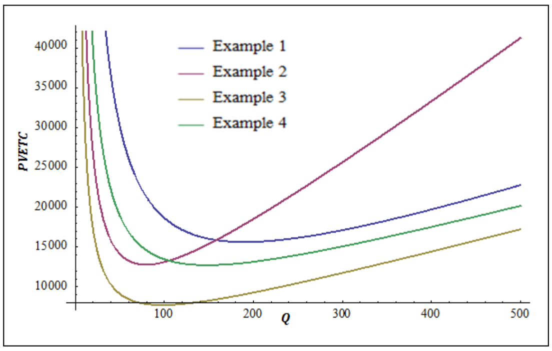

Figure 5.6: The convexity of expected cost function $(PVETC)$ with respect to Q

5.5.1 Sensitivity analysis

To obtain insights of the behavior of the model, a brief sensitivity analysis is conducted in this section by varying several model-parameters. The sensitivity analysis is performed based on the parameter-values of Example 3. The optimal decisions that minimize the net present value of the expected total cost of the supply chain are found for three different scenarios - high demand deviation $(\sigma_l = 50)$ scenario, medium demand deviation $(\sigma_l = 30)$ scenario, and low demand deviation $(\sigma_l = 10)$ scenario. For these scenarios, we study the cost minimizing decision variables and the expected total cost for a varying productivity improvement cost (S).

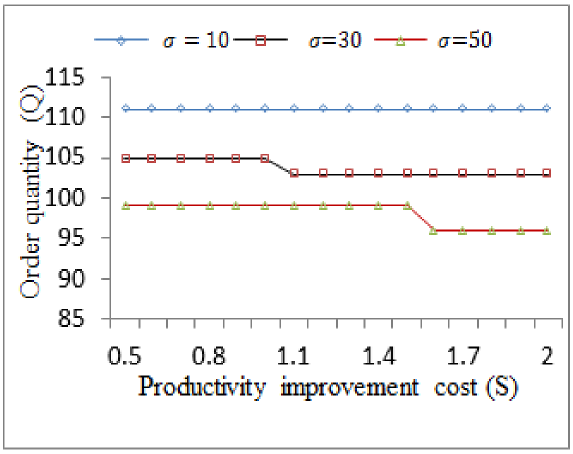

Figure 5.7: Impact of productivity improvement cost (S) on order quantity (Q)

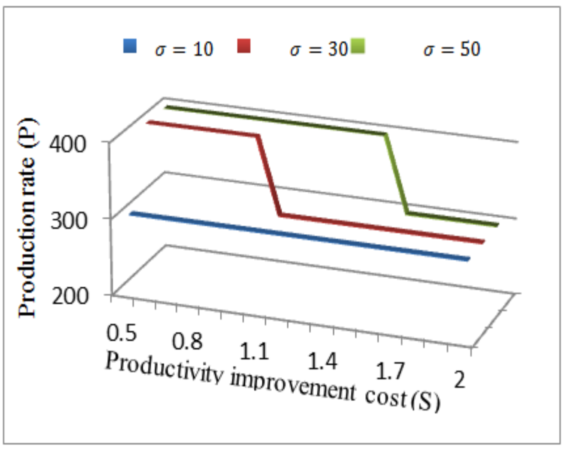

Figure 5.8: Impact of productivity improvement cost (S) on production rate (P)

An increasing value of demand deviation increases the chance of stock-out probability. Therefore, the present value of the expected total cost increases for all three scenarios. In Figure 5.7, it is observed that the order quantity is insensitive to productivity improvement cost for low demand deviation. This is due to the fact that for low demand deviation, there is no need to invest in order to improve the productivity of the system (see Figure 5.8). However, for high and medium demand deviations, order quantities are sensitive to productivity improvement cost and a step-wise decrease in order quantity can be seen in Figure 5.7. Therefore, for the case of medium and high demand deviations, it is beneficial to decrease the replenishment lead-time by ordering less quantity.

The effect of variation in productivity improvement cost (S) on the production rate (P) is illustrated in Figure 5.8. It is observed that, for low demand deviation, the production rate is kept constant at it's minimum level, i.e, $P = 300$, which means that there is no need to increase the production rate for low demand deviation as in this case the chance of stock-out is less. However, for the the case of medium and high demand deviations, the production rate is kept at it's maximum level i.e., $P = 400$ until S reaches 1 and 1.5, and P is decreased afterwards as well. This result is practical, as the demand deviation increases the stock-out probability increases which pushes the supply chain manager to run the production system with maximum production rate to reduce the replenishment lead-time and prevent inventory stock-out.

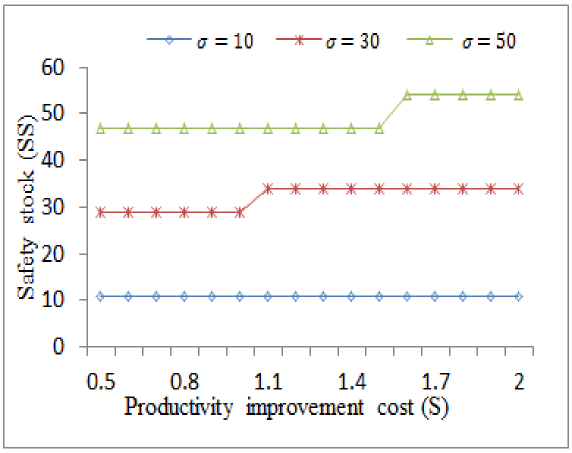

Figure 5.9: Impact of productivity improvement cost (S) on safety stock (SS)

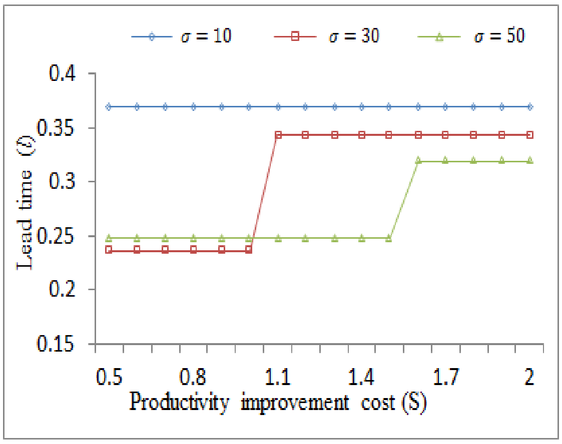

Figure 5.10: Impact of productivity improvement cost (S) on lead-time l

Figures 5.9 and 5.10 exhibit the effects of productivity improvement cost (S) on safety stock level and replenishment lead-time. From Figure 5.9, we see that, for the case of high demand uncertainty, the safety stock level should be higher than medium and low demand uncertainties in order to defend the system from the risk of stock-out probability. Figure 5.10 illustrates that replenishment lead-time for low demand uncertainty is high and constant compared to medium and high demand uncertainties, respectively. This behavior is due to the fact that, for low demand uncertainty, the order quantity is high (see Figure 5.7) and production rate is minimum (see Figure 5.8) which leads to higher replenishment lead-time. On the other hand, for the case of medium and high demand uncertainties, the replenishment lead-time increases after a certain time due to minimum production rate.

Figure 5.11 presents the effect of coefficient of backorder rate (α) ranging from 0 to 15 on the expected total cost and backorder rate. Figure 5.11 shows that the coefficient of backorder rate is proportional to the total supply chain cost and inversely proportional to the backorder rate. A higher value of α increases the mean waiting time, thereby it decreases the backorder rate and hence it increases the total supply chain cost.

Figure 5.12 illustrates the effects of the buyer's holding cost on optimal decisions. Figure 5.12 depicts that with higher holding cost, both the order quantity and safety stock level decrease. The buyer's replenishment lead-time decreases due to decrease in order quantity. It is observed that the buyer's holding cost plays an important role in deciding the optimal safety stock level. The more the safety stock is, the higher is the holding cost, and hence higher is the supply chain cost.

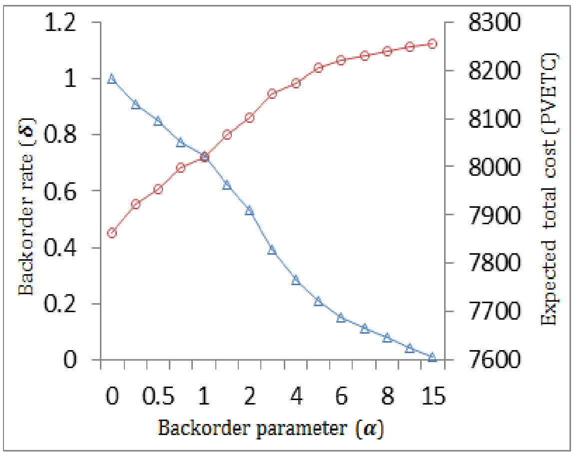

Figure 5.11: Impact of backorder parameter (α) on backorder rate (δ) and expected cost $(PVETC)$

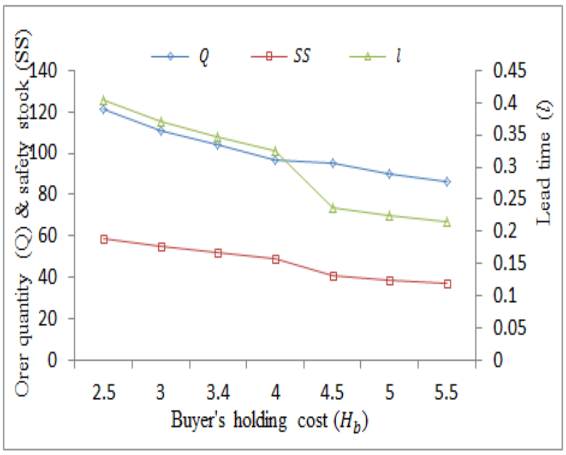

Figure 5.12: Impact of holding cost (H_b) on order quantity (Q), safety stock (SS) and lead-time (l)

5.5.2 Managerial insights

I. In real-life situations, it is quite natural that the demand is random. When the demand deviation is high, the chance of inventory stock-out increases. To overcome this situation, supply chain managers should store more safety stocks. Moreover, when the demand deviation is high, it is always preferable to cut down the replenishment lead-time through by increasing the production rate.

II. Reorder point is an important decision in the continuous review supply chain systems. Very early order placement can increase the inventory holding cost whereas a very late order placement can put the supply chain system in stock-out situation. Supply chain managers can determine the optimum reorder point following the strategies of the proposed model.

III. From the computational results, it is observed that if the holding cost starts to increase, the total cost of the supply chain shoots rapidly. So, the supply chain managers must monitor the safety stock and the order quantity for this condition. When the buyer's holding cost is high, it is preferable to store less safety stock.

IV. It is profitable to run the production system with a low production rate during low demand deviation. However, in the case of medium and high demand deviations, it is suggested to run the production system with a high production rate.

5.6 Conclusions

In reality, the market demand is highly dependent on the delivery lead-time; a little change in lead-time affects extremely on the market demand. So, deciding the optimal delivery lead-time plays a vital role in optimizing the total cost of a supply chain. In this chapter, we develop a two-echelon supply chain model where the buyer faces stochastic lead-time demand from the customers and the lead-time is assumed to be a function of order quantity and production rate. The vendor has the option that he can produce the order quantity through a maximum or minimum production rate. The backloging rate at the buyer is a function of replenishment lead-time. Therefore, if the buyer wants to increase the backorder rate, he can reduce the replenishment lead-time by some additional investment. From the numerical study, we have found that lead-time demand deviation has an impressive effect on selecting the optimal production rate. The results of sensitivity analysis revealed that the waiting time of the customer decreases the backorder rate and increases the supply chain cost. Additionally, it is seen that, for comparatively high additional cost, it is not profitable to increase the production rate.

Chapter 6

An integrated two-echelon supply chain model with controllable lead-time and trade-credit financing

6.1 Introduction

This chapter* studies a lead-time reduction strategy for a single-manufacturer single-retailer integrated inventory system with controllable backorder rate and trade-credit financing. Here the lead-time is decomposed into several components, each with a minimum and maximum/normal duration. Moreover, its is considered that each lead-time component is reducible to its minimum duration with some crashing cost which is an increasing function of reduced lead-time. First, the lead-time demand at the retailer is assumed to be normally distributed, and then it is considered distribution-free. Shortages in the retailer's inventory, if occur, are partially backlogged and the backlogging rate depends on the lead-time. The manufacturer offers the retailer a credit period that is less than the reorder interval. Min-max approach is adopted to solve the model when lead-time demand is distribution-free.

6.2 Preliminary aspects

We consider a single item for a single-buyer and single-vendor SC problem. The customer's demand is stochastic. Following continuous review (Q, r) inventory replenishment policy, the buyer places an order of Q units to the vendor as soon as the

*This chapter is based on the work published in *RAIRO Operations Research*, 55 (2021) S673-S698.

inventory level falls to the reorder point r. The vendor follows the single setup multiple delivery (SSMD) policy to meet the demand. The buyer receives each shipment after a certain time, specified by the replenishment lead-time, which is deterministic and fixed and can be controlled by some additional investment. We formulate the problem to minimize the expected joint total cost per unit time for the vendor-buyer integrated system.

6.2.1 Notation and assumptions

We use the following notation to develop the proposed model.

- **Decision variables**

Q	retailer's ordered quantity (units)
L	retailer's lead-time (week)
k	safety factor
β	fraction of demand which is backordered during stock-out period, $\beta \in [0, 1]$
m	number of deliveries from manufacturer to retailer

- **Parameters**

D	annual demand at the retailer (units/year)
S	manufacturer's setup cost per setup (\$/setup)
A	retailer's ordering cost per order (\$/order)
r_b	retailer's holding cost rate per unit per unit time
r_v	manufacturer's holding cost rate per unit per unit time
π_0	retailer's marginal profit (\$/unit)
π	unit shortage cost at the retailer (\$/unit)
$C(L)$	lead-time crashing cost function
r	reorder point at the retailer
t_c	retailer's trade-credit period (year)
c_b	purchasing price (\$/item)
c_s	selling price (\$/item)
c_v	unit production cost at the manufacturer (\$/item)
I_c	fixed interest rate at which the retailer has to pay to the bank for the remaining amount of stock during the period t_c to Q/D (\$\$/year)
I_d	fixed interest rate for the revenue earned by the retailer (\$/\$/year)

I_v	fixed interest rate for calculating the manufacturer's interest (opportunity) loss due to trade-credit offer (\$/\$/year)
σ	standard deviation of the lead-time demand
u_i	i-th component of lead-time with u_i as minimum duration (days), $i = 1, 2, ..., n$
v_i	i-th component of lead-time with v_i as (days), normal duration $i = 1, 2, ..., n$
m_i	i-th component of lead-time with m_i as crashing cost per day, $i = 1, 2, 3, ...n$
X	lead-time demand having distribution function F, finite mean DL and standard deviation $\sigma\sqrt{L}$
$E(X)$	mathematical expectation of X
x^+	max $\{x, 0\}$
$E(X-r)^+$	expected shortage quantity at the end of the cycle
ETC_b	expected average cost for the retailer
ETC_v	average cost for the manufacturer
ETC^N	expected average cost of the integrated system in normal distribution case
ETC^W	expected average cost of the integrated system in distribution-free case

We make the following assumptions to develop the model:

1. A supply chain consisting a single-manufacturer and a single-retailer deals with a single type of item.

2. The retailer places an order of size mQ which the manufacturer produces with a finite production rate $P(> D)$ in a single setup but ships the entire quantity to the retailer over m deliveries of equal size.

3. The retailer's inventory is continuously monitored. Replenishment is planned whenever the inventory level drops to the reorder point r. The reorder point r is defined by $r = DL + k\sigma\sqrt{L}$, where DL = expected demand during lead-time, $k\sigma\sqrt{L}$ = safety stock.

4. The lead-time L consists of n mutually independent components. The i-th component has a minimum duration u_i days, normal duration v_i days, and a crashing cost c_i per day. Furthermore, we rearrange c_i as $c_1 \leq c_2 \leq c_3 .... \leq c_n$. Then, it is clear that the reduction of lead-time should first occur in component 1 (because it has the minimum unit crashing cost), and then component 2, and so on.

5. We take $L_0 = \sum_{j=1}^{n} v_j$ and L_i as (Liao and Shyu, 1991)

$$L_i = \sum_{j=1}^{n} v_j - \sum_{j=1}^{i} (v_j - u_j)$$

where $i = 1, 2, 3, ..., n$, and the lead-time crashing cost function $C(L)$ as

$$C(L) = m_i(L_{i-1} - L) + \sum_{j=1}^{i-1} m_j(v_j - u_j)$$

6. The manufacturer provides a permissible delay period to the retailer.

7. The offered credit period is less than the reorder interval, which means that the credit period cannot be longer than the time at which another order is placed. This is in agreement with the usual practice.

8. The backorder rate is variable and it is a function of lead-time.

6.3 Model development

As mentioned in assumption (3), whenever the inventory level drops to the reorder point r, the retailer requests the manufacturer for delivery. The manufacturer produces mQ units (where m is an integer) at one setup. Therefore, the average cycle time for the manufacturer is $\frac{mQ}{D}$ and the average length of a replenishment cycle is $\frac{Q}{D}$. The average ordering cost and the average lead-time crashing cost are $\frac{AD}{Q}$ and $\frac{DC(L)}{Q}$, respectively.

According to our assumption, the lead-time demand X has a probability density function $f(x)$ with mean DL and standard deviation $\sigma\sqrt{L}$ and the reorder point $r = DL + k\sigma\sqrt{L}$. Shortages occur when $X > r$. The retailer's expected shortage quantity at the end of a replenishment cycle is $E(X-r)^+$ and hence, the expected backorder quantity is $\beta E(X-r)^+$. Therefore, the expected loss in sales per shipment cycle is $(1-\beta)E(X-r)^+$ and the expected stock-out cost per replenishment cycle is $[\pi + \pi_0(1-\beta)]\, E(X-r)^+$.

Further, at the beginning of each replenishment cycle, the retailer's expected net inventory is the safety stock $(r - DL)$ plus the previous replenishment cycle's lost sales $(1-\beta)E(X-r)^+$, and the expected net inventory level immediately after a replenishment is $Q + r - DL + (1-\beta)E(X-r)^+$. Therefore, the average inventory over a replenishment cycle is $\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+$. Hence, the retailer's holding cost per unit time is $h_b[\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+]$. Further, the safety stock plus the previous replenishment cycle's lost sales is $r - DL + (1-\beta)E(X-r)^+$ which is carried throughout the replenishment cycle. Therefore, the total interest charged at a rate I_c by the manufacturer to the retailer for this amount of stock is $c_b I_c[r - DL + (1-\beta)E(X-r)^+]$.

We assume that the permissible delay period is t_c which is less than the reorder interval. This assumption is realistic, as the payment for the earlier order should be cleared before another order is placed. Here, the retailer earns interest on the sales revenue at the rate I_d during the time period $(0, t_c)$. Therefore, the retailer's interest earn per unit time is $\frac{c_s I_d D}{Q}\int_0^{t_c} Dtdt = \frac{D^2 t_c^2 c_s I_d}{2Q}$. Additionally, the previous replenishment cycle's backlogged items are cleared at the beginning of the cycle. Therefore, the interest earned per unit time from the backlogged items is $\frac{c_s I_d t_c D}{Q}\beta E(X-r)^+$. The retailer still has some inventory $(Q - Dt_c)$ after the credit period t_c. If he takes a short term loan from the bank at an interest rate I_c for the duration $(t_c, \frac{Q}{D})$ to finance the unsold stock then his opportunity cost (due to payment of interest) per unit time is $\frac{c_b I_c D}{Q}\int_{t_c}^{Q/D}(Q - Dt)dt = \frac{(Q-Dt_c)^2 c_b I_c}{2Q}$.

We take the backorder rate β as a variable and define it as

$$\beta = \frac{1}{1 + \alpha E(X-r)^+} \tag{6.1}$$

α $(0 < \alpha < \infty)$ being constant. From (6.1), we see that the backorder rate is a decreasing function of shortage quantity. Further, as $\alpha \to \infty$, we have $\beta \to 0$ (complete lost sale case) and as $\alpha \to 0$, we have $\beta \to 1$ (complete backordered case).
The retailer's expected total cost per unit time is

$$\begin{aligned}
ETC_b(Q, r, L) &= \text{ordering cost + holding cost + safety stock plus previous cycle's lost} \\
&\quad \text{sale cost + stock out cost + opportunity (interest) cost - interest earned} \\
&= \frac{AD}{Q} + \frac{r_b c_b Q}{2} + c_b(r_b + I_c)\left[r - DL + (1-\beta)E(X-r)^+\right] \\
&+ \frac{D}{Q}\left[\pi + \pi_0(1-\beta)\right]E(X-r)^+ + \frac{(Q-Dt_c)^2 c_b I_c}{2Q} \\
&- \frac{D^2 t_c^2 c_s I_d}{2Q} - \frac{c_s t_c I_d D\beta}{Q}E(X-r)^+
\end{aligned} \tag{6.2}$$

On the other hand, the manufacturer's total cost is

$$TC_v(m) = \text{setup cost} + \text{holding cost} + \text{opportunity (interest) cost}$$

The manufacturer's setup cost per unit time is $\frac{SD}{mQ}$.
Therefore, the manufacturer's average inventory is

$$\begin{aligned} & \frac{D}{mQ}\left[\left\{mQ\left(\frac{Q}{P}+\frac{Q}{D}(m-1)\right)-\frac{m^2Q^2}{2P}\right\}-\left\{\frac{Q^2}{D}(1+2+...+(m-1))\right\}\right] \\ = & \frac{Q}{2}\left[(m-1)-(m-2)\frac{D}{P}\right] \end{aligned}$$

Therefore, the manufacturer's holding cost per unit time is

$$= \frac{r_v c_v Q}{2}\left[(m-1)-(m-2)\frac{D}{P}\right]$$

Hence, the manufacturer's total cost per unit time is

$$TC_v(m) = \frac{SD}{mQ} + \frac{r_v c_v Q}{2}\left\{(m-1)-(m-2)\frac{D}{P}\right\} + I_v c_b t_c D \tag{6.3}$$

Therefore, the expected average total cost of the supply chain is the sum of the retailer's expected average total cost given by (6.2) and the manufacturer's average total cost given by (6.3), i.e.,

$$\begin{aligned} ETC(Q,r,L,m) \;=\; & \frac{D}{Q}\left[A+C(L)+\frac{S}{m}\right]+\frac{r_b c_b Q}{2}+\frac{D}{Q}\left[\pi+\pi_0(1-\beta)\right]E(X-r)^+ \\ + \; & c_b(r_b+I_c)\left[r-DL+(1-\beta)E(X-r)^+\right]+\frac{(Q-Dt_c)^2 c_b I_c}{2Q} \\ - \; & \frac{D^2 t_c^2 c_s I_d}{2Q}-\frac{D c_s t_c I_d}{Q}\beta E(X-r)^+ + I_v c_b t_c D \\ + \; & \frac{r_v c_v Q}{2}\left\{(m-1)-(m-2)\frac{D}{P}\right\} \end{aligned} \tag{6.4}$$

6.3.1 Solution procedure

6.3.1.1 Lead-time demand follows normal distribution

In this sub-section, we assume that the lead-time demand X is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$. We note that $r = DL + k\sigma\sqrt{L}$ and the expected shortage quantity at the end of a cycle is

$$\begin{aligned} E(X-r)^{+} &= \int_{r}^{\infty} (x-r)\, f(x) dx \\ &= \int_{DL+k\sigma\sqrt{L}}^{\infty} \left\{x - \left(DL + k\sigma\sqrt{L}\right)\right\} \frac{1}{\sigma\sqrt{L}\sqrt{2\pi}} e^{-\frac{(x-DL)^2}{2\sigma^2 L}} dx \end{aligned}$$

After some calculations, the above expression reduces to (see Pan and Yang, 2002; Ouyang, Wu, and Ho, 2004)

$$E(X-r)^{+} = \sigma\sqrt{L}\Psi(k) \tag{6.5}$$

where $\Psi(k) = \phi(k) - k[1 - \Phi(k)]$, and ϕ and Φ denote the standard normal probability density function and distribution function, respectively.
Substituting the value of $E(X-r)^{+}$ in (6.1), we get

$$\beta = \frac{1}{1 + \alpha\sigma\sqrt{L}\Psi(k)} \tag{6.6}$$

Therefore, when lead-time demand follows normal distribution, the expected average total cost of the supply chain can be obtained by using (6.5) and (6.6) in (6.4) as

$$\begin{aligned} ETC^{N}(Q,k,L,m) &= \frac{D}{Q}[G(m) + C(L)] + c_b(r_b + I_c)k\sigma\sqrt{L} + \frac{Q}{2}H(m) \\ &+ \left\{\frac{D}{Q}\left(\pi - \frac{c_s t_c I_d}{1 + \alpha\sigma\sqrt{L}\Psi(k)}\right) + \frac{\alpha\sigma\sqrt{L}\Psi(k)M(Q)}{1 + \alpha\sigma\sqrt{L}\Psi(k)}\right\} \sigma\sqrt{L}\Psi(k) \\ &+ \frac{(Q - Dt_c)^2 c_b I_c}{2Q} - \frac{D^2 t_c^2 c_s I_d}{2Q} + I_v c_b t_c D \qquad (6.7) \\ \text{where } G(m) &= A + \frac{S}{m} \\ M(Q) &= \frac{D\pi_0}{Q} + c_b(r_b + I_c) \\ H(m) &= r_b c_b + r_v c_v \left[(m-1) - (m-2)\frac{D}{P}\right] \end{aligned}$$

Note 1. It is clear that if $I_d = 0, I_c = 0, I_v = 0$ and $\alpha = 0$, i.e., the case when trade-credit is not allowed and shortages are fully backlogged, (6.7) reducers to

$$ETC^{N}(Q,k,L,m) = \frac{D}{Q}[G(m) + \pi\sigma\sqrt{L}\Psi(k) + C(L)] + \frac{Q}{2}H(m) + r_b c_b k\sigma\sqrt{L} \tag{6.8}$$

which is the same as of Ouyang, Wu, and Ho, 2004. Therefore, Ouyang, Wu, and Ho, 2004 model is a special case of our model.

Note 2. If we take $I_d = 0$ and $I_c = 0$ then the retailer's cost equation (6.2) becomes

$$\begin{aligned} ETC_b(Q,L) &= \frac{D}{Q}[A + R(L)] + r_b c_b\left(\frac{Q}{2} + k\sigma\sqrt{L}\right) \\ &+ \left\{\frac{r_b c_b \alpha\sigma\sqrt{L}\Psi(k)}{1+\alpha\sigma\sqrt{L}\Psi(k)} + \frac{D}{Q}\left[\pi + \frac{\pi_0\alpha\sigma\sqrt{L}\Psi(k)}{1\alpha\sigma\sqrt{L}\Psi(k)}\right]\right\}\sigma\sqrt{L}\Psi(k) \quad (6.9)\end{aligned}$$

which is same as the expected average cost derived by Ouyang and Chuang, 2001 (taking $r_b c_b = h$, $\Psi(k) = G(k)$, and $C(L) = R(L)$). This indicates that Ouyang and Chuang, 2001's model is also a special case of our model.

To show that the expected cost function (6.7) is strictly convex i.e., it has a unique minimum, we derive the following propositions:

Proposition 6.1. *For given values of $Q, k,$ and m, $ETC^N(Q,k,L,m)$ is concave in $L \in [L_i, L'_{i-1}]$, where $L'_{i-1} = min\left\{L_{i-1}, \left(\frac{\sqrt{\pi c_s t_c I_d} - \pi}{\pi\alpha\sigma\Psi(k)}\right)^2\right\}$.*

Proof. For fixed m, taking the first order partial derivative of $ETC^N(Q,k,L,m)$ in (6.7) with respect to $L \in [L_i, L_{i-1}]$ we have

$$\begin{aligned} \frac{\partial ETC^N}{\partial L} &= \frac{c_b(r_b + I_c)k\sigma L^{-1/2}}{2} + M(Q)\frac{(2+v)v^2}{2\alpha L(1+v)^2} \\ &+ \frac{Dv}{2\alpha LQ}\left\{\pi - \frac{c_s t_c I_d}{(1+v)^2}\right\} - \frac{D}{Q}c_i \end{aligned}$$

The second order partial derivative of $ETC^N(Q,k,L,m)$ with respect to $L \in [L_i, L_{i-1}]$ is

$$\begin{aligned} \frac{\partial^2 ETC^N}{\partial L^2} &= -\frac{1}{4}c_b(r_b + I_c)k\sigma L^{-3/2} - M(Q)\frac{(3+v)v^3}{4\alpha L^2(1+v)^3} \\ &- \frac{D\pi v}{4\alpha L^2 Q}\left\{\pi - \frac{c_s t_c I_d(1+3v)}{(1+v)^3}\right\} < 0 \end{aligned}$$

if $L < \left(\frac{\sqrt{\pi c_s t_c I_d} - \pi}{\pi\alpha\sigma\Psi(k)}\right)^2$.

Hence, $ETC^N(Q,k,L,m)$ is concave in $L \in [L_i, L'_{i-1}]$ where $L'_{i-1} = \min\left\{L_{i-1}, \left(\frac{\sqrt{\pi c_s t_c I_d} - \pi}{\pi\alpha\sigma\Psi(k)}\right)^2\right\}$. Therefore, Proposition 6.1 is proved. □

Hence, the minimum value of the expected total cost $ETC^N(Q,k,L,m)$ will occur at the end points of the interval $[L_i, L'_{i-1}]$ (see Liao and Shyu, 1991, Ouyang, Wu, and Ho, 2004).

Proposition 6.2. *If $I_c > \frac{c_s I_d}{c_b}$, then for a fixed value of m and $L \in [L_i, L_{i-1}]$, $ETC^N(Q,k,L,m)$ is convex in Q and k for all $Q > 0$ and $k > 0$ such that $\Psi(k) > \frac{c_s t_c I_d}{\pi_0 \alpha \sigma \sqrt{L}}$.*

Proof. For fixed m and $L \in [L_i, L_{i-1}]$, taking first and second order partial derivatives of $ETC^N(Q,k,L,m)$ with respect to Q and k, we have

$$\begin{aligned}
\frac{\partial ETC^N}{\partial Q} &= -\frac{D}{Q^2}[G(m)+C(L)] - \frac{D^2 t_c^2 c_b I_c}{2Q^2} + \frac{D^2 t_c^2 c_s I_d}{2Q^2} \\
&- \frac{Dv}{\alpha Q^2}\left\{\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right\} + \frac{H(m)}{2} \\
\frac{\partial ETC^N}{\partial k} &= c_b(r_b + I_c)\sigma\sqrt{L} + \alpha\sigma^2 L \Psi(k)\lambda(k)\left\{\frac{2+v}{(1+v)^2}\right\} M(Q) \\
&+ \frac{D\lambda(k)\sigma\sqrt{L}}{Q}\left(\pi - \frac{c_s t_c I_d}{(1+v)^2}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 ETC^N}{\partial Q^2} &= \frac{2D}{Q^3}\left[G(m)+C(L)+\sigma\sqrt{L}\Psi(k)\left\{\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right\}\right] \\
&+ \frac{D^2 t_c^2}{Q^3}(c_b I_c - c_s I_d) > 0 \text{ if } I_c > \frac{c_s I_d}{c_b} \text{ and } \Psi(k) > \frac{c_s t_c I_d}{\pi_0 \alpha \sigma \sqrt{L}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 ETC^N}{\partial k^2} &= (r_b + I_c)c_b\left(\frac{v\sigma\sqrt{L}\phi(k)(2+v)}{(1+v)^2}\right) + \frac{2\alpha\sigma^2 L[\Phi(k)-1]^2}{(1+v)^3} \\
&\left\{\frac{D}{Q}(\pi_0 + c_s t_c I_d) + c_b(I_c + r_b)\right\} + \frac{D\sigma\sqrt{L}\phi(k)}{Q} \\
&\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right) > 0 \text{ if } \Psi(k) > \frac{c_s t_c I_d}{\pi_0 \alpha \sigma \sqrt{L}}
\end{aligned}$$

where $\overline{\pi} = \pi + \frac{\pi_0 v}{1+v}$, $\lambda(k) = \Phi(k) - 1$, $v = \alpha\sigma\sqrt{L}\Psi(k)$.

Hence if $\frac{\partial^2 ETC^N}{\partial Q^2} > 0$ and $\frac{\partial^2 ETC^N}{\partial k^2} > 0$, we can say that $ETC^N(Q,k,L,m)$ is convex in Q and k for all $Q > 0$ and $k > 0$ such that $\Psi(k) > \frac{c_s t_c I_d}{\pi_0 \alpha \sigma \sqrt{L}}$. Hence Proposition 6.2 is proved. □

Proposition 6.3. *For given values of m and* $L \in [L_i, L_{i-1}]$, $ETC^N(Q, k, L, m)$ *has a unique minimum provided that* $I_c > \frac{c_s I_d}{c_b}$ *and* $k > 0$ *such that* $\Psi(k) > \frac{c_s t_c I_d}{\pi_0 \alpha \sigma \sqrt{L}}$, *and the corresponding values of Q and k are given by*

$$Q = \sqrt{\frac{2D[G(m) + \sigma\sqrt{L}\Psi(k)\Delta(k, L) + C(L)] + F_1}{F_2 + H(m)}} \tag{6.10}$$

$$k = \Phi^{-1}\left(1 - \frac{F_2[V(k, L)]^2 Q}{v(k, L)QM(Q)[1 + V(k, L)] + D\pi[V(k, L)]^2 - Dst_c I_d}\right) \tag{6.11}$$

where $F_1 = D^2 t_c^2 (c_b I_c - c_s I_d)$, $F_2 = c_b(r_b + I_c)$, $\Delta(k, L) = \overline{\pi} - \frac{c_s t_c I_d}{1+v(k,L)}$, $V(k, L) = 1 + v(k, L)$.

Proof. For convexity of the cost function given in (6.7), the Hessian matrix for ETC^N must be positive definite. The Hessian matrix is given by

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 ETC^N(\cdot)}{\partial Q^2} & \frac{\partial^2 ETC^N(\cdot)}{\partial Q \partial k} \\ \frac{\partial^2 ETC^N(\cdot)}{\partial k \partial Q} & \frac{\partial^2 ETC^N(\cdot)}{\partial k^2} \end{bmatrix}$$

where $\frac{\partial^2 ETC^N(\cdot)}{\partial Q^2}$ and $\frac{\partial^2 ETC^N(\cdot)}{\partial k^2}$ are given above, and

$$\frac{\partial^2 ETC^N}{\partial Q \partial k} = \frac{\partial^2 ETC^N}{\partial k \partial Q} = \frac{D\sigma\sqrt{L}[1 - \Phi(k)]}{Q^2}\left[\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right]$$

Now

$$\begin{aligned}
|H| &= \frac{\partial^2 ETC^N(\cdot)}{\partial Q^2} \times \frac{\partial^2 ETC^N(\cdot)}{\partial k^2} - \left[\frac{\partial^2 ETC^N(\cdot)}{\partial Q \partial k}\right]^2 \\
&= \left\{\frac{2D}{Q^3}\left[G(m) + C(L) + \sigma\sqrt{L}\Psi(k)\left\{\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right\}\right] + \frac{D^2 t_c^2}{Q^3}(c_b I_c - c_s I_d)\right\} \\
&\times \left\{(r_b + I_c)c_b\left(\frac{v\sigma\sqrt{L}\phi(k)(2+v)}{(1+v)^2}\right) + \frac{2\alpha\sigma^2 L[\Phi(k) - 1]^2}{(1+v)^3}\left\{\frac{D}{Q}(\pi_0 + c_s t_c I_d)\right.\right. \\
&+ \left.\left. c_b(I_c + r_b)\right\} + \frac{D\sigma\sqrt{L}\phi(k)}{Q}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right)\right\} \\
&- \left\{\frac{D\sigma\sqrt{L}[1 - \Phi(k)]}{Q^2}\left[\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right]\right\}^2
\end{aligned}$$

$$
\begin{aligned}
&> \frac{2D}{Q^3}\sigma\sqrt{L}\Psi(k)\left\{\overline{\pi}-\frac{c_st_cI_d}{1+v}\right\}\frac{D\sigma\sqrt{L}\phi(k)}{Q}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)\\
&- \left\{\frac{D\sigma\sqrt{L}[1-\Phi(k)]}{Q^2}\left[\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right]\right\}^2\\
&= \frac{2D^2\sigma^2L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi}-\frac{c_st_cI_d}{1+v}\right)\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)\\
&- \frac{D^2\sigma^2L[1-\Phi(k)]^2}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)^2\\
&= \frac{2D^2\sigma^2L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)\\
&- \frac{2D^2\sigma^2L\phi(k)\Psi(k)}{Q^4}\left(\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}+\frac{c_st_cI_d}{1+v}\right)\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)\\
&- \frac{D^2\sigma^2L[1-\Phi(k)]^2}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)^2\\
&< \frac{2D^2\sigma^2L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)^2+\frac{2D^2\sigma^2L\phi(k)\Psi(k)}{Q^4}\\
&\times \left(\frac{c_st_cI_d}{(1+v)^2}\right)\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)-\frac{D^2\sigma^2L[1-\Phi(k)]^2}{Q^4}\\
&\times \left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)^2\\
&> \frac{2D^2\sigma^2L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)^2-\frac{D^2\sigma^2L[1-\Phi(k)]^2}{Q^4}\\
&\times \left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)^2\\
&= \frac{2D^2\sigma^2L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_st_cI_d}{(1+v)^2}\right)^2\{2\phi(k)\Psi(k)-[\Phi(k)-1]^2\}\\
&> 0,
\end{aligned}
$$

because $\phi(k) > 0, \Psi(k) > 0$ and $2\phi(k)\Psi(k) - [\Phi(k)-1]^2 > 0$, for all $k > 0$ (Ouyang, Chen, and Chang, 1999). Hence, the Proposition 6.3 is proved. □

Therefore, if the Hessian matrix for $ETC^N(Q,k,L,m)$ is positive definite then there exists a unique optimal solution which can be obtained from the first order necessary conditions $\frac{\partial ETC^N}{\partial Q} = 0$ and $\frac{\partial ETC^N}{\partial k} = 0$ as in (6.10) and (6.11.)

The optimal value of m i.e. m^* can be obtained from

$$ETC^N(m^*-1) \geq ETC^N(m^*) \leq ETC^N(m^*+1) \tag{6.12}$$

The following algorithm is suggested to obtain numerically the optimal values of Q and k for specific values of m and L.

Algorithm 1

Step 1 Set $m = 1$.

Step 2 For each L_i, $i = 0, 1, 2, ..., n$, perform 2a to 2c.

2a Set $k_{i1} = 0$ (implies $\Psi(k_{i1}) = 0.39894$).

2b Substituting $\Psi(k_{i1})$ into (10), evaluate Q_{i1}.

2c Utilize Q_{i1} to obtain the value of k_{i2} from (6.11), by checking the normal table and evaluate $\Psi(k_{i2})$.

2d Repeat 2b to 2c until no change occurs in the values of Q_i and k_i.

Step 3 For each set of values (Q_i, k_i, L_i, m), find $ETC^N(Q_i, k_i, L_i, m), i = 1, 2, ..., n$.

Step 4 Find $\min_{i=0,1,2,...,n} ETC^N(Q_i, k_i, , L_i, m)$.
If $ETC^N(Q^*_m, k^*_m, L^*_m, m) = \min_{i=0,1,2,...,n} ETC^N(Q_i, k_i, L_i, m)$, then (Q^*_m, k^*_m, L^*_m, m) is the optimal solution for fixed m.

Step 5 Set $m = m + 1$, repeat steps (2), (3), and (4) to get $ETC^N(Q^*_m, k^*_m, L^*_m, m)$.

Step 6 If $ETC^N(Q^*_m, k^*_m, L^*_m, m) \le ETC^N(Q^*_{m-1}, k^*_{m-1}, L^*_{m-1}, m)$, then go to Step 5; otherwise, go to Step 7.

Step 7 Set $ETC^N(Q^*_m, k^*_m, L^*_m, m) = ETC^N(Q^*_{m-1}, k^*_{m-1}, L^*_{m-1}, m)$. Then (Q^*, k^*, L^*, m) is the optimal solution.

After substituting the values of k^* and L^*, the optimal backorder rate and the reorder point can be obtained as

$$\beta^* = \frac{1}{1 + \alpha\sigma\sqrt{L^*}\Psi(k^*)} \text{ and } r^* = DL^* + k^*\sigma\sqrt{L^*}.$$

6.3.1.2 Lead-time demand is distribution-free

In many practical situations, the information about the probability distribution of lead-time demand is limited or unavailable. In this sub-section, we relax the assumption of normally distributed lead-time demand. We assume that the density function of the lead-time demand belongs to Ω with finite mean DL and standard deviation $\sigma\sqrt{L}$. If the distributional form of lead-time demand X is unknown, the exact value of $E(X-r)^+$ cannot be determined. Therefore, the min-max distribution-free approach is used to solve this problem (Gallego and Moon, 1993):

$$\text{Min Max}_{F\in\Omega} \quad ETC^W(Q,k,L,m) \tag{6.13}$$

The following proposition, which was proposed by Gallego and Moon, 1993 is used to approximate the value of $E(X-r)^+$.

Proposition 6.4. *For any $F \in \Omega$,*

$$E(X-r)^+ \leq \frac{1}{2}\left\{\sqrt{\sigma^2 L + (r-DL)^2} - (r-DL)\right\} \tag{6.14}$$

Substituting $r = DL + k\sigma\sqrt{L}$ in (6.14), the following inequality is obtained.

$$E(X-r)^+ \leq \frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right) \tag{6.15}$$

Using the above inequality, the backorder rate β can be expressed as

$$\beta \geq \frac{1}{1+\frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)} \tag{6.16}$$

Using (6.4) and (6.16), equation (6.13) becomes

$$\begin{aligned} ETC^W(Q,L,k,m) &= \frac{D}{Q}[G(m)+C(L)] + c_b(r_b+I_c)k\sigma\sqrt{L} + \frac{r_b c_b Q}{2} + \frac{Q}{2}H(m) \\ &+ \left[\left(\frac{\frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)}{1+\frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)}\right)M(Q)\right.\end{aligned}$$

$$+ \quad \frac{D}{Q}\left(\pi - \frac{c_s t_c I_d}{1+\frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)}\right)\Bigg]\frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)$$
$$+ \quad \frac{(Q-Dt_c)^2 c_b I_c}{2Q} - \frac{D^2 t_c^2 c_s I_d}{2Q} + I_v c_b t_c D \tag{6.17}$$

Similar to the case of normally distributed demand, it can be easily verified that, for fixed (Q,k,m), $ETC^W(Q,L,k,m)$ is convex in $L \in [L_i, L_{i-1}]$. Therefore, the minimum expected average cost will occur at the end point of the interval $[L_i, L_{i-1}]$. Keeping m and $L \in [L_i, L_{i-1}]$ fixed, it can also be verified that $ETC^W(Q,L,k,m)$ is convex in Q and k. Therefore, for fixed values of m and $L \in [L_i, L_{i-1}]$, the average cost will be minimum at the point (Q,k) which satisfies $\partial ETC^W(Q,L,k,m)/\partial Q = 0$ and $\partial ETC^W(Q,L,k,m)/\partial k = 0$, simultaneously. This gives

$$Q = \sqrt{\frac{2D[G(m)+\sigma\sqrt{L}\Psi(k)\Upsilon(k,L)+C(L)]+F_1}{F_2+H(m)}} \tag{6.18}$$

$$[1+\omega(k,L)]^2 \quad = \quad \frac{\omega[QM(Q)+DsI_d t_c]}{D\omega(\pi+\pi_0)-QF_2\left(\omega+\alpha k\sigma\sqrt{L}\right)} \tag{6.19}$$

where $\omega(k,L) = \frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)$, $\hat{\pi} = \pi + \frac{\pi_0\omega(k,L)}{1+\omega(k,L)}$ and $\Upsilon(k,L) = \hat{\pi} - \frac{c_s t_c I_d}{1+\omega(k,L)}$.

The following algorithm is developed to obtain the optimal values of Q and k for specific values of m and $L \in [L_i, L_{i-1}]$.

Algorithm 2

Step 1 Set $m = 1$.

Step 2 For each L_i, $i = 0,1,2,...,n$, perform 2a to 2c.

2a Set $k_{i1} = 0$.

2b Evaluate Q_{i1} from (6.18).

2c Utilize Q_{i1} to obtain the value of k_{i2} from (6.19).

2d Repeat 2b and 2c until no change occurs in the values of Q_i and k_i.

Step 3 For each set of values (Q_i, k_i, L_i, m), compute $ETC^W(Q_i, k_i, L_i, m), i = 1,2,....,n$.

Step 4 Find $\min_{i=0,1,2,\ldots,n} ETC^W(Q_i, k_i, , L_i, m)$. If $ETC^W(Q_m^{**}, k_m^{**}, L_m^{**}, m) = \min_{i=0,1,2,\ldots,n} ETC^W(Q_i, k_i, L_i, m)$, then $(Q_m^{**}, k_m^{**}, L_m^{**}, m)$ is the optimal solution for fixed m.

Step 5 Set $m = m + 1$, repeat steps $(2) - (4)$ to get $ETC^W(Q_m^{**}, k_m^{**}, L_m^{**}, m)$.

Step 6 If $ETC^W(Q_m^{**}, k_m^{**}, L_m^{**}, m) \leq ETC^W(Q_{m-1}^{**}, k_{m-1}^{**}, L_{m-1}^{**}, m)$, then go to Step 5; otherwise, go to Step 7.

Step 7 Set $ETC^W(Q_m^{**}, k_m^{**}, L_m^{**}, m) = ETC^W(Q_{m-1}^{**}, k_{m-1}^{**}, L_{m-1}^{**}, m)$. Then $(Q^{**}, k^{**}, L^{**}, m)$ is the optimal solution.

After substituting the values of k^{**} and L^{**}, the optimal backorder rate and the reorder point can be obtained as

$$\beta^{**} = \frac{1}{1 + \frac{1}{2}\alpha\sigma\sqrt{L^{**}}\left(\sqrt{1 + k^{**2}} - k^{**}\right)} \text{ and } r^{**} = DL^{**} + k^{**}\sigma\sqrt{L^{**}}.$$

6.4 Numerical examples

Example 1. In order to illustrate the solution procedure of the model, we consider in Table 6.1 the data used by Ouyang, Wu, and Ho, 2004. For controllable backorder rate and trade-credit financing, we take some additional parameter-values as: $\pi_0 = \$150/\text{unit}$, $\alpha = 0.1$, $t_c = 0.2$ years, $I_d = \$0.04/\$/$ year, $I_c = \$0.08/\$/$ year, $I_v = \$0.04/\$/$ year. The lead-time has three components, with data given in Table 6.2. Using the lead-time data and Algorithm 1, we obtain the results for the case when lead-time demand follows normal distribution. The summary of optimal results is given in Table 6.3. Variation of the expected average cost with respect to the number of shipments m is depicted in Figure 6.1. From Table 6.3, we obtain the optimal order quantity $Q^* = 136$ units, safety

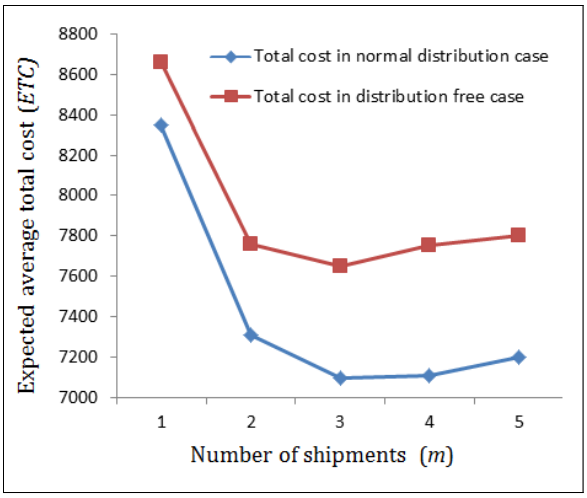

Figure 6.1: Expected total cost (ETC) vs: number of shipments (m)

stock $k^* = 1.31$, reorder point $r^* = 64$ units, lead-time $L = 4$ weeks, number of lots delivered from the manufacturer to the retailer $m^* = 3$, backorder rate $\beta^* = 0.94$, and the minimum expected average cost $ETC^N = \$7094$.

Parameters	Values	Parameters	Values	Parameters	Values
D	600 units/year	P	2000 units	S	\$1500/set up
A	\$200/order	r_b	\$0.2/unit/year	r_v	\$0.2/unit/year
c_b	\$100/unit/year	c_s	\$110/unit/year	c_v	\$70/unit/year
π_0	\$150/unit	π	\$50/unit	σ	7 units/weeke

Table 6.1: Parameter values

Lead-time component i	Normal duration v_i (days)	Minimum duration u_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 6.2: Lead-time data

We now examine the case when both parties make decisions independently to determine their own optimal policies. When the retailer makes decision independently, his optimal policy is as follows: order quantity $Q_b^* = 112$ units, safety stock $k_b^* = 1.39$, reorder point $r_b^* = 66$ units, lead-time $L_b^* = 4$ weeks, backorder rate $\beta^* = 0.95$, and the minimum expected average cost is \$2735.67. Also, the manufacturer's optimal production quantity is $m_v^* Q_b^* = 448$ units and the minimum average cost \$4370.53. Therefore, when the manufacturer and the retailer do not cooperate with each other, the expected average cost of the supply chain is \$7106, see Table 6.4. However, when both parties cooperate with each other, the expected average cost is \$7094, which is less than the expected average cost of the supply chain in the decentralized system. From Table 6.4, we can observe that the retailer's expected average cost in the decentralized model is lower than that of the integrated model, which implies that the retailer may not prefer an integrated decision-making model unless there is some cost sharing mechanism.

m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$ETC^N(Q_m^*, k_m^*, L_m^*, m)$
1	3	1.00	47	264	0.91	\$8349
2	4	1.20	63	174	0.93	\$7311
3	**4**	**1.31**	**64**	**136**	**0.94**	**\$7094**
4	4	1.39	66	114	0.95	\$7105

Table 6.3: Optimal results in Example 1.

Goyal, 1977 suggested the following method to allocate the expected cost among the retailer and the manufacturer:

retailer's cost $= \rho \times ETC^N(Q^*, r^*, L^*)$ and manufacturer's cost $= (1-\rho) \times ETC^N$ (Q^*, r^*, L^*), where

$$\rho = \frac{ETC_b(Q_b^*, k_b^*, L_b^*)}{ETC_b(Q_b^*, k_b^*, L_b^*) + ETC_v(Q_b^* m_v^*)}, \tag{6.20}$$

The allocated costs for the retailer and the manufacturer are shown in Table 6.4.

Model type	Buyer		Manufacturer	
Independent	Order quantity	112	Number of shipments	4
	Lead-time (weeks)	4	Production quantity	448
	Safety factor	1.39		
	Reorder point	66		
	Backorder rate	0.95		
	Expected average cost	\$2735.67	Average cost	\$4370.53
Integrated	Order quantity	136	Number of shipments	3
	Lead-time (weeks)	4	Production quantity	408
	Safety factor	1.31		
	Reorder point	64		
	Backorder rate	0.94		
	Expected average cost	\$2789.92	Average cost	\$4304.28
	Allocated average cost	\$2731.00	Allocated average cost	\$4363.20

Table 6.4: Allocation of expected average cost

We now investigate the effects of the controllable backorder and trade-credit financing on the average costs of the manufacturer and the retailer. In Table 6.5, we compare the results of our model with those of Ouyang, Wu, and Ho, 2004 where shortages were fully backlogged and trade-credit financing was not considered. From Table 6.5, we observe that the expected average cost of the supply chain is

	Integrated model		Independent model	
	Present model	Ouyang et al. (2004) ($I_d = 0, I_c = 0,$ $I_v = 0, \alpha = 0$)	Present model	Ouyang et al. (2004) ($I_d = 0, I_c = 0,$ $I_v = 0, \alpha = 0$)
Buyer's cost	\$2789.92	\$2862.7	\$2735.67	\$2832.0
Manufacturer's cost	\$4304.28	\$3797.7	\$4370.53	\$3893.9
Joint cost	\$7094.20	\$6660.4	\$7106.20	\$6725.9
Buyer's allocated cost	\$2731.00	\$2804.4	–	–
Manufacturer's allocated cost	\$4363.20	\$3856.0	–	–

Table 6.5: A comparative study

greater than that of the Ouyang, Wu, and Ho, 2004's model by 6.11%. This is due to consideration of variable backorder and trade-credit financing. In our model, the retailer's allocated cost which is 60% of the integrated cost is 3% more than that of Ouyang, Wu, and Ho, 2004's model. On the other hand, the manufacturer's allocated cost which is 40% of the integrated cost is 3% less than that of Ouyang, Wu, and Ho, 2004's model. This indicates that the manufacturer is beneficial in our model.

Example 2. In this example, we use the same data as given in Table 6.1. Applying Algorithm 2, we obtain the results of the model when lead-time demand does not follow any specific distribution. The results are given in Table 6.6.

m	L_m^{**}	k_m^{**}	R_m^{**}	Q_m^{**}	β_m^{**}	$ETC^N(Q_m^{**}, k_m^{**}, L_m^{**}, m)$
1	3	1.16	49	271	0.82	\$8658
2	3	1.44	52	184	0.84	\$7760
3	**3**	**1.62**	**54**	**146**	**0.85**	**\$7652**
4	3	1.77	56	124	0.86	\$7754

Table 6.6: Optimal results in distribution-free case (Example 2)

ϵ	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$ETC^N(Q_m^*, k_m^*, L_m^*, m)$
0.00	3	1.58	68	136	0.97	\$7208
2.00	3	1.78	71	136	0.98	\$7252
10.0	3	1.89	73	136	0.98	\$7292
20.0	3	1.89	73	136	0.98	\$7303
40.0	3	1.87	72	136	0.98	\$7305
80.0	3	1.87	72	136	0.98	\$7305
∞	3	1.87	72	136	0.98	\$7305

Table 6.7: Summary of results for negative exponential backorder rate

6.4.1 Evaluation of EVAI

Now, we compare the results of the distribution-free model with those of the normal distribution model. We see from Tables 6.3 and 6.6 that, in the normal distribution model, the set of optimal values of the decision variables is $(Q^*, k^*, L^*, m^*) = (136, 1.31, 4, 3)$, and that in the distribution-free model is $(Q^{**}, k^{**}, L^{**}, m^{**}) = (146, 1.62, 3, 3)$. If we utilize the solution obtained by the distribution-free approach instead of utilizing the normal distribution model, then the added cost will be $ETC^N(Q^{**}, k^{**}, L^{**}, m^{**}) - ETC^N(Q^*, k^*, L^*, m^*) = ETC^N(146, 1.62, 3, 3) - ETC^N(136, 1.31, 4, 3) = 7200 - 7094 = \$106$. This amount is said to be the expected value of additional information (EVAI) for the retailer that he would be willing to pay to collect the information to understand the form of lead-time demand distribution.

Additionally, we consider the same problem with negative exponential backorder rate as $\beta = \theta e^{-\epsilon B(r)}, B(r) = E(X - r)^{+}$ (Lee, Wu, and Hsu, 2006). The results are given in Table 6.7.

6.4.2 Sensitivity analysis

In this section, we perform sensitivity analysis to investigate the effects of the key parameters on the optimal solutions.

- *Effect of trade-credit period* (t_c)
  Table 6.8 contributes the effect of credit period t_c ranging from 0.1 to 0.9 on optimal solutions. From Table 6.8, it is observed that a higher value of credit period increases the retailer's order quantity. Safety factor and reorder point both tend to decrease as credit period increases. Furthermore, the expected average cost of the supply chain tends to decrease for $t_c \in [0.1, 0.3]$ and increase for $t_c \in [0.4, 0.9]$. The expected average cost and order quantity are more sensitive for higher value of t_c, whereas the reorder point and the safety factor are less sensitive to t_c (see Figures 6.2 and 6.3).

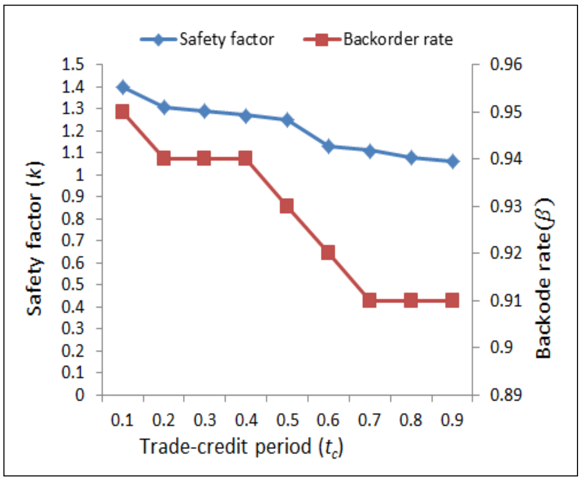

Figure 6.2: Trade-credit period (t_c) vs: safety factor (k) and backorder rate (β)

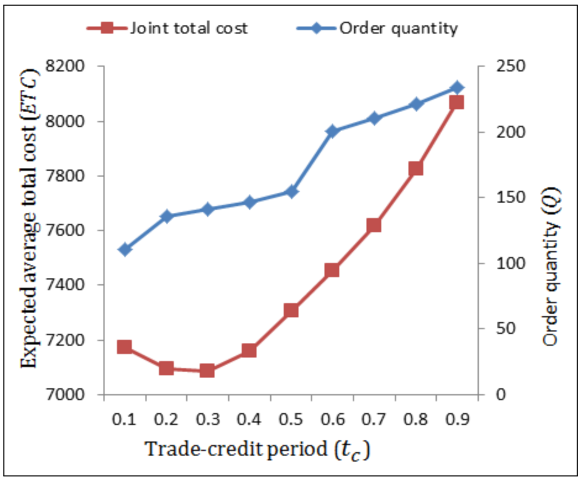

Figure 6.3: Trade-credit period (t_c) vs: cost (ETC) and order quantity (Q)

- *Effect of backorder parameter* (α)
  Table 6.9 indicates that an increase in the value of α increases the expected average cost whereas it decreases the backorder rate. This is because as shortage quantity becomes more sensitive to backorder parameter α, shortage quantity increases resulting an increase in the average cost. Even for a small value of α, the expected average cost, safety factor, and the reorder point are highly sensitive. When α takes a very high value, the expected average cost represents the

t_c	m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$ETC^N(Q_m^*, k_m^*, L_m^*, m)$
0.1	4	4	1.40	66	111	0.95	\$7174
0.2	3	4	1.31	64	136	0.94	\$7094
0.3	3	4	1.29	64	141	0.94	\$7087
0.4	3	4	1.27	64	147	0.94	\$7160
0.5	3	4	1.25	64	155	0.93	\$7306
0.6	2	4	1.13	62	201	0.92	\$7452
0.7	2	4	1.11	62	211	0.91	\$7619
0.8	2	4	1.08	61	222	0.91	\$7827
0.9	2	4	1.06	61	234	0.91	\$8070

Table 6.8: Effects of tarde-credit period t_c on optimal results

lost sale case (i.e., $\beta \to 1$), and when α takes a very small value, the expected average cost represents the fully backorder case (i.e., $\beta \to 0$). However, α has no effect on lead-time and number of shipments (see Figures 6.4 and 6.5).

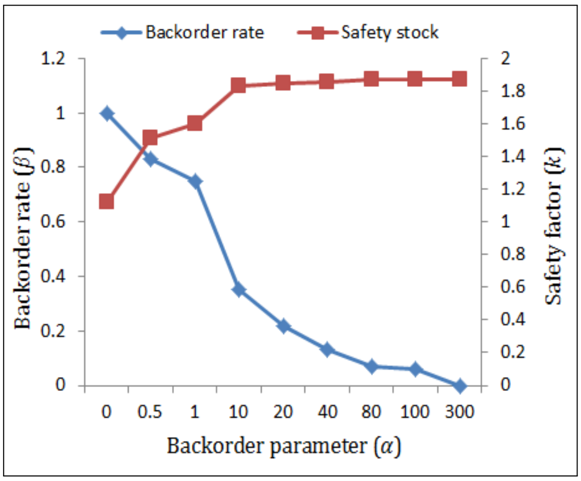

Figure 6.4: Backorder parameter (α) vs: safety factor (k) and backorder rate (β)

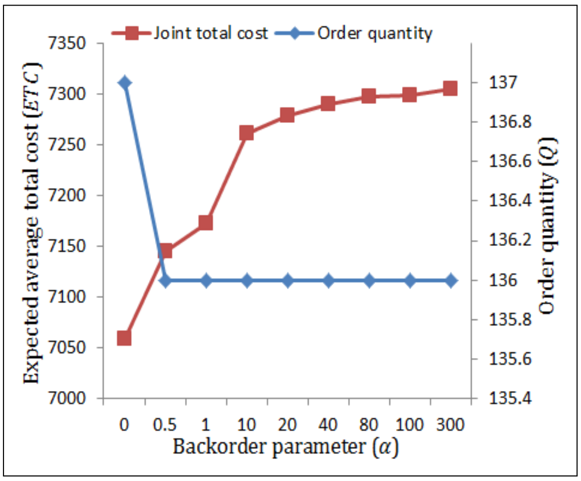

Figure 6.5: Backorder parameter (α) vs: expected total cost (ETC) and order quantity (Q)

α	m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$ETC^N(Q_m^*, k_m^*, L_m^*, m)$
0.00	3	4	1.12	62	137	1.00	\$7059
0.50	3	4	1.51	67	136	0.83	\$7145
1.00	3	4	1.60	68	136	0.75	\$7173
10.0	3	4	1.83	72	136	0.35	\$7261
20.0	3	4	1.85	72	136	0.22	\$7278
40.0	3	4	1.86	72	136	0.13	\$7290
80.0	3	4	1.87	72	136	0.07	\$7297
100	3	4	1.87	72	136	0.06	\$7299
∞	3	4	1.87	72	136	0.00	\$7305

Table 6.9: Effects of backorder parameter α on optimal results

- *Effect of lead-time demand standard deviation* (σ)
  We investigate the effect of lead-time standard deviation on the optimal results. From Table 6.10, we see that an increase in the value of σ decreases the backorder rate. This is because a higher value of σ implies a higher amount of shortages which decreases the backorder rate. Also, we see that, as σ increases, the order quantity and the safety factor also increase. This is because shortages increase for a higher value of σ, which leads to larger order quantity and safety stock. Figures 6.6 and 6.7 also indicate that σ impacts the expected average cost and safety factor significantly.

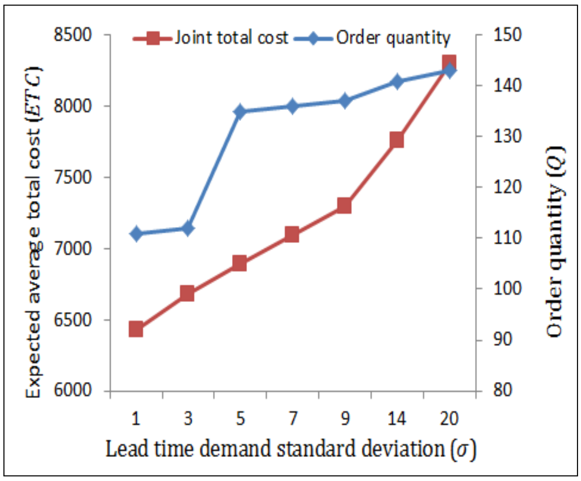

Figure 6.6: Lead-time demand deviation (σ) vs: expected total cost (ETC) and order quality (Q)

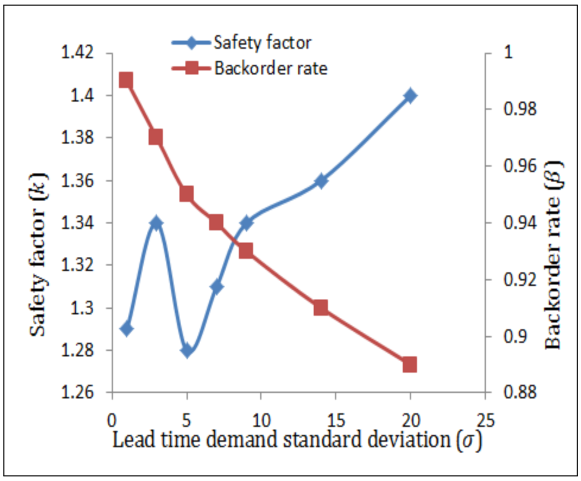

Figure 6.7: Lead-time demand deviation (σ) vs: safety factor (k) and backorder rate (β)

σ	m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$ETC^N(Q_m^*, k_m^*, L_m^*, m)$
1	4	6	1.29	72	111	0.99	\$6430
3	4	6	1.34	79	112	0.97	\$6676
5	3	4	1.28	59	135	0.95	\$6896
7	3	4	1.31	64	136	0.94	\$7094
9	3	4	1.34	70	137	0.93	\$7295
14	3	3	1.36	67	141	0.91	\$7762
20	3	3	1.40	83	143	0.89	\$8297

Table 6.10: Effects of standard deviation σ on optimal results

6.4.3 Conclusions

In this chapter, we develop a two-echelon supply chain model with a single-retailer and a single-manufacturer considering lead-time dependent backorder rate and delay-in-payment offer from the manufacturer. The uncertain lead-time demand faced by the retailer is first adapted with a normal distribution and then treated with a distribution-free approach. Further, it is considered that the lead time is controllable. Two separate algorithms are developed for two cases to find the optimal results. It is shown that Ouyang, Wu, and Ho, 2004's model is a special case of the present model. Numerical studies also show that an increased lead-time demand rate requires an increased safety stock to increase the backorder rate.

Chapter 7

Two-echelon supply chain model with price and effort dependent demand under a service level constraint

7.1 Introduction

In terms of global and intense competition, product diversity, and increasingly high alternatives, lack of discretion at the service level can often be the reason for frequent stock-out, which can cause the customer to lose or even can transfer customers to the competitors. Sometimes, the opportunity stock-out cost becomes too expensive for enterprises. Sometimes, it is hard to define a specific stock-out cost. Therefore, managing guaranteed service level becomes a vital issue for SC managers. In this chapter*, we aim to develop a SC model that focuses on ordering, price, effort, and lead-time decisions to coordinate amongst SC participants when a service level restriction exists. We model the problem under decentralized, centralized, and coordinated structures. The price reduction contract serves as the foundation for coordination.

*This chapter is based on the work submitted to *European Journal of Operational Research*, 2021.

7.2 Preliminary aspects

We present a two-echelon SC model where a single vendor delivers a single type of product to a single buyer. The buyer encounters price and effort dependent stochastic demand with a known mean and standard deviation. The buyer implements a continuous review (Q, r) inventory replenishment policy and issues an order of Q units to the vendor as soon as the inventory level falls to the reorder point r. The vendor follows the single setup multiple delivery (SSMD) policy to meet the buyer's demand. The buyer receives each shipment after a certain time, specified by the replenishment lead-time, which is a decision variable and can be controlled by some additional investment. The vendor's production system is imperfect, therefore each shipment delivered to the buyer carries some defective items. Each batch received by the buyer undergoes a sub-lot inspection process to identify the defective items. Discovered defectives are sold in the secondary market with a reduced price, while the fraction of undiscovered ones enters in the buyer's inventory to meet demand. The problem is to establish the production-inventory replenishment decision under centralized and decentralized that maximizes the expected total profit per time unit under a service level constraint on fill rate taking into account variable lead-time. Moreover, a coordination model is formulated under a price discount scheme.

7.2.1 Notation and assumptions

We use the following notation to develop the model.

Decision variables

Q	buyer's order quantity (units)
p	buyer's selling price (\$/item)
y	buyer's effort level
R	buyer's reorder point (quantity units)
k	safety factor, an equivalent decision variable to R (units)
L	length of the lead-time (time unit)
m	number of lots delivered from vendor to buyer, a positive integer

Parameters

$D(p, y)$	annual constant demand at the buyer's end (units/year)
P	annual production rate of the vendor (units/year)
a	scaling factor of the demand function
b	index of price elasticity of the demand function
μ	index of promotional effort elasticity
A	buyer's ordering cost (\$/order)
S	vendor's setup cost (\$/setup)

H_v	vendor's holding cost ($/unit/year)
H_b	buyer's holding cost ($/unit/year)
w	purchasing price for the buyer ($/item)
ν	salvage price ($/item)
k	safety factor
β	fill rate
σ	lead-time demand deviation (units)
F	promotional effort cost efficiency coefficient
f	proportion of quantity inspected per shipment
u_0	penalty cost for un-inspected defective items ($/item)
θ	fraction of defective items in every batch
X	lead-time demand with finite mean DL and standard deviation $\sigma\sqrt{L}$
$E(x)$	mathematical expectation of x
x^+	max $\{x, 0\}$
$C(L)$	lead-time crashing cost function
$C(y)$	promotional effort cost function
$E(X-r)^+$	expected shortage quantity at the end of the cycle

We assume the following assumptions to develop our model.

1. Single vendor and single buyer deal with a single type of item.

2. The demand rate for the buyer is a decreasing function of the selling price (Qin, Tang, and Guo, 2007). The promotion effort by the buyer also affects the demand function via a multiplying effect i.e., when the buyer takes part in a promotion effort, the demand is modified by a multiplier y. We have the demand function $D(p, y) = (a - bp)y^{\mu}$, where $a > 0$ is a scaling factor, $b > 0$ is the price elasticity of demand, and $\mu > 0$ is the promotional sales effort elasticity.

3. Inventory is continuously monitored by the buyer. Replenishment is made whenever the inventory level drops to the reorder point r.

4. The reorder point is defined by $R = DL + k\sigma\sqrt{L}$, where DL = expected demand during the lead-time, $k\sigma\sqrt{L}$ = safety stock.

5. We assume that the required service level is more than 50% and all excess demand is back-ordered (complete back-ordering) (Tajbakhsh, 2010).

6. The buyer places an order of size Q and the vendor produces mQ units with a finite production rate $P(> D)$ in a single setup but ships the products to the buyer over m times.

7. The buyer incurs a promotional effort cost $C(y) = F(y-1)^2$ which is convex, increasing, and continuously differentiable with respect to the promotional effort (g) for any $g > 1$ (Krishnan, Kapuscinski, and Butz, 2004; Pal, Sana, and Chaudhuri, 2015). This promotional effort cost can be viewed as the buyer's investment in advertising, enroll sales forces, reserve shelves for the product to increase its sales revenue.

8. Each lot received by the buyer contains some defective items. Hence, an error-free and non-destructive sampled inspection is performed at the buyers' end. The inspection is processed so quickly that the length of inspection is negligible (Wu, Ouyang, and Ho, 2007).

9. The expected number of good quality items in each shipment, $Q - Z$ is equal to the demand during the order cycle T, i.e., $Q - Z = D(p, y)T$.

10. The lead-time crashing cost which is determined by the length of lead-time satisfies $C(L) = Ue^{-\gamma L}$, where U and γ are positive constants (Chandra and Grabis, 2008.)

7.3 Model development

To model the problem, we first discuss the demand function. Generally, there are different kinds of demand functions out of which the most used demand function is the linear demand function. Normally, the linear demand function is formed as $D(p, y) = (a - bp)$, where $a > 0$ is a scaling factor, $b > 0$ is the price elasticity of demand. To reflect current market competition behavior, we adopt the nonlinear multiplicative form of sales effort to the basic liner price-dependent demand rate to highlight their interacted effects on the market demand. Therefore, when the buyer deploys a sales effort, the demand is modified by a multiplier y^μ, where $\mu > 0$ is the promotional sales effort elasticity and there is no sales effort when $\mu = 0$. Then the demand function with the joint effect of price and sales effort is defined as

$$D(p, y) = (a - bp)y^\mu \tag{7.1}$$

The buyer incurs a promotional effort cost $C(y) = F(y-1)^2$ which is convex, increasing, and continuously differentiable with respect to the promotional effort (y) for any $y > 1$ (Krishnan, Kapuscinski, and Butz, 2004).

7.3.1 Expected cycle length

For a given defective rate θ in the entire lot, the number of defective units in the sub-lot sampled is a random variable z, which has a hypergeometric distribution with parameters Q, f and θ. That is, z has a hypergeometric probability mass function (p.m.f.).

$$Pr(Z|\theta) = \frac{C_z^{Q\theta} C_{fQ-z}^{Q-\theta Q}}{C_{fQ}^{Q}}$$

where $0 \leq z \leq \min\{fQ, Q\theta\}$.
In this case,

$$E(Z|\theta) = fQ\theta$$

and

$$Var(Z|\theta) = \frac{f(1-f)\theta(1-\theta)Q^2}{Q-1}$$

Hence, unconditioning on θ, we have

$$E(Z) = \int_0^1 E(Z|\theta)g(\theta)d\theta = fQE(\theta)$$

and

$$\begin{aligned} E(Z^2) &= \int_0^1 E(Z^2|\theta)g(\theta)d\theta \\ &= \int_0^1 \left\{[E(Z|\theta)]^2 + Var(Z|\theta)\right\} g(\theta)d\theta \\ &= f^2Q^2E(\theta^2) + \frac{f(1-f)E[\theta(1-\theta)]Q^2}{Q-1} \end{aligned}$$

The expected length of cycle time is

$$E(T) = \frac{E(Q-Z)}{D(p,y)} = \frac{Q[1-fE(\theta)]}{D(p,y)} \tag{7.2}$$

7.3.2 Buyer's expected total profit per time unit

The buyer's expected net inventory level just before new order arrives is the safety stock, i.e. $R - LD(p,y)$ and the expected net inventory level at the beginning of the cycle, given that there are $Q - z$ items entering in inventory among the inspected units fQ is $(Q-z) + R - LD(p,y)$. Following Hadley and Whitin, 1963, the buyer's average inventory is

$$= \left(\frac{Q-z}{D(p,y)}\right)\left(\frac{Q-z}{2} + R - LD(p,y)\right) \quad (7.3)$$

Therefore, the buyer's total cost per cycle is

$$\begin{aligned} TC &= \text{ordering cost} + \text{purchasing cost} + \text{holding cost} + \text{inspection cost} \\ &\quad + \text{uninspected defective penalty cost} + \text{lead-time crashing cost} \\ &\quad + \text{promotional effort cost} \\ &= A + wQ + H_b \frac{(Q-z)}{D(p,y)}\left[\frac{Q-z}{2} + R - LD(p,y)\right] + i_c Qf + u_0(\theta Q - z) \\ &+ C(L) + C(y)\frac{(Q-z)}{D(p,y)} \end{aligned} \quad (7.4)$$

Now the buyer's expected total cost is

$$\begin{aligned} ETC(Q,L,R,p,y,) &= A + wQ + \frac{H_b}{D(p,y)}\left[\frac{Q^2 - 2QE[Z] + E[Z^2]}{2} + (Q - E[Z])\right. \\ &\times \left.(R - LD(p,y))\right] + C(y)\frac{(Q-E[Z])}{D(p,y)} + i_c Qf \\ &+ u_0(QE[\theta] - E[Z]) + C(L) \end{aligned} \quad (7.5)$$

The buyer's expected total revenue (from good quality and the imperfect quality items) per cycle is

$$ETR(Q,p) = pE(Q-Z) + vE(Z) \quad (7.6)$$

Therefore, the buyer's expected total profit per cycle is

$$\begin{aligned} ETP(Q,L,R,p,y) &= \frac{ETR(Q,p) - ETC(Q,L,R,p,y)}{E[T]} \\ &= pD(p,y) + \frac{\nu E(Z)D(p,y)}{E(Q-Z)} - \frac{AD(p,y)}{E(Q-Z)} - \frac{D(p,y)wQ}{E(Q-Z)} - C(y) \\ &- H_b\left[R - LD(p,y)\right] - \frac{H_b}{2}\left[\frac{Q^2 - 2QE[Z] + E[Z^2]}{Q - E[Z]}\right] - \frac{i_c QfD(p,y)}{E(Q-Z)} \\ &- \frac{u_0(QE[\theta] - E[Z])D(p,y)}{E(Q-Z)} - \frac{D(p,y)C(L)}{E(Q-Z)} \end{aligned} \tag{7.7}$$

which can be further written as

$$\begin{aligned} ETP(Q,L,R,p,y) &= pD(p,y) + \frac{\nu fE[\theta]D(p,y)}{1 - fE[\theta]} - \frac{AD(p,y)}{Q[1 - fE[\theta]]} - \frac{D(p,y)w}{1 - fE[\theta]} - C(y) \\ &- \frac{H_b}{2}\left[Q\left(\frac{1 - 2fE[\theta] + f^2E[\theta^2]}{1 - fE[\theta]}\right) + \frac{f(1-f)E[\theta(1-\theta)]Q}{(1 - fE[\theta])(Q-1)}\right] \\ &- H_b[R - LD(p,y)] - \frac{i_c fD(p,y)}{1 - fE[\theta]} - \frac{u_0 E[\theta](1-f)D(p,y)}{1 - fE[\theta]} \\ &- C(L)\frac{D(p,y)}{Q[1 - fE[\theta]]} \end{aligned} \tag{7.8}$$

Letting

$$\begin{aligned} E_1 &= \frac{fE(\theta)}{1 - fE(\theta)} \\ E_2 &= 1 - fE(\theta) \\ E_3 &= \frac{1 - 2fE(\theta) + f^2E(\theta^2)}{1 - fE(\theta)} \\ E_4 &= \frac{f(1-f)E[\theta(1-\theta)]}{[1 - fE(\theta)]} \end{aligned}$$

we have

$$\begin{aligned} ETP(Q,L,R,p,y) &= \left[p + E_1\left\{\nu - u_0\left(\frac{1}{f} - 1\right)\right\}\right]D(p,y) - \left(A + Ue^{-\gamma L}\right)\frac{D(p,y)}{QE_2} \\ &- (w + fi_c)\frac{D(p,y)}{E_2} - \frac{H_b}{2}\left(QE_3 + E_4\frac{Q}{Q-1}\right) \\ &- H_b[R - LD(p,y)] - F(y-1)^2 \end{aligned} \tag{7.9}$$

7.3.3 Vendor's expected total profit per time unit

The vendor's total inventory in a cycle can be written as (Wu, Ouyang, and Ho, 2007)

$$\begin{aligned} I_v(Q,m) &= mQ\left(\frac{Q}{P}+(m-1)T\right)-\frac{mQ(mQ/P)}{2}-T(Q+2Q+\ldots\ldots+(m-1)Q) \\ &= mQ\left[\frac{Q}{P}+(m-1)\left(\frac{Q-Z}{D}\right)\right]-\frac{m^2Q^2}{2P}-\left(\frac{Q-Z}{D}\right)\frac{m(m-1)Q}{2} \\ &= \frac{mQ^2}{P}+\frac{mQ(m-1)}{2}\left(\frac{Q-Z}{D}\right)-\frac{m^2Q^2}{2P} \end{aligned} \tag{7.10}$$

The vendor's expected total profit per production cycle, ETP_v, is revenue minus the sum of setup and carrying costs i.e.,

$$ETP_v(m) = wmQ - S - H_v\left[\frac{mQ^2}{P}+\frac{mQ(m-1)}{2}\left(\frac{Q-Z}{D}\right)-\frac{m^2Q^2}{2P}\right] \tag{7.11}$$

Now, the vendor's expected total profit per unit time can be obtained by dividing (7.11) by cycle length $mE[T]$ as

$$\begin{aligned} ETP_v(m) &= \frac{ETP_v}{mE[T]} \\ &= \frac{wD(p,y)}{E_2}-\frac{SD(p,y)}{mQE_2}-\frac{H_v}{2}\left[(m-1)-(m-2)\frac{D(p,y)}{PE_2}\right] \end{aligned} \tag{7.12}$$

7.3.4 Problem formulation and optimization

Shortages occur when lead-time demand (X) is larger than reorder point (R). The buyer's expected shortage quantity at the end of a replenishment cycle is $E(X-R)^+$, which are fully backlogged, by assumption. Due to the abstract influence of shortages, usually, it is not easy to measure the penalty cost associated with shortages. Therefore, various authors developed their models by assuming a target service level (fill rate) corresponding to the fraction of total demand to be met through immediate stock available at hand (Tajbakhsh, 2010). Therefore, the fill rate constraint puts a limit on the proportion of demands that are met from stock. This performance measure, denoted by β, is obtained by:

$$\beta = \frac{\text{expected demand satisfied per replenishment cycle}}{\text{expected demand per replenishment cycle}}$$

According to Tajbakhsh, 2010, not only does β consider the probability of stockouts but also it takes the size of shortages into account. This is why the fill rate is considerably appealing to practitioners. Now, the fill rate is given by

$$\beta = 1 - \frac{E(X-R)^+}{Q} \tag{7.13}$$

7.3.4.1 Decentralized policy

The problem is to find the order quantity, lead-time, selling price, and promotional effort, and reorder point of the buyer that maximize the expected total profit per time unit of the buyer, considering a service level constraint. In this case, the problem can be formulated as follows:

$$\max_{(Q_\mathrm{d}, R_\mathrm{d}, L_\mathrm{d}, p_\mathrm{d}, y_\mathrm{d})} \quad \Psi_\mathrm{b}^{\mathrm{decn}}(Q_\mathrm{d}, R_\mathrm{d}, L_\mathrm{d}, p_\mathrm{d}, y_\mathrm{d}) \tag{7.14}$$

$$\text{s.t.} \quad \frac{E(X-R_\mathrm{d})^+}{Q_\mathrm{d}} \leq 1-\beta \tag{7.15}$$

where $\Psi_\mathrm{b}^{\mathrm{decn}}$ is the expected total profit of the buyer under decentralized model which is given in (7.9). The subscript "d" in each variable denotes decentralized model.

To reflect the practical circumstance in which the lead-time demand distribution is not exactly known, we assume that the distribution u of X is not specified. The only available information regarding u is the finite mean and variance. In particular, we thus have $u \in \Omega$. Since the family of u is not specified, problem (7.14) under the constraint cannot be solved directly. In fact, being u not known, $E(X-R_\mathrm{d})^+$ cannot be calculated.

To overcome this issue, we adopt the maximax principle. According to this principle, we choose u as the most unfavourable probability density function in Ω, for each vector $(Q_\mathrm{d}, R_\mathrm{d}, L_\mathrm{d}, p_\mathrm{d}, y_\mathrm{d})$, and then we maximize in $(Q_\mathrm{d}, R_\mathrm{d}, L_\mathrm{d}, p_\mathrm{d}, y_\mathrm{d})$. This is a conservative approach, but there are supporting arguments to it. Firstly, it can be easily applied in practice; secondly, it leads to analytically tractable results; thirdly, it is optimal under some conditions, and, finally, it has been extensively used in inventory management literature.

In place of problem (7.14), we thus consider the problem

$$\min_{(Q_\text{d},R_\text{d},L_\text{d},p_\text{d},y_\text{d})} \max_{u\in\Omega} \quad \Psi_\text{b}^\text{dcen}(Q_\text{d},R_\text{d},L_\text{d},p_\text{d},y_\text{d}) \tag{7.16}$$

$$\text{s.t.} \quad \frac{E(X-R_\text{d})^+}{Q_\text{d}} \leq 1-\beta \tag{7.17}$$

This optimization problem can be approached by means of the following proposition (Gallego and Moon, 1993).

Proposition 7.1.

$$E(X-R_\text{d})^+ \leq \frac{1}{2}\left\{\sqrt{\sigma^2 L_\text{d} + [R_\text{d} - D(p_\text{d},y_\text{d})L_\text{d}]^2} - [R_\text{d} - D(p_\text{d},y_\text{d})L_\text{d}]\right\} \tag{7.18}$$

Moreover, for every R_d, there exists a distribution $u \in \Omega$ such that the above bound is tight.

According to Proposition 7.1 and to the definition of R_d (see assumption No. 4), problem (7.16) becomes

$$\min_{(Q_\text{d},L_\text{d},p_\text{d},y_\text{d},k_\text{d})} \max_{u\in\Omega} \quad \Psi_\text{b}^\text{dcen}(Q_\text{d},L_\text{d},p_\text{d},y_\text{d},k_\text{d}) \tag{7.19}$$

$$\text{s.t.} \quad \frac{\frac{1}{2}\sigma\sqrt{L_\text{d}}\left[\sqrt{1+k_\text{d}^2}-k_\text{d}\right]}{Q} \leq 1-\beta \tag{7.20}$$

where

$$\begin{aligned}\Psi_\text{b}^\text{dcen}(Q_\text{d},L_\text{d},p_\text{d},y_\text{d},k_\text{d}) &= \left[p + E_1\left\{v - u_0\left(\frac{1}{f}-1\right)\right\}\right] D(p_\text{d},y_\text{d}) - (w+fi_c) \\ &\times \frac{D(p_\text{d},y_\text{d})}{E_2} - \left(A + Ue^{-\gamma L_\text{d}}\right)\frac{D(p_\text{d},y_\text{d})}{Q_\text{d}E_2} - H_b k_\text{d}\sigma\sqrt{L_\text{d}} \\ &- \frac{H_b}{2}\left(Q_\text{d}E_3 + E_4\frac{Q_\text{d}}{Q_\text{d}-1}\right) - F(y_\text{d}-1)^2 \end{aligned} \tag{7.21}$$

This optimization problem includes an inequality constraint. The first step consists of solving the problem ignoring the inequality constraint, i.e., setting the constraint as inactive. If the solution satisfies the constraint, it solves the problem. Otherwise, we have to evaluate the solution with active constraint.

However, one can note that the cost function is globally unbounded in k_d ($\Psi_\text{b}^\text{dcen}$ follows a linear law in k_d). In other words, the problem of minimizing Ψ_b in k_d with constraint as inactive does not admit any solution. For this reason, the optimization in k_d requires that the constraint be active.

That is, we turn to solve the following problem:

$$\max_{(Q_\mathrm{d},L_\mathrm{d},p_\mathrm{d},y_\mathrm{d},k_\mathrm{d})} \quad \Psi_\mathrm{b}^{\mathrm{decn}}(Q_\mathrm{d},L_\mathrm{d},p_\mathrm{d},y_\mathrm{d},k_\mathrm{d}) \tag{7.22}$$

$$\text{s.t.} \quad \frac{\frac{1}{2}\sigma\sqrt{L_\mathrm{d}}\left[\sqrt{1+k_\mathrm{d}^2}-k_\mathrm{d}\right]}{Q_\mathrm{d}} = 1-\beta \tag{7.23}$$

From the constraint in (7.22) we have

$$k_\mathrm{d} = \frac{1}{\sigma\sqrt{L_\mathrm{d}}}\left(\frac{\sigma^2 L_\mathrm{d}}{4(1-\beta)Q_\mathrm{d}} - (1-\beta)Q_\mathrm{d}\right) \tag{7.24}$$

Substituting the value of k_d we have

$$\begin{aligned}\Psi_\mathrm{b}^{\mathrm{decn}}(Q_\mathrm{d},L_\mathrm{d},p_\mathrm{d},y_\mathrm{d}) &= \left[p_\mathrm{d}+E_1\left\{\nu-u_0\left(\frac{1}{f}-1\right)\right\}\right]D(p_\mathrm{d},y_\mathrm{d}) - \left(A+Ue^{-\gamma L_\mathrm{d}}\right)\\ &\times \frac{D(p_\mathrm{d},y_\mathrm{d})}{Q_\mathrm{d}E_2} - (w+fi_c)\frac{D(p_\mathrm{d},y_\mathrm{d})}{E_2} - \frac{H_b}{2}\left(Q_\mathrm{d}E_3+\frac{E_4Q_\mathrm{d}}{Q_\mathrm{d}-1}\right)\\ &-H_b\left(\frac{\sigma^2L_\mathrm{d}}{4(1-\beta)Q_\mathrm{d}}-(1-\beta)Q_\mathrm{d}\right)-F(y_\mathrm{d}-1)^2 \end{aligned} \tag{7.25}$$

For large value of Q_d, we can approximate $\frac{Q_\mathrm{d}}{Q_\mathrm{d}-1} \approx 1$, and hence (7.25) can be approximated as follows:

$$\begin{aligned}\Psi_\mathrm{b}^{\mathrm{decn}}(Q_\mathrm{d},L_\mathrm{d},p_\mathrm{d},y_\mathrm{d}) &= \left[p_\mathrm{d}+E_1\left\{\nu-u_0\left(\frac{1}{f}-1\right)\right\}\right]D(p_\mathrm{d},y_\mathrm{d}) - \left(A+Ue^{-\gamma L_\mathrm{d}}\right)\\ &\times \frac{D(p_\mathrm{d},y_\mathrm{d})}{Q_\mathrm{d}E_2} - (w+fi_c)\frac{D(p_\mathrm{d},y_\mathrm{d})}{E_2} - \frac{H_b}{2}\left(Q_\mathrm{d}E_3+E_4\right)\\ &- \quad H_b\left(\frac{\sigma^2L_\mathrm{d}}{4(1-\beta)Q_\mathrm{d}}-(1-\beta)Q_\mathrm{d}\right)-F(y_\mathrm{d}-1)^2 \end{aligned} \tag{7.26}$$

Now, before proceeding with the optimization procedure, we need to introduce the following conjecture:

Conjecture 2. *The inequality*

$$\omega = \frac{1}{2}\frac{1-2fE\left(\theta\right)+f^2E\left(\theta^2\right)}{1-fE\left(\theta\right)} - (1-\beta) > 0 \tag{7.27}$$

is satisfied for typical parameter values adopted in practice.

Under Conjecture 2, we have the following properties:

Proposition 7.2. *1. For fixed $(Q_\mathrm{d}, p_\mathrm{d}, y_\mathrm{d})$, $\Psi_\mathrm{b}^\mathrm{decn}$ is concave in L_d. Moreover, the first order condition for optimality with respect to L_d gives*

$$L_\mathrm{d}^* = \frac{1}{\gamma}\ln\left(\frac{4U\gamma(1-\beta)D(p_\mathrm{d}, y_\mathrm{d})}{\sigma^2 H_b E_2}\right) \tag{7.28}$$

2. For fixed $(p_\mathrm{d}, y_\mathrm{d})$ with $L_\mathrm{d} = L_\mathrm{d}^*$, $\Psi_\mathrm{b}^\mathrm{decn}$ is concave in Q_d. Moreover, the first order condition for optimality with respect to Q_d gives

$$Q_\mathrm{d}^* = \sqrt{\frac{4A\gamma(1-\beta)D(p_\mathrm{d}, y_\mathrm{d}) + \sigma^2 H_b E_2\left[1 + \ln\left(\frac{4U\gamma(1-\beta)D(p_\mathrm{d}, y_\mathrm{d})}{\sigma^2 H_b E_2}\right)\right]}{4\gamma E_2 H_b(1-\beta)\left(\beta + \frac{E_3}{2} - 1\right)}} \tag{7.29}$$

Proof. We now prove the concavity of $\Psi_\mathrm{b}^\mathrm{decn}$, for fixed $(Q_\mathrm{d}, p_\mathrm{d}, y_\mathrm{d})$. Noting that

$$\frac{\partial^2 \Psi_\mathrm{b}^\mathrm{decn}}{\partial L_\mathrm{d}^2} = -\frac{1}{Q_\mathrm{d} E_2}\left[U\gamma^2 e^{-\gamma L_\mathrm{d}} D(p_\mathrm{d}, y_\mathrm{d})\right] < 0,$$

we can thus deduce the concavity of $\Psi_\mathrm{b}^\mathrm{decn}$, for fixed $(Q_\mathrm{d}, p_\mathrm{d}, y_\mathrm{d})$. Then, the value L_d^* is obtained solving in L_d the equation

$$\frac{\partial \Psi_\mathrm{b}^\mathrm{decn}}{\partial L_\mathrm{d}} = \frac{4D(p_\mathrm{d}, y_\mathrm{d})U\gamma e^{-\gamma L_\mathrm{d}}}{E_2} + \frac{\sigma^2 H_b}{\beta - 1} = 0.$$

We now prove the concavity of $\Psi_\mathrm{b}^\mathrm{decn}$ in Q_d, for fixed $(p_\mathrm{d}, y_\mathrm{d})$ with $L_\mathrm{d} = L_\mathrm{d}^*$. We have

$$\frac{\partial^2 \Psi_\mathrm{b}^\mathrm{decn}}{\partial Q_\mathrm{d}^2} = -\frac{1}{2Q_\mathrm{d}^3}\left[4\left(A + Ue^{-\gamma L_\mathrm{d}}\right)\frac{D(p_\mathrm{d}, y_\mathrm{d})}{E_2} + \frac{H_b \sigma^2 L_\mathrm{d}}{1-\beta}\right] < 0$$

which indicates that $\Psi_\mathrm{b}^\mathrm{decn}$ is concave in Q_d, for fixed $(p_\mathrm{d}, y_\mathrm{d})$ with $L_\mathrm{d} = L_\mathrm{d}^*$. The value $Q_{\mathrm{d}^*}^*$ obtained solving in Q_d the equation

$$\frac{\partial \Psi_\mathrm{b}^\mathrm{decn}}{\partial Q_\mathrm{d}} = \frac{\left[A + Ue^{-\gamma L_\mathrm{d}}\right] D(p_\mathrm{d}, y_\mathrm{d})}{E_2 Q_\mathrm{d}^2} - H_b\left(\beta - 1 - \frac{\sigma^2 L_\mathrm{d}}{4Q_\mathrm{d}^2(1-\beta)} + \frac{E_3}{2}\right) = 0$$

□

Substituting the value of L_{d}^* and Q_{d}^* from (7.28) and (7.29) in (7.26), we obtain

$$\begin{aligned}\Psi_{\mathrm{b}}^{\mathrm{decn}}(p_{\mathrm{d}},y_{\mathrm{d}}) &= \left[p_{\mathrm{d}}+E_1\left\{\nu-u_0\left(\frac{1}{f}-1\right)\right\}\right]D(p_{\mathrm{d}},y_{\mathrm{d}})-\frac{(w+fi_c)D(p_{\mathrm{d}},y_{\mathrm{d}})}{E_2}\\ &- \frac{H_bE_4}{2}-F(y_{\mathrm{d}}-1)^2-2\left[\left(\frac{H_bE_3}{2}-H_b(1-\beta)\right)\left(\frac{AD(p_{\mathrm{d}},y_{\mathrm{d}})}{E_2}\right.\right.\\ &+ \left.\left.\frac{\sigma^2H_b}{4\gamma(1-\beta)}\left[1+\ln\left(\frac{4U\gamma(1-\beta)D(p_{\mathrm{d}},y_{\mathrm{d}})}{\sigma^2H_bE_2}\right)\right]\right)\right]^{1/2} \qquad (7.30)\end{aligned}$$

Now, the conditions that have to be satisfied simultaneously to maximize the buyer's expected profit are $\frac{\partial\Psi_{\mathrm{b}}^{\mathrm{decn}}(p_{\mathrm{d}},y_{\mathrm{d}})}{\partial p_{\mathrm{d}}}=0$ and $\frac{\partial\Psi_{\mathrm{b}}^{\mathrm{decn}}(p_{\mathrm{d}},y_{\mathrm{d}})}{\partial y_{\mathrm{d}}}=0$, i.e.,

$$\begin{aligned}\frac{d\Psi_{\mathrm{b}}^{\mathrm{decn}}(p_{\mathrm{d}},y_{\mathrm{d}})}{dp_{\mathrm{d}}} &= D(p_{\mathrm{d}},y_{\mathrm{d}})-by_{\mathrm{d}}^{\mu}\left(p_{\mathrm{d}}+E_1\left\{\nu-u_0\left(\frac{1}{f}-1\right)\right\}\right)+\frac{b(w+fi_c)y_{\mathrm{d}}^{\mu}}{E_2}\\ &+ \left(\left(\frac{H_bE_3}{2}-H_b(1-\beta)\right)\left(\frac{Aby_{\mathrm{d}}^{\mu}}{E_2}+\frac{b\sigma^2H_b}{4D(p_{\mathrm{d}},y_{\mathrm{d}})(1-\beta)\gamma}\right)\right)\Big/\left\{\left(\frac{AD(p_{\mathrm{d}},y_{\mathrm{d}})y_{\mathrm{d}}^{\mu}}{E_2}\right.\right.\\ &+ \left.\left.\frac{\sigma^2H_b}{4\gamma(1-\beta)}\left[1+\ln\left(\frac{4D(p_{\mathrm{d}},y_{\mathrm{d}})(1-\beta)\gamma U}{\sigma^2H_bE_2}\right)\right]\right)\left(\frac{H_bE_3}{2}-H_b(1-\beta)\right)\right\}^{1/2}=0 \qquad (7.31)\end{aligned}$$

$$\begin{aligned}\frac{d\Psi_{\mathrm{b}}^{\mathrm{decn}}(p_{\mathrm{d}},y_{\mathrm{d}})}{dy_{\mathrm{d}}} &= \left[\left(p_{\mathrm{d}}+E_1\left\{\nu-u_0\left(\frac{1}{f}-1\right)\right\}\right)-\frac{fi_c+w}{E_2}\right]D(p_{\mathrm{d}},y_{\mathrm{d}})\mu y_{\mathrm{d}}^{(\mu-1)}\\ &-2F(1-y_{\mathrm{d}})+\left(\left(\frac{H_bE_3}{2}-H_b(1-\beta)\right)\left(\frac{Aby_{\mathrm{d}}^{\mu}}{E_2}+\frac{b\sigma^2H_b}{4D(p_{\mathrm{d}},y_{\mathrm{d}})(1-\beta)\gamma}\right)\right)\Big/\\ &\left\{\left(\frac{H_bE_3}{2}-H_b(1-\beta)\right)\left(\frac{AD(p_{\mathrm{d}},y_{\mathrm{d}})y_{\mathrm{d}}^{\mu}}{E_2}+\frac{\sigma^2H_b}{4\gamma(1-\beta)}\right.\right.\\ &\left.\left.\left[1+\ln\left(\frac{4D(p_{\mathrm{d}},y_{\mathrm{d}})(1-\beta)\gamma U}{\sigma^2H_bE_2}\right)\right]\right)\right\}^{1/2}=0 \qquad (7.32)\end{aligned}$$

The solutions may be found by numerical methods. For example, the NSolve procedure of MATHEMATICA could be applied to solve the equations (7.31) and (7.32), as it was done in this chapter.

It is important to note that the information on the buyer's order quantity Q_{d}, selling price p_{d}, and effort level y_{d} are required for the vendor to determine the number of shipments to the buyer. Here it is assumed that the buyer acts as a leader, which means that the buyer imposes his/her optimal decisions that maximize his/her profit. The vendor, instead, takes this as provided and determines the optimal number of shipments m_{d} that maximizes his/her profit.

Proposition 7.3. *The vendor's annual decentralized profit function* $\Psi_{\mathrm{v}}^{\mathrm{decn}}(m_{\mathrm{d}})$ *is concave in* m_{d}.

Proof. We have the second order derivative of vendor's annual profit function as

$$\frac{d^2\Psi_{\mathrm{v}}^{\mathrm{decn}}(m_d|_{Q_{\mathrm{d}},L_{\mathrm{d}},p_{\mathrm{d}},y_{\mathrm{d}}})}{dm_d^2} = -\frac{2D(p_{\mathrm{d}},y_{\mathrm{d}})S}{m_d^3 Q_{\mathrm{d}} E_2}$$

Clearly, the second-order derivative of the vendor's annual profit function is negative. Therefore, the vendor's annual profit function is concave in m_{d}.
Therefore, the unique value of m_{d} can be obtained by solving $\frac{d\Psi_{\mathrm{v}}^{\mathrm{decn}}}{dm_{\mathrm{d}}} = 0$, i.e.,

$$m^d_{Decimal} = \frac{1}{Q_{\mathrm{d}}}\sqrt{\frac{2D(p_{\mathrm{d}},y_{\mathrm{d}})PS}{H_v\left[PE_2 - D(p_{\mathrm{d}},y_{\mathrm{d}})\right]}} \tag{7.33}$$

□

It should be noted that the number of shipments must be an integer value; however, (7.33) may give a decimal value. Therefore, following relation is proposed to calculate the suitable value of m^d that optimizes the vendor's decentralized annual profit.

$$m_{\mathrm{d}}^* = \begin{cases} \lfloor m_{\mathrm{d}}^{\mathrm{Decimal}} \rfloor & \text{if } \Psi_{\mathrm{v}}^{\mathrm{decn}}(\lfloor m_{\mathrm{d}}^{\mathrm{Decimal}} \rfloor) > \Psi_{\mathrm{v}}^{\mathrm{decn}}(\lfloor m_{\mathrm{d}}^{\mathrm{Decimal}} \rfloor + 1) \\ \lfloor m_{\mathrm{d}}^{\mathrm{Decimal}} \rfloor + 1 & \text{if } \Psi_{\mathrm{v}}^{\mathrm{decn}}(\lfloor m_{\mathrm{d}}^{\mathrm{Decimal}} \rfloor) \le \Psi_{\mathrm{v}}^{\mathrm{decn}}(\lfloor m_{\mathrm{d}}^{\mathrm{Decimal}} \rfloor + 1) \end{cases} \tag{7.34}$$

The expected joint total annual profit of the supply chain under the decentralized optimization scheme $\Psi_{\mathrm{sc}}^{\mathrm{decn}}(Q_{\mathrm{d}}, L_{\mathrm{d}}, p_{\mathrm{d}}, y_{\mathrm{d}}, m_{\mathrm{d}})$ is obtained by summing the buyer's and the vendor's individual net annual profits i.e.,$\Psi_{\mathrm{sc}}^{\mathrm{decn}}(Q_{\mathrm{d}}, L_{\mathrm{d}}, p_{\mathrm{d}}, y_{\mathrm{d}}, m_{\mathrm{d}}) = \Psi_{\mathrm{b}}^{\mathrm{decn}}(Q_{\mathrm{d}}, L_{\mathrm{d}}, p_{\mathrm{d}}, y_{\mathrm{d}}) + \Psi_{\mathrm{v}}^{\mathrm{decn}}(m_{\mathrm{d}})$.

7.3.4.2 Centralized policy

In this framework, supply chain members react jointly; a single decision-maker tries to optimize the profit of the whole supply chain system rather than focusing on individual members' profits. Although perfect coordination is not possible in real life, a centralized decision-making system can produce better profit than a decentralized decision-making system. Some examples of the centralized system can be found in the restaurant, electronics business, and online businesses where the core important decisions are taken by those at a higher level of authority and after the decision has been taken, it is reported to lower level employees who are expected to follow the order. Here, in our model both the vendor and the buyer coordinate to make the decisions. The goal is to maximize the total profit of the entire supply chain. The cost stemming from the purchasing cost is an internal transfer of money from one supply chain member (the vendor) to another supply chain member (the buyer).

With similar arguments to those discussed in Section 4.4.1, the problem can be formulated as follows:

$$\max_{(Q_c, L_c, p_c, y_c, k_c, m_c)} \quad \Psi_{sc}^{cent}(Q_c, L_c, p_c, y_c, k_c, m_c) \tag{7.35}$$

$$\text{s.t.} \quad \frac{\frac{1}{2}\sigma\sqrt{L_c}\left[\sqrt{1+k_c^2}-k_c\right]}{Q_c} = 1-\beta \tag{7.36}$$

where $\Psi_{sc}^{cent}(Q_c, L_c, p_c, y_c, k_c, m_c)$ can be obtained by adding (7.9) and (7.12) as follows:

$$\begin{aligned}
\Psi_{sc}^{cent}(Q_c, L_c, p_c, y_c, k_c, m_c) &= \left[p_c + E_1\left\{v - u_0\left(\frac{1}{f}-1\right)\right\}\right] D(p_c, y_c) - F(y_c-1)^2 \\
&- \left(A + Ue^{-\gamma L_c}\right)\frac{D(p_c, y_c)}{Q_c E_2} - \frac{D(p_c, y_c) f i_c}{E_2} - \frac{S D(p_c, y_c)}{m_c Q_c E_2}
\end{aligned}$$

$$\begin{aligned}
&- \frac{H_b}{2}\left(Q_c E_3 + E_4 \frac{Q_c}{Q_c - 1}\right) - H_b k_c \sigma\sqrt{L_c} - \frac{H_v}{2}\Bigg[(m_c-1) \\
&- (m_c-2)\frac{D(p_c, y_c)}{P E_2}\Bigg]
\end{aligned} \tag{7.37}$$

From the constraint in (7.35) we have

$$k_c = \frac{1}{\sigma\sqrt{L_c}}\left(\frac{\sigma^2 L_c}{4(1-\beta)Q_c} - (1-\beta)Q_c\right) \tag{7.38}$$

Substituting the value of k_c in (7.37) we have

$$\begin{aligned}\Psi_{sc}^{cent}(Q_c, L_c, p_c, y_c, m_c) &= \left[p_c + E_1\left\{\nu - u_0\left(\frac{1}{f} - 1\right)\right\}\right] D(p_c, y_c) - \frac{D(p_c, y_c)}{Q_c E_2} \\ &\times \left(A + Ue^{-\gamma L_c}\right) - \frac{D(p_c, y_c) f i_c}{E_2}\frac{H_b}{2}\left(Q_c E_3 + E_4 \frac{Q_c}{Q_c - 1}\right) \\ &- H_b\left(\frac{\sigma^2 L_c}{4(1-\beta)Q_c} - (1-\beta)Q_c\right) - F(y_c - 1)^2 \\ &- \frac{S D(p_c, y_c)}{m_c Q_c E_2} - \frac{H_v}{2}\left[(m_c - 1) - (m_c - 2)\frac{D(p_c, y_c)}{P E_2}\right] \quad (7.39)\end{aligned}$$

For large value of Q_c, we can approximate $\frac{Q_c}{Q_c - 1} \approx 1$, and hence (7.39) can be approximated as follows:

$$\begin{aligned}\Psi_{sc}^{cent}(Q_c, L_c, p_c, y_c, m_c) &= \left[p_c + E_1\left\{\nu - u_0\left(\frac{1}{f} - 1\right)\right\}\right] D(p_c, y_c) - \left(A + Ue^{-\gamma L_c}\right) \\ &\frac{D(p_c, y_c)}{Q_c E_2} - \frac{D(p_c, y_c) f i_c}{E_2} - \frac{H_b}{2}(Q_c E_3 + E_4) - F(y_c - 1)^2 \\ &- H_b\left(\frac{\sigma^2 L_c}{4(1-\beta)Q_c} - (1-\beta)Q_c\right) - \frac{S D(p_c, y_c)}{m_c Q_c E_2} \\ &- \frac{H_v}{2}\left[(m_c - 1) - (m_c - 2)\frac{D(p_c, y_c)}{P E_2}\right] \quad (7.40)\end{aligned}$$

Proposition 7.4. *1. For fixed (Q_c, p_c, y_c, m_c), Ψ_{sc}^{cent} is concave in L_c. Moreover, the first order condition for optimality with respect to L_c gives*

$$L_c^* = \frac{1}{\gamma}\ln\left(\frac{4U\gamma(1-\beta)D(p_c, y_c)}{\sigma^2 H_b E_2}\right) \quad (7.41)$$

2. For fixed (p_c, y_c, m_c) with $L_c = L_c^*$, Ψ_{sc}^{cent} is concave in Q_c. Moreover, the first order condition for optimality with respect to Q_c gives

$$Q_c^* = \sqrt{\frac{4\left(A + \frac{S}{m_c}\right)(1-\beta)D(p_c, y_c)\gamma + \sigma^2 H_b E_2\left[1 + \ln\left(\frac{4D(1-\beta)U\gamma}{\sigma^2 H_b E_2}\right)\right]}{4\gamma(1-\beta)E_2\left\{H_b\left(\beta + \frac{E_3}{2} - 1\right) + \frac{H_v}{2}\left[\frac{(2-m_c)D(p_c, y_c)}{P E_2} + m_c - 1\right]\right\}}} \quad (7.42)$$

Proof. We now prove the concavity of Ψ_{sc}^{cent}, for fixed (Q_c, p_c, y_c, m_c). Noting that

$$\frac{\partial^2 \Psi_{sc}^{cent}}{\partial L_c^2} = -\frac{1}{Q_c E_2}\left[U\gamma^2 e^{-\gamma L_c} D(p_c, y_c)\right] < 0,$$

we can thus deduce the concavity of $\Psi_{\text{sc}}^{\text{cent}}$, for fixed $(Q_{\text{c}}, p_{\text{c}}, y_{\text{c}}, m_{\text{c}})$. Then, the value L_{c}^* is obtained solving in L_{c} the equation

$$\frac{\partial \Psi_{\text{sc}}^{\text{cent}}}{\partial L_{\text{c}}} = \frac{4D(p_{\text{c}}, y_{\text{c}})U\gamma e^{-\gamma L_{\text{c}}}}{E_2} + \frac{\sigma^2 H_b}{\beta - 1} = 0.$$

We now prove the concavity of $\Psi_{\text{sc}}^{\text{cent}}$ in Q_{c}, for fixed $(p_{\text{c}}, y_{\text{c}}, m_{\text{c}})$ with $L_{\text{c}} = L_{\text{c}}^*$. We have

$$\frac{\partial^2 \Psi_{\text{sc}}^{\text{cent}}}{\partial Q_{\text{c}}^2} = -\frac{4AD(p_{\text{c}},y_{\text{c}})\left(A+\frac{S}{m_{\text{c}}}\right)(1-\beta)\gamma + H_b E_2 \sigma^2 \left(1+\ln\left(\frac{4D(p_{\text{c}},y_{\text{c}})(1-\beta)\gamma U}{\sigma^2 H_b E_2}\right)\right)}{2Q_{\text{c}}^3(1-\beta)\gamma E_2} < 0$$

which indicates that $\Psi_{\text{sc}}^{\text{cent}}$ is concave in Q_{c}, for fixed $(p_{\text{c}}, y_{\text{c}}, m_{\text{c}})$ with $L_{\text{c}} = L_{\text{c}}^*$. The value Q_{c}^* obtained solving in Q_{c} the equation

$$\begin{aligned}\frac{\partial \Psi_{\text{sc}}^{\text{cent}}}{\partial Q_{\text{c}}} &= \frac{D(p_{\text{c}}, y_{\text{c}})}{Q_{\text{c}}^2 E_2}\left(A + \frac{S}{m_{\text{c}}}\right) + H_b\left(1 - \beta - \frac{E_3}{2}\right) \\ &+ H_b E_2 \sigma^2 \left(1 + \ln\left(\frac{4D(p_{\text{c}}, y_{\text{c}})(1-\beta)\gamma U}{H_b \sigma^2 E_2}\right)\right) = 0\end{aligned}$$

□

Using the values of L_{c} and Q_{c} from (7.41) and (7.42) into (7.40) and after rearranging, we have

$$\begin{aligned}\Psi_{\text{b}}^{\text{decn}}(p_{\text{c}}, y_{\text{c}}, m_{\text{c}}) &= \left(p_{\text{c}} + E_1\left\{\nu - u_0\left(\frac{1}{f} - 1\right)\right\}\right) D(p_{\text{c}}, y_{\text{c}}) - \frac{fi_{\text{c}} D(p_{\text{c}}, y_{\text{c}})}{E_2} \\ &- \frac{H_b E_4}{2} - F(y_{\text{c}} - 1)^2 -\end{aligned}$$

$$\begin{aligned}&2\left[\left\{H_b\left(\frac{E_3}{2} + \beta - 1\right) + \frac{H_v}{2}\left(\frac{(2 - m_{\text{c}})D(p_{\text{c}}, y_{\text{c}})}{PE_2} + m_{\text{c}} - 1\right)\right\}\right. \\ \times &\left.\left\{\frac{\sigma^2 H_b}{4\gamma(1-\beta)}\left[1 + \ln\left(\frac{4D(p_{\text{c}}, y_{\text{c}})(1-\beta)\gamma U}{\sigma^2 H_b E_2}\right)\right] + \left(A + \frac{S}{m_{\text{c}}}\right)\frac{D(p_{\text{c}}, y_{\text{c}})}{E_2}\right\}\right]^{1/2} \quad (7.43)\end{aligned}$$

For given values of p_c and y_c , maximizing $\Psi_{sc}^{cent}(p_c, y_c, m_c)$ with respect to m_c is equivalent to minimizing the following expression:

$$\begin{aligned}\Im'(m_c) &= \left\{ H_b \left(\frac{E_3}{2} + \beta - 1 \right) + \frac{H_v}{2} \left(\frac{(2-m_c)D(p_c,y_c)}{PE_2} + m_c - 1 \right) \right\} \\ &\times \left\{ \frac{\sigma^2 H_b}{4\gamma(1-\beta)} \left[1 + \ln\left(\frac{4D(p_c,y_c)(1-\beta)\gamma U}{\sigma^2 H_b E_2} \right) \right] \right. \\ &+ \left. \left(A + \frac{S}{m_c} \right) \frac{D(p_c,y_c)}{E_2} \right\} \end{aligned} \tag{7.44}$$

Proposition 7.5.

For fixed (p_c, y_c) with $L_c = L_c^*$ and $Q_c = Q_c^*$, $\Im'(m_c)$ is convex in m_c. Moreover, the first order condition for optimality with respect to m_c gives

$$m_c^{Decimal} = \sqrt{\frac{\left[H_b\left(\frac{E_3}{2}+\beta-1\right)+\frac{1}{2}H_v\left(\frac{2D(p_c,y_c)}{PE_2}-1\right)\right]\frac{SD(p_c,y_c)}{E_2}}{\frac{1}{2}H_v\left(1-\frac{D(p_c,y_c)}{PE_2}\right)\left[\frac{AD(p_c,y_c)}{E_2}+\frac{\sigma^2 H_b}{4\gamma(1-\beta)}\left\{1+\ln\left(\frac{4D(p_c,y_c)(1-\beta)\gamma U}{\sigma^2 H_b E_2}\right)\right\}\right]}} \tag{7.45}$$

Proof. We have the second order partial derivative of (7.44) as

$$\frac{\partial^2 \Im'}{\partial m_c^2} = \frac{2D(p_c,y_c)S\left[\frac{1}{2}H_v\left(-1+\frac{2D(p_c,y_c)}{PE_2}\right)+H_b\left(\beta-1+\frac{E_3}{2}\right)\right]}{m_c^3 E_2} \tag{7.46}$$

For $\beta > \frac{H_v(PE_2-2D(p_c,y_c))+H_bPE_2(2-E_3)}{2H_bPE_2}$, we have $\frac{d^2\Im'(m_c)}{dm_c^2} > 0$. Hence, $\Im'(m_c)$ is convex in m_c. Therefore, the unique value of m (denoted by m_c^*) can be obtained by solving $\frac{d\Im'(m_c)}{dm_c} = 0$. □

It should be noted that the number of shipments must be an integer value; however, (7.45) may give a decimal value. Therefore, following relation is proposed to calculate the suitable value of m_c^* that optimizes the vendor's decentralized annual profit.

$$m_c^* = \begin{cases} \lfloor m_c^{Decimal} \rfloor & \text{if } \Psi_b^{decn}(\lfloor m_c^{Decimal} \rfloor) > \Psi_b^{decn}(\lfloor m_c^{Decimal} \rfloor + 1) \\ \lfloor m_c^{Decimal} \rfloor + 1 & \text{if } \Psi_b^{decn}(\lfloor m_c^{Decimal} \rfloor) \leq \Psi_b^{decn}(\lfloor m_c^{Decimal} \rfloor + 1) \end{cases} \tag{7.47}$$

For given m_c, the conditions that have to be satisfied simultaneously to maximize the centralized annual profit in 7.43 are $\frac{\partial \Psi_b^{decn}}{\partial p_c} = 0$ and $\frac{\partial \Psi_b^{decn}}{\partial y_c} = 0$, i.e.,

$$\frac{\partial \Psi_{\text{sc}}^{\text{decn}}}{\partial p_c} = D(p_c, y_c) - b y_c^{\mu} \left(p_c + E_1 \left\{ \nu - u_0 \left(\frac{1}{f} - 1 \right) \right\} \right) + \frac{b f i_c y_c^{\mu}}{E_2}$$
$$- \left\{ \left(\frac{H_v}{2} \left(\frac{(2-m_c) D(p_c, y_c)}{P E_2} + m_c - 1 \right) + H_b \left(\frac{E_3}{2} - 1 + \beta \right) \right) \left(- \frac{b \left(A + \frac{S}{m_c} \right) y_c^{\mu}}{E_2} \right. \right.$$
$$- \left. \frac{b \sigma^2 H_b}{4 D(p_c, y_c)(1-\beta)\gamma} \right) - \frac{b H_v (2-m_c) y_c^{\mu}}{2 P E_2} \left(\frac{\sigma^2 H_b \left(1 + \ln \left(\frac{4 D(p_c, y_c)(1-\beta)\gamma\lambda}{\sigma^2 H_b E_2} \right) \right)}{4(1-\beta)\gamma} \right.$$
$$+ \left. \left. \frac{D(p_c, y_c) \left(A + \frac{S}{m_c} \right)}{E_2} \right) \right\} \Bigg/ \left\{ \left(\frac{H_v}{2} \left(\frac{(2-m_c) D(p_c, y_c)}{P E_2} + m_c - 1 \right) + H_b \left(\frac{E_3}{2} - 1 \right. \right. \right.$$
$$+ \left. \left. \left. \beta \right) \right) \left(\frac{D(p_c, y_c) \left(A + \frac{S}{m_c} \right)}{E_2} + \frac{\sigma^2 H_b \left(1 + \ln \left(\frac{4 D(p_c, y_c)(1-\beta)\gamma\lambda}{\sigma^2 H_b E_2} \right) \right)}{4(1-\beta)\gamma} \right) \right\}^{1/2} = 0 \quad (7.48)$$

$$\frac{\partial \Psi_{\text{sc}}^{\text{cent}}}{\partial y_c} = 2F(1-y_c) + D(p_c, y_c) \left[\left(p_c + E_1 \left\{ \nu - u_0 \left(\frac{1}{f} - 1 \right) \right\} \right) - \frac{f i_c + w}{E_2} \right] \frac{\mu}{y_c}$$
$$- \left\{ \left(\frac{H_v}{2} \left(\frac{(2-m_c) D(p_c, y_c)}{P E_2} + m_c - 1 \right) + H_b \left(\frac{E_3}{2} - 1 + \beta \right) \right) \left(\frac{\mu \sigma^2 H_b}{4 y_c (1-\beta)\gamma} \right. \right.$$
$$+ \left. \frac{\left(A + \frac{S}{m_c} \right) D(p_c, y_c) \mu}{E_2 y_c} \right) + \frac{H_v (2-m_c) D(p_c, y_c) \mu}{2 P E_2 y_c} \left(\frac{\sigma^2 H_b \left(1 + \ln \left(\frac{4 D(p_c, y_c)(1-\beta)\gamma\lambda}{\sigma^2 H_b E_2} \right) \right)}{4(1-\beta)\gamma} \right.$$
$$+ \left. \left. \frac{D(p_c, y_c) \left(A + \frac{S}{m_c} \right)}{E_2} \right) \right\} \Bigg/ \left\{ \left(\frac{H_v}{2} \left(\frac{(2-m_c) D(p_c, y_c)}{P E_2} + m_c - 1 \right) + H_b \left(\frac{E_3}{2} - 1 \right. \right. \right.$$
$$+ \left. \left. \left. \beta \right) \right) \left(\frac{D(p_c, y_c) \left(A + \frac{S}{m_c} \right)}{E_2} + \frac{\sigma^2 H_b \left(1 + \ln \left(\frac{4 D(p_c, y_c)(1-\beta)\gamma\lambda}{\sigma^2 H_b E_2} \right) \right)}{4(1-\beta)\gamma} \right) \right\}^{1/2} = 0 \qquad (7.49)$$

The solutions may be found by numerical methods. For example, the NSolve procedure of MATHEMATICA could be applied to solve the equations (7.48) and (7.49), as it was done in this chapter.

7.3.4.3 Coordinated decision-making model

In this sub-section, we propose and explain a price discount contract between the SC members. Under this agreement, the SC members are contracted in such a way that the vendor reduces the wholesale price w and instead the buyer changes his decision on $Q, L, p,$ and y with respect to the centralized optimal decision.

The contract is signed under the following form:

$$w^{'} = \begin{cases} w & \text{if } Q < Q_c \\ w(1-\alpha) & \text{if } Q \geq Q_c \end{cases}$$

Let us recall the decentralized profit function of the buyer and the vendor:

$$\begin{aligned} \Im_b^d(Q_d, L_d, p_d, y_d) &= \left(p_d - \frac{w}{E_2} + E_1\left(\nu + \frac{u_0(f-1)}{f}\right)\right) D(p_d, y_d) - F(y_d - 1)^2 \\ &- \frac{H_b}{2}(Q_d E_3 + E_4) - \left(A + Q_d f i_c + U e^{-\gamma L_d}\right)\frac{D(p_d, y_d)}{Q_d E_2} \\ &- H_b\left(\frac{\sigma^2 L_d}{4(1-\beta)Q_d} - (1-\beta)Q_d\right) \end{aligned} \tag{7.50}$$

and

$$\Im_v^d(m_d) = \frac{wD(p_d, y_d)}{E_2} - \frac{SD(p_d, y_d)}{m_d Q_d E_2} - \frac{H_v Q_d}{2}\left[\frac{(2-m_d)D(y_d, p_d)}{PE_2} + m_d - 1\right] \tag{7.51}$$

After accepting the price discount contract, the buyer's and the vendor's newly generated decentralized profit functions are

$$\begin{aligned} \Im_b^{co}(Q_c, L_c, p_c, y_c) &= \left(p_c - \frac{w(1-\alpha)}{E_2} + E_1\left(\nu + \frac{u_0(f-1)}{f}\right)\right) D(p_c, y_c) \\ &- \frac{H_b}{2}(Q_c E_3 + E_4) - \left(A + Q_c f i_c + U e^{-\gamma L_c}\right)\frac{D(p_c, y_c)}{Q_c E_2} \\ &- H_b\left(\frac{\sigma^2 L_c}{4(1-\beta)Q_c} - (1-\beta)Q_c\right) - F(y_c - 1)^2 \end{aligned} \tag{7.52}$$

and

$$\Im_v^{co}(m_c) = \frac{w(1-\alpha)D(p_c, y_c)}{E_2} - \frac{SD(p_c, y_c)}{m_c Q_c E_2} - \frac{H_v Q_c}{2}\left[\frac{(2-m_c)D(p_c, y_c)}{PE_2} + m_c - 1\right] \tag{7.53}$$

- *Lower bound of price discount*

Given (7.50) and (7.52), the target of the buyer is to mark the minimum discount level so that the profit after implementation of discount contract is greater or equal to the decentralized profit without discount contract. Therefore, in determining the minimum discount level, the following condition must hold:

$$\Im_b^{co}(Q_c, L_c, p_c, y_c, \alpha) \geq \Im_b(Q_d, L_d, p_d, y_d) \tag{7.54}$$

Based on the above inequality, the buyer's profit under a price discount contract should be greater or equal to the buyer's profit without employing discount contract. Otherwise, the buyer does not accept the vendor's discount offer.
After substituting (7.50) and (7.52) in the inequality (7.54), we have

$$\left(p_c - \frac{w(1-\alpha)}{E_2} + E_1\left(\nu + \frac{u_0(f-1)}{f}\right)\right) D(p_c, y_c) - \left(A + Q_c f i_c + U e^{-\gamma L_c}\right)$$
$$\frac{D(p_c, y_c)}{Q_c E_2} - \frac{H_b}{2}\left(Q_c E_3 + E_4\right) - F(y_c - 1)^2 - H_b\left(\frac{\sigma^2 L_c}{4(1-\beta)Q_c} - (1-\beta)Q_c\right) \geq$$
$$\left(p_d - \frac{w}{E_2} + E_1\left(\nu + \frac{u_0(f-1)}{f}\right)\right) D(p_d, y_d) - \left(A + Q_d f i_c + U e^{-\gamma L_d}\right)$$
$$\frac{D(p_d, y_d)}{Q_d E_2} - H_b\left(\frac{\sigma^2 L_d}{4(1-\beta)Q_d} - (1-\beta)Q_d\right) - \frac{H_b}{2}\left(Q_d E_3 + E_4\right) - F(y_d - 1)^2 \quad (7.55)$$

Solving (7.55), we have the optimal discount schedule as

$$\begin{aligned}
\alpha_{min} &= \frac{E_2}{wD(p_c, y_c)}\Bigg[p_d D(p_d, y_d) - p_c D(p_c, y_c) + \left\{E_1\left(\nu + \frac{u_0(f-1)}{f}\right) - \frac{w + f i_c}{E_2}\right\} \\
&\times \left[D(p_d, y_d) - D(p_c, y_c)\right] + \frac{A}{E_2}\left(\frac{D(p_c, y_c)}{Q_c} - \frac{D(p_d, y_d)}{Q_d}\right) + H_b\left(\frac{E3}{2} + \beta - 1\right) \\
&\times (Q_c - Q_d) + \frac{\sigma^2 H_b}{4(1-\beta)}\left(\frac{L_c}{Q_c} - \frac{L_d}{Q_d}\right) + F\left(y_c - 1\right)^2 - F(y_d - 1)^2 \\
&+ \frac{U}{E_2}\left(\frac{D(p_c, y_c)e^{-\gamma L_c}}{Q_c} - \frac{D(p_d, y_d)e^{-\gamma L_d}}{Q_d}\right)\Bigg]
\end{aligned} \quad (7.56)$$

- *Upper bound of price discount*

Given (7.51) and (7.53), the target of the vendor is to mark the maximum discount level so that the profit after implementation of discount contract is greater or equal to the decentralized profit with out discount contract. Therefore, in determining the maximum discount level, the following condition must hold:

$$\Im_v^{co}(m_c, \alpha) \geq \Im_v^{d}(m_d) \quad (7.57)$$

Based on the above inequality, the vendor's profit under a price discount contract should be greater or equal to the profit without employing discount contract. Otherwise, the vendor does not provide discount offer. After substituting (7.51) and (7.53) in the inequality (7.57), we have

$$\frac{w(1-\alpha)D(p_c, y_c)}{E_2} - \frac{SD(p_c, y_c)}{m_c Q_c E_2} - \frac{H_v Q_c}{2}\left[\frac{(2-m_c)D(p_c, y_c)}{PE_2} + m_c - 1\right]$$

$$\geq \frac{wD(p_\mathrm{d},y_\mathrm{d})}{E_2} - \frac{SD(p_\mathrm{d},y_\mathrm{d})}{m_\mathrm{d}Q_\mathrm{d}E_2} - \frac{H_vQ_\mathrm{d}}{2}\left[\frac{(2-m_\mathrm{d})D(y_\mathrm{d},p_\mathrm{d})}{PE_2} + m_\mathrm{d} - 1\right] \quad (7.58)$$

Solving (7.58), we have the maximum discount schedule as

$$\begin{aligned}\alpha_{\max} &= \frac{E_2}{wd_c}\Bigg[\frac{w}{E_2}[D(p_\mathrm{c},y_\mathrm{c}) - D(p_\mathrm{d},y_\mathrm{d})] - \frac{H_v}{2}\Bigg\{Q_c\left(\frac{(2-m_c)D(p_\mathrm{c},y_\mathrm{c})}{PE_2} + m_\mathrm{c} - 1\right)\\ &- Q_\mathrm{d}\left(\frac{(2-m_\mathrm{d})D(p_\mathrm{d},y_\mathrm{d})}{PE_2} + m_\mathrm{d} - 1\right)\Bigg\} - \frac{S}{E_2}\left(\frac{D(p_\mathrm{c},y_\mathrm{c})}{m_cQ_c} - \frac{D(p_\mathrm{d},y_\mathrm{d})}{m_dQ_d}\right)\Bigg] \quad (7.59)\end{aligned}$$

Equations (7.56) and (7.59) provide the range of the price discount to execute the proposed coordination policy. $[\alpha_{min}, \alpha_{max}]$ is the interval within which each value of α represents how both the SC members will share the profit. As α moves closer to α_{max}, the buyer will be more benefited. On the other hand, as α approaches closer toward α_{min}, the vendor will be more benefited.

7.4 Numerical experiment

This section investigates numerical experiments and the sensitivity of previously developed models. The aim is to draw insights into how optimal inventory decisions should be modified according to decentralized, centralized, and coordinated decision-making policies. We consider the following data for numerical experiment: $a = 280, b = 1.2$, $P = 500$, $A = 200$, $S = 1000$, $w = 25$, $v = 20$, $H_b = 2$, $H_v = 1$, $\sigma = 15$, $\mu = 0.4$, $\beta = 0.95$, $U = 156$, $\gamma = 0.8$, $F = 500$, $f = 0.15$, $i_c = 0.5$, $u_0 = 40$, $I_e = 0.11$, $\beta_1 = 1, \beta_2 = 8.0$. The defective rate θ follows the Beta distribution with parameters $\beta_1 = 1$ and $\beta_2 = 9$ (Wu, Ouyang, and Ho, 2007). That is, the pdf of θ is given by

$$h(\theta) = 9(1-\theta)^8, 0 < \theta < 1.$$

Hence $E(\theta) = \beta_1/(\beta_1+\beta_2) = 1/10$ and $E(\theta^2) = \beta_1(\beta_1+1)/[(\beta_1+\beta_2)(\beta_1+\beta_2+1)] = 1/55$.

In this example, we analyze the optimal ordering, pricing, effort, lead-time, and shipment decisions under a service level constraint for three decision-making policies: (1) decentralized, (2) centralized, and (3) coordinated. In the decentralized model, the buyer decides the optimal order quantity, selling price, effort level, lead-time and the vendor decides the number of shipments. On the other hand, in the centralized model, the decisions are taken jointly by the buyer and the vendor keeping in mind the profit growth of the entire supply chain. Finally, in the coordinated model, the vendor offers to the buyer some price discount so that the buyer accepts the centralized decisions.

Parameters	Decentralized SC	Centralized SC	Coordinate SC (price discount)
Q	238.012	247.736	247.736
p	131.625	119.152	119.152
y	3.38782	3.80627	3.80627
L	3.02167	3.2245	3.2245
k	0.0913	0.0838	0.0838
m	3	4	4
$D(p,y)$	198.841	233.871	233.871
α_{min}	–	–	0.0742805
α_{max}	–	–	0.146302
α	–	–	0.110291
$\Im_b$	17134.	–	17349.9
$\Im_v$	4582.2	–	4796.31
$\Im_{chain}$	21716.2	22146.2	22146.2

Table 7.1: Optimal solutions under the decentralized, centralized and coordinated scenarios

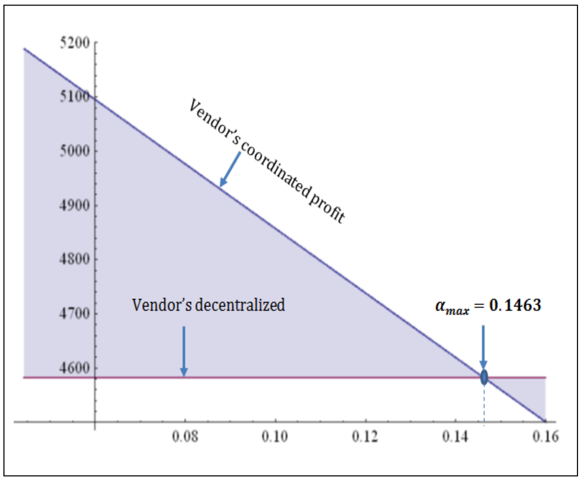

Figure 7.1: Buyer's decentralized profit vs. coordinated profit

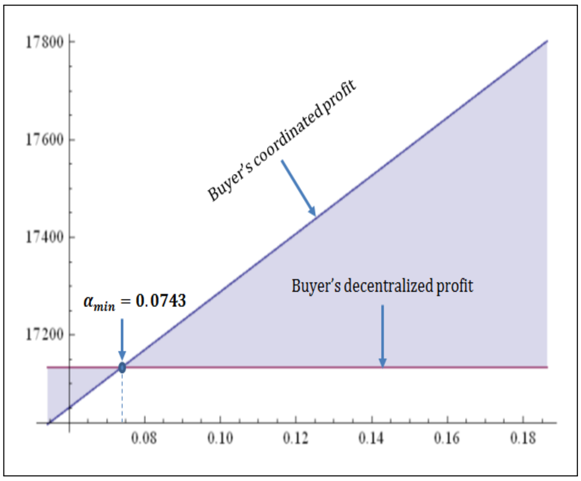

Figure 7.2: Vendor's decentralized profit vs. coordinated profit

The optimal outcomes of the models for decentralized, centralized and coordinated scenarios are provided in Table 7.1. Note that, the expected total annual profit in the centralized model is \$430 more than the total expected annual profit in the decentralized scenario. The centralized policy is able to increase the demand by 17.61% compared to the decentralized model. Note that, customers get more benefit (lower product price and better customer service) in the centralized channel compared to the decentralized one. However, the centralized policy does not necessarily increase the economic gain of all SC members. In the centralized policy, due to the larger order volume, the buyer's profit margin decreases compared to the decentralized policy, so the buyer may not be willing to participate in the centralized policy. However, as shown in Table 7.1, the proposed price discount coordination policy is able to increase the total SC profit as well as the individual SC member's profit as compared to the decentralized policy. Thus, this collaboration policy guarantees the participation of both SC members in the practice. The minimum and maximum range of the proposed price discount coordination policy i.e., $[\alpha_{min}, \alpha_{max}]$ is obtained as $[0.0742805, 0.146302]$. Under this range, the buyer's and the vendor's profit changes within $[17134, 17350]$ and $[4582.2, 4796.31]$.

Figures 7.1 and 7.2 show the changes in buyer's and vendor's profits with price discount policy. Significant improvement in buyer's and vendor's profits in coordinated policy is observed compared to decentralized policy with changes in price discount rate α. Looking at Figures 7.1 and 7.2, it can be seen that while the price discount is minimum, i.e., $\alpha_{min} = 0.0743$, the buyer's coordinated profit is same as it's decentralized profit and thereafter gradually increases with increasing price discount. Therefore, in the worst possible case, even if the vendor provides a minimum price discount, the buyer will still be willing to accept the centralized decision. On the other hand, from the vendor's point of view, the situation is going in the opposite direction and gives the maximum profit at $\alpha_{min} = 0.0743$ whereas the minimum profit (which is same as decentralized profit) at $\alpha_{max} = 0.1463$. So it is clear that the coordination model is able to increase the profits of of the SC members as compared to decentralized model. The above observation reveals that both parties will agree to coordinate if $\alpha \in [\alpha_{min}, \alpha_{max}]$; α will be determined based on the bargaining power of the members. In our case, we consider α as the average of α_{min} and α_{max}.

7.4.1 Sensitivity analysis

This subsection is dedicated to analyze the sensitivity of the parameters on the decision-making. Sensitivity analysis helps to recognize the high, moderate and low sensitive parameters. To check the effect of the parameters, we change one parameter at a time. That is, all other parameters remain the same when we check the sensitivity of a parameter.

7.4.1.1 Effect of price elasticity parameter b

The changes in order quantity (Q), selling price (p), effort level (y), and lead time (L) with changes in price elasticity coefficient b has been shown in Table 7.2 and depicted in Figure 7.3. The price elasticity parameter b has a high impact on the optimal selling price (p).

		Decentralized SC				Coordinated SC				
		(Q,p,y,L,m)	Ψ_b^{decn}	Ψ_v^{decn}	Ψ_{sc}^{decn}	(Q,p,y,L,m)	Ψ_b^{co}	Ψ_v^{co}	Ψ_{sc}^{co}	$\alpha_{min},\alpha_{max},\alpha$
b	0.6	(265,248,5.24,3.33,4)	46275	5958	52233	(274,236,5.57,3.43,4)	46407	6090	52497	0.0759,0.0383,0.0571
	0.8	(254,190,4.38,3.21,4)	31045	5385	36430	(262,177,4.74,3.34,4)	31208	5547	36754	0.0998,0.0504,0.0751
	1.0	(246,155,3.81,3.11,4)	22511	4944	27455	(254,143,4.19,3.28,4)	22702	5133	27834	0.1232,0.0624,0.0928
	1.2	(238,132,3.39,3.02,3)	17134	4587	21721	(248,119,3.81,3.22,4)	17348	4799	22147	0.1452,0.0739,0.1096
	1.4	(231,115,3.07,2.94,3)	13477	4277	17754	(242,103,3.51,3.18,4)	13715	4514	18229	0.1684,0.0860,0.1272

Table 7.2: Effect of price elasticity parameter b on optimal solution

The optimal selling price decreases in both the coordinated and decentralized scenarios when the value of b increases. However, in decentralized decision making policy selling price is higher than coordinated policy, resulting in less demand in decentralized situations which results in the loss of some of its end customers. This indicates that if the market demand for the product becomes more price sensitive then the focus should be on reducing the selling price of the product instead of increasing the promotional effort level.

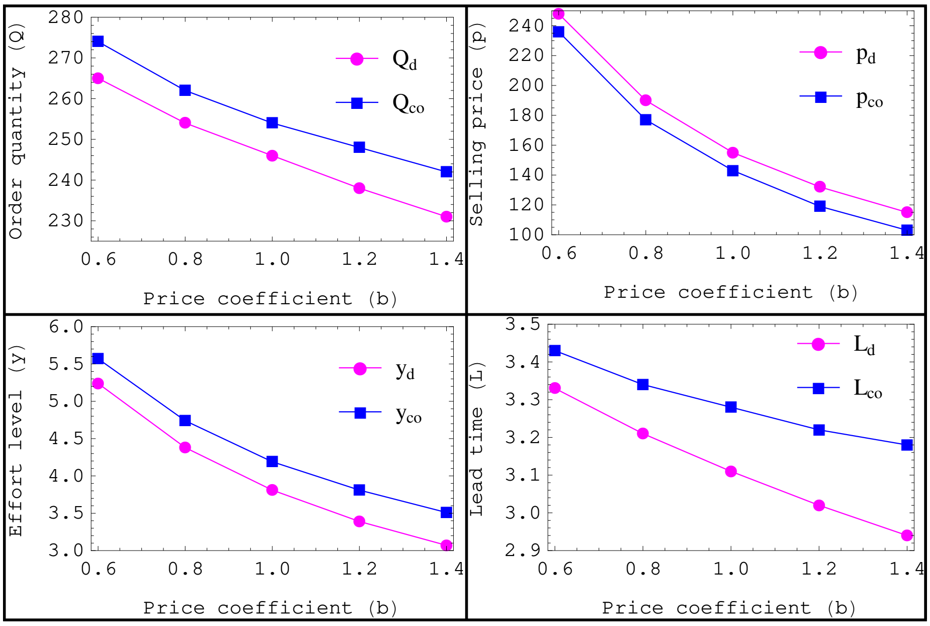

Figure 7.3: The trend of optimal decisions Q, p, y, L with respect to price elasticity coefficient b

In short, when the demand for the product becomes more price sensitive, the manager should reduce the selling price of the product in order to attract the customer. Such a situation could occur when there are many competing companies for the same product in the market, there is a tendency among the customers to cancel the product based on the selling price. In such a situation the coordination policy becomes important as it is able to reduce the selling price of the product as we can see from Figure 7.3. An adverse effect of parameter b on the order quantity has been observed from Figure 7.3. In general, increasing the sensitivity of consumers' towards the price of the product greatly reduces the sales of the product in the market. So in such situation it is wise to reduce the order quantity and Figure 7.3 indicates that. As the price sensitivity level of the product increases, the quantity of orders decreases, which in turn leads to a reduction in lead time. Firm's profit is decreasing significantly with the increasing value of b. However, if we compare the profit increment between decentralized and coordinated model, it is seen that the coordinated decision making policy is profitable where demand is more price sensitive.

		Decentralized SC				Coordinated SC				
		(Q,p,y,L,m)	Ψ_b^{decn}	Ψ_v^{decn}	Ψ_{sc}^{decn}	(Q,p,y,L,m)	Ψ_b^{co}	Ψ_v^{co}	Ψ_{sc}^{co}	$\alpha_{min},\alpha_{max},\alpha$
μ	0.1	(198,132,1.75,2.48,3)	12615	2885	15500	(225,120,1.88,2.63,3)	12715	2984	15699	0.0547,0.1083,0.0815
	0.2	(207,132,2.28,2.62,3)	13584	3244	16828	(239,120,2.50,2.78,3)	13708	3366	17074	0.0599,0.1184,0.0891
	0.3	(220,132,2.81,2.80,3)	15049	3788	18837	(260,119,3.11,2.98,3)	15208	3946	19154	0.0661,0.1306,0.0983
	0.4	(238,132,3.39,3.02,3)	17134	4582	21716	(248,119,3.81,3.22,4)	17350	4796	22146	0.0743,0.1463,0.1103
	0.5	(261,132,4.08,3.29,4)	20095	5767	25862	(257,119,4.65,3.52,5)	20398	6069	26468	0.0850,0.1652,0.1251
	0.6	(293,132,4.94,3.61,5)	24381	7575	31956	(281,118,5.73,3.87,7)	24850	8043	32893	0.1004,0.1939,0.1472

Table 7.3: Effect of promotional effort parameter μ on optimal solution

7.4.1.2 Effect of promotional effort parameter μ

Table 7.4 and Figure 7.4 show the impact of unit effort parameter μ on optimal solutions. The parameter μ has high impact on the optimal Q, y, and L. Promotional effort level of the retailer increases heavily with the increase of the parameter, μ. The level of promotional effort in the coordinated model is always higher than in the decentralized model. The increase in the promotional effort level has resulted

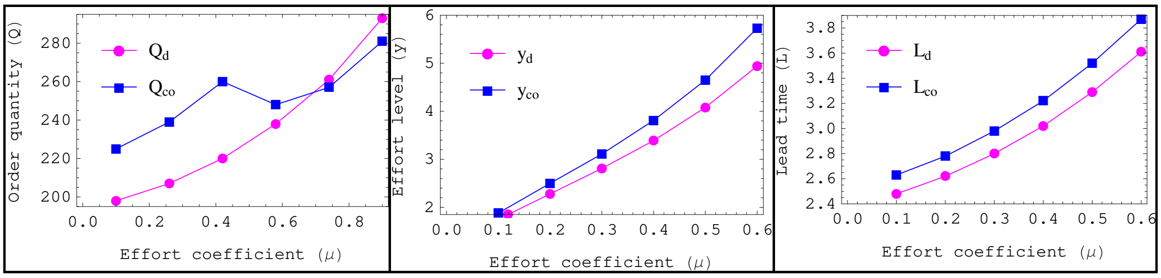

Figure 7.4: The trend of optimal decisions Q, y, L with respect to effort elasticity coefficient μ

in a favorable effect on demand which increases the order quantity. Figure 7.4 indicates that lead time tends to increase as order quantity increases. The profit of each of the channel members increases when μ increases. Note that, both α_{min} and α_{max} increase if μ increase. Importantly, the bargaining space between the retailer and the supplier increases when μ increases. This happens as the rate of increment of α_{max} is more than that of α_{min}.

7.4.1.3 Effect of purchasing price w

Table 7.4 and Figure 7.5 illustrate the impact of purchasing cost on optimal solutions. The optimal order quantity decreases and selling price increases as purchase cost w for the buyer increases. This is a very normal observation because when the selling

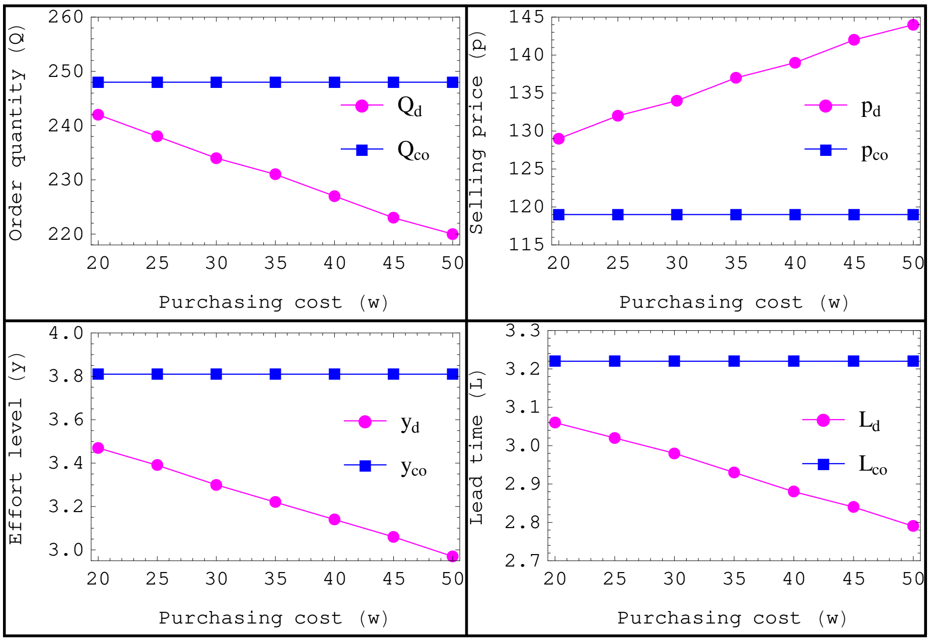

Figure 7.5: The trend of optimal decisions Q, p, y, L with respect to purchasing price w

price of the product is high, the sales of the product will naturally decrease which will result in a decrease in the order quantity and demand. Further, it is seen that the total profit decreases as the purchasing cost increases because of the decreases in the annual demand. The results also show that when the purchasing cost is high, the interval of price discount $[\alpha_{min}, \alpha_{max}]$ becomes wider. Therefore, it can be concluded that the proposed model is much more profitable when the purchasing cost is high.

		Decentralized SC				Coordinated SC				
		(Q, p, y, L, m)	Ψ_b^{decn}	Ψ_v^{decn}	Ψ_{sc}^{decn}	(Q, p, y, L, m)	Ψ_b^{co}	Ψ_v^{co}	Ψ_{sc}^{co}	$\alpha_{min}, \alpha_{max}, \alpha$
w	20	(242,129,3.47,3.06,4)	18163	3709	21872	(248,119,3.81,3.22,4)	18301	3845	22146	0.0591,0.1164,0.0878
	25	(238,132,3.39,3.02,3)	17134	4582	21716	(248,119,3.81,3.22,4)	17350	4796	22146	0.1463,0.0743,0.1103
	30	(234,134,3.30,2.98,3)	16141	5389	21530	(248,119,3.81,3.22,4)	16450	5696	22146	0.0894,0.1755,0.1324
	35	(231,137,3.22,2.93,3)	15182	6128	21311	(248,119,3.81,3.22,4)	15601	6545	22146	0.1044,0.2044,0.1544
	40	(227,139,3.14,2.88,3)	14258	6802	21060	(248,119,3.81,3.22,4)	14803	7343	22146	0.1192,0.2331,0.1762
	45	(223,142,3.06,2.84,3)	13368	7411	20779	(248,119,3.81,3.22,4)	14054	8093	22147	0.1340,0.2614,0.1977
	50	(220,144,2.97,2.79,3)	12512	7957	20469	(248,119,3.81,3.22,4)	13353	8793	22146	0.1487,0.2894,0.2190

Table 7.4: Effect of purchasing price w on optimal solution

7.4.1.4 Effect of buyer's holding cost H_b

The optimal Q and L decrease as the retailer's non-defective holding cost (H_b) increases. The result is quite normal because when the holding cost is high, retail will try to reduce the holding cost by storing less items. Moreover, the buyer will order

		Decentralized SC				Coordinated SC				
		(Q,p,y,L,m)	Ψ_b^{decn}	Ψ_v^{decn}	Ψ_{sc}^{decn}	(Q,p,y,L,m)	Ψ_b^{co}	Ψ_v^{co}	Ψ_{sc}^{co}	$\alpha_{min},\alpha_{max},\alpha$
H_b	1.1	(310,132,3.39,3.77,3)	17254	4598	21852	(327,119,3.81,3.97,3)	17466	4810	22276	0.0730,0.1442,0.1086
	1.4	(278,132,3.39,3.47,3)	17210	4596	21806	(312,119,3.81,3.67,3)	17422	4807	22229	0.0730,0.1440,0.1085
	1.7	(255,132,3.39,3.22,3)	17171	4590	21761	(299,119,3.80,3.43,3)	17384	4801	22185	0.0732,0.1444,0.1088
	2	(238,132,3.39,3.02,3)	17134	4582	21716	(248,119,3.81,3.22,4)	17350	4796	22146	0.1463,0.0743,0.1103
	2.3	(224,132,3.39,2.85,4)	17099	4582	21681	(241,119,3.80,3.05,4)	17315	4795	22110	0.0742,0.1460,0.1101
	2.6	(213,132,3.38,2.69,4)	17066	4580	21646	(234,119,3.80,2.90,4)	17281	4793	22074	0.0743,0.1458,0.1101
	2.9	(203,132,3.38,2.56,4)	17034	4578	21612	(228,119,3.80,2.76,4)	17249	4790	22039	0.0744,0.1458,0.1100

Table 7.5: Effect of buyer's holding cost H_b on optimal solution

frequently with small lot size when its holding cost per item per year increase. Also the optimal lead time decreases as holding cost increases. This is because for low values of Q, the production lead time decreases, which in turn, leads to smaller lead time. The optimal results are given in Table 7.5 and depicted in Figure 7.6.

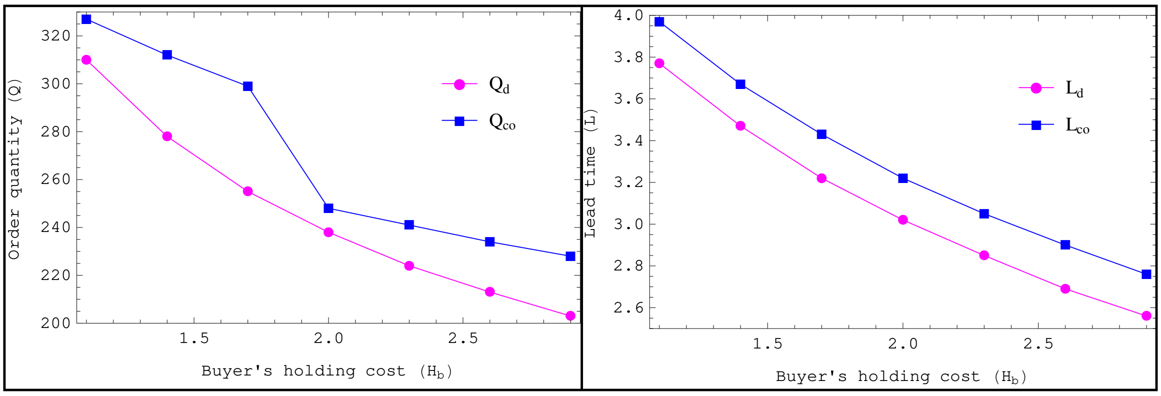

Figure 7.6: The trend of optimal decisions Q, L with respect to buyer's holding cost H_b

7.4.1.5 Effect of vendor's holding cost H_v

The vendor's holding cost (H_v) has no impact on buyer's optimal decisions and its profit. Profit of the vendor decreases when H_v increases. Both α_{min} and α_{max} decrease if H_v increase. When the vendor's holding cost increases the buyer's profit remains almost the same while the buyer's profit decreases as a result total channel profit decreases. That is why, the bargaining space between the retailer and the supplier decreases when H_v increases. The optimal results are given in Table 7.6.

		Decentralized SC				Coordinated SC				
		(Q,p,y,L,m)	Ψ_b^{decn}	Ψ_v^{decn}	Ψ_{sc}^{decn}	(Q,p,y,L,m)	Ψ_b^{co}	Ψ_v^{co}	Ψ_{sc}^{co}	$\alpha_{min},\alpha_{max},\alpha$
H_v	0.4	(238,132,3.39,3.02,5)	17134	4753	21887	(255,119,3.81,3.236)	17355	4972	22327	0.0759,0.1495,0.1127
	0.6	(238,132,3.39,3.02,5)	17134	4686	21820	(252,119,3.81,3.22,5)	17353	4904	22257	0.0753,0.1483,0.1118
	0.8	(238,132,3.39,3.02,4)	17134	4634	21768	(26,119,3.81,3.22,4)	17350	4848	22198	0.0744,0.1464,0.1104
	1.0	(238,132,3.39,3.02,3)	17134	4582	21716	(248,119,3.81,3.22,4)	17350	4796	22146	0.1463,0.0743,0.1103
	1.2	(238,132,3.39,3.02,3)	17134	4544	21678	(276,119,3.80,3.22,3)	17345	4754	22099	0.0729,0.1435,0.1082
	1.4	(238,132,3.39,3.02,3)	17134	4506	21640	(265,119,3.80,3.22,3)	17344	4714	22058	0.0725,0.1426,0.1075
	1.6	(238,132,3.39,3.02,3)	17134	4468	21602	(255,119,3.80,3.22,3)	17343	4675	22018	0.0722,0.1421,0.1071

Table 7.6: Effect of vendor's holding cost H_v on optimal solution

7.4.1.6 Effect of standard deviation σ

It can be observed that with an increase in lead-time demand deviation (σ), the order quantity also increases and number of shipment decreases. We can interpret the results in such a way that as the demand deviation increases, the probability of stock out increases which leads the system to a larger order quantity and smaller lot size. As expected, a significant reduction in lead time has been observed with the

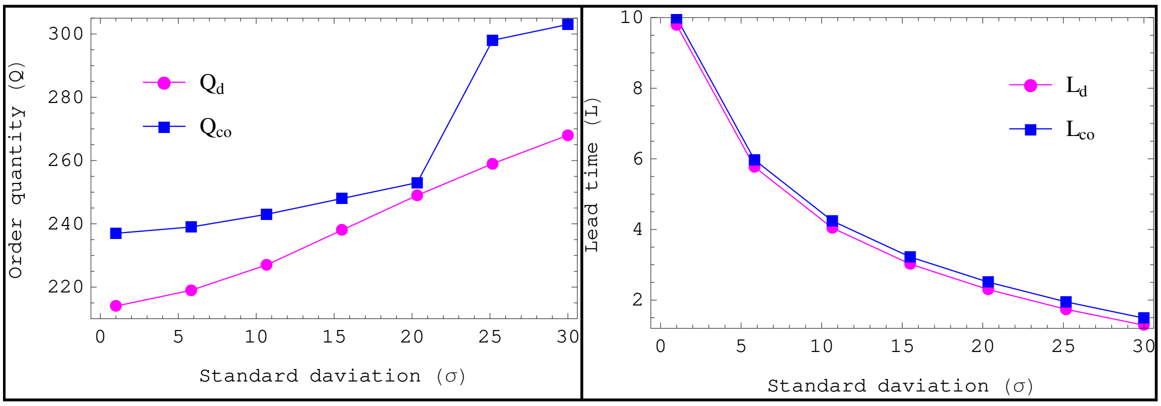

Figure 7.7: The trend of optimal decisions Q, L with respect to standard deviation σ

increase in demand deviation. This indicates that the lead time reduction is a key parameter for a supply chain when demand deviation is high. The optimal results are given in Table 7.7 and depicted in Figure 7.7.

		Decentralized SC				Coordinated SC				
		(Q,p,y,L,m)	Ψ_b^{decn}	Ψ_v^{decn}	Ψ_{sc}^{decn}	(Q,p,y,L,m)	Ψ_b^{co}	Ψ_v^{co}	Ψ_{sc}^{co}	$\alpha_{min},\alpha_{max},\alpha$
σ	1	(214,132,3.39,9.79,4)	17176	4583	21759	(237,119,3.81,9.99,4)	17390	4797	22187	0.0738,0.1458,0.1098
	5	(219,132,3.39,5.77,4)	17168	4584	21752	(239,119,3.81,5.97,4)	17383	4797	22180	0.0739,0.1458,0.1098
	10	(227,132,3.39,4.04,4)	17153	4584	21737	(243,119,3.81,4.24,4)	17367	4798	22165	0.0740,0.1460,0.1100
	15	(238,132,3.39,3.02,3)	17134	4582	21716	(248,119,3.81,3.22,4)	17350	4796	22146	0.0743,0.1463,0.1102
	20	(249,132,3.39,2.30,3)	17115	4586	21701	(253,119,3.81,2.51,4)	17330	4797	22127	0.0746,0.1457,0.1101
	25	(259,132,3.39,1.74,3)	17097	4588	21685	(298,119,3.81,1.95,3)	17311	4798	22109	0.0738,0.1443,0.1090
	30	(268,132,3.39,1.29,3)	17082	4588	21670	(303,119,3.80,1.49,3)	17296	4796	22093	0.0740,0.1443,0.1092

Table 7.7: Effect of standard deviation σ on optimal solution

7.4.1.7 Effect of effort cost efficiency coefficient F

The optimal Q, y, and L of the retailer are highly sensitive on F. The optimal order quantity, effort level, and lead time decrease when the value of F increases. The retailer's optimal selling price remains almost the same when the value of F increases. The profit of both the channel members decrease in both the decentralized and coordinated scenarios if F increases. Both α_{min} and α_{max} as well bargaining space between the retailer and the supplier decrease if F increases. The optimal results are given in Table 7.8 and depicted in Figure 7.8.

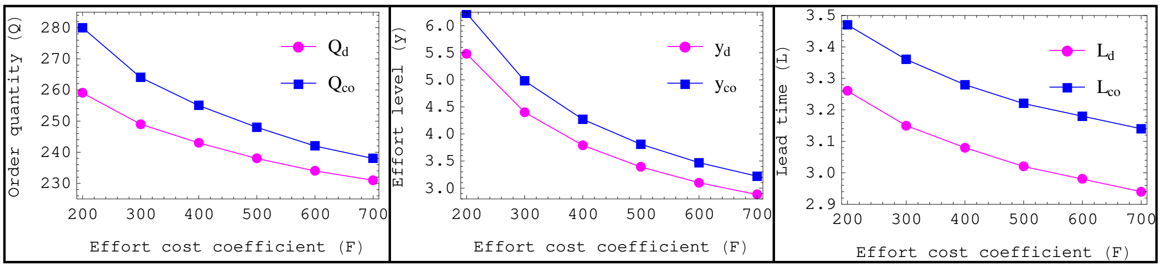

Figure 7.8: The trend of optimal decisions Q, y, L with respect to effort cost coefficient F

		Decentralized SC				Coordinated SC				
		(Q, p, y, L, m)	Ψ_b^{decn}	Ψ_v^{decn}	Ψ_{sc}^{decn}	(Q, p, y, L, m)	Ψ_b^{co}	Ψ_v^{co}	Ψ_{sc}^{co}	$\alpha_{min}, \alpha_{max}, \alpha$
	200	(259,132,5.48,3.26,4)	20263	5631	25894	(280,119,6.23,3.47,4)	20535	5903	26438	0.0773,0.1522,0.1148
	300	(249,132,4.40,3.15,4)	18747	5128	24365	(264,119,4.98,3.36,4)	18992	5372	24364	0.07610.1498,0.1129,
	400	(243,132,3.79,3.08,4)	17800	4808	22608	(255,119,4.27,3.28,4)	18028	5036	23064	0.0751,0.1479,0.1115
F	500	(238,132,3.39,3.02,3)	17134	4582	21716	(248,119,3.81,3.22,4)	17350	4796	22146	0.0743,0.1463,0.1102
	600	(234,132,3.10,2.98,3)	16633	4412	21045	(242,119,3.47,3.18,4)	16839	4615	21454	0.0736,0.1446,0.1091
	700	(231,132,2.88,2.94,3)	16239	4276	20515	(238,119,3.22,3.14,4)	16436	4471	20907	0.0780,0.1431,0.1080

Table 7.8: Effect of effort cost efficiency coefficient F on optimal solution

7.4.1.8 Effect of inspection fraction f

The optimal order quantity increases when the value of the retailer's proportion of quantity inspected per shipment (f) increases. The result is quite reasonable because when a large number of products are inspected it is very common to find a large number of defective products which are unable to meet the demand of the customers. Therefore, in this case the order quantity should be increased in such a way that even after the defective items are rejected there are good enough items which are able to meet the demand of the customers. The retailer's optimal selling price decreases when the value of f increases. It is seen that The profit of both the channel members increase in both the decentralized and coordinated scenarios if f increases. Thus, the result clearly states that if time and other circumstances for inspection permits then the system will prefer the full inspection instead of the partial

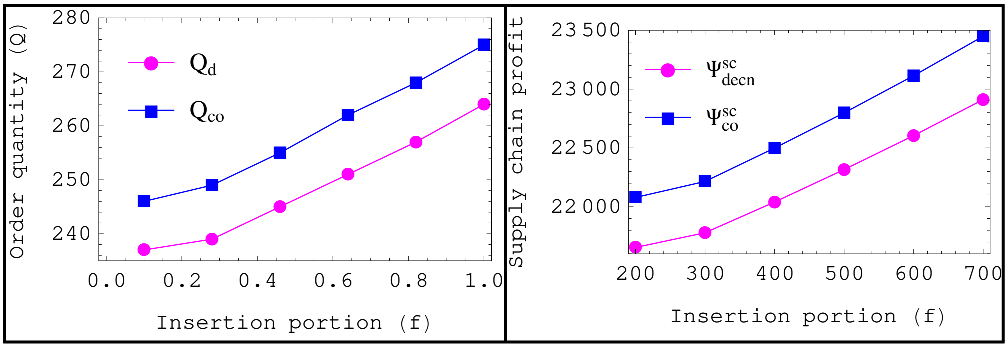

Figure 7.9: The trend of optimal decisions Q, Ψ with respect to inspection portion f

inspection. Both α_{min} and α_{max} as well bargaining space between the retailer and the supplier increase if f increases. The optimal results are given in Table 7.9 and depicted in Figure 7.9.

		Decentralized SC				Coordinated SC				
		(Q,p,y,L,m)	Ψ^{decn}_b	Ψ^{decn}_v	Ψ^{decn}_{sc}	(Q,p,y,L,m)	Ψ^{co}_b	Ψ^{co}_v	Ψ^{co}_{sc}	$\alpha_{min}, \alpha_{max}, \alpha$
	0.1	(237,132,3.85,3.01,4)	17104	4550	21654	(246,119,3.80,3.22,4)	17318	4761	22079	0.0739,0.1458,0.1097
	0.2	(239,132,3.39,3.03,3)	17164	4615	21779	(249,119,3.81,3.23,4)	17383	4832	22215	0.0746,0.1470,0.1108
	0.4	(245,131,3.40,3.06,3)	17289	4751	22040	(255,118,3.83,3.27,4)	17519	4980	22499	0.0762,0.1500,0.1131
f	0.6	(251,131,3.41,3.10,3)	17420	4896	22316	(262,118,3.85,3.31,4)	17659	5139	22798	0.0775,0.1522,0.1148
	0.8	(257,131,3.42,3.14,3)	17557	5048	22605	(268,117,3.88,3.35,4)	17812	5302	23114	0.0794,0.1564,0.1179
	1.0	(264,130,3.43,3.17,3)	17702	5209	22911	(275,116,3.90,3.39,4)	17972	5477	23449	0.0812,0.1599,0.1205

Table 7.9: Effect of inspection portion f on optimal solution

7.5 Conclusions

In this chapter, the SC model is extended by considering variable demand and lead-time under a service level constraint. In the investigated SC, the buyer faced stochastic price and effort dependent demand and followed (Q, r) inventory system. The vendor's production system is considered to be imperfect resulting in defective items. Each lot delivered by the vendor carries some defective item. It was assumed that the buyer applied a sub-lot inspection on arrival of each lot to identify the defective products. This model has included service level constraints instead of determining the shortages cost. In both the centralized and decentralized policies, we formulated the total profit function and optimized using distribution-free approach. We proposed a number of properties that were proved to be satisfied by the profit function. Finally, a price discount policy was proposed to establish the coordination between the buyer and the vendor. We then obtained the minimum and maximum satisfactory price discounts which enable us to establish a win-win situation among the buyer and the vendor. Several experiments were finally carried out through sensitivity analysis to draw insights into how optimal production-inventory decisions were modified in presence of service level constraint.

Chapter 8

Conclusion and future prospects

Numerous challenges viz., optimal ordering policy, backorder quantity, safety stock, pricing, lead-time, imperfect quality items etc., that are faced by organizations while dealing with a two-echelon supply chain have been addressed in the current thesis. We have developed mathematical models for several realistic situations, and solved then through optimization frameworks. In addition, models are validated with numerical examples, and in-depth sensitivity analysis has been performed for important managerial insights and better decisions.

In Chapter 1, a general introduction has been provided for the supply chain and inventory management. Moreover, the scope and overview of this thesis have been presented.

Chapter 2 presents the literature review on supply chain models.

Chapter 3 integrates sub-lot sampled inspection policy into a two-echelon supply chain model with price and green sensitive demand subject to random defective items. As not every product is being inspected before sold to the market, a penalty cost is charged to the retailer. Three supply chain scenarios, namely decentralized, centralized, and coordinated, have been developed. A simple trade-credit coordination mechanism is developed that enables to share the profit among the SC members in such a way that both members can earn more profit than their individual decentralized profits. Some theoretical results along with numerical analysis have been conducted to validate the proposed model. It is seen that the coordination among the SC members has resulted in an improvement in the level of the greening of the product and profits of the supply chain.

In Chapter 4, an integrated vendor-buyer SC model with variable lead-time and uncertain market demand has been developed. Unlike the traditional integrated supply chain model, we have assumed lead-time as a function of production time, setup time, and transportation time. Further, the replenishment lead-time is shortened by reducing transportation time. From the numerical analysis, it is found that the reorder point and safety stock can be reduced by reducing the replenishment lead-time. It is found that high demand uncertainty influences the SC members to reduce the replenishment lead-time in order to lessen the stock-out probability.

In Chapter 5, a two-echelon supply chain model has been developed where lead-time is assumed to be a function of order quantity and production rate. Lead-time is reduced through controlling production rate and order quantity. The backlogging rate is considered as a function of lead-time. The proposed model is formulated to obtain the net present value (NPV) of the expected total cost of the integrated system through optimization of (i) the buyer's order quantity, (2) the buyer's safety factor, and (3) the vendor's production rate.

In Chapter 6, a lead-time reduction strategy has been proposed for a single-manufacturer single retailer integrated inventory system with controllable backorder rate and trade-credit financing. Initially, we have assumed the lead time demand at the retailer as normally distributed and then it is considered as distribution-free. The model allows shortages which are partially backlogged with lead-time dependent backlogging rate. Min-max approach is adopted to solve the model when lead-time demand is distribution-free. The effects of controllable lead-time and backorder rate along with trade-credit financing are illustrated through numerical examples.

In Chapter 7, a supply chain model is developed that focuses on ordering, price, effort, and lead-time decisions to coordinate amongst the SC participants with a service level constraint. The model is developed under centralized, decentralized, and coordinated decision-making policy. Demand rate for the retailer is considered as stochastic and also dependent on both price and effort. The lead-time is treated as an extra control parameter that could be slashed by a cost that is a negative exponential function of lead-time. Given that the distribution of lead-time demand is unknown, we have developed the models using a distribution-free methodology.

The most common way to develop a SC model is to assume a single-buyer single-vendor scenario which limits its applicability to the industry with multiple buyers or multiple manufacturers. As an illustration, future research can analyze a SC model consisting of multiple buyers or multiple manufacturers. Moreover, it would be interesting to include routing decisions and, hence, Green House Gas (GHG) emissions from the transportation process as well. In addition, it can be noted that the effort cost is paid only by the retailer. Therefore, there is an option to include a cost sharing agreement that divides the cost of the investment among the SC members. It will be interesting to consider information asymmetry among channel members and compare results with those of the existing ones.

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1. **Sumon Sarkar**, N. M. Modak, Bibhas C. Giri, Ashis Kumar Sarkar, L. E. Cárdenas-Barrón (**2022**)(**Accepted**) Coordinating a supplier-retailer JELS model considering product quality assessment and green retailing, ***Journal of Cleaner Production***, (**I.F.-9.297**) (**SCIE**) (**Elsevier**), DOI: 10.1016/j.jclepro.2022.131658.

2. **Sumon Sarkar**, S. Tiwari, Bibhas C. Giri (**2021**) Impact of uncertain demand and lead-time reduction on two-echelon supply chain, ***Annals of Operations Research*** (**I.F.-4.854**) (**SCIE**) (**Springer**) https://doi.org/10.1007/s10479-021-04105-0.

3. **Sumon Sarkar**, Bibhas C. Giri, Ashis Kumar Sarkar (**2020**) A vendor-buyer inventory model with lot-size and production rate dependent lead-time under time value of money, ***RAIRO-Operations Research***, 54 (4), 961-979 (**SCIE**) (**I.F.-1.393**) (**EDP Sciences**).

4. **Sumon Sarkar**, Bibhas C. Giri (**2021**). Optimal ordering policy in a two-echelon supply chain model with variable backorder and demand uncertainty, ***RAIRO-Operations Research***, 55 (2021) S673-S698 (**SCIE**) (**I.F.-1.393**) (**EDP Sciences**).

5. **Sumon Sarkar**, Ashis Kumar Sarkar, Bibhas C. Giri (2021) Pricing, effort, and lead-time decision in a coordinated joint economic lot-size model under imperfect production and service level constraint (**communicated**).

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Coordinating a supplier–retailer JELS model considering product quality assessment and green retailing

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ABSTRACT

Environmental awareness among the people is constantly increasing, which has resulted in massive pressure on companies from various stakeholders including the government and consumers to mitigate the detrimental effects on the environment. This work deals with a joint economic lot size (JELS) model to enhance the greening efforts of a product that flows along a two-level supply chain (supplier–retailer). Both selling price and greening effort level of the product influence the market demand. Here we assume that every individual lot shipped to the retailer carries some random defective items. Each lot received by the retailer goes through an error-free sub-lot sampled inspection process to remove the defective items. A fraction of faulty items are classified as usable and are sold at a salvage value and the rest items are disposed of. The retailer has to bear per item penalty cost for selling the uninspected defective items. We develop the profit function under three decision-making scenarios: centralized, decentralized, and coordinated. Coordination is made based on a trade-credit scheme under which the retailer changes his/her optimal decisions according to a centralized policy. We obtain a minimum and maximum credit period duration which encourages both the retailer and the supplier to follow the coordinated decision-making policy. The coordinated model suggests that more emphasis should be given to the greening effort level for higher profit. It is observed that, in many cases, a sub-lot inspection gives better results compared to a full lot inspection.

1. Introduction and motivation

Global market diversity, such as rapid product development, the drop in oil prices, short product life cycle, and higher product variety, has intensified competition among the companies, making it more challenging to maintain the desired level of profitability. One of the most effective tools for companies to survive is to optimize and improve their supply chain (SC) performance. A collaborative instance of management is an important source of competitive advantage, as it can enhance the effectiveness and efficiency of the SC. As the collaboration between SC players increases, the total cost decreases by up to 30% (Lee et al., 1997). Therefore, companies are trying to integrate and work together in a chain. A joint decision-making policy makes the supply chain system stronger and flexible to deal with the market's fluctuating demand. The issue of optimization of integrated inventory replacement policies across several items is commonly referred to as a "joint economic lot size" (JELS) problem. This paper falls into the class of researches aiming at investigating JELS problem, taking into account ordering, pricing, greening, backordering, and replenishment decisions under sub-lot sampling inspection.

More and more consumers are paying attention to issues such as sustainability, climate change, and other environmental issues. Consumers' increased awareness of the environment has increased the demand for green products in the market (Peterson and Michalek, 2013; Zhao et al., 2016). Retailers are currently keen to show their concerns about environmental sustainability. This involves taking steps to lessen a retailer's impact on the environment. Therefore, retailers are seizing the opportunity to use the concept of green retailing to improve their business as more and more consumers demand eco-friendly products and services. The retailer can transform its business into green retailing by switching to energy-efficient lights and equipment, using jute made or paper made recycled and reusable bags, sending e-receipts instead of a paper receipt, encouraging its supplier to deliver products in

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S.I. : BUSINESS ANALYTICS AND OPERATIONS RESEARCH

Impact of uncertain demand and lead-time reduction on two-echelon supply chain

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Abstract

This paper develops a continuous-review vendor-buyer supply chain (SC) model wherein the lead-time (taken as replenished) is considered as a factor affected upon by the time stamp required for setup and production followed by transportation. Here, the production time indicates the interaction between the lot-size and lead-time. Assuming the existence of an opportunity with the buyer of reducing the replenishment lead-time. The buyer receives normally distributed stochastic lead-time demands from its customers. Due to the stochastic nature of lead-time demand, shortages may arise at the buyer's side which is fully backlogged. We presume imperfection production at the vendor's end, which leads to the generation of a certain ratio/percentage of defective products, which results in additional warranty costs for the vendor. This study intends to uncover the best policy that minimizes the system's total expected cost. A solution algorithm with some lemmas is provided which helped in finding the optimal solution and to prove the uniqueness of the solutions. Findings demonstrate that a reduction in lead-time can effectively lower safety stock as well as the total cost.

Keywords Lead-time reduction · Demand uncertainty · Two-echelon supply chain · Safety stock · Investment

1 Introduction

In the recent past, integrated supply chain models considering various practical assumptions have attracted much consideration among researchers. The reason behind this is that the policy of integrated decision making strengthens the SC system which results in both buyer

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A VENDOR–BUYER INVENTORY MODEL WITH LOT-SIZE AND PRODUCTION RATE DEPENDENT LEAD TIME UNDER TIME VALUE OF MONEY

Sumon Sarkar*, Bibhas Chandra Giri and Ashis Kumar Sarkar

Abstract. The paper studies an integrated vendor–buyer model with shortages under stochastic lead time which is assumed to be variable but depends on the buyer's order size and the vendor's production rate. The replenishment lead time and the market demand uncertainty are assumed to be reduced by changing the regular production rate of the vendor at the risk of paying additional cost. Shortages are partially backlogged and the backlogging rate depends on the length of the buyer's replenishment lead time. The proposed model is formulated to obtain the net present value (NPV) of the expected total cost of the integrated system through optimization of (i) the buyer's order quantity, (2) the buyer's safety factor, and (3) the vendor's production rate. Theoretical results are derived to demonstrate the existence and uniqueness of the optimal solution. Through extensive numerical study, some valuable managerial insights are obtained.

Mathematics Subject Classification. 90B05, 90B06.

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1. Introduction

Today's uncertain economy forces the supply chain managers to search for an alternative way to stay one step ahead from their competitors. It becomes very difficult for big retail companies to stand a chance without appropriate inventory control model. For many years, economic order quantity (EOQ) and reorder point have been used to make their decisions. An EOQ could help the company managers in order to take decision about the best optimal order quantity. On the other hand, the reorder point instructs when to place an order for particular products based on historical demand (Ben-Daya and Hariga [3], Ho and Hsiao [13], Tiwari *et al.* [49], Sarkar and Giri [43]). Additionally, the reorder point enables sufficient stock of products at hand *i.e.*, safety stock to fulfill the customer's demand while the next order arrives due to the lead time. Almost all integrated inventory models are developed based on the assumption that replenishment lead time is either zero or constant (Wee and Widyadana [51]; Das [7]) or a stochastic variable (Sajadieh and Jokar [42]; Zhou *et al.* [53]; Hossain *et al.* [14]) which is not subjected to control. According to Tersine [48], lead time involves order preparation time, order shipment/delivery time, set-up time, *etc.* Recognizing that manufacturing lead time is so much

Keywords. Supply chain, variable production rate, NPV method, lead time reduction, backordering, demand uncertainty.

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OPTIMAL ORDERING POLICY IN A TWO-ECHELON SUPPLY CHAIN MODEL WITH VARIABLE BACKORDER AND DEMAND UNCERTAINTY

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Abstract. The paper investigates a two-echelon production-delivery supply chain model for products with stochastic demand and backorder-lost sales mixture under trade-credit financing. The manufacturer delivers the retailer's order quantity in a number of equal-sized shipments. The replenishment lead-time is such that it can be crashed to a minimum duration at an additional cost that can be treated as an investment. Shortages in the retailer's inventory are allowed to occur and are partially backlogged with a backlogging rate dependent on customer's waiting time. Moreover, the manufacturer offers the retailer a credit period which is less than the reorder interval. The model is formulated to find the optimal solutions for order quantity, safety factor, lead time, and the number of shipments from the manufacturer to the retailer in light of both distribution-free and known distribution functions. Two solution algorithms are provided to obtain the optimal decisions for the integrated system. The effects of controllable lead time, backorder rate and trade-credit financing on optimal decisions are illustrated through numerical examples.

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1. Introduction

Supply chain (SC) management is concerned with the coordination of material, information, and money along with a network of companies whose purpose is to achieve better performance. Supply chain can be classified into two categories-integrated (or centralized) and non-integrated (decentralized) supply chains. In a non-integrated supply chain each member decides based on its own policy, which can lead to inefficient decisions (Katok and Wu [21]). According to Giannoccaro and Pontrandolfo [10], co-ordination strategy incentivises each supply chain member in such a way that the decisions taken jointly by the members are optimal from a centralized supply chain perspective to increase the chain profit (Weng [46]). Coordination strategies involve mechanization of a company's replenishment processes as well as the connection of buyer and supplier communities with real-time forecast, inventory on-hand, optimal lot sizing, quality improvements, inspections, and shipment information to reduce inventory and eliminate unnecessary expenses. The so-called integrated supply chain models simulate today's business practices (*e.g.*, automotive, apparel, grocery) where there exists a long relationship between buyers and suppliers.

Keywords. Integrated model, lead time reduction, controllable backorder, trade-credit financing, distribution-free approach.

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