Problems and Solutions of a Class of Stochastic Green Supply Chain Management Models

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DECLARATION

This thesis is submitted at Jadavpur University, Kolkata 700032, India for the degree "*Doctor of Philosophy*" in science. The research described herein is conducted under the supervision of Dr. Narayan Chandra Majee, Department of Mathematics, Jadavpur University and Dr. Vaskar Sarkar, Assistant Professor, Department of Mathematics, School of Basic and Applied Sciences, Adamas University between the time period April, 2014 and August, 2022.

This research work is original to the best of my knowledge except where the references and acknowledgments are made to the previous works. Neither this nor any substantially similar research work has been or is being submitted for any other degree, diploma or other qualification at any other university.

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Date: 16.08.2022 Rajib Chakrabarty

CERTIFICATE FROM THE SUPERVISOR

This is to certify that this thesis entitled "*Problems and Solutions of a Class of Stochastic Green Supply Chain Management Models*" submitted by Mr. Rajib Chakrabarty who got his name registered on 23.04.2014 for the award of Ph.D (Science) degree of Jadavpur University, is absolutely based upon his own work under the supervision of Dr. Narayan Chandra Majee, Associate Professor, Department of Mathematics, Jadavpur University, and Dr. Vaskar Sarkar, Assistant Professor, Department of Mathematics, School of Basic and Applied Sciences, Adamas University and that neither this thesis nor any part of it has been submitted for either any degree/ diploma or any other academic award anywhere before under my knowledge.

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ABSTRACT

So far, the manufacturing industries have adapted the carbon pricing policies to maximize the financial benefits and for better environmental protection. In the thesis, a stochastic deteriorating and stochastic demand item production inventory model in the environment of two echelon supply chain management (SCM) is proposed. We assessed the model through a numerical example and graphical demonstration and also comparison with different scenarios is provided. Our model is more generalized than other peer models because, both the deterioration rate of the item and the demand are stochastic rather than deterministic in our model. We found the stochastic nature of various parameters is more realistic than deterministic nature and the carbon emission cost is applied to optimize the whole SCM cost, which has a significant role in the reduction of carbon footprint.

In the thesis considered a two echelon supply chain (SC) under aspect of carbon cap and trade, where the two players of the SC are manufacturer, distributors. We have considered price and carbon emission dependent stochastic demand for single manufacturer and many distributors. We found the optimal values of distributor's base selling price, carbon foot print (emission) after carbon emission mitigation and manufacturer's wholesale price for unit product.

In the thesis, also considered the significant role of wireless communications for monitoring and to controlling supply-chain management of any organization.The system (we named it WARKS) can be implemented in home, industrial, hospital, farms, forest, agriculture and many more. To verify the system capabilities and work performance, we performed the experiment in indoor and outdoor using required hardware and software.

Also, in the thesis we have taken a single item probabilistic deterioration and considered two types of continuous probabilistic demand along with a fixed demand. Also we have considered carbon emission cost per delivery. Here we estimated minimum production quantity under probabilistic environment and used algebraic method to determine the minimum cost associated with the whole SCM. The objective of this paper

is to find the minimum production quantity, minimum cost and optimum lot size with integer valued deliveries.

We developed a multi-retailer supply chain model, where single vendor supplies products to multiple buyers to satisfy stochastic and carbon emission dependent demand. The production rate and production cost are considered as a variable quantities. We have considered production cost as a function of production rate, which developed a special ‘U’-shaped function and is used to develop the model. We have considered a single-setup-multi-delivery (SSMD) policy for delivering products to buyers from supplier. At the buyer’s end, partial backorder is considered for shortages. The lead time demand is considered as stochastic and follows a normal distribution. Considering the market demand is affected by consumer’s environmental awareness in addition form, the manufacturer considered the technological up-gradation under carbon cap and trade mechanism. Considering all such aspects this paper proposes the methods to determine the optimal order quantity and the optimal level of carbon emissions through model optimization.

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CHAPTER 1

Introduction

In the supply chain(s) (SC), the devastating environmental impacts have resulted in government law formulation, customer awareness and pressure from various stakeholders to implement environmentally sustainable strategies in SC. A sustainable SC requires management policies that minimize the SC cost and reduce consumption of resources and environmental pollution. Therefore carbon emission cost (28; 73; 77; 83; 64; 59) has to be incorporated to develop any new SC model. In real life, it is quite common for different inventory items, such as milk, fruits, blood, pharmaceutical products, vegetables etc, to perish or depreciate over time. Therefore, it is important to study the behaviour of such decaying and deteriorating items (44), toward the formulation of appropriate inventory control policies that explicitly take such behaviour into account. The deterioration rate is different among various products and depends on several physical and environmental situations. Thus, the deterioration rate is stochastic (44; 35; 74) in nature, rather than deterministic. This feature has to be considered when modelling the real life SC scenario. In reality the demand of any product has a seasonal effect. Thus, in SC the demand has to be considered in two factors: one part is constant and another is stochastic. In our proposed model we have considered the above features to develop a realistic model that can be adopted by different SC managers and practitioners for achieving utmost economic goals and minimum environmental impacts i.e., carbon footprints.

The inventory system is taking an important part of cost controlling in business and organization. Inventory consists of usable but idle resources which are materials and goods. The volume of material, a company has in stock at a specific time, is known as inventory or in terms of money it can be defined as the total capital investment over all the materials stocked in the company at any specific time. Inventory may be in the form of raw material inventory, in process inventory, finished goods inventory, among others. Inventory management is a key component in any production environment. This has been recognized not only in the operations research literature but also in the management science and industrial engineering domains. Inventory Control (IC) is the supervision of supply, storage and accessibility of items to ensure an adequate supply without excessive oversupply. The integrated inventory management system is a common practice in the global markets and provides economic advantages for both the vendor and the buyer. Globalization of market and increased competition force organizations to rely on effective supply chains to improve their overall performance. The goal in many research and development efforts that are related to the SC management (SCM) is to have practical models that can effectively reduce operational costs.

Transportation has a major role in SC management. The ever growing volume of such activity not only benefits the growth and sustainability of international economy and globalization but also has its own consequences, particularly those pertaining to the environment. Transportation activities are considerable sources of air pollution and greenhouse gas emissions, with the former known to have harmful effects on human health and the latter is responsible for global warming. These issues have raised concerns on reducing the amount of emissions worldwide. In this respect, many countries, including both developed countries and developing countries, have set strict targets on reducing their carbon emissions now and in future. There is a large amount of literature on supply chain management regarding environmental issues through the emerging concept "green supply chain management" (GrSCM) (72). In the carbon emissions reduction literatures, the decision problem of carbon emissions reduction in supply chain is a hot topic. Benjaafar et. al. (32) illustrated how carbon emission concerns could be integrated into operational decision making and analyzed the impact of operational decisions on carbon emissions.

Deterioration inventory model research has received increasing attentions in recent years, extending existing models with a variety of deterioration patterns, demand functions and back- ordering policies. In most cases, deterioration is assumed to be a constant fraction of total on-hand inventory. But the deterioration rate taken as stochastic by Ghare and Schrader (44) and Chakrabarty et al. (33) among several researchers. The Weibull distribution has been used to model item decay by Chakrabarty et al. (33) and some attention has been focused on deteriorating items with expiration dates by Hsu et al. (52) and Lo et al. (55).

In inventory and SC models, several researchers considered the demand as constant, but in reality the demand has some seasonal effects. As a result, the total demand is a combination of a constant demand, which is constant throughout the year, and a variable demand, which is stochastic in nature. For example, the demand of garments throughout the year.

In recent years global warming is a foremost hazard to our planet. It's having severe threat to the human health, and well-being and nature. It has many disastrous effects, such as rise of sea level, flood, drought, storm (44), disruption in ecosystems and increased clear-air turbulence (7). The world ecosystem, environment and human society facing the most severe problems of eco-environment deterioration and imbalance. The main reason behind this crisis is excessive emission of greenhouse gases. To control and reduce the emission of greenhouse gases effectively, many countries and regions (44; 35) have been proposed some legislations and regulations. By cap-and-trade mechanisms (74; 58) i.e., the emission trading scheme is one of the widely accepted mechanisms to make possible the enterprises for conservation of energy and reduction of emission of the greenhouse gases. Some countries already have taken remarkable measures to reduce the emission of green house gases. The Kyoto Protocol, in 1997, have issued by United Nations (UN), which get on a scheme to involved enterprises members' production process under the regulation of cap-and-trade (69). In 2005, European Union Trading Systems (EU-ETS) was initiated the biggest international scheme for the trading of greenhouse gasses. European Union, covering more than 31 counties which contribute almost half of the carbon emission, controls the emission of greenhouse gases through the implementation of a series of financial policies which includes CCA (Climate Change Agreements), CCT (Climate Change Tax) and CPS (Carbon Price Support)(56). Being a large scale manufacturing country China has taken continuous drives to reduce emission of greenhouse gases. At some provinces and cities like Beijing, Tianjin, the trading market of carbon emission was framed, in 2013. The emission reduction and efficiency of energy saving is promoted by the carbon emission trading scheme. It is evident that, irrespective of developed or developing country, adapting the measures for reduction of carbon emission. Therefore, with the low-carbon economy becoming a global trend and national strategic behaviour. Use of the cap-and trade regulation encourage enterprises to raise the environmental performance which has the characteristics of external public interests. Therefore, being the primary carrier of greenhouse gases, enterprises have been faced a challenge, due to carbon emission limit and also put in many restrictions in the operations. Due to consumer awareness, they are inclined to the choice to purchase a low-carbon and environment friendly products. Under this situation, the enterprises focus on the technological up-gradation and improved operational management of enterprises to comply with cap-and-trade regulation (60). Also profit disagreement among the players (upstream and downstream) in supply chain is increasing seriously, since the technological up-gradation in the carbon emission mitigation incurred huge investment. Here we consider that the market demand is stochastic and also affected by selling price and carbon footprint with linear relationship.

Wireless devices have grown tremendously within recent few years due to the appearance of global adoption, various functionality and wide applications. Future WiFi probably dedicate broadband speeds. Wireless access is consider to both indoor and outdoor environments. For high quality of service (QoS), WiFi enabled devices (i.e., smartphones, tablets, laptops, smart TVs, cameras, sensors etc.) improve user experience and attract them to this technologies. The communication technologies currently used in D2D (device-to-device) communications, to interconnect multiple sensor nodes spread into a particular area. Access to communication technology can play a pivotal role in social and economic development. The choice of technology to achieve this a significant aspect (157). If the network has to cover a larger area than router is not capable of transmitting to, or if signals have to penetrate through obstacles, performance will take a hit. Interference is also a big issue, signals from other wireless networks and electronics can impact speeds.

The goal of this work is to analyse WiFi feasibility and evaluate its performance and applications in different fields by using a WiFi network with sensors, kits and multimedia support. This work improve D2D communications in a significant way, appointing intermediate multi hop WiFi nodes with different network topology. The objective of this work to retain the WiFi communications without boosting the signals for both static and dynamic cases (maintaining the order and range of WiFi nodes). For static purpose this network is usable in house, colony, factory, hospital, office, hostel, restaurant, forest etc. Also for dynamic purpose this network is usable in agriculture, animal monitoring, mountain, train, army and rivers etc. Authors intention to establish a online/offline data processing enable network for communication (158).

WiFi deals with the specification of an unlicensed bands worldwide use in wireless local area network supporting a set of scenarios based on number of devices, range, and energy constraints. WiFi offers a simple, robust, and efficient solution in the industrial, scientific, and medical radio band (ISM band) compared with other existing technologies. WiFi technology enables devices to exchange information and perform actions without human intervention. Due to their short wireless range and high obstruction losses, current WiFi require the use of intermediate nodes, to reduce complexity of the network.

In the modern marketing environment, it is more relevant to optimize the total system cost jointly for all parties involved in the supply chain system than to optimize the individual cost of each party (114). Currently, a vendor or manufacturer typically delivers products to numerous buyers. Many vendors build their own retail outlet to deliver products to multiple buyers. Thus, a single-vendor multi-buyer model is applicable in many cases. Goyal (101) proposed the integrated inventory model with coordination between a single buyer and a single vendor as a pioneering approach. Banerjee (87) extended Goyal's (101) model by assuming a lot-for-lot policy, which was again extended by Goyal (103) with SSMD policy. Goyel (103) suggested a supply chain model, where the vendor's production quantity is an integer multiple of the buyer's order quantity. Ha and Kim (105) developed a lot splitting supply chain model with a single retailer and a single supplier. Ouyang et al. (112) investigated an integrated vendor-buyer cooperative model with controllable lead time and stochastic demand. Sarkar and Majumder (114) developed an integrated inventory model with vendor's setup cost reduction and solved by a distribution free-approach. Cardenas-Barron et al. (91) surveyed a number of research articles regarding economic order quantity model. Cardenas-Barron and Sana (91) investigated the channel coordination of a two-echelon supply chain, where the demand pattern is dependent on sales' teams initiatives. Moon et al. (110) introduced a service level constrain in a continuous review model with variable stochastic lead time. Regarding backorder rate, Sarkar et al. (116) introduced random defective production rate with variable backorder rate. Sarkar et al. (118) introduced fill rate in a continuous review inventory model to minimize the total system cost with setup cost reduction. Sarkar and Mahapatra [M33] developed a periodic review inventory model with fuzzy demand to minimize the total cost by considering setup cost reduction of vendor. Sarkar et al. (118) considered backorder price discount in an integrated inventory model, where they developed two models with lead time demand as normally distributed and without having any distribution. Sarkar et al. (121) introduced product specific (products having fixed lifetime) back-ordering policy in a two-echelon supply chain model with coordination between the supply chain players. Based on the imperfect quality of products, Sarkar et al. (122) discussed the way to improve the quality by additional investment.

Banerjee and Burton (89) discussed a comparison between coordinated and independent replenishment policies in a single-vendor multi-buyer supply chain model. Banerjee and Banerjee (88) developed a multi-buyer inventory model using electronic data interchange with an order-up-to inventory control policy. Sarmah et al. (123) considered a single-supplier multi-buyer coordinated supply chain model with a trade credit policy. Hoque (106) discussed three different single-vendor multi-buyer models by synchronizing the production ow with equal and unequal sized batch transfer for the first two models and the last model, respectively. Guan and Zhao (104) developed a multi-retailer inventory system with a continuous review policy, which optimizes the decisions of pricing and inventory management with the aim of maximizing profit. Jha and Shankar (107) developed a singlevendor multi-buyer constrained non-linear model under service level constraint and solved it using a Lagrangian multiplier method. Cardenas-Barron and Trevi no-Garza (91) developed a three-echelon supply chain model with multiple products and multiple periods. Glock and Kim (100) studied the effect of forward integration in a multi-retailer supply chain under retailer competition. Cardenas-Barron and Sana (94) studied a two-layer supply chain model with multiple items and a promotional effort.

To improve customer service and to reduce stockout loss, it is important to reduce lead time. Liao and Shyu (109) first incorporated a probabilistic inventory model assuming lead time as a unique decision variable. Ben- Daya and Rauf (90) considered an inventory model as an extension of Liao and Shyu's (109) model, where lead time is one of the decision variables. Ben-Daya and Rauf's (90) model dealt with no shortage and continuous lead time. Ouyang et al. (111) extended Ben-Daya and Rauf's (90)] model by assuming discrete lead time and shortages. Pan and Yang (113) analyzed an integrated inventory model with lead time in a controllable manner. Annadurai and Uthayakumar (86) developed a periodic review inventory model under controllable lead time and lost sales reduction. Gholami-Qadikolaei et al. (98) developed a probabilistic inventory model with lead time and ordering cost reduction under budget and space constraint. Shin et al. (124) studied an integrated inventory model with controllable lead time and a service level constraint. You (126) studied an inventory model with partial backorder under vertical shift demand. Chung (96) assumed an integrated production-inventory model with backorder and used the method of comprising cost difference rate. Sarkar and Moon (116) developed an inventory model with variable backorder and deduced the procedure to reduce setup cost and quality improvement. The production rate is assumed to be constant in the classical supply chain model, however, in many cases, the machine production rate may change (108). Conard and McClamrock's (97) analysis stated that a 10 percent change in processing rate resulting a 50 percent change in machine tool cost. Moreover, the possibility of failure in the production process gradually increases with increasing production rate. As a result, the product quality may deteriorate at some rate. Thus, it is reasonable to consider the production rate as a decision variable not constant. Unit production cost also depends on the production rate and should be treated as one of the decision variables. Giri and Dohi (99) considered a generalized extended EMQ model with variable production rate by assuming stochastic machine breakdown and repair. Chang et al. (95) developed an EMQ model with variable production rate for a two-stage assembly system. Soni and Patel (125) studied an integrated single-supplier single-retailer inventory model with variable production rate and trade-credit policy.

Also, to control and reduce carbon emissions is a global problem. With "United Nations Framework Convention on Climate Change" and "Kyoto Protocol" signed and coming into force, carbon emissions reduction and the cap-and-trade system have become the consensus and inevitable trend. Enterprises are the main bodies of carbon emissions reduction; however the development of modern market economy makes any enterprise be placed in some supply chain systems. Then if we control carbon emissions from the perspective of single enterprise, the spillover effects generated by other enterprises' carbon emissions control in the supply chain will be ignored. Only when we solve the problem of carbon emissions reduction cooperation and coordination among supply chain enterprises effectively can we fundamentally promote carbon emissions reduction work. Otherwise it will be difficult for us to achieve the goal of carbon emissions reduction. With the improvement of cap-and-trade system, the carbon trading price, as the most direct market signal,will regulate the manufacturer's carbon emissions in his production process, thereby affecting the manufacturer's per unit carbon emissions. For example, Baosteel Group, one of the largest steel enterprises in China, has launched two carbon trading projects from 2007 to 2012, aiming to reduce per-unit carbon emissions through energy saving and emissions reduction technologies (127). Then Baosteel Group sold excess carbon credits at the appropriate price in the carbon trading market and profited more than 150 million RMB (128). In addition, with the development of social civilization and low-carbon education, there are more environmentally conscious consumers who prefer low-carbon products. How to meet consumers' low-carbon preference has become a key guarantee to win over in the fierce market competition (129). Therefore, it is of great value and practical significance to study the carbon emissions reduction in the supply chain with considering carbon trading price and consumers' environmental awareness (CEA). The investigated supply chain in this paper consists of single vendor supplies products to multiple buyers to satisfy stochastic and carbon emission dependent demand. At the beginning of production, the manufacturer obtains an initial carbon emissions limit from the government that has established a carbon emissions center. The manufacturer can buy carbon emissions rights from the carbon emissions center if he exceeds his initial carbon emissions limit at the end of production cycle; otherwise, he can sell his unused carbon emissions rights to the carbon emissions center. The manufacturer also can upgrade his production technology to reduce carbon emissions; then he needs to bear the cost of carbon emissions reduction. The consumers have environmental awareness, which means the products' carbon emissions will influence the market demand.

CHAPTER 2

Literature review

In the field of production–inventory literature, articles considering carbon policies are few but increasing rapidly. Dobos (84) introduced limit of carbon trading permits in the well-known Arrow–Karlin production–inventory model. Also, Dobos (38) further extended his earlier work considering the aspects of time-dependent unit tradable permit and calculated the immediate procurement cost of environmental licenses. The basic EOQ model with carbon emission mitigation investment, has extended by Toptal et. al. (14), under carbon cap, carbon tax and cap-and-trade policies. An optimal model for an SC to minimize opportunity cost considering carbon cap developed by Diabat and Simchi-Levi (79). A model for a green SC, developed by Abdallah et. al. (16), to determine lot sizes of production and shipment for raw materials and finished products assuming the constraints over carbon emission. Benjaafar et. al. (17) considered carbon tax, strict cap on emission, cap-and-trade and carbon-offset policies to determine optimal production, inventory, backorder quantity and amount of carbon traded to minimize the total SC cost for a single firm and also extended their models for multiple firms with or without coordination. Chen et al (41) depicted how with operational adjustments emissions can be reduced without significant increase in cost under different carbon policies. Wahab et. al. (29) incorporated screening and holding cost of defective items in their model taking the fixed and variable carbon costs in their model. Du et al. (20) developed a model using game theoretical approach for centralized and decentralized decisions of the manufacturer and the retailer in an SC where both the parties induce low-carbon efforts.

Mostly the papers in the field of production-inventory and supply chain management with carbon policies have considered deterministic demand. But in real life scenario, enterprises regularly experience stochastic demand due to shrinking product life cycles, seasonality, buying patterns of customers, and other relevant issues (31). Few research articles recently considered random or stochastic demand to reflect realistic aspects while considering different carbon policies. Rosic and Jammernegg (80) optimized a dual sourcing problem with carbon tax and carbon cap-and-trade policies considering stochastic demand in the newsboy environment. Dong et. al. (37) suggested a profit maximization multi-stage SC model considering carbon-cap-and-trade policy and stochastic demand in single-period planning horizon. Zhang and Xu (53) also considered stochastic demand at the time of developing a multi-item production planning model to maximize profit with carbon trading cost, and assumed newsvendor-type products while developed the model. Arikan and Jammernegg (36) suggested a single-period inventory model considering carbon footprint constraint and demand as a positive random variable.

Deteriorating item inventory models has drawn a remarkable attention after the work developed by of Ghare and Schrader (44). In real life scenario, it is impossible to prevent deterioration of highly volatile and/or explosive items (e.g., alcohol, fuels), degradable items (such as blood), several pharmaceutical products and some daily consumable products (viz., milk, fruit, vegetables, etc.) for decaying or deteriorating the nature over time. Hence, it is very imperative to explicitly consider the characteristics of the decaying and deteriorating items in the notion of the formulation of apposite SCM models. Ghare and Schrader (44) addressed the characteristics of those deteriorating items as "inventory decay", that described the direct spoilage, physical depletion and deterioration. They had not considered the entire SCM, but only developed a general EOQ (economic order quantity) model in which demand considered as constant and decay rate following the exponential distribution. Considering the Weibull and gamma distribution as deterioration rate, the work developed by Ghare and Schrader (44) was then further extended by Covert and Philip (1973) (35) and Tadikamalla (1978) (74), respectively. They had considered instantaneous replenishment, i.e., inventory growth of Just-In-Time (JIT) and did not allow back orders or shortages. Misra (1975) (58) suggested EOQ type models with finite production rates without shortages or backorders and discussed the cases of varying and constant deterioration rates. Exponential and Weibull deterioration rates along with the backorders even though with instantaneous replenishment (JIT) had been developed by Shah (1977) (69). Mak (1982) (56) suggested a production lot size inventory framework with backorders for the items following exponentially deterioration rate.

Research on the inventory models with deteriorating item has cached growing attention during the recent years and extended the different models with the diversified deterioration patterns, demand functions along with the shortages, and different backordering policies. In most cases, deterioration has been considered as a constant fraction of total on-hand inventory. Raafat (60) conducted a literature survey on the existing models for deteriorating the items. The (T, S_i) model was extended by Dave and Patel (41) considering the time proportional demand and deterioration. In 1998, Chakrabarty et al.(33) used Weibull distribution to model the item decay. Hsu et al.(52) in 2006 and

Lo et al.(55) in 2007 worked on deteriorating items that have the expiration dates, while the deteriorating item economic ordering policy over an infinite time horizon was adopted by Goyal (47). Researchers such as B. Sarkar (62; 61), Datta and Pal (40), Hariga (49), Goyal and Giri (48), Skouri and Papachristos (71), Goswami and Chaudhuri (45), Skouri et al. (70), T. Sarkar et. al. (65; 66; 67; 68) etc. have expanded the literature of the inventory models in the direction of various types of deterioration rates. In 1930, Harris (50) introduced the basic but now well-known EOQ model using differential calculus. In 1934, Wilson (79) derived the same type of model. In the literature of inventory models, differentiation method has been applied by most of the researchers to develop various types of algorithms for the inventory model in SCM milieu. In the SCM system, there are different parties (viz., buyers, suppliers, producers, distributors and retailers, etc.) that take different roles of the customers. Furthermore, SCM consisted of carbon emission cost for transportation of items. In the study of SCM, the purpose was to optimize the entire system simultaneously. In 1975 Goyal (46) developed a single supplier-single buyer problem for an integrated inventory model. In 1986, Banerjee (29) introduced a joint economic lot size model for the vendor and purchaser. The integrated production inventory model in the context of single vendor-single buyer as generalized policy was extended by Hill (51). The coordinating supply chain inventory through common replenishment epochs was discussed by Viswanathan and Piplani (76). The EOQ for deteriorating items in the multi-item lot size integrated vendor-buyer inventory model with JIT environment model with out differentiation was discussed by Yang and Wee (81; 82). Using the analytic geometry and algebraical methods, Cardenas-Barron (30; 31) discussed a multi-stage multi-customer supply chain model for optimizing the inventory decisions and the derived EPQ/EOQ inventory models with multiple backordering costs. In the same paper the author Cardenas-Barron suggested a series of optimization procedure for different class of algebraic functions using basic algebra. Items with constant deterioration was considered and extended by Yan et al. (80). Khouja (53) discussed optimization procedure of a multi-stage multi-customer supply chain model. Recently Duan et.al. derived a mathematical model for buffer capacity optimization was established and optimized using an extended vector universal generating function (UGF) and an improved adaptive non-dominated sorting genetic algorithm II (NSGA-II) (42).

In reference (36), Cardenas-Barron developed a model using optimum manufacturing batch-size in a single-stage production system with rework. Imperfect quality and quantity discounts in an EOQ model was noted by Cardenas-Barron (37). Some outstanding notes to find the integer valued lot-size in the basic EOQ and EPQ model were suggested by Garcia-Laguna et al. (84) and Cardenas-Barron et al. (38). Using arithmetic geometric inequality the vendor–buyer inventory model was extended by

Cardenas-Barron et al. (39). For the EOQ and EPQ inventory models with the linear and fixed backorder costs (63), a complete solution procedure was suggested by Chung and Cardenas-Barron (34). For rework and stochastic preventive maintenance time with deteriorating items, an EPQ model was presented by Wee and Widyadana (78). An EOQ model for buyer-distributor-vendor supply chain with backlogging without using derivatives was extended by Teng et al. (75).

Recently, by addressing the environmental effect as a part of the supply chain, the transportation cost and carbon emission cost due to the transportation gained attention by the researchers in inventory and supply chain modeling. In practice, SC decisions are strongly affected by the flow paths and mode of transportation and transportation frequency (or quantity) of the inventory items. Regardless of its significance, very few researchers have developed SC and close loop supply chain (CLSC) models considering transportation costs and carbon emission cost. Some of the works related to the transportation costs and carbon emission cost were developed by Andriolo et al.(28), Swenseth and Godfrey (73), Vroblefski et al. (77), Zhao et al. (83), Sarkar et al. (64). Paksoy et al. (59) and Sarkar et al. (64) considered transportation costs and carbon emission cost in CLSC model.

It is a basic work to evaluate and manage supply chain's carbon footprint to control carbon emissions in supply chain. Braithwaite and Knivett (130) proposed a methodology to evaluate supply chain's carbon footprint by introducing the "carbon-to-serve" concept. Cholette and Venkat (131) found that different supply chain configurations can result in vastly different energy and emissions' profiles by using a web-based tool to calculate carbon emissions. Sundarakani et al. (132) examine the carbon footprint across supply chains and use transport methods to construct the analytical model. Lam et al. (133) presented a newmethod for supply chain synthesis and used a demand-driven approach to assess the feasible ways for transferring energy from renewable sources to customers in a given region. Xu and Fan (134) established the carbon footprint calculation models of fixed emissions sources and dynamic emissions sources in the supply chain. Compared with these surveyed papers, ourwork is based on these studies and focuses on the impacts of carbon trading price and CEA on carbon emissions reduction. In the carbon emissions reduction literatures, the decision problem of carbon emissions reduction in supply chain is a hot topic. Benjaafar et al. (135) illustrated how carbon emission concerns could be integrated into operational decision making and analyzed the impact of operational decisions on carbon emissions. Rosiˇc and Jammernegg (136) extended the dual sourcing model based on the newsvendor framework by considering the environmental impact of transport with emission taxes. Song and Leng (137) investigated the classical single-period problem under different carbon emissions policies. Du et al. (138) focused on the impact of "cap-and-trade" mechanism, proposed a game-theoretical analytical model, and derived a unique Nash equilibrium. Jaber et al. (139) presented a supply chain coordination model while accounting for greenhouse gas emissions from manufacturing processes. Xu et al. (140) proposed three differential game models with the emission reduction of the product and the retailer's promotion dependent demand. Li et al. (141) proposed a carbon trading model discussed between enterprises under strict carbon cap. Zhao et al. (142) developed a retailer-driven revenue-sharing contract to coordinate the supply chain with the constraint of product carbon emissions. Lu and Chen (143) studied the supply chain coordination with buyback contract under different carbon emissions policies and found buyback contract can coordinate supply chain. Compared with the surveyed papers in this scope, our work focuses on the coordination mechanism about carbon emissions reduction and order quantity in the supply chain.

CEA affects market demand, which has an impact on the enterprise's carbon emissions. Liu et al. (144) found that asCEA increases, retailers and manufacturers with superior ecofriendly operations will benefit. Zhang et al. (145) analyzed the impact of CEA on order quantities and channel coordination, in which the manufacturer produces the environmental products and the traditional products. Wang and Zhao (146) studied how to determine the optimum order quantity and which situation the supplier should choose to reduce carbon emissions. Xie et al. (147) studied carbon emission reduction and sharing decision-making of supply chain system with the carbon emissions level dependent demand. Chen (148) proposed several important topics for future research based on a brief overview of the current research in the field of low-carbon supply chain management, including the impacts of CEA on carbon emissions reduction. Compared with the literatures in this scope, ourworknot only considers the CEA, but also considers the carbon trading price. Through coordination, supply chain can fully exploit its potential and improve its performance (149). There are many works in the field of supply chain coordination. Heydari and Norouzinasab (150) proposed a coordination mechanism to coordinate both pricing and ordering decisions simultaneously based on quantity discount. Chakraborty and Chatterjee (151) developed the economics of surcharge pricing as a supply chain coordinating mechanism under JIT environment.

The above-mentioned literatures all carry out in-depth studies on the supply chain carbon emissions reduction. They put forward many carbon emissions control methods and models, which are a great contribution to the field of carbon emissions reduction. Meanwhile it is also needed to be pointed out that there are three issues that should be explored in depth: (1) There are few literatures concerning the impacts of carbon trading price on the supply chain carbon emissions reduction. In fact, carbon trading price has important impact on the supply chain carbon emissions reduction and the carbon emissions allocation. (2)There are also few literatures that take both the carbon trading price andCEAinto account. It has become an important problem to study the impacts of carbon trading price on supply chain carbon emissions reduction while taking CEA into account. (3) It is necessary to further study the coordination mechanism in supply chain carbon emissions reduction. Under the constraint of carbon emissions, if the supply chain members try to optimize their own profits to reduce their carbon emissions, carbon emissions reduction of the whole supply chain will often not be the optimal; that is, there is a “marginal double” effect. So it is also needed to explore how to achieve the supply chain carbon emissions reduction coordination.

CHAPTER 3

Product pricing in a two-echelon supply chain with stochastic demand under carbon cap and trade regulations

3.1 Introduction

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In recent years global warming is a foremost hazard to our planet. It's having severe threat to the human health, and well-being and nature. It has many disastrous effects, such as rise of sea level, flood, drought, storm (44), disruption in ecosystems and increased clear-air turbulence (7). The world ecosystem, environment and human society facing the most severe problems of eco-environment deterioration and imbalance. The main reason behind this crisis is excessive emission of greenhouse gases. To control and reduce the emission of greenhouse gases effectively, many countries and regions (44; 35) have been proposed some legislations and regulations. By cap-and-trade mechanisms (74; 58) i.e., the emission trading scheme is one of the widely accepted mechanisms to make possible the enterprises for conservation of energy and reduction of emission of the greenhouse gases. Some countries already have taken remarkable measures to reduce the emission of green house gases. The Kyoto Protocol, in 1997, have issued by United Nations (UN), which get on a scheme to involved enterprises members' production process under the regulation of cap-and-trade (69). In 2005, European Union Trading Systems (EU-ETS) was initiated the biggest international scheme for the trading of greenhouse gasses. European Union, covering more than 31 counties which contribute almost half of the carbon emission, controls the emission of greenhouse gases through the implementation of a series of financial policies which includes CCA (Climate Change Agreements), CCT (Climate Change Tax) and CPS (Carbon Price Support)(56). Being a large scale manufacturing country China has taken continuous drives to reduce emission of greenhouse gases. At some provinces and cities like Beijing, Tianjin, the trading market of carbon emis-

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sion was framed, in 2013. The emission reduction and efficiency of energy saving is promoted by the carbon emission trading scheme. It is evident that, irrespective of developed or developing country, adapting the measures for reduction of carbon emission. Therefore, with the low-carbon economy becoming a global trend and national strategic behaviour. Use of the cap-and trade regulation encourage enterprises to raise the environmental performance which has the characteristics of external public interests. Therefore, being the primary carrier of greenhouse gases, enterprises have been faced a challenge, due to carbon emission limit and also put in many restrictions in the operations. Due to consumer awareness, they are inclined to the choice to purchase a low-carbon and environment friendly products. Under this situation, the enterprises focus on the technological up-gradation and improved operational management of enterprises to comply with cap-and-trade regulation (60). Also profit disagreement among the players (upstream and downstream) in supply chain is increasing seriously, since the technological up-gradation in the carbon emission mitigation incurred huge investment. Here we consider that the market demand is stochastic and also affected by selling price and carbon footprint with linear relationship.

3.2 Literature review

In the field of production–inventory literature, articles considering carbon policies are few but increasing rapidly. Dobos (84) introduced limit of carbon trading permits in the well-known Arrow–Karlin production–inventory model. Also, Dobos (38) further extended his earlier work considering the aspects of time-dependent unit tradable permit and calculated the immediate procurement cost of environmental licenses. The basic EOQ model with carbon emission mitigation investment, has extended by Toptal et. al. (14), under carbon cap, carbon tax and cap-and-trade policies. An optimal model for an SC to minimize opportunity cost considering carbon cap developed by Diabat and Simchi-Levi (79). A model for a green SC, developed by Abdallah et. al. (16), to determine lot sizes of production and shipment for raw materials and finished products assuming the constraints over carbon emission. Benjaafar et. al. (17) considered carbon tax, strict cap on emission, cap-and-trade and carbon-offset policies to determine optimal production, inventory, backorder quantity and amount of carbon traded to minimize the total SC cost for a single firm and also extended their models for multiple firms with or without coordination. Chen et al (41) depicted how with operational adjustments emissions can be reduced without significant increase in cost under different carbon policies. Wahab et. al. (29) incorporated screening and holding cost of defective items in their model taking the fixed and variable carbon costs in their model. Du et al. (20) developed a model using game theoretical approach for centralized and decentralized decisions of the manufacturer and the retailer in an SC where both the parties induce low-carbon efforts.

Mostly the papers in the field of production-inventory and supply chain management with carbon policies have considered deterministic demand. But in real life scenario, enterprises regularly experience stochastic demand due to shrinking product life cycles, seasonality, buying patterns of customers, and other relevant issues (31). Few research articles recently considered random or stochastic demand to reflect realistic aspects while considering different carbon policies. Rosic and Jammernegg (80) optimized a dual sourcing problem with carbon tax and carbon cap-and-trade policies considering stochastic demand in the newsboy environment. Dong et. al. (37) suggested a profit maximization multi-stage SC model considering carbon-cap-and-trade policy and stochastic demand in single-period planning horizon. Zhang and Xu (53) also considered stochastic demand at the time of developing a multi-item production planning model to maximize profit with carbon trading cost, and assumed newsvendor-type products while developed the model. Arikan and Jammernegg (36) suggested a single-period inventory model considering carbon footprint constraint and demand as a positive random variable.

The literature review reveals that the authors have considered stochastic demand with different carbon policies either in single-period or multi-period time horizon. To our knowledge, only a few authors in the literature considered carbon policies with random/stochastic demand in infinite planning horizon.

3.3 Problem definition

In this paper, we considered infinite planning horizon, stochastic demand in a two-echelon supply chain under cap-and-trade regulation, which consists of single manufacturer and many distributors. The manufacturer uses raw materials to produce product with a wholesale price and adopts emission reduction technologies to reduce carbon footprint. Meanwhile, the distributors buy the single type of product. The relationship between the manufacturer and distributors in supply chain can be considered as two-stage Stackelberg game, where the manufacturer and the distributors having same power or someone is leader and someone is follower. Under cap-and-trade mechanism, the government or regulatory body directs the related laws and regulations to impose on manufacturer during production process. Further, the manufacturer trades with other firms in carbon emission market, which has surplus or insufficient carbon emission quotas. Also we have considered that consumers have an environmental awareness to pay more buying interest for products with lower carbon footprint.

3.4 Notations and assumptions

3.4.1 Notations

In this paper we have used following notations to develop the model:

Parameters

- d_i Potential market demand for i^{th} distributor
- M_C Manufacturer's production cost for unit product
- e_0 Initial amount of carbon emission for unit product
- μ Cost coefficient in carbon emission mitigation
- E Total carbon emission quota allowed (carbon cap) by government
- P_e Unit trading price for carbon credit (carbon quota)
- ξ_i Sensitivity coefficient of consumer's awareness to carbon emission mitigation for i^{th} distributor
- a_i Sensitivity coefficient of consumer's preference to selling price for i^{th} distributor
- Π_j^i Profit in different scenarios, $i \in C, D$ refers to centralize and decentralize models, $j \in m, d, sc$ refers to manufacturer, distributor and supply chain.
- n Number of distributor
- x_d Percentage of minimal ensured or restricted profit of the distributors (for the existence of the business process), in decentralized model where manufacturer is leader
- x_m Percentage of minimal ensured or restricted profit of the manufacturer (for the existence of the business process), in decentralized model where distributor is leader

  Decision variables

  - P_d Distributors' base selling price for unit product

- e Carbon foot print (emission) for unit product after carbon emission mitigation
- w Manufacturer's wholesale price for unit product

3.4.2 Assumptions

(1) In this supply chain we have two players, namely, manufacturer, distributers. Here first we consider different competitive scenario of profit maximizing between manufacturer and distributors.
(2) Here we assume that d_i is a random variable following some distribution function with known mean and standard deviation (s.d.). Also referring to the related literature, such as, (60; 51) we assume that the market demand is affected by selling price and carbon foot print, which indicates the consumer's preference for selling price and carbon foot print (emission level) will generate a negative contribution effect on market demand. With out loss of generality the demand function for i^{th} distributor can be modelled as $D_i = d_i - a_i P_d - \xi_i e$.
(3) As in the existing literature (76; 81) the initial amount of carbon emission for unit product is e_0, is a known constant. After technological upgradation adopted by manufacturer the carbon foot print reduced from e_0 to e , where $0 < e \leq e_0$, for unit product. The cost function for such technological up-gradation is given by $C(e) = \frac{1}{2}\mu(e_0 - e)^2$.
(4) The carbon credit under the cap and trade mechanism i.e., the total cap of carbon emission E during production process is allowed by the government, which is called carbon credit (82; 30). Here we have assumed that after adopting carbon emission mitigation the carbon foot print is e. So, the total carbon emission is $e\sum_{i=1}^{n} D_i$. Thus the cost of surplus or deficit carbon credit is $P_e[E - e\sum_{i=1}^{n}(d_i - a_i P_d - \xi_i e)]$.
(5) In the real life scenario, the carbon emission mitigation investment results in diseconomy of scale. That is through the changes in products design and manufacturing process the mitigation of carbon emission will come easily and the mitigation of carbon emission is more difficult with diminishing profit.Therefor we assume that the coefficient for carbon emission mitigation μ should be large enough and the conditions

(a) $\frac{1}{2\sum_{i=1}^{n} a_i}(\sum_{i=1}^{n}(\xi_i + P_e a_i))^2 - \mu < 0$,

(b) $\frac{1}{4\sum_{i=1}^{n} a_i}(\sum_{i=1}^{n}(\xi_i + P_e a_i))^2 - \mu < 0$,

(c) $2P_e \sum_{i=1}^{n} \xi_i - \mu < 0$.

(d) $(\mu - P_e \sum_{i=1}^{n} \xi_i)(\sum_{i=1}^{n} \xi_i - \mu \sum_{i=1}^{n} a_i) > 0$.

3.5 Mathematical modelling of the system considering manufacturer and distributors

Here depending up on the dominating scenario of the manufacturer and the distributors, we derived two types of models namely centralized model and decentralized model.

3.5.1 Centralized model

In the centralized scenario, the model consider the manufacturer and the distributors of entire supply chain as one system, where the manufacturer and distributors jointly make the optimal decision with selling price P_d of distributors and carbon emission e to maximize the total profit Π_{sc}^C. Here we consider that on mutual understanding basis the total profit Π_{sc}^C is equally shared among the manufacturer and distributors. So, the model of the supply chain system under centralized scenario can be expressed as follows:

$Max\ \Pi_{sc}^C(P_d, e) = (P_d - M_C)\sum_{i=1}^n D_i + P_e(E - e\sum_{i=1}^n D_i) - C(e)$, i.e.,

$$\begin{aligned} Max\ \Pi_{sc}^C(P_d, e) = (P_d - M_C)\sum_{i=1}^n (d_i - a_iP_d - \xi_i e) \\ +P_e[E - e\sum_{i=1}^n (d_i - a_iP_d - \xi_i e)] \\ -\frac{1}{2}\mu(e_0 - e)^2 \end{aligned} \tag{3.1}$$

Proposition 1: In the model under centralized scenario the optimal selling price P_r and carbon emission e are as follows:

$$P_d^C = \frac{1}{2\sum_{i=1}^n a_i}[\sum_{i=1}^n (d_i + M_C a_i) - e\sum_{i=1}^n (\xi_i + P_e a_i)] \tag{3.2}$$

$$e^C = \frac{\sum_{i=1}^n (\xi_i - P_e a_i)\sum_{i=1}^n (d_i - M_C a_i) - 2\mu e_0 \sum_{i=1}^n a_i}{(\sum_{i=1}^n (\xi_i - P_e a_i))^2 - 2\mu\sum_{i=1}^n a_i} \tag{3.3}$$

with the condition

$$\frac{1}{2\sum_{i=1}^n a_i}(\sum_{i=1}^n (\xi_i + P_e a_i))^2 - \mu < 0 \tag{3.4}$$

Proof: Here first we find the optimal value of P_d and put that value in Π_{sc}^C and then find the optimal value of e as follows:

$\frac{\delta}{\delta P_d}\Pi_{sc}^c = \sum_{i=1}^n (d_i - a_iP_d - \xi_i e) + (P_d - M_C)(-\sum_{i=1}^n a_i) + eP_e\sum_{i=1}^n a_i$ and $\frac{\delta^2}{\delta P_d^2}\Pi_{sc}^c = -2\sum_{i=1}^n a_i < 0$.

Now, $\frac{\delta}{\delta P_d}\Pi_{sc}^c = 0$ gives, $P_d = \frac{1}{2\sum_{i=1}^n a_i}[\sum_{i=1}^n (d_i + M_C a_i) - e\sum_{i=1}^n (\xi_i - P_e a_i)]$.

Now putting P_d in Π_{sc}^c we get,

$\pi_{sc}^c = \frac{1}{4\sum_{i=1}^n a_i}[(\sum_{i=1}^n (d_i - e\xi_i - M_C a_i))^2 - (eP_e\sum_{i=1}^n a_i)^2] + P_e[E - \frac{1}{2}\sum_{i=1}^n (ed_i - e^2\xi_i - eM_C a_i - e^2P_e a_i)] - \frac{1}{2}\mu(e_0 - e)^2$

and $\frac{\delta}{\delta e}\Pi_{sc}^{c} = 0$ gives, $e = \frac{\sum_{i=1}^{n}(\xi_i - P_e a_i)\sum_{i=1}^{n}(d_i - M_C a_i) - 2\mu e_0 \sum_{i=1}^{n} a_i}{(\sum_{i=1}^{n}(\xi_i - P_e a_i))^2 - 2\mu \sum_{i=1}^{n} a_i}$.
Also, $\frac{\delta^2}{\delta e^2}\Pi_{sc}^{c} = \frac{1}{2\sum_{i=1}^{n} a_i}(\sum_{i=1}^{n}(\xi_i + P_e a_i))^2 - \mu < 0$ provided,
$\mu > (\sum_{i=1}^{n}(\xi_i + P_e a_i))^2$.

3.5.2 Decentralized model

In a supply chain it is not always possible to take centralized decision. Thus the decentralized scenario is unavoidable. Within the supply chain it is not always possible that manufacturer and distributors are of equal power. Some times, distributors (retailers) are more power full, like different shopping malls, some times manufacturer is more power full, like Microsoft. Then under decentralized scenarios, it is required to assume some one as leader and some one as follower based on the dominating nature of the players. So for the decentralized model each one is considered as leader and each one is considered as follower and optimize the supply chain cost to determine the best possible combination. Also we consider all distributors (retailers) are independent and have same power, i.e., either they together act as leader or as follower. So we classify the decentralized model as following two cases:

3.5.2.1 Case 1: Manufacturer is leader and distributors are follower

While manufacturer is leader and distributors are follower in the Stackelberg game of supply chain which maximize their own profits as the goal. Thus the game between members is as follows:
First the manufacture determines the wholesale price w and emission level e for unit product and then the distributors decides the optimal strategy of selling price P_d accordingly to the best response of manufacturer. We can easily find the dynamic game under complete information where subgame should be perfect Nash equilibrium. Also to protect the minimal profit of the distributors (for the existence of the business process) a minimum x_d percentage of profit is ensured or restricted. So the decision models of manufacturer and distributors can be expresses as follows:
$Max\,\Pi_m^D(w,e) = (w - M_C)\sum_{i=1}^{n} D_i + P_e(E - e\sum_{i=1}^{n} D_i) - C(e)$, i.e.,

$$\begin{aligned} Max\,\Pi_m^D(w,e) = (w - M_C)\sum_{i=1}^{n}(d_i - P_d a_i - e\xi_i) \\ +P_e[E - e\sum_{i=1}^{n}(d_i - P_d a_i - e\xi_i)] \\ -\frac{1}{2}\mu(e_0 - e)^2 \end{aligned} \tag{3.5}$$

and $Max\,\Pi_d^D(P_d) = (P_d - w)\sum_{i=1}^{n} D_i$ i.e.,

$$Max\,\Pi_d^D(P_d) = (P_d - w)\sum_{i=1}^{n}(d_i - P_d a_i - e\xi_i), \tag{3.6}$$

subject to the constraint,$P_d \geq (1 + \frac{x_d}{100})w$.

Proposition 2: In the model under decentralized scenario, where manufacturer is leader and distributors are follower, the optimal selling price P_d, wholesale price w and carbon emission e are as follows:

$P_d^D = \frac{1}{2\sum_{i=1}^n a_i}\sum_{i=1}^n (d_i + wa_i - e\xi_i)$

$w^D = \frac{1}{2\sum_{i=1}^n a_i}\sum_{i=1}^n [d_i + M_C a_i - e(\xi_i - P_e a_i)]$

$e^D = \frac{\sum_{i=1}^n (d_i - M_C a_i)\sum_{i=1}^n (\xi_i + P_e a_i) - 4\mu e_0 \sum_{i=1}^n a_i}{(\sum_{i=1}^n (P_e + \xi_i))^2 - 4\mu\sum_{i=1}^n a_i}$

with the condition

$\frac{1}{4\sum_{i=1}^n a_i}(\sum_{i=1}^n (\xi_i + P_e a_i))^2 - \mu < 0$

Proof: Here first we find the optimal value of P_d for the maximum value of $\Pi_d^D(P_d)$ and we substitute P_d in $\Pi_m^D(w, e)$. For maximum value of $\Pi_m^D(w, e)$ we find the optimal value of w and substitute it in $\Pi_m^D(w, e)$ and derive the reduced function $\Pi_m^D(e)$ to find the optimal value of e for maximum $\Pi_m^D(e)$ as follows:

Here, $\frac{\delta}{\delta P_d}\Pi_d^D = \sum_{i=1}^n (d_i - P_d a_i - e\xi_i) - (P_d - w)\sum_{i=1}^n a_i$

and $\frac{\delta}{\delta P_d}\Pi_d^D = -2\sum_{i=1}^n a_i < 0$.

Now $\frac{\delta}{\delta P_d}\Pi_d^D = 0$ gives, $P_d = \frac{1}{2\sum_{i=1}^n a_i}\sum_{i=1}^n (d_i + wa_i - e\xi_i)$.

3.5.2.2 Case 2: Distributors are leaders and Manufacturer is follower

While distributors are leaders and manufacturer is follower in the Stackelberg game of supply chain which maximizes their own profits as the goal. Thus the game between members is as follows: First the distributors determine the base selling price P_d and wholesale price w for unit product and then the manufacturer decides the emission level e for unit product. As before we can easily find dynamic game under complete information where subgame should be perfect Nash equilibrium and also to protect the minimal profit of the manufacturer (for the existence of the business process) a minimum x_m percentage of profit is ensured or restricted. So the decision models of manufacturer and retailers can be expresses as follows:

$Max\,\Pi_d^D(P_d, w) = (P_d - w)\sum_{i=1}^n D_i$ i.e.,

$$Max\,\Pi_d^D(P_d, w) = (P_d - w)\sum_{i=1}^n (d_i - P_d a_i - e\xi_i) \tag{3.7}$$

and $Max\,\Pi_m^D(e) = (w - M_C)\sum_{i=1}^n D_i + P_e(E - e\sum_{i=1}^n D_i) - C(e)$, i.e.,

$$\begin{aligned} Max\,\Pi_m^D(e) = (w - M_C)&\sum_{i=1}^n (d_i - P_d a_i - e\xi_i) \\ &+P_e[E - e\sum_{i=1}^n (d_i - P_d a_i - e\xi_i)] \\ &\qquad -\frac{1}{2}\mu(e_0 - e)^2, \end{aligned} \tag{3.8}$$

subject to the constraint,$w \geq (1 + \frac{x_m}{100})M_C$.

Proposition 3: In the model under decentralized scenario, where retailers are leaders and manufacturer is follower, the optimal selling price P_r, wholesale price w and carbon emission e are as follows:

$e^D = \frac{(w-M_C)\sum_{i=1}^n \xi_i - P_e \sum_{i=1}^n (P_r a_i - d_i) - \mu e_0}{2P_e \sum_{i=1}^n \xi_i - \mu}$

$w^D = \frac{\sum_{i=1}^n (d_i - a_i P_r)\sum_{i=1}^n (2P_e \xi_i - \mu) + (M_C + P_r)(\sum_{i=1}^n \xi_i)^2 + e_0 \mu \sum_{i=1}^n \xi_i}{2(\sum_{i=1}^n \xi_i)^2}$

$P_d^D = (-\mu \sum_{i=1}^n a_i \sum_{i=1}^n d_i(\mu - P_e \sum_{i=1}^n \xi_i)) + \sum_{i=1}^n a_i \sum_{i=1}^n \xi_i(e_0\mu + \sum_{i=1}^n d_i P_e + M_C \sum_{i=1}^n \xi)(\mu - P_e \sum_{i=1}^n \xi_i) + \mu \sum_{i=1}^n d_i(-\mu \sum_{i=1}^n a_i + \sum_{i=1}^n \xi_i)(\sum_{i=1}^n a_i P_e + \sum_{i=1}^n \xi_i) - \sum_{i=1}^n \xi_i(e_0\mu + \sum_{i=1}^n d_i P_e - \sum_{i=1}^n \xi_i(w + M_C))$

$- \frac{-\mu \sum_{i=1}^n a_i + \sum_{i=1}^n \xi_i(\sum_{i=1}^n a_i P_e + \sum_{i=1}^n \xi_i)}{2\sum_{i=1}^n a_i(\mu - P_e \sum_{i=}^n \xi_i)(-\mu \sum_{i=1}^n a_i + \sum_{i=1}^n \xi_i(\sum_{i=1}^n a_i P_e + \sum_{i=1}^n \xi_i))}$

with the condition,

$2P_e \sum_{i=1}^n \xi_i - \mu < 0$ and

$(\mu - P_e \sum_{i=1}^n \xi_i)(\sum_{i=1}^n \xi_i - \mu \sum_{i=1}^n a_i) > 0.$

Proof: Here first we find the optimal value of P_d and w taking $\frac{\delta}{\delta P_d}\Pi_d^D = 0$ and $\frac{\delta}{\delta w}\Pi_d^D = 0$ for the maximum value of $\Pi_d^D(P_d, w)$. We put the optimal values of P_d and w in Π_m^D, which will become a function of e only. Then we find the optimal value of e taking $\frac{\delta}{\delta e}\Pi_m^D(e) = 0$ for which $\Pi_m^D(e)$ is minimum.

3.6 Numerical example

In this section, we present some numerical examples to demonstrate the impact of consumers' awareness and initial carbon emission on the performance and social welfare, as well as present the performances of coordination contract under centralized scenario. Also we consider the both decentralized scenarios with the same input data and compare all the evaluated results. We consider that d_i follows uniform distribution with known parameters and consider the specific values for different parameters as follows (all data are with usual unit): $n = 3, M_C = 2, e_0 = 2.5, \mu = 5, E = 5, P_e 0.5, \xi_1 = 0.2, \xi_2 = 0.2, \xi_3 = 0.1, a_1 = 0.2, a_2 = 0.3, a_3 = 0.1, d_i \sim Uniform(m_i, n_i), m_1 = 5, n_1 = 3, m_2 = 5, n_2 = 2, m_3 = 4, n_3 = 3$ and for decentralized models $x_m = 30$ and $x_d = 30$. We replace d_i by the mean of d_i i.e., by $E[d_i] = \frac{m_i + n_i}{2}$.

We have solved the models using Mathematica 7.0, using the above input data. In Table 1, the obtained results are given.

Case Type	Manufacturer's total profit	Distributors' total profit	Manufacturer's Wholesale Price /unit	Distributors' base selling price/ unit	Emission level/unit after mitigation
Centralized (Manufacturer and distributors having same power)	15.44	15.44	NA	9.94	1.34
Decentralized (Manufacturer is leader and distributors are follower)	15.89	7.07	9.84	13.27	1.95
Decentralized (Distributors are leaders and Manufacturer is follower)	2.58	29.26	2.60	9.40	2.12

Table 3.1 Result analysis

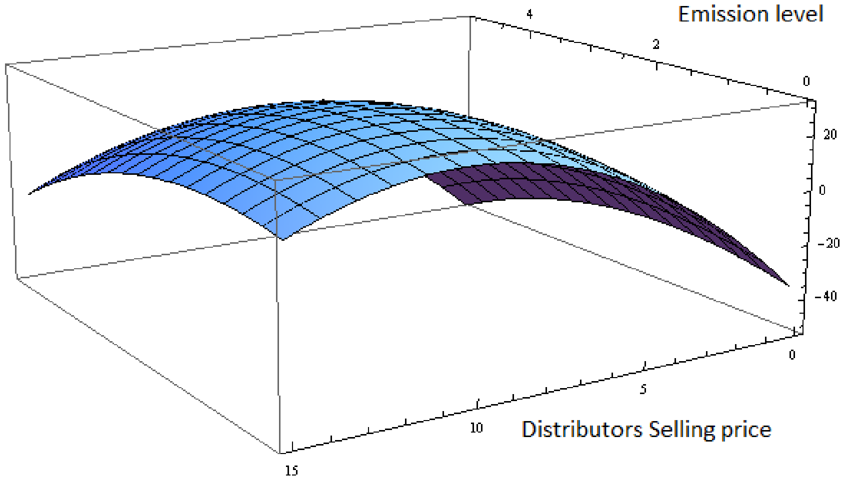

Fig. 3.1 Distributors Selling price Vs Emission level in Centralized Model

3.7 Result analysis and conclusion

In the centralized model, when manufacturer and distributors having same power, the total profit is distributed equally within manufacturer and distributors, where distributers directly sell the products. We can observe in Figure 1, that the selling price of distributors increases as the carbon emission level decreases, which results in over all maximum profit shared among manufacturer and distributors.

In the decentralized model, when manufacturer is leader and distributors are follower, the emission level after mitigation is higher than that of centralized model. Also, as the distributors base selling price increases and affect the total sell, which results in remarkably less profit of the distributors. In the Figure 2, we also observed that,as the emission level decreases the manufacturer's wholesale price decreases.

Again, in the decentralized model, when distributors are leaders and manufacturer is follower, the emission level after mitigation is highest than that of all the models. It indicates that being the leader under decentralized scenario, the distributors have not taken care of about carbon emission mitigation. Also the manufacturer's profit decreased remarkably and the distributors profit increased unnaturally. In the Figure 3, the distributors dominated the manufacturer by keeping the manufacturer's wholesale Price as less as possible and hiking the distributors base selling price for more profit making, ignoring the carbon emission effects.

Thus, it has been found that the centralized model worked best under the coordination system. In centralized model the distributors selling price is

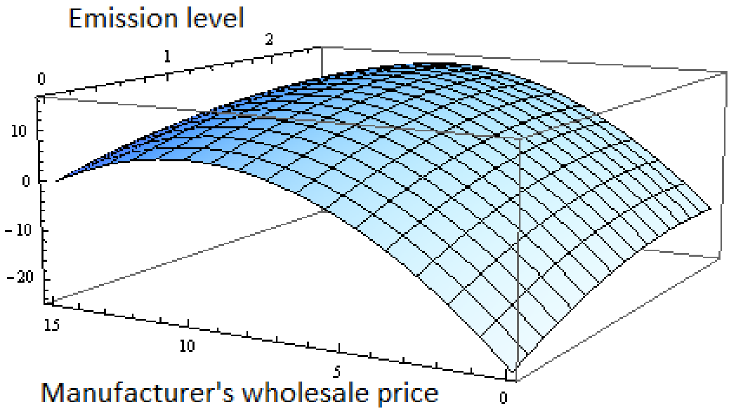

Fig. 3.2 Manufacturer's wholesale price Vs Emission level in Decentralized Model(when manufacturer is leader and distributors are follower)

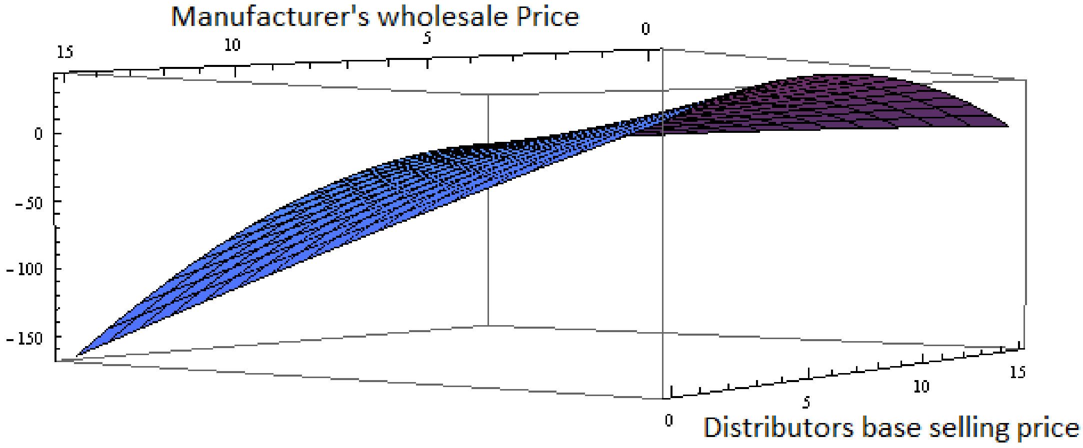

Fig. 3.3 Distributors base selling price Vs manufacturer's wholesale Price in Decentralized Model(when distributors are leaders and manufacturer is follower)

under control and also the emission level after mitigation is the least due to joint contribution of both manufacturer and distributors.

This model can be further be extended to three-echelon supply chain model by introducing the retailers with auction concept as buyers of the distributors.

CHAPTER 4

Development of WARKS for Accessing Supply-chain management

4.1 Introduction

[0]

Wireless devices have grown tremendously within recent few years due to the appearance of global adoption, various functionality and wide applications. Future WiFi probably dedicate broadband speeds. Wireless access is consider to both indoor and outdoor environments. For high quality of service (QoS), WiFi enabled devices (i.e., smartphones, tablets, laptops, smart TVs, cameras, sensors etc.) improve user experience and attract them to this technologies. The communication technologies currently used in D2D (device-to-device) communications, to interconnect multiple sensor nodes spread into a particular area. Access to communication technology can play a pivotal role in social and economic development. The choice of technology to achieve this a significant aspect (157). If the network has to cover a larger area than router is not capable of transmitting to, or if signals have to penetrate through obstacles, performance will take a hit. Interference is also a big issue, signals from other wireless networks and electronics can impact speeds.

The goal of this work is to analyse WiFi feasibility and evaluate its performance and applications in different fields by using a WiFi network with sensors, kits and multimedia support. This work improve D2D communications in a significant way, appointing intermediate multi hop WiFi nodes with different network topology. The objective of this work to retain the WiFi communications without boosting the signals for both static and dy-

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namic cases (maintaining the order and range of WiFi nodes). For static purpose this network is usable in house, colony, factory, hospital, office, hostel, restaurant, forest etc. Also for dynamic purpose this network is usable in agriculture, animal monitoring, mountain, train, army and rivers etc. Authors intention to establish a online/offline data processing enable network for communication (158).

WiFi deals with the specification of an unlicensed bands worldwide use in wireless local area network supporting a set of scenarios based on number of devices, range, and energy constraints. WiFi offers a simple, robust, and efficient solution in the industrial, scientific, and medical radio band (ISM band) compared with other existing technologies. WiFi technology enables devices to exchange information and perform actions without human intervention. Due to their short wireless range and high obstruction losses, current WiFi require the use of intermediate nodes, to reduce complexity of the network.

This paper upsurge our experiences, should create a revolution in offline WiFi network and keep the impression at peoples mind (159). In the following sections we describe four different perspective relevant to the WiFi networks,

- Introduction
- Network Models
- Network Applications
- Conclusion and Future Plan
- Acknowledgment
- Reference

This network will be adoptable for future communication technologies to establish a convenient environment.

4.2 Network Models

Authors intention to construct a network which can be use in different field and for different purpose with various functionality. Which produce a secure, reliable, energy shaven, portable and simple local area network, that favor in both cellular and ad hoc compositions.

4.2.0.1 Concepts

Multiple walkie talkies use a single radio channel, and only one radio on the channel can transmit at a time, and others walkie talkies receive it. The goal of authors is to build a network, which can work as both cellular and ad hoc, with multi hop connection. Which can use in home for static network and in outdoor dynamic network for natural disaster. By walkie talkie the voice data can transfer, but this WiFi network must include various sensors, kits and enable multimedia support. So the concepts are increase the capability of network with different functions. Such that it is possible to form different topology by this network.

4.2.0.2 Opportunities

This technique implement for various field of applications. By this network it is possible multi hop nodes communication. If any person control a device/machine of 7^{th} node with mobile connection from 1^{st} node Fig. 4.1, then it is also possible to monitor another device/machine of 4^{th} node with mobile connection from 8^{th} node, and so on simultaneously. In this way multiple device can be control by one clients. WiFi devices are easily portable, so they are use in both static and dynamic purpose. This technique increase the range and the number of connections. Also both short and long range WiFi connection may use to provide good services.

4.2.0.3 Hardware

In this network an AP, a STA, a Controller, various sensors and kits have been used to model the WiFi network. AP and STA are connected each other with wire connections, STA/Master connected to a Controller and a display with wire. At each nodes different type of sensors and kits have used, which are connected with wire to Controller. AP, STA and Controller are connected with power bank by USB (Universal Serial Bus). In each node same type of WiFi device have been used for AP and STA, AP

Table 4.1 comparison with existing technologies

Technology	WiFi	Bluetooth LE	ZigBee	WiMAX	LoRa	LTE	4G
Standard	IEEE 802.11	IEEE 802.15.1	IEEE 802.15.4a	IEEE 802.16a	IEEE 802.15.4g	IEEE 802.16	IEEE 802.16m
Frequency	2.4 GHz, 5 GHz	2.4 GHz	2.4 GHz, 868 MHz, 915 MHz	2.5−5.8 GHz	433, 868, 780, 915 MHz	824−1990 MHz	1.8−2.5 GHz
Bandwidth	20−40 MHz	1 MHz	1 MHz	1.25−20 MHz	125−500 KHz	1.4 MHz	5−20 MHz
Range	10−150 m	10−50 m	10−100 m	5−30 km	2−15 km	10−30 km	10−30 km
Protocol	CSMA/CA	BR/EDR	CSMA	CSMA/CA	ALOHA	TCP/IP	TCP/IP
Modulation	BPSK, QPSK, QAM	GFSK	BPSK, QPSK	CSK	CSS	QPSK, QAM	PSK, QAM
Data Rate	54 Mb/s, 6.75 Gb/s	1 Mb/s	20−250 Kb/s	30−40 Mb/s	50 Kb/s	50−100 Mb/s	50−100 Mb/s
Network Topology	Star	Star, P2P	Star, Tree, Mesh	Star, Tree, Mesh	Star	Cellular	Cellular
Band	ISM	ISM	Wireless	Wireless	Wireless	Wireless	Wireless
Duplex	Half	Half	Half	Full	Half	Full	Full
Spectrum	Unlicense	Unlicense	Unlicense	License	Unlicense	License	License
Transmission Technique	OFDM	FHSS	DSSS	CDMA	OFDM	OFDMA	OFDMA
Latency	3 ms	6 ms	20 ms	5 ms	2 ms	2 ms	2 ms
Power Use	6 W	0.01−0.5 W	Low	High	Low	High	High
Battery Life	Months to Years	Days to Weeks	Months to Years	Months to Years	Months to Years	Days to Weeks	Days to Weeks
Transmit Power	1−100 mW	1−10 mW	1−100 mW	200−20000 mW	10−2000 mW	200−20000 mW	200−20000 mW
Sensitivity	-95 dBm	-97 dBm	-100 dBm	-100 dBm	-149 dBm	-120 dBm	-122 dBm
Packet Length	100 bytes	200 bytes	100 bytes	100 bytes	200 bytes	1200 bytes	1240 bytes
Security	WEP, WPA, WPS	128 bit AES with counter mode CBC MAC	Low	Low	Low	Low	Low
Module Price	$10	$10	$40	$50	$60	$70	$80

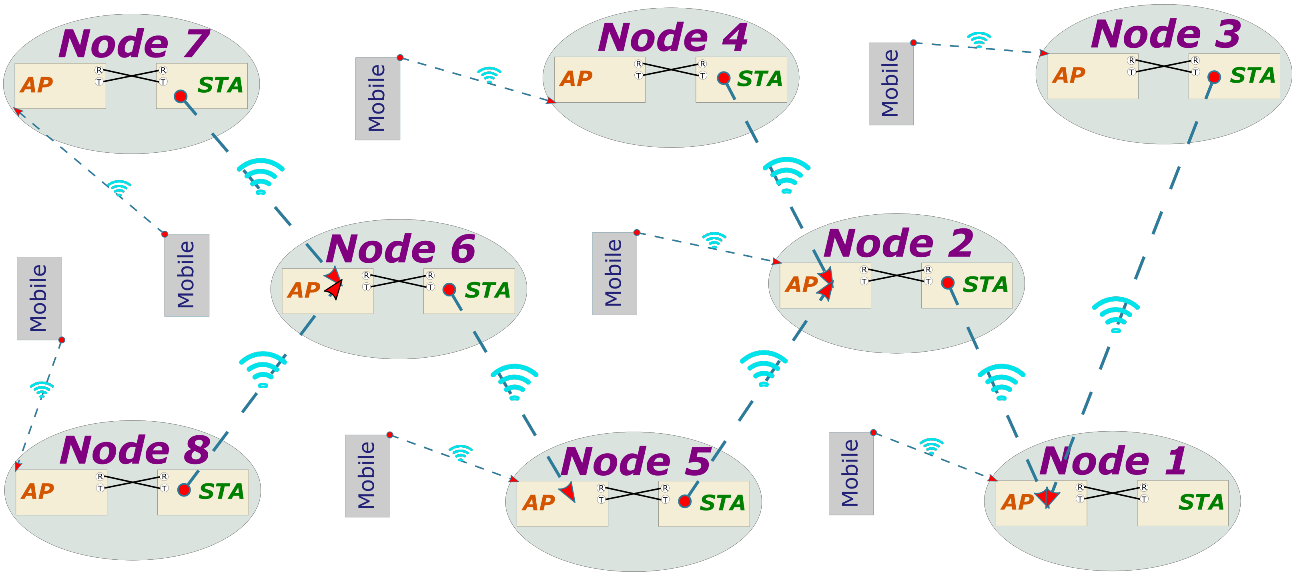


Fig. 4.1 WiFi Communications using WARKS

works as Server and STA as Client.

4.2.0.4 Duplex

Four type of half-duplex protocol have been used in this network. WiFi communication between AP and STA is half-duplex, UART communication between AP and STA is half-duplex, I2C communication between STA and Controller is half-duplex and HTTP communication between AP and Client (mobile) is also half-duplex. This means no device can send and receive, or upload and download, simultaneously.

4.2.0.5 Mapping

By mathematics, since ‘STAs’ and ‘APs’ are two sets and by some given rule, element of STAs corresponds to a unique element of APs, so the rule is called a mapping of ‘STAs’ into ‘APs’. Here ‘STAs/ Clients’ set include mobile, laptop, wireless sensors etc. and ‘APs’ set include all Servers/APs in the network. Mapping are two types “one one” and “many one”. Since many STAs can connect with one Server therefore the map are many one mapping. And STAs/Clients connected to a Server/AP of a node with wire connection is a many one mapping, Similarly Controllers connected to a STA with wire in this node also a many one mapping, Sensors/Kits are

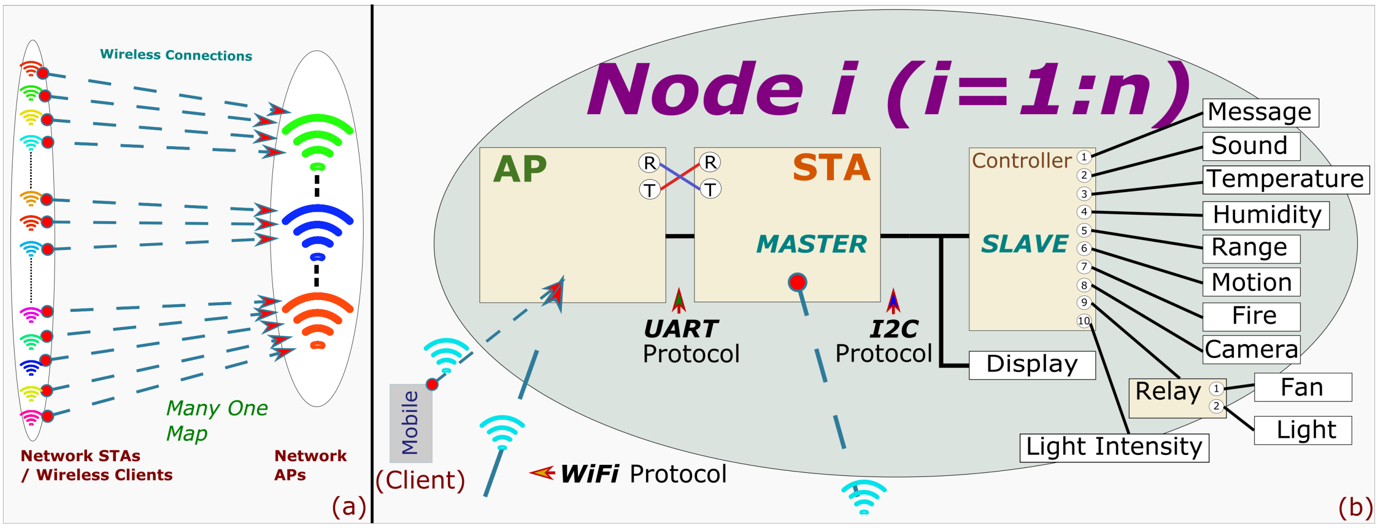


Fig. 4.2 (a) STA/Clients map into APs/Servers, (b) Hardware design of ith Node with communication protocol and sensors/kits of the Network

connected to a Controller with wire is also form a many one mapping.

4.2.0.6 Topology

The topology of this network can form tree, star, bus, line, chain or combination of them. But that nodes of this network should not form ring or mesh topology. Because the data will forward from one node to all others nodes. Since each node forward data to all others nodes, which nodes are connected to it. Therefore if some nodes form a ring or mesh topology of this network then the data will turn around in, which happen continuously and will hang this network. For n nodes network, total $2n$ numbers of AP and STA are need.

4.2.0.7 Security

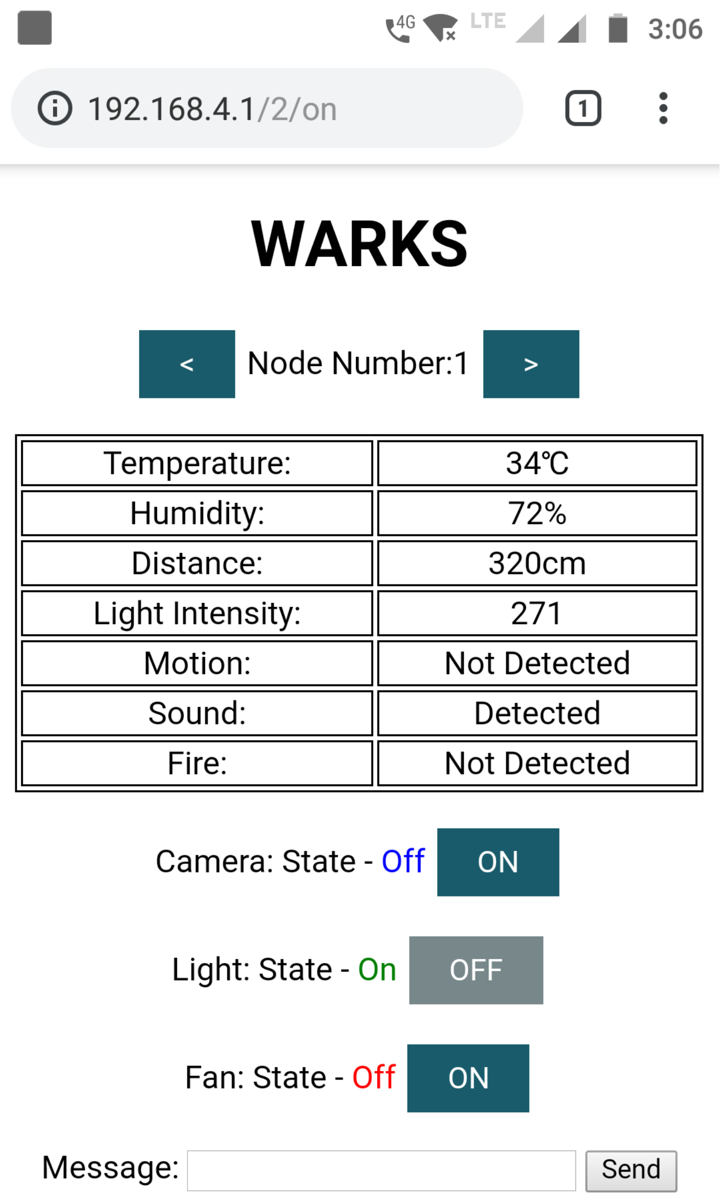
For establish a connection to a node of this network by mobile or computer, peoples should go through some authentication password. After network authentication, users also should go through Second password for access specific node of this network. A web server has been design for receive/request instruction of specific node of this network from mobile/computer. By this web server peoples can control or monitor specific node's relay, sensors, devices and also can do, multimedia communications online/offline.

4.2.0.8 Experiment

The experiment has done in the Hostel and in the Department of Jadavpur University. Which cover different floors of building with distributed rooms for indoor purpose. This experiment also has done in the playground of University for outdoor purpose, where each node placed at 20 meters distance from others. All external clients (mobile) $Fig.4.1$ are connect to the AP/Server of a node of this network for monitoring or controlling any sensor/kits of other nodes. AP of any node receive and request the instruction from clients and forward to all STA and AP (both wire and wireless) by these Serial and WiFi connections. Total eight nodes are used, each node have one AP, one STA and one Controller. STA work like a Client, one display and one controller are connected to it. Controller works like a Slave of STA/Master. Various sensors/kits are connected to the Slave by wire. Seven mobile are used as an external clients. Both command and request are done simultaneously. Message of any client flash to the screen of display. In case of connection lost of any node to this network, this also flash to all nodes display that which node lost the connection. The connection speed was approximately 112kbps for using seven mobiles to receive/request instructions of eight nodes network. The receive/request instruction also work if maximum three STA/Clients connected to one AP and access to other different nodes.

4.2.0.9 Challenges

For static/dynamic situation, if some nodes lost connection, then it automatically reconnect to its previous node, that it was connected to in this network. Since STA can connect only one AP and many station can connected to a AP $Fig.4.1$. If some cases, connection is not define (i.e STA can randomly select an AP). In this type of cases if connection lost, STA of any node search random AP within its range. If it get another node (not previously connected node) and connect it's AP. Then the network may form two or more disconnected sub network with different topology. Same node STA and AP can connect each other. This is a great challenge to this network. Since one STA can connect only one AP wirelessly therefore the whole system will suffer for it. So for random connection, should make a manual connection configuration.



WARKS

Node Number:1

Temperature:	34℃
Humidity:	72%
Distance:	320cm
Light Intensity:	271
Motion:	Not Detected
Sound:	Detected
Fire:	Not Detected

Camera: State - Off

Light: State - On

Fan: State - Off

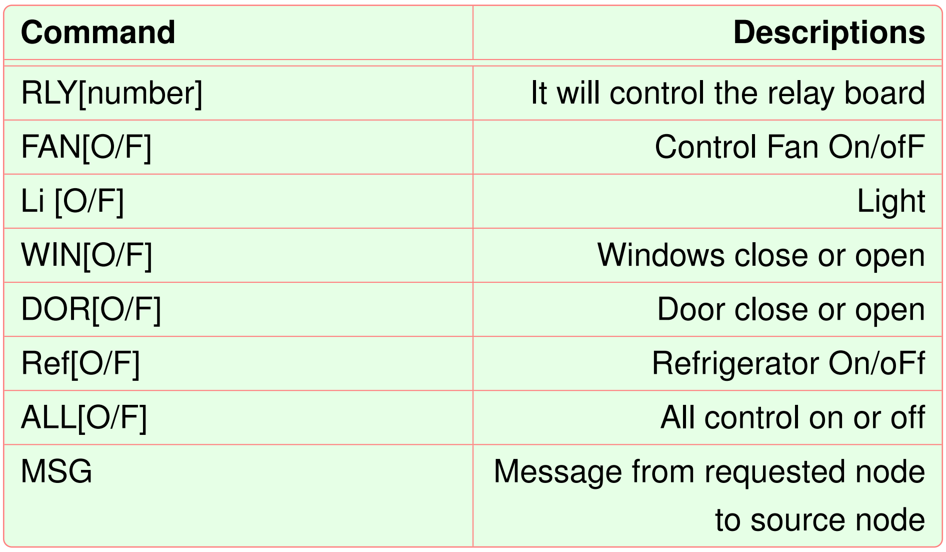
Message:

Fig. 4.3 WARKS user interface

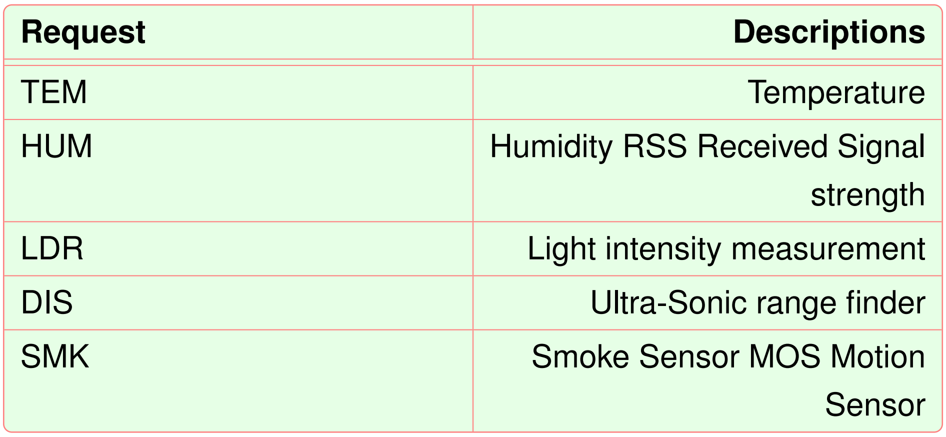
String Processing					
DNID	SNID	CMD	REQ	MSG	TERM
16 bits	16 bits	32 bits	32 bits	N bits	8 bits

Table 4.2 Comparison of different board we used for this system

Specs/Board	ESP32	ESP8266	Arduino Uno
Number of Cores	2	1	1
Architecture	32 Bit	32 Bit	8 Bit
CPU Frequency	160 MHz	80 MHz	16 MHz
WiFi	Yes	Yes	No
Bluetooth	Yes	No	No
RAM	512 Kb	160 Kb	2 Kb
Flash	16 Mb	16 Mb	32 Mb
GPIO Pins	36	17	14
Busses	SPI, I2C, UART, I2S, CAN	SPI, I2C, UART, I2S	SPI, I2C, UART
ADC Pines	18	1	6
DAC Pines	2	0	0



Command	Descriptions
RLY[number]	It will control the relay board
FAN[O/F]	Control Fan On/ofF
Li [O/F]	Light
WIN[O/F]	Windows close or open
DOR[O/F]	Door close or open
Ref[O/F]	Refrigerator On/oFf
ALL[O/F]	All control on or off
MSG	Message from requested node to source node



Request	Descriptions
TEM	Temperature
HUM	Humidity RSS Received Signal strength
LDR	Light intensity measurement
DIS	Ultra-Sonic range finder
SMK	Smoke Sensor MOS Motion Sensor

4.3 Network Applications

Several scope of applications (160) of this network are present in multiple fields $Fig.4.3$, some of them are describe as follows,

4.3.1 Industry

Automatic control has played a vital role in the advance of engineering and science and it has become an important and integral part of modern manufacturing and industrial processes. It makes the process easier and time shaven. One of the main ingredients of automation is undoubtedly control, which means information being collected, processed and delivered back to each sensors. Workers of a factory can easily control various machine with their mobile, using WiFi connections. Also they can receive real time temperature, humidity, gas, radio frequency, fire, smoke, alarm etc., to their mobile app. Moving robots or vehicles can be control. Workers can get access to this network by connecting moving robots or vehicles, they act as multi hop nodes of WiFi network. From first WiFi connection from mobile, workers can control a robot and by second connection from mobile to robot's node (this time robot's node work as multi hop node) can monitor/control another device or machine. All data will store to the servers through multi hop connections.

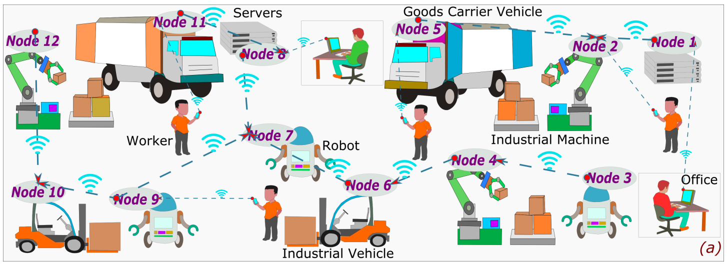


Fig. 4.4 Applications in Industry

4.3.2 Train

WiFi network connections enable operators to manage carriage to carriage and train to ground communications with increased efficiency and create attractive onboard multimedia services that give passengers safe and environmentally friendly transportation. At each carriage of a train should have some WiFi nodes which have some sensors. Passengers can access this network using mobile by connecting to a single node of the network.

They can receive information about temperature, humidity, fire, smoke, alcohol etc., can access movie, music to their mobile from the network servers. Passenger can do, chat to other coaches passengers, order foods, complaints, take necessary helps from other passengers or securities. This network should distributed among authority, security, kitchen and coaches. Train authority can control air conditioner, doors, send massage, alarms to passengers. The authority can collect all information to server by this multi hop nodes. In this way restaurants, shopping mall can provide services to their customers.

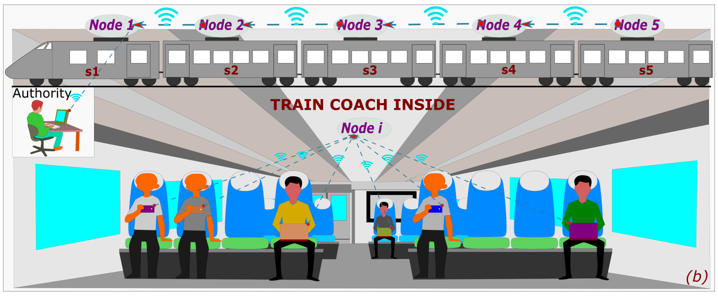


Fig. 4.5 Applications in Train

4.3.3 Hospital

In a hospital constant health care monitoring system is necessary for the patient's physiological parameters. In case of emergency patient should need medical treatment or shifted to other hospital. For this purpose a communication network need among patient, doctors, patient relatives, nurses, inquiry, pharmacy, ambulances and offices. This WiFi network full fill all functionality and take care about hospital emergency management. Patients are placed in different ward, it is difficult to look after all patients if lack of doctors or nurses in hospital. So by WiFi sensors, camera and various devices, it is easy to monitoring to all patients physical conditions from different places with mobile or computer by this network. Using real time data this system will be faster.

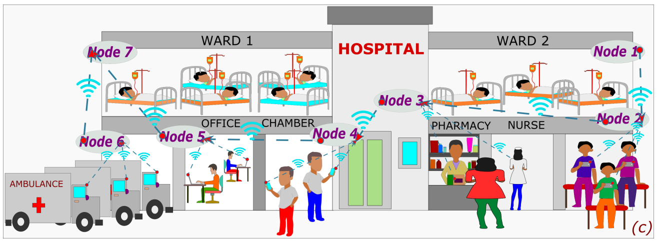


Fig. 4.6 Applications in Hospital

4.3.4 Agriculture

WiFi reduce effort and time for monitoring agricultural environment (161). The utilization of WiFi technology allow for remote measurement of temperature, humidity, crop condition, atmospheric pressure, soil moisture, water level and insects detection, observation etc. The WiFi system will reduce the cost. Sensor location can easily repositioning. Also enhance the flexibility and mobility of sensing points. Greenhouse farming growth, germination, sprouting, flowering and fruit development monitoring with WiFi reduce the labor of farmers. This proposed idea introduce together a controlling and monitoring system which activate or deactivate automatic irrigation, reaper, seed planter and agricultural vehicles of an agricultural fields with short and long range multi hop WiFi nodes. A farmer can fly a drone by mobile, together with drone node WiFi (i.e. three hop connection) the farmer can communicate with other farmers. Any animals, birds or human activity to the crop field can easily identify from any corner or share data to each other by multi hop WiFi network. Dairy farming easily look after with deploying various WiFi sensors. For short of connection farmers can access to network with moving agricultural vehicles. Fishermen can communicate each other by this WiFi network from fishing boats.

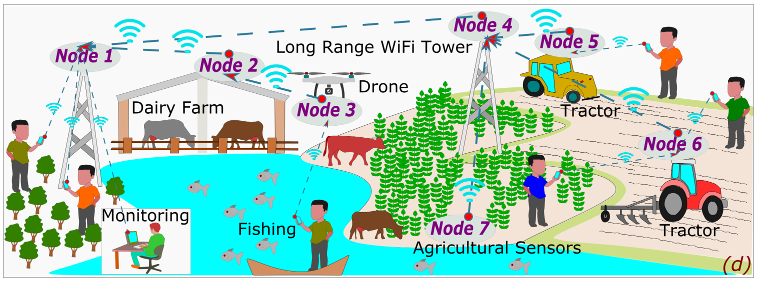


Fig. 4.7 Applications in Agriculture

4.3.5 Mountain

Mountaineering is the set of activities which involves ascending mountains. Mountaineering related activities include traditional outdoor climbing (rock and ice), hiking, backpacking, skiing, and traversing via ferratas. Mountaineer are travel long distance for climb up to horn of the mountain. Sometime they leave behind their accompany or needs to medical helps. As there is no network coverage, so it is impossible to communicate each other. For this purpose they should need both short and long range WiFi communication networks. Long range WiFi (162) are deploy like mobile tower (base station) and short range WiFi communicate among the group of mountaineer. If any group want to communicate another group to whom they leave behind, simply connect to long range WiFi network, by mobile WiFi and can communicate with each other. In case of medical emergency, can get help from administrator or rescue camp by connecting to long range WiFi network. Administrator or mountaineer can share any problem with others by connecting one node of this network such that rest of group can avoid this difficulties. Since each and everyone are connected to the long range WiFi network. So by RSSI values (163) one group can find other groups node location to the mountain and how far away this group traveling from a WiFi tower can be identify. In the same way, this technology can be use in desert.

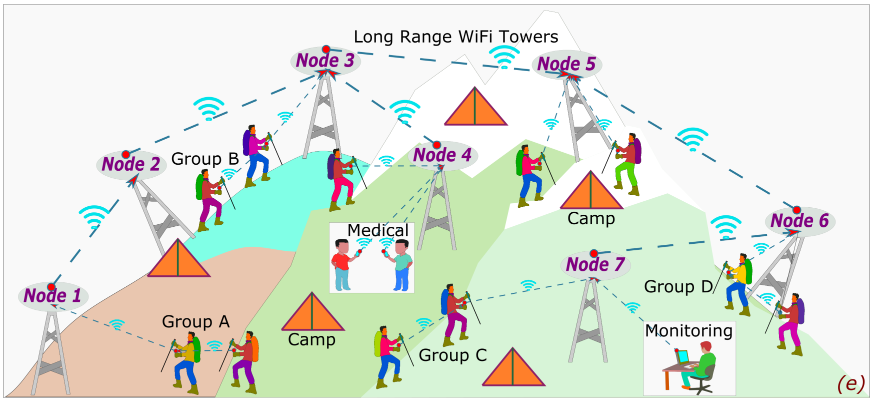


Fig. 4.8 Applications in Mountain

4.3.6 Forest

The forest is a complex ecosystem consisting mainly of trees that buffer the earth and support a myriad of life form. Wild animals graze in forest, sometime herd of several wild animals or birds migrate together one forest to another. They are need monitoring from different place of forest. Also the forest needs security from poaching, forest fire etc. To get away from this problem this WiFi network is a good solution. Each node of this WiFi network should have camera, fire detector, motion sensors, sound detector etc., and will be a combination of short and long range WiFi network. By this network, animal, birds or any unauthorized human activity can easily track for taking necessary steps.

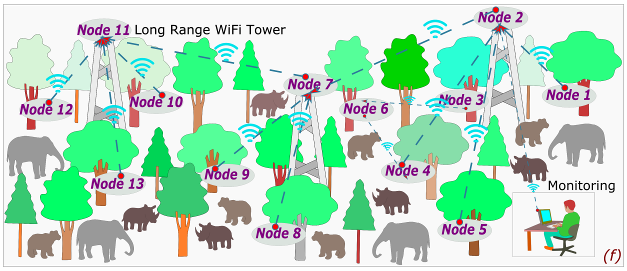


Fig. 4.9 Applications in Forest

4.3.7 Army

Military automation has changed the face of battle, and WiFi devices have played a key role. The air force, the army and the navy all get benefit and improve their function. Army communicate each other for battlefield operations. It may be take place in forest, hill, sea, urban or rural areas. Army can communicate among them, using drone or vehicles to aerial vehicles by WiFi network. Even inside the army camp this technology may use. In case of natural disaster, i.e. earthquake, flood, fire etc. this technology will use. The advantage of this network is, it can use dynamic, static and both together.

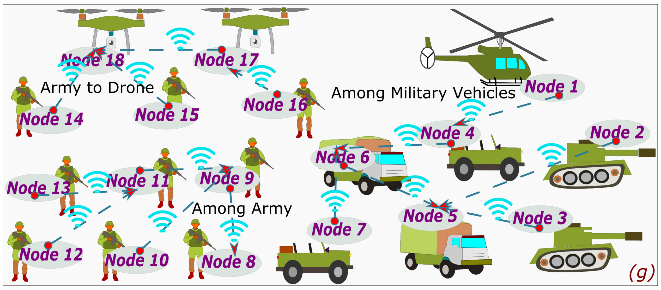


Fig. 4.10 Applications in Army

4.3.8 Home

A home automation system will control TV, light, fan, door, window, climate, entertainment etc (164). It may also include home security, such as alarm, motion, gas, fire, smoke, baby care systems. All of this sensor devices can control and monitor connecting with one node by mobile, present in any place of network coverage area. Use this multi hop WiFi connection people can communicate among multi floors with multi buildings. Any person can send message to any people or a group of people by this WiFi networks. Also can control their cars from multi floor building using this network. The same technique may use in office buildings.

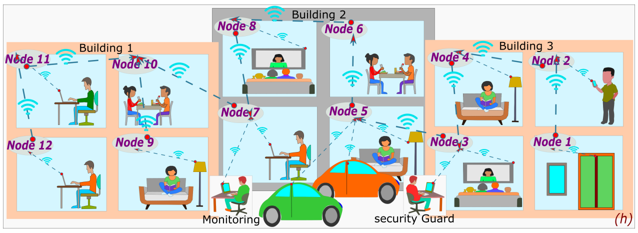


Fig. 4.11 Applications in Home

4.4 Conclusion and future plan

This work is done to make the living environment easier for controlling and monitoring remotely using mobile or computer. The proposed system can be used in agriculture, home automation, industrial automation, smart cities, army, fishing, mountain tracking etc. Distance can be find using RSSI technique (171; 165; 166; 167; 168; 173). In this model all the AP has variant SSID but unique IP address and port number. Making unique SSID for all AP and variant IP address is under process.

Our future plan is to implement this system into swarm robotics to create dynamic formation using wireless communication to solve the complex problem which can be finding shortest path of path following robot (159; 169; 170; 172), rescue people from danger, battle field fighting technique and many more. To improve the hardwares, we have to use the LiFi or future technology to create a super speed wireless data transfer. To improve softwares, we plan to develop a new programming language to configure required wireless network hardwares easily.

CHAPTER 5

Mathematical models for optimization of economic and environmental effects in a two-echelon supply chain management system with stochastic deterioration, seasonal stochastic demand and carbon emission cost

5.1 Introduction

[0]

In the supply chain(s) (SC), the devastating environmental impacts have resulted in government law formulation, customer awareness and pressure from various stakeholders to implement environmentally sustainable strategies in SC. A sustainable SC requires management policies that minimize the SC cost and reduce consumption of resources and environmental pollution. Therefore carbon emission cost (28; 73; 77; 83; 64; 59) has to be incorporated to develop any new SC model. In real life, it is quite common for different inventory items, such as milk, fruits, blood, pharmaceutical products, vegetables etc, to perish or depreciate over time. Therefore, it is important to study the behaviour of such decaying and deteriorating items (44), toward the formulation of appropriate inventory control policies that explicitly take such behaviour into account. The deterioration rate is different among various products and depends on several physical and environmental situations. Thus, the deterioration rate is stochastic (44; 35; 74) in nature, rather than deterministic. This feature has to be considered when modelling the real life SC scenario. In reality the demand of any product has a seasonal effect. Thus, in SC the demand has to be considered in two factors: one part is constant and another is stochastic. In our proposed model we have considered the above features to develop a realistic model that can be adopted by different SC managers and practi-

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[0] This paper communicated to the journal Operational Research - An International Journal (ORIJ) , Electronic ISSN: 1866-1505

tioners for achieving utmost economic goals and minimum environmental impacts i.e., carbon footprints.
The inventory system is taking an important part of cost controlling in business and organization. Inventory consists of usable but idle resources which are materials and goods. The volume of material, a company has in stock at a specific time, is known as inventory or in terms of money it can be defined as the total capital investment over all the materials stocked in the company at any specific time. Inventory may be in the form of raw material inventory, in process inventory, finished goods inventory, among others. Inventory management is a key component in any production environment. This has been recognized not only in the operations research literature but also in the management science and industrial engineering domains. Inventory Control (IC) is the supervision of supply, storage and accessibility of items to ensure an adequate supply without excessive oversupply. The integrated inventory management system is a common practice in the global markets and provides economic advantages for both the vendor and the buyer. Globalization of market and increased competition force organizations to rely on effective supply chains to improve their overall performance. The goal in many research and development efforts that are related to the SC management (SCM) is to have practical models that can effectively reduce operational costs.

Transportation has a major role in SC management. The ever growing volume of such activity not only benefits the growth and sustainability of international economy and globalization but also has its own consequences, particularly those pertaining to the environment. Transportation activities are considerable sources of air pollution and greenhouse gas emissions, with the former known to have harmful effects on human health and the latter is responsible for global warming. These issues have raised concerns on reducing the amount of emissions worldwide. In this respect, many countries, including both developed countries and developing countries, have set strict targets on reducing their carbon emissions now and in future. There is a large amount of literature on supply chain management regarding environmental issues through the emerging concept “green supply chain management” (GrSCM) (72). In the carbon emissions reduction literatures, the decision problem of carbon emissions reduction in supply chain is a hot topic. Benjaafar et. al. (32) illustrated how carbon emission concerns could be integrated into operational decision making and analyzed the impact of operational decisions on carbon emissions.

Deterioration inventory model research has received increasing attentions in recent years, extending existing models with a variety of deterioration patterns, demand functions and back- ordering policies. In most cases, deterioration is assumed to be a constant fraction of total on-hand inventory. But the deterioration rate taken as stochastic by Ghare and Schrader (44) and Chakrabarty et al. (33) among several researchers. The Weibull distribution has been used to model item decay by Chakrabarty et al. (33) and some attention has been focused on deteriorating items with expiration dates by Hsu et al. (52) and Lo et al. (55).

In inventory and SC models, several researchers considered the demand as constant, but in reality the demand has some seasonal effects. As a result, the total demand is a combination of a constant demand, which is constant throughout the year, and a variable demand, which is stochastic in nature. For example, the demand of garments throughout the year.

In this paper, we proposed a SC model with stochastic deterioration rate. We considered a part of the demand as stochastic. We have also considered carbon emission cost due to the shipment of the items over a period of time. We estimated minimum production rate under probabilistic environment and used algebraic method to determine the minimum cost associated with the whole SCM. The objective of this article is to find the minimum production quantity, minimum SC cost and optimum lot size with optimum integer valued number of deliveries. In sections 1 and 2, we had introduction and literature survey, and defined different parameters under some assumptions. In section 3, we described the proposed model. Sections 4 and 5 described the integrated SCM cost and customization of that under different probabilistic scenario. In section 6, we discussed the technique and graphical representation of the solutions along with the sensitivity analysis of the solution in terms of changes in different parameters. Finally, section 7 concludes the article.

5.2 Literature Survey

Deteriorating item inventory models has drawn a remarkable attention after the work developed by of Ghare and Schrader (44). In real life scenario, it is impossible to prevent deterioration of highly volatile and/or explosive items (e.g., alcohol, fuels), degradable items (such as blood), several pharmaceutical products and some daily consumable products (viz., milk, fruit, vegetables, etc.) for decaying or deteriorating the nature over time.

Hence, it is very imperative to explicitly consider the characteristics of the decaying and deteriorating items in the notion of the formulation of apposite SCM models. Ghare and Schrader (44) addressed the characteristics of those deteriorating items as "inventory decay", that described the direct spoilage, physical depletion and deterioration. They had not considered the entire SCM, but only developed a general EOQ (economic order quantity) model in which demand considered as constant and decay rate following the exponential distribution. Considering the Weibull and gamma distribution as deterioration rate, the work developed by Ghare and Schrader (44) was then further extended by Covert and Philip (1973) (35) and Tadikamalla (1978) (74), respectively. They had considered instantaneous replenishment, i.e., inventory growth of Just-In-Time (JIT) and did not allow back orders or shortages. Misra (1975) (58) suggested EOQ type models with finite production rates without shortages or backorders and discussed the cases of varying and constant deterioration rates. Exponential and Weibull deterioration rates along with the backorders even though with instantaneous replenishment (JIT) had been developed by Shah (1977) (69). Mak (1982) (56) suggested a production lot size inventory framework with backorders for the items following exponentially deterioration rate.

Research on the inventory models with deteriorating item has cached growing attention during the recent years and extended the different models with the diversified deterioration patterns, demand functions along with the shortages, and different backordering policies. In most cases, deterioration has been considered as a constant fraction of total on-hand inventory. Raafat (60) conducted a literature survey on the existing models for deteriorating the items. The (T, S_i) model was extended by Dave and Patel (41) considering the time proportional demand and deterioration. In 1998, Chakrabarty et al.(33) used Weibull distribution to model the item decay. Hsu et al.(52) in 2006 and Lo et al.(55) in 2007 worked on deteriorating items that have the expiration dates, while the deteriorating item economic ordering policy over an infinite time horizon was adopted by Goyal (47). Researchers such as B. Sarkar (62; 61), Datta and Pal (40), Hariga (49), Goyal and Giri (48), Skouri and Papachristos (71), Goswami and Chaudhuri (45), Skouri et al. (70), T. Sarkar et. al. (65; 66; 67; 68) etc. have expanded the literature of the inventory models in the direction of various types of deterioration rates. In 1930, Harris (50) introduced the basic but now well-known EOQ model using differential calculus. In 1934, Wilson

(79) derived the same type of model. In the literature of inventory models, differentiation method has been applied by most of the researchers to develop various types of algorithms for the inventory model in SCM milieu. In the SCM system, there are different parties (viz., buyers, suppliers, producers, distributors and retailers, etc.) that take different roles of the customers. Furthermore, SCM consisted of carbon emission cost for transportation of items. In the study of SCM, the purpose was to optimize the entire system simultaneously. In 1975 Goyal (46) developed a single supplier-single buyer problem for an integrated inventory model. In 1986, Banerjee (29) introduced a joint economic lot size model for the vendor and purchaser. The integrated production inventory model in the context of single vendor-single buyer as generalized policy was extended by Hill (51). The coordinating supply chain inventory through common replenishment epochs was discussed by Viswanathan and Piplani (76). The EOQ for deteriorating items in the multi-item lot size integrated vendor-buyer inventory model with JIT environment model with out differentiation was discussed by Yang and Wee (81; 82). Using the analytic geometry and algebraical methods, Cardenas-Barron (30; 31) discussed a multi-stage multi-customer supply chain model for optimizing the inventory decisions and the derived EPQ/EOQ inventory models with multiple backordering costs. In the same paper the author Cardenas-Barron suggested a series of optimization procedure for different class of algebraic functions using basic algebra. Items with constant deterioration was considered and extended by Yan et al. (80). Khouja (53) discussed optimization procedure of a multi-stage multi-customer supply chain model. Recently Duan et.al. derived a mathematical model for buffer capacity optimization was established and optimized using an extended vector universal generating function (UGF) and an improved adaptive non-dominated sorting genetic algorithm II (NSGA-II) (42).

In reference (36), Cardenas-Barron developed a model using optimum manufacturing batch-size in a single-stage production system with rework. Imperfect quality and quantity discounts in an EOQ model was noted by Cardenas-Barron (37). Some outstanding notes to find the integer valued lot-size in the basic EOQ and EPQ model were suggested by Garcia-Laguna et al. (84) and Cardenas-Barron et al. (38). Using arithmetic geometric inequality the vendor–buyer inventory model was extended by Cardenas-Barron et al. (39). For the EOQ and EPQ inventory models with the linear and fixed backorder costs (63), a complete solution procedure was suggested by Chung and Cardenas-Barron (34). For rework and stochastic preventive maintenance time with deteriorating items, an EPQ model was presented by Wee and Widyadana (78). An EOQ model for buyer-distributor-vendor supply chain with backlogging without using derivatives was extended by Teng et al. (75).

Recently, by addressing the environmental effect as a part of the supply chain, the transportation cost and carbon emission cost due to the transportation gained attention by the researchers in inventory and supply chain modeling. In practice, SC decisions are strongly affected by the flow paths and mode of transportation and transportation frequency (or quantity) of the inventory items. Regardless of its significance, very few researchers have developed SC and close loop supply chain (CLSC) models considering transportation costs and carbon emission cost. Some of the works related to the transportation costs and carbon emission cost were developed by Andriolo et al.(28), Swenseth and Godfrey (73), Vroblefski et al. (77), Zhao et al. (83), Sarkar et al. (64). Paksoy et al. (59) and Sarkar et al. (64) considered transportation costs and carbon emission cost in CLSC model. Of note, our proposed work is an extension of Yan et al. (80) and Sarkar (62) with Cardenas-Barron's (31) algebraic procedure under various types of probabilistic deterioration function and probabilistic demand. In this article, we derived a SCM model to identify the optimum lot size and the integral number of deliveries per production batch cycle which minimizes the total cost of the entire SCM. Yan et al. (80) and Sarkar (62) considered constant demand in their model, while in numerical example, Sarkar (62) considered an arbitrary production quantity which affected the total cost associated with entire SCM and leads to a conditional optimal solution instead of the absolute optimal solution. However, the most realistic scenario is that part of the demand is stochastic considering the seasonal demand rate. In our article, the stochastic seasonal demand as a part of demand is considered, while the requirement of a minimum production quantity with a certain aspiration level is obtained. In addition, Yan et al. (80) and Sarkar (62) had not considered carbon emission cost in their models. The carbon emission cost depends on several factors including distance between SC members, fuel and engine efficiency, the amount of emission per unit of fuel, among others. Ultimately all these above factors can be accumulated and reflected as the number of shipment or deliveries. We have also considered carbon emission cost, which has a significant contribution in the total cost, associated with the SCM. We derived the minimum production quantity and the expression for the total cost and optimum lot-size and the number of deliveries (integer valued) per production batch cycle using algebraic method without using calculus.

5.2.1 Notations and assumptions

Notations: In this paper, we have used the following notations in the development of our model, which is similarly used by Yan et. al. (80) and Sarkar (62):

P	Production rate ($ /unit time)
C_S	Setup cost for production batch ($ / batch)
H_S	Holding cost for the supplier ($ / unit / unit time)
I_S	Area under the supplier's inventory level
C_O	Ordering cost for the buyer
C_E	Derived per unit time carbon emission cost per delivery ($ / delivery / unit time)
D	Stochastic demand
H_B	Holding cost for buyer ($ / unit / unit time)
I_B	Area under the buyer's inventory level
T_r	Constant transportation cost per delivery ($ / delivery)
V_c	Unit variable cost for order handling and receiving ($ / unit)
δ	Deterioration rate
T	Duration of the inventory cycle
C_δ	Deterioration cost per unit ($ / unit)
N	Number of deliveries per production batch (N $\geq$ 1)
Q	Production lot-size per batch cycle (units)
q	Delivery lot-size (units)
T_P	Production time duration for the supplier
T_{NP}	Non production time duration for the supplier
T_g	Duration between two successive deliveries
TC	Total cost of the system

Assumptions:

(i) The stochastic operation environment is considered.
(ii) The system of the production inventory consists of single supplier single buyer for a single type of item.

(iii) Supplier always updated with the information of demand and buyer's inventory position.
(iv) Shortages and back ordering are not allowed.
(v) Buyer pays transportation and other handling costs.
(vi) Production rate is constant and greater than the demand rate i.e.,
$P > D$ with at least probability $(1-\alpha)$.
(vii) Cash and / or quantity discount is not allowed.
(viii) Demand $D = D_1 + D_2$, where D_1 is constant and D_2 is a random variable with known probability density function $f(x)$.
(ix) Deterioration rate of the inventory item is a random fraction (δ) of the present inventory, where δ is a random variable with continuous probability distribution. Here we have considered
$\delta \sim Gamma\ distribution.$
(x) Deterioration rate is sufficiently small, such that it's square and higher powers can be ignored.

5.2.2 Determination of minimum production rate

Before we calculate different inventory costs, we find the expression for minimum production rate $P^* = MinP$ satisfying the condition $P > D$. Here $D = D_1 + D_2$, where D_1 is constant and deterministic but D_2 is a random variable with known probability density function $f(x)$. We consider two cases: (1) D_2 follows normal distribution and (2) D_2 follows probability distribution other than normal.

5.2.2.1 D_2 follows normal distribution

Here we assume that $D_2 \sim N(\mu), \sigma)$ distribution. Again, though production rate P of the supplier is assumed as constant but we also assumed that $P > D = D_1 + D_2$ with at least probability $(1-\alpha)$, for a given α. Therefore,
$Prob[P > D] \geq 1-\alpha$
i.e., $Prob[D_2 < P - D_1] \geq 1-\alpha$
i.e., $Prob[\frac{D_2-E(D_2)}{s.d.(D_2)} < \frac{P-D_1-E(D_2)}{s.d.(D_2)}] \geq 1-\alpha$
i.e., $Prob[Z < \frac{P-D_1-E(D_2)}{s.d.(D_2)}] \geq (1-\alpha)$, where, $Z \sim N(0,1)$
i.e., $\int_{-\infty}^{\tau} \Phi(z)dz \geq \int_{-\infty}^{K_\alpha} \Phi(t)dt$, where, $\tau = \frac{P-D_1-E(D_2)}{s.d.(D_2)}$ and the value of K_α can be obtained from normal distribution table. Thus, $\tau \geq K_\alpha$
i.e., $P \geq K_\alpha s.d.(D_2) + D_1 + E(D_2)$.

To ensure that $P \geq D$ with at least probability $(1-\alpha)$, the minimum value of P is $P^* = K_\alpha s.d.(D_2) + D_1 + E(D_2)$. Thus, instead of P, we use expected optimum value of P, i.e., P^*.

5.2.2.2 D_2 follows probability distribution other than normal

Here, we consider D_2 being a random variable with probability density function $f(x)$ and not a normal random variate. Then, to ensure $P > D = D_1 + D_2$ with at least probability $(1-\alpha)$, for a given α, we consider $Prob[P > D] \geq 1 - \alpha$,
i.e., $Prob[D_2 < P - D_1] \geq 1 - \alpha$
i.e., $\int_{-\infty}^{P-D_1} f(x)dx \geq 1 - \alpha$ i.e., $P \geq g(D_1, \alpha)$.
Thus, the minimum value of P is $P^* = g(D_1, \alpha)$.

5.3 Proposed mathematical model

A single supply multiple delivery (SSMD) system is that, a product produced once and delivered multiple times with a constant time gap, i.e., the buyer's order quantity of a product is manufactured once and to be supplied by the supplier to the buyer's place (store or warehouse) with a regular interval. We consider it as JIT environment(54), i.e., the supplier delivers the item just in that time, when previously delivered items are exhausted.

In our proposed model we have considered a SSMD system with probabilistic demand of deteriorating items with probabilistic deterioration rate. Also, we have incorporated carbon emission cost in the proposed model to minimize the carbon footprint. Here, we have separately shown the inventory models for buyer and supplier in the figure 5.1 and figure 5.2, respectively (62). We considered T as total inventory cycle time and divided it into two segments depending upon the scenarios as follows (figure 5.2):

- $*$ T_P being the time when supplier produces the product,
- $*$ T_{NP} being the time when supplier does not produce the product.

In addition, the time gap between successive deliveries is defined as T_g.

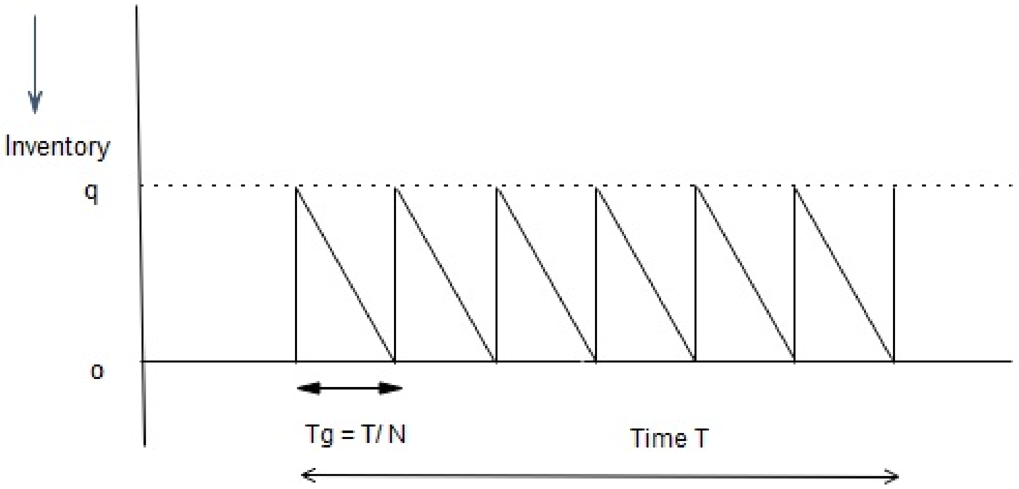

Fig. 5.1 Buyer's inventory model: inventory versus time.

Fig. 5.2 Supplier's inventory model: inventory versus time.

5.3.1 Buyer's inventory cost

The different types of costs associated with buyer's inventory are as follows:

(i) Ordering cost for the buyer per unit time $= \frac{C_O}{T}$,

(ii) Holding cost for the buyer per unit time $= \frac{H_B.I_B}{T}$,

(iii) Deterioration cost for the buyer per unit time $= \frac{C_\delta.(\delta.I_B)}{T}$,

(iv) Transportation and handling cost for the buyer per unit time $= \frac{N.T_r+V_c.N.q}{T}$.

Let us assume that η be the number of deteriorating items during time T_g and $q =$ number of deteriorating items in time T_g + consumption during T_g. Then, q will be $q = \eta + (D_1 + E(D_2)).T_g = \eta + D_1.T_g + \mu.T_g$. As per the assumption (5.2.1), deterioration rate is small, and we here neglect quadratic terms and higher order terms. Therefore, η is the deterioration of q units of items in the time interval T_g for the buyer. So, $\eta = \delta.\frac{q.T_g}{2}$. Thus, $q = D_1T_g + \mu T_g + \delta.\frac{q.T_g}{2} = T_g.(\Delta + \frac{\delta.q}{2})$, whereas $D_1 + \mu = \Delta$. Now putting $T_g = \frac{T}{N}$ (see figure 5.1), we obtain the following:

$$q = \frac{T}{N}.(\Delta + \frac{\delta.q}{2}) \tag{5.1}$$

So, $T = \frac{2.N.q}{2\Delta+\delta.q}$.
i.e.,

$$\frac{1}{T} = \frac{1}{q.N}.(\Delta + \frac{\delta.q}{2}) = \frac{\Delta}{q.N} + \frac{\delta}{2.N} \qquad (5.2)$$

Also at buyer's end, total deterioration is given by
$\delta.I_B = N.q - \Delta.T$ i.e., $I_B = \frac{N.q}{\delta} - \frac{\Delta.T}{\delta}$
or, $\frac{I_B}{T} = \frac{N.q}{\delta T} - \frac{\Delta}{\delta}$.
So using (6.1),

$$\frac{I_B}{T} = \frac{q}{2}. \qquad (5.3)$$

Thus, the inventory cost function for buyer is constructed as follows:
$TC_B(q,N) = \frac{C_O}{T} + \frac{H_B.I_B}{T} + \frac{N.T_r+V_c.N.q}{T} + \frac{C_\delta.\delta.I_B}{T}$.
Now using (6.2) and (6.3), we have

$$TC_B(q,N) = (\frac{\Delta}{q.N} + \frac{\delta}{2N})(C_O + N.T_r + V_c.N.q) + \frac{q}{2}(H_B + \delta.C_\delta) \qquad (5.4)$$

5.3.2 Supplier inventory cost

Different types of cost associated with supplier's inventory are as follows:

(i) Per unit time setup cost for supplier $= \frac{C_S}{T}$,

(ii) Per unit time holding cost for supplier $= \frac{H_S.I_S}{T}$,

(iii) Per unit time deterioration cost for supplier $= \frac{C_\delta.\delta.I_S}{T}$,

(iv) Per unit time carbon emission cost for N deliveries $= N.C_E$.

Let ξ be the number of deteriorating units of the item for the supplier. Therefore, $\xi = \delta.I_S$ i.e., $I_S = \frac{\xi}{\delta}$.
Hence in the whole SCM, the total number of deteriorating units $= \xi + \delta.\frac{q.T}{2}$.
Since $Q = N.q + \xi$ and $T_P = \frac{Q}{P} = \frac{N.q+\xi}{P}$, considering the average inventory of entire SCM and initial inventory we have the following:
$\xi + \delta.\frac{q.T}{2} = \delta.T(\frac{1}{2}T_P(P-\Delta) + \frac{\Delta}{P}.q)$, where $\Delta = D_1 + \mu$.
i.e., $\xi + \delta.\frac{q.T}{2} = \delta.T(\frac{\Delta}{P}.q + \frac{N.q+\xi}{P}.\frac{P-\Delta}{2})$.
To ensure $P > D$, with at least probability $(1-\alpha)$, the minimum value of P is P^*, (see section 5.2.2). Thus, we have $\xi + \delta.\frac{q.T}{2} = \delta.T(\frac{\Delta}{P^*}.q + \frac{N.q+\xi}{P^*}.\frac{P^*-\Delta}{2})$.
After simplification, we obtain, $\xi + \delta.\frac{q.T}{2} = \delta.T(\frac{\Delta}{P^*}.q + \frac{Q(1-\frac{\Delta}{P^*})}{2})$
i.e., $\xi + \delta.\frac{q.T}{2} = \delta.T(\frac{\Delta}{P^*}.q + (\frac{N}{2} - \frac{\Delta N}{2P^*})q)$

i.e., $I_S = \frac{\xi}{\delta} = qT(\frac{\Delta}{P^*} + \frac{N-1}{2} - \frac{\Delta N}{2P^*})$ i.e.,

$$\frac{I_S}{T} = q[(1 - \frac{N}{2})(\frac{\Delta}{P^*} - 1) + \frac{1}{2}] \tag{5.5}$$

Then, total cost function for supplier, using (6.1) and (6.5), can be written as
$TC_S(q, N) = \frac{C_S}{T} + \frac{H_S.I_S}{T} + \frac{C_\delta.\delta.I_S}{T} + N.C_E$ (see equation 6.6)

$$i.e., TC_S(q, N) = (\frac{\Delta}{q.N} + \frac{\delta}{2N})C_S + q[(1 - \frac{N}{2})(\frac{\Delta}{P*} - 1) + \frac{1}{2}](H_S + \delta.C_\delta) + N.C_E \tag{5.6}$$

5.4 Integrated inventory cost for the entire system

The average total cost associated with the integrated production inventory model for the complete SCM calculated as cumulative average cost of the supplier and buyer is as follows (for instance, see Yan et. al. (80) and Sarkar (62)): $TC(q, N) = TC_S(q, N) + TC_B(q, N)$ (see equation 6.7).

$$TC(q, N) = (\frac{\Delta}{q.N} + \frac{\delta}{2N})(C_S + C_O + N.T_r + V_c.N.q) +$$
$$\frac{q}{2}[(H_B + \delta.C_\delta) + (H_S + \delta.C_\delta)((2 - N).\frac{\Delta}{P^*} + N - 1] + N.C_E \tag{5.7}$$

For zero deterioration, i.e., $\delta = 0$ and $C_E = 0$ and deterministic demand this production inventory model reduced to the JIT (62) model without deterioration.

5.5 Customization of the total cost function as per the probability distribution of δ and D_2

In our model, deterioration δ follows gamma distribution, which is a two-parameter family of continuous probability distribution. In the total cost function we replace δ with $E[\delta]$. In our model we have considered the demand $D = D_1 + D_2$, where D_1 is constant while D_2 follows is stochastic distribution i.e., a random variable with (a) normal distribution and (b) exponential distribution. We consider that $P > D$, with at least probability $(1 - \alpha)$ and find minimum value of P, denoted by P^*.

5.5.1 Total cost function for $D_2 \sim N(m,\sigma)$ and $\delta \sim gamma(k,\theta), k > 0$ and $\theta > 0$

Here $D_2 \sim N(\mu,\sigma)$, so $\mu = E[D_2]$ and $s.d.(D_2) = \sigma$. Therefore $\Delta = D_1 + \mu$ and to comply $Prob(P > D) \geq (1-\alpha)$, we have minimum value of P as $P^* = K_\alpha.\sigma + D_1 + \mu$. As $\delta \sim gamma(k,\theta)$, $E(\delta) = k.\theta$. Now from (6.7) we have

$$\begin{aligned}TC(q,N) = (\frac{D_1+\mu}{q.N} + \frac{k\theta}{2N})(C_S + C_O + N.T_r + V_c.N.q)\\ +\frac{q}{2}[(H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)\\ ((2-N)\frac{D_1+\mu}{K_\alpha.\sigma + D_1 + \mu}) + N - 1] + N.C_E\end{aligned} \qquad (5.8)$$

For given N equation (5.8) becomes,
$TC(q)|_N = \frac{q}{2}[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)((2-N)\frac{D_1+\mu}{K_\alpha\sigma + D_1 + \mu} + N - 1)]$
$+\frac{1}{q}[(\frac{D_1+\mu}{N})(C_S + C_O + N.T_r)] + [(D_1+\mu)V_c + \frac{k.\theta}{2N}(C_S + C_O + N.T_r)] + N.C_E$
Therefore,

$$TC(q)|_N = q.b_1 + \frac{1}{q}.b_2 + b_3, \qquad (5.9)$$

where
$b_1 = \frac{1}{2}[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)((2-N)\frac{D_1+\mu}{K_\alpha\sigma + D_1 + \mu} + N - 1)]$,
$b_2 = [(\frac{D_1+\mu}{N})(C_S + C_O + N.T_r)]$,
$b_3 = [(D_1+\mu)V_c + \frac{k.\theta}{2N}(C_S + C_O + N.T_r)] + N.C_E$.

Now let us consider the expression $\psi(q) = q.b_1 + \frac{1}{q}.b_2 + b_3$, which can be written as $\psi(q) = b_1 q(1 - \frac{1}{q}\sqrt{\frac{b_2}{b_1}})^2 + 2\sqrt{b_1 b_2} + b_3$ and it attains it's minimum for $q = \sqrt{\frac{b_2}{b_1}}$ and $\psi_{min} = 2\sqrt{b_1 b_2} + b_3$, (see Cárdenas-Barrón (31)).
Therefore, $TC(q)|_N$ is minimum when q is as given in equation (5.10) and the minimum cost for given N is $TC(q)|_N = 2\sqrt{b_1 b_2} + b_3$, i.e., $TC(q)|_N$ is as given in equation (5.11).

$$q = \sqrt{\frac{b_2}{b_1}} = \sqrt{\frac{(D_1+\mu)(C_S + C_O + N.T_r)}{\frac{N}{2}[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)((2-N)\frac{D_1+\mu}{K_\alpha.\sigma + D_1 + \mu} + N - 1]}} \qquad (5.10)$$

$$\begin{aligned}TC(q)|_N = \sqrt{2[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)((2-N)\frac{D_1+\mu}{K_\alpha\sigma + D_1 + \mu} + N - 1)][(\frac{D_1+\mu}{N})(C_S + C_O + N.T_r)]}\\ +[(D_1+\mu)V_c + \frac{k.\theta}{2N}(C_S + C_O + N.T_r)] + N.C_E\end{aligned} \qquad (5.11)$$

Now given q, equation (5.8) becomes

$$TC(N)|_q = 2\sqrt{[\frac{q}{2}(H_S + k.\theta.C_\delta)(1 - \frac{D_1 + \mu}{K_\alpha\sigma + D_1 + \mu}) + C_E](\frac{D_1 + \mu}{q} + \frac{k.\theta}{2})(C_S + C_O)}$$
$$+[(\frac{D_1 + \mu}{q} + \frac{k\theta}{2})(T_r + q.V_c) + \frac{q}{2}(H_B + k.\theta.C_\delta) + q(H_S + k.\theta.C_\delta)(\frac{D_1 + \mu}{K_\alpha\sigma + D_1 + \mu} - \frac{1}{2}) \qquad (5.1$$

$$\begin{aligned} TC(N)|_q = N[\frac{q}{2}(H_S + k.\theta C_\delta)(1 - \frac{D_1 + \mu}{K_\alpha\sigma + D_1 + \mu}) + C_E] \\ + \frac{1}{N}(\frac{D_1 + \mu}{q} + \frac{k.\theta}{2})(C_S + C_O) \\ +[(\frac{D_1 + \mu}{q} + \frac{k\theta}{2})(T_r + q.V_c) + \frac{q}{2}(H_B + k.\theta C_\delta) + \\ q(H_S + k.\theta C_\delta)(\frac{D_1 + \mu}{K_\alpha\sigma + D_1 + \mu}) - \frac{q}{2}(H_B + k.\theta C_\delta)] \end{aligned}$$

$$i.e., TC(N)|_q = b_4 N + b_5 \frac{1}{N} + b_6 \tag{5.12}$$

where, $b_4 = [\frac{q}{2}(H_S + k.\theta C_\delta)(1 - \frac{D_1+\mu}{K_\alpha\sigma+D_1+\mu}) + C_E]$,
$b_5 = (\frac{D_1+\mu}{q} + \frac{k.\theta}{2})(C_S + C_O)$,
$b_6 = [(\frac{D_1+\mu}{q} + \frac{k\theta}{2})(T_r + q.V_c) + \frac{q}{2}(H_B + k.\theta C_\delta) + q(H_S + k.\theta C_\delta)(\frac{D_1+\mu}{K_\alpha\sigma+D_1+\mu}) - \frac{q}{2}(H_B + k.\theta C_\delta)]$
Proceeding as previously, $TC(N)|_q$ is minimum when $N = \sqrt{\frac{b_5}{b_4}}$, i.e.

$$N = \sqrt{\frac{(\frac{D_1+\mu}{q} + \frac{k.\theta}{2})(C_S + C_O)}{[\frac{q}{2}(H_S + k.\theta C_\delta)(1 - \frac{D_1+\mu}{K_\alpha\sigma+D_1+\mu}) + C_E]}} \tag{5.13}$$

and the minimum cost for given q is $TC(N)|_q = 2\sqrt{b_4 b_5} + b_6$, i.e., q is as given in equation (5.14).

5.5.1.1 Optimal range of lot-size q

As per our assumption $N \geq 1$. Therefore, from the expression of q given in (5.10), the maximum value of q is at $N = 1$. Again, from the expression of q, it decreases as N increases. Thus, the bounds of q are as given in the equation (15).

$$q \leq \sqrt{\frac{(D_1+\mu)(C_S+C_O+T_r)}{\frac{1}{2}[k.\theta.V_c+(H_B+k.\theta.C_\delta)+(H_S+k.\theta.C_\delta)(\frac{D_1+\mu}{K_\alpha.\sigma+D_1+\mu}]}}$$

$$q \geq \sqrt{\frac{(D_1+\mu)(C_S+C_O+N.T_r)}{\frac{N}{2}[k.\theta.V_c+(H_B+k.\theta.C_\delta)+(H_S+k.\theta.C_\delta)((2-N)\frac{D_1+\mu}{K_\alpha.\sigma+D_1+\mu}+N-1)]}} \tag{5.15}$$

$$\begin{aligned} TC(q,N) = (\frac{D_1+\frac{1}{\lambda}}{q.N}+\frac{k\theta}{2N})(C_S+C_O+N.T_r+V_c.N.q) \\ +\frac{q}{2}[(H_B+k.\theta.C_\delta)+(H_S+k.\theta.C_\delta) \\ ((2-N)\frac{D_1+\frac{1}{\lambda}}{D_1-\frac{1}{\lambda}log(\alpha)})+N-1]+N.C_E \end{aligned} \tag{5.16}$$

5.5.2 Total cost function for $D_2 \sim exponential(\lambda)$ and $\delta \sim gamma(k,\theta), k > 0$ and $\theta > 0$

Here $D_2 \sim exponential(\lambda)$, so $E(D_2) = \frac{1}{\lambda}$ and $s.d.(D_2) = \frac{1}{\lambda}$. Therefore, $\Delta = D_1 + \frac{1}{\lambda}$ and to comply $Prob(P > D) \geq (1-\alpha)$, we have minimum value of P as $P^* = D_1 - \frac{1}{\lambda}log(\alpha)$. As $\delta \sim gamma(k,\theta)$, $E(\delta) = k.\theta$. Hence, from equation (6.7), we have $TC(q,N)$ as given in equation (16).

As previously, for a given N, the equation (5.16) reduced to equation (5.17). Here $TC(q)|_N$ is minimum when q is given as given in equation (5.18) and minimum cost for given N is as given $TC(q)|_N$ in equation (5.19).

Now for given q, equation (5.16) reduces to equation (20).

Here, $TC(N)|_q$ is minimum when N is given by equation (21).

$$N = \sqrt{\frac{(\frac{D_1+\frac{1}{\lambda}}{q}+\frac{k.\theta}{2})(C_S+C_O)}{[\frac{q}{2}(H_S+k.\theta C_\delta)(1-\frac{D_1+\frac{1}{\lambda}}{D_1-\frac{1}{\lambda}log\alpha})+C_E]}} \tag{5.21}$$

The minimum cost for given q is $TC(N)|_q$ as given in equation (22).

$$\begin{aligned} TC(q)|_N = \frac{q}{2}[k.\theta.V_c+(H_B+k.\theta.C_\delta)+(H_S+k.\theta.C_\delta)((2-N)\frac{D_1+\frac{1}{\lambda}}{D_1-\frac{1}{\lambda}log(\alpha)}+N-1)] \\ +\frac{1}{q}[(\frac{D_1+\frac{1}{\lambda}}{N})(C_S+C_O+N.T_r)]+[(D_1+\frac{1}{\lambda})V_c+\frac{k.\theta}{2N}(C_S+C_O+N.T_r)+N.C_E] \end{aligned} \tag{5.17}$$

$$q = \sqrt{\frac{(D_1 + \frac{1}{\lambda})(C_S + C_O + N.T_r)}{\frac{N}{2}[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)((2-N)\frac{D_1+\frac{1}{\lambda}}{D_1-\frac{1}{\lambda}log(\alpha)} + N - 1)]}} \tag{5.18}$$

$$\begin{aligned} TC(q)|_N = & \sqrt{2[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)((2-N)\frac{D_1 + \frac{1}{\lambda}}{D_1 - \frac{1}{\lambda}log(\alpha)} + N - 1)]\frac{(D_1 + \frac{1}{\lambda}}{N}(C_S + C_O + N.T_r)} \\ & +[(D_1 + \frac{1}{\lambda})V_c + \frac{k.\theta}{2N}(C_S + C_O + N.T_r) + N.C_E] \end{aligned} \tag{5.19}$$

5.5.2.1 Optimal range of lot size q

As per our assumption $N \geq 1$. Therefore, from the expression of q given in (5.18), the maximum value of q is when $N = 1$. Again, from the expression of q, it decreases as N increases. Thus, the bounds of q are as given in equation (5.23).

5.6 Solution technique

In our proposed model, we will find optimal value of q and N, where N is integer value. We solve the eq. (5.8) , eq. (5.9) and eq. (5.12) for the case considered in section 5.5.1 and eq. (5.16), eq. (5.17) and eq. (5.20) for the case considered in section 5.5.2 to determine optimal solution, where N may be non-integral. In these solutions, if N is found to be integral value, we keep the solutions as it is. However, when N is not an integral value, we examine the value of TC for the value greater or less than N (i.e., ceiling or floor) and consider that integral value of N as the optimal integral value of N for which TC is minimum and recalculate values of q and associated TC using the eq. 5.10 and eq. 5.11 for the case considered in Section 5.5.1 and eq. (5.18) and eq. (5.19) for the case considered in Section 5.5.2. Finally, we summarize the realistic modified optimal solutions N^* (integral valued) , q^* and TC^* for each of the cases, separately, and also consider different graphs and sensitivity analysis of the solution.

$$\begin{aligned} TC(N)|_q = & N[\frac{q}{2}(H_S + k.\theta C_\delta)(1 - \frac{D_1 + \frac{1}{\lambda}}{D_1 - \frac{1}{\lambda}log\alpha}) + C_E] + \frac{1}{N}(\frac{D_1 + \frac{1}{\lambda}}{q} + \frac{k.\theta}{2})(C_S + C_O) \\ & +[(\frac{D_1 + \frac{1}{\lambda}}{q} + \frac{k\theta}{2})(T_r + q.V_c) + \frac{q}{2}(H_B + k.\theta C_\delta) + q(H_S + k.\theta C_\delta)(\frac{D_1 + \frac{1}{\lambda}}{D_1 - \frac{1}{\lambda}log\alpha}) - \\ & \frac{q}{2}(H_B + k.\theta C_\delta)] \end{aligned} \tag{5.20}$$

$$TC(N)|_q = 2\sqrt{[\frac{q}{2}(H_S + k.\theta C_\delta)(1 - \frac{D_1 + \frac{1}{\lambda}}{D_1 - \frac{1}{\lambda}log\alpha}) + C_E](\frac{D_1 + \frac{1}{\lambda}}{q} + \frac{k.\theta}{2})(C_S + C_O)}$$

$$+[(\frac{D_1 + \frac{1}{\lambda}}{q} + \frac{k\theta}{2})(T_r + q.V_c) + \frac{q}{2}(H_B + k.\theta C_\delta) + q(H_S + k.\theta C_\delta)(\frac{D_1 + \frac{1}{\lambda}}{D_1 - \frac{1}{\lambda}log\alpha} - \frac{1}{2})] \quad (5.22)$$

$$q \leq \sqrt{\frac{(D_1 + \frac{1}{\lambda})(C_S + C_O + T_r)}{\frac{1}{2}[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)(\frac{D_1+\frac{1}{\lambda}}{D_1-\frac{1}{\lambda}log\alpha})]}}$$

$$q \geq \sqrt{\frac{(D_1 + \mu)(C_S + C_O + N.T_r)}{\frac{N}{2}[k.\theta.V_c + (H_B + k.\theta.C_\delta) + (H_S + k.\theta.C_\delta)((2 - N)\frac{D_1+\mu}{K_\alpha.\sigma+D_1+\mu} + N - 1]}} \quad (5.23)$$

5.6.1 Numerical analysis and graphical representation

We used Mathematica 7.0 (57) to solve the different numerical examples. In our examples, some parameter values (data sets) are considered from the article by Sarkar (62). The parameter values are considered with units as described in the section (5.2.1), where notations are described.

5.6.1.1 Example 1

This example is associated with the case considered in section (5.5.1). Different parameter values are: $D_1 = 5000; \mu = 1000; \sigma = 60; K_\alpha = 1.65; P_O = K_\alpha * \sigma + D_1 + \mu; \Delta = D_1 + \mu; \delta = 0.4; C_S = 800; H_B = 7; H_S = 6; C_0 = 25; T_r = 50; V_C = 1; C_\delta = 50; C_E = 50$. After this solving, we had $TC = 15175.4, q = 129.906$ and $N = 22.2342$. Since N is not integral valued, we recalculated the value of TC for $N = 22$ and $N = 23$ and found that for $N = 22$ the value of TC is minimum. Thus, the optimal solution is $N^* = 22$ / production batch cycle, $TC^* = \$15175.6$ / year, $q^* = 130.304$ units (see Figure 5.3).

5.6.1.2 Example 2

This example is associated with the case considered in section (5.5.2). Different parameter values are: $D_1 = 5000; \lambda = 0.02; \alpha = 0.1; P_O = D_1 - log\alpha\frac{1}{\lambda}; \Delta = D_1 + \frac{1}{\lambda}; \delta = 0.4; C_S = 800; H_B = 7; H_S = 6; C_0 = 25; T_R = 50; V_C = 1; C_\delta = 50; C_E = 50$. After this solving, we obtained $TC = 13450.5, q = 121.001$ and $N = 22.2268$. As N is not an integral

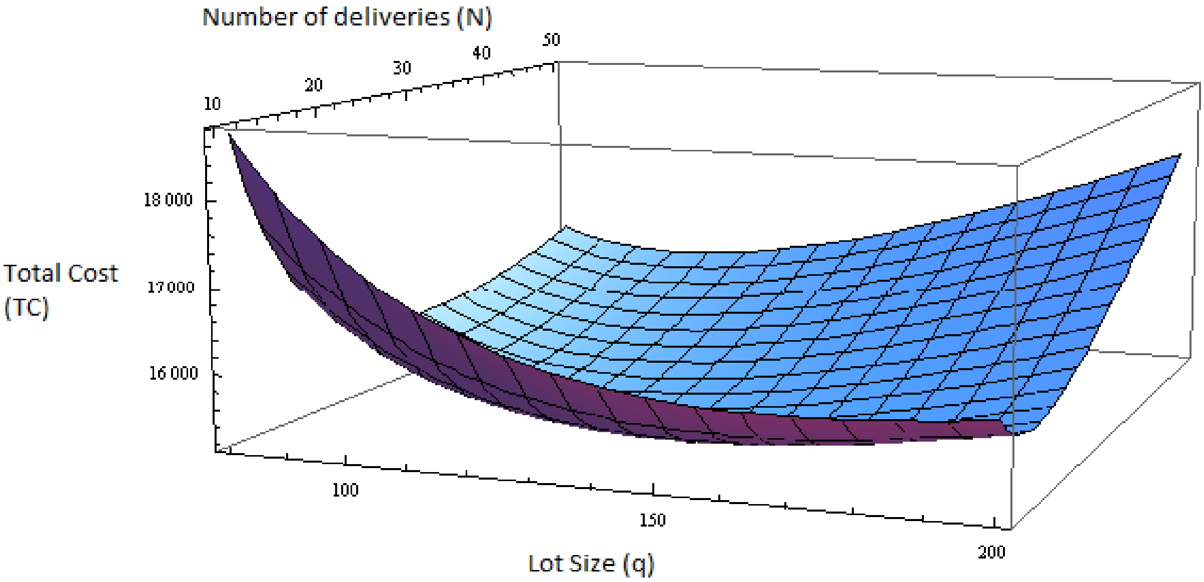

Fig. 5.3 Number of deliveries per production and lot size versus total cost as the case considered in section (5.5.1).

value, we recalculated the value of TC for $N = 22$ and $N = 23$ and identified that for $N = 22$, the value of TC is minimum. Thus, the optimal solution is $N^* = 22$ / production batch cycle, $TC^* = \$13450.6$ / year, $q^* = 121.343$ units (see Figure 5.4).

5.6.1.3 Cost and lot-size comparison of the models for different values of N with combined graphical representation

Here, we mark Case-1 for the model scenario referred in section 5.5.1 and Case-2 for the model scenario referred in section 5.5.2. Table 5.1 shows the analysis of the cost and lot size. It has been found that for both the cases the minimum total cost (TC) is achieved for $N = 22$ and for Case - 1 the minimum cost is $15175.6 / year with lot-size is 130.304 units and for Case - 2 the minimum cost is $13450.6 / year with lot-size is 121.343 units. A graphical representation is given in Figure 5.5.

5.6.1.4 Managerial insights in the view of sensitivity analysis for change in parameter value

The sensitivity analysis for change in different parameter values has been given in Tables 5.2 and 5.3. In this analysis, the following features have been observed, some of which are almost the same as those observed by

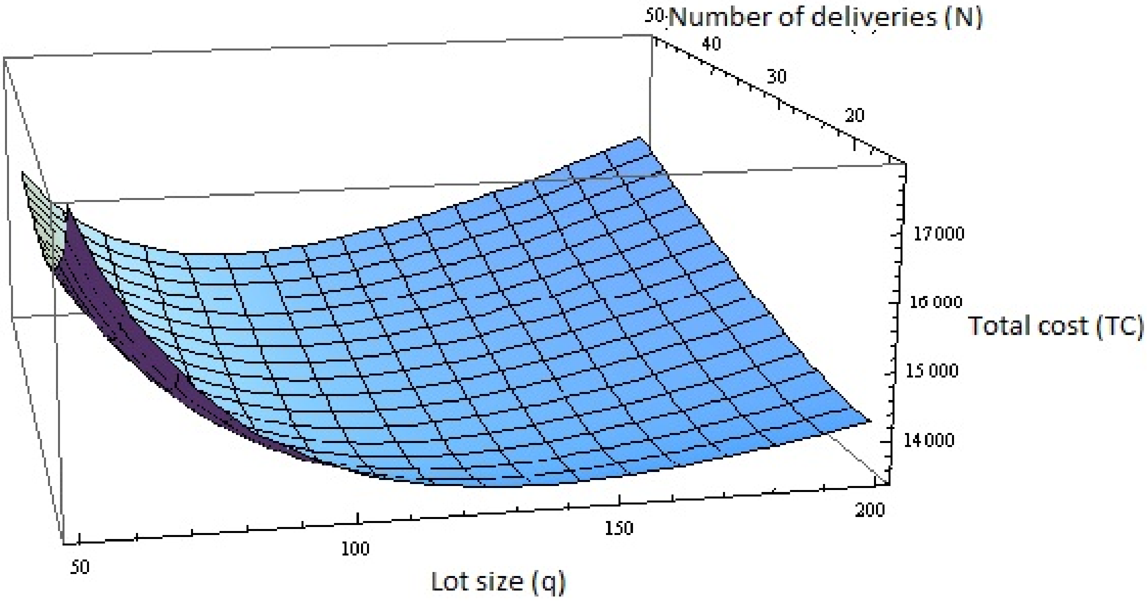

Fig. 5.4 Total cost versus number of deliveries per production batch and lot-size the case considered in section (5.5.2).

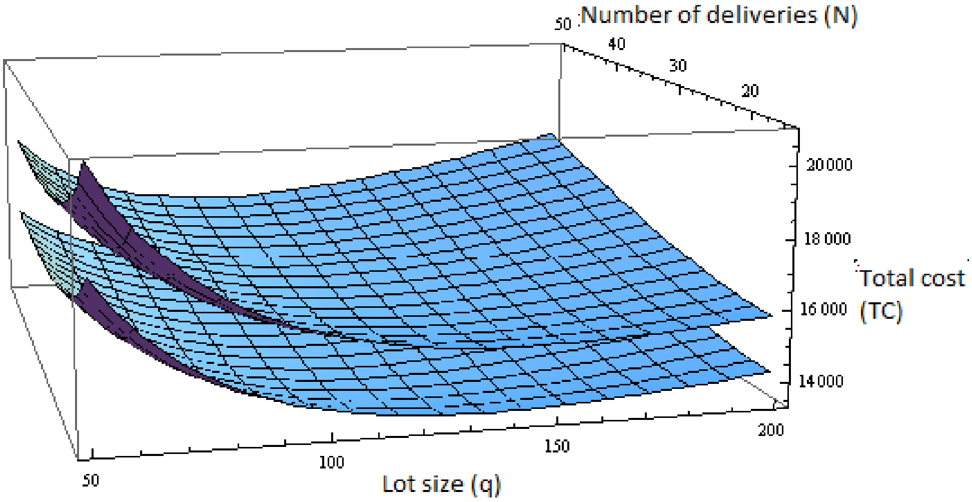

Fig. 5.5 Comparison of number of deliveries versus total cost per production and lot size for the models.

Table 5.1 Comparison of cost and lot size

Number of deliveries	Case 1		Case 2	
	Lot size (q)	Total cost (TC)	Lot size (q)	Total cost (TC)
1	445.192	29810.3	408.079	26931.3
2	322.386	23407.9	295.764	21036.3
3	269.186	20703.1	247.167	18545.5
4	238.093	19166.4	218.799	17130.0
5	217.245	18169. 0	199.806	16211.1
6	202.098	17470.7	186.025	15567.6
7	190.491	16957.8	175.481	15094.7
8	181.252	16568.4	167.102	14735.8
9	173.685	16266.2	160.25	14457.1
10	167.346	16027.8	154.521	14237.1
11	161.94	15837.7	149.643	14061.8
12	157.26	15685.2	145.427	13921.0
13	153.159	15562.4	141.739	13807.7
14	149.526	15463.7	138.478	13716.6
15	146.277	15384.8	135.567	13643.7
16	143.35	15322.2	132.949	13585.9
17	140.694	15273.4	130.576	13540.9
18	138.268	15236.3	128.414	13506.6
19	136.04	15209.3	126.431	13481.7
20	133.984	15190.8	124.605	13464.6
21	132.078	15179.9	122.914	13454.6
22	***130.304***	***15175.6***	***121.343***	***13450.6*** [a]
23	128.646	15177. 0	119.877	13452.0
24	127.091	15183.6	118.504	13458.1
25	125.629	15194.7	117.215	13468.4

---

[a] The values in bold indicate the optimal solution.

Sarkar (62):

- The increase in production quantity increases the total cost.
- As we have used the optimum value of the production quantity P^*, very low percentage of decrease in production value is possible in the system (this variation is allowed by the system due to consideration of standard deviation of stochastic demand).
- Total cost increases when we increase the setup cost.
- Increasing value of the holding cost, the total cost for the whole system increases.
- If the other costs (e.g., demand, ordering cost of the buyer, transportation cost, handling and receiving cost) as well as deterioration cost increase, the total cost of the system increases.
- Holding cost of the supplier is less than the holding cost of the buyer. Hence supplier will try to produce as much as possible and buyer

Table 5.2 Sensitivity analysis of the parameters

Parameter	**Parameter change (%)**	**% change in TC (Case 1)**	**% change in TC (Case 2)**
P	$<$ - 3.1	No feasible solution	$<$-2.8 % change no feasible solution
	-3	-9.13	
	-2	-5.28	-5.60
	-1	-2.39	-2.53
	1	2.07	2.18
	2	3.92	4.10
	3	5.56	5.86
	10	14.26	14.92
	20	22.19	23.16
	25	25.12	26.19
	50	34.70	36.14
C_S	-50	-6.46	-6.60
	-25	-2.95	-3.00
	25	2.59	2.64
	50	4.93	5.02
H_B	-50	-1.53	-1.60
	-25	-0.76	-0.80
	25	0.74	0.78
	50	1.47	1.55
H_S	-50	-1.73	-1.73
	-25	-0.86	-0.86
	25	0.85	0.84
	50	1.68	1.67
C_O	-50	-0.17	-0.18
	-25	-0.09	-0.09
	25	0.08	0.09
	50	0.17	0.17
T_R	-50	-8.49	-8.63
	-25	-4.01	-4.08
	25	3.66	3.74
	50	7.06	7.22

Table 5.3 Continued table 5.2: Sensitivity analysis of the parameters

Paramete	**Parameter change (%)**	**% change in TC (Case 1)**	**% change in TC (Case 2)**
V_C	-50	-19.86	-18.86
	-25	-9.93	-9.43
	25	9.93	9.43
	50	19.86	18.86
C_δ	-50	-11.21	-11.43
	-25	-5.26	-5.36
	25	4.76	4.84
	50	9.13	9.30

will seek for multiple deliveries. Furthermore, as the suppliers responsible for paying carbon emission cost for each delivery, the suppliers need to minimize the number of deliveries. So a trade off is made considering all these aspects.

5.7 Conclusion

Our proposed model is the extended version of the existing literature in SCM with stochastic seasonal demand (i.e., a part of demand is stochastic) and stochastic deterioration rate with algebraic procedure. Here, we have derived the optimal production quantity complying that the production quantity is always greater than stochastic demand satisfying an aspiration level. We have considered the carbon emission cost to reduce the environmental effects due to the supply chain activity. The demand is taken in two segments, one part is deterministic and another part is stochastic following gamma distribution. We have considered two types of stochastic deterioration rate: (a) normal probability distribution and (b) exponential probability distribution. For different cases, we have already identified the integral number of deliveries, optimal lot-size and minimum total system cost associated with the entire SCM. We have also considered the comparison of the models along with the graphical representation and sensitivity analysis in the view of managerial insight. Instead of using differential calculus for finding optimal system solution, we have used algebraic method to make the calculation procedure easiest. We suggest various possible extensions in future work. These future extension include other probability distribution for the stochastic parameters, shortages / backlog, effect of inflation, etc.. This model can be extended to the multi-item multi-supplier inventory model for the two-echelon SCM considering the aspects of carbon trading.

CHAPTER 6

A multi-retailer supply chain model with stochastic demand, backorder and production rate dependent production cost under aspect of carbon cap and trade

6.1 Introduction

In the modern marketing environment, it is more relevant to optimize the total system cost jointly for all parties involved in the supply chain system than to optimize the individual cost of each party (114). Currently, a vendor or manufacturer typically delivers products to numerous buyers. Many vendors build their own retail outlet to deliver products to multiple buyers. Thus, a single-vendor multi-buyer model is applicable in many cases. Goyal (101) proposed the integrated inventory model with coordination between a single buyer and a single vendor as a pioneering approach. Banerjee (87) extended Goyal's (101) model by assuming a lot-for-lot policy, which was again extended by Goyal (103) with SSMD policy. Goyel (103) suggested a supply chain model, where the vendor's production quantity is an integer multiple of the buyer's order quantity. Ha and Kim (105) developed a lot splitting supply chain model with a single retailer and a single supplier. Ouyang et al. (112) investigated an integrated vendor-buyer cooperative model with controllable lead time and stochastic demand. Sarkar and Majumder (114) developed an integrated inventory model with vendor's setup cost reduction and solved by a distribution free-approach. Cardenas-Barron et al. (91) surveyed a number of research articles regarding economic order quantity model. Cardenas-Barron and Sana (91) investigated the channel coordination of a two-echelon supply chain, where the demand pattern is dependent on sales' teams initiatives. Moon et al. (110) introduced a service level constrain in a continuous review model with variable stochastic lead time. Regarding backorder rate, Sarkar et al. (116) introduced random defective production rate with variable backorder rate. Sarkar et al. (118) introduced fill rate in a continuous review inventory model to minimize the total system cost with setup cost reduction. Sarkar and Mahapatra [M33] developed a periodic review inventory model with fuzzy demand to minimize the total cost by considering setup cost reduction of vendor. Sarkar et al. (118) considered backorder price discount in an integrated inventory model, where they developed two models with lead time demand as normally distributed and without having any distribution. Sarkar et al. (121) introduced product specific (products having fixed lifetime) back-ordering policy in a two-echelon supply chain model with coordination between the supply chain players. Based on the imperfect quality of products, Sarkar et al. (122) discussed the way to improve the quality by additional investment.

Banerjee and Burton (89) discussed a comparison between coordinated and independent replenishment policies in a single-vendor multi-buyer supply chain model. Banerjee and Banerjee (88) developed a multi-buyer inventory model using electronic data interchange with an order-up-to inventory control policy. Sarmah et al. (123) considered a single-supplier multi-buyer coordinated supply chain model with a trade credit policy. Hoque (106) discussed three different single-vendor multi-buyer models by synchronizing the production ow with equal and unequal sized batch transfer for the first two models and the last model, respectively. Guan and Zhao (104) developed a multi-retailer inventory system with a continuous review policy, which optimizes the decisions of pricing and inventory management with the aim of maximizing profit. Jha and Shankar (107) developed a singlevendor multi-buyer constrained non-linear model under service level constraint and solved it using a Lagrangian multiplier method. Cardenas-Barron and Trevi no-Garza (91) developed a three-echelon supply chain model with multiple products and multiple periods. Glock and Kim (100) studied the effect of forward integration in a multi-retailer supply chain under retailer competition. Cardenas-Barron and Sana (94) studied a two-layer supply chain model with multiple items and a promotional effort.

To improve customer service and to reduce stockout loss, it is important to reduce lead time. Liao and Shyu (109) first incorporated a probabilistic inventory model assuming lead time as a unique decision variable. Ben-Daya and Rauf (90) considered an inventory model as an extension of Liao and Shyu's (109) model, where lead time is one of the decision variables. Ben-Daya and Rauf's (90) model dealt with no shortage and continuous lead time. Ouyang et al. (111) extended Ben-Daya and Rauf's (90)] model by assuming discrete lead time and shortages. Pan and Yang (113) analyzed an integrated inventory model with lead time in a controllable manner. Annadurai and Uthayakumar (86) developed a periodic review inventory model under controllable lead time and lost sales reduction. Gholami-Qadikolaei et al. (98) developed a probabilistic inventory model with lead time and ordering cost reduction under budget and space constraint. Shin et al. (124) studied an integrated inventory model with controllable lead time and a service level constraint. You (126) studied an inventory model with partial backorder under vertical shift demand. Chung (96) assumed an integrated production-inventory model with backorder and used the method of comprising cost difference rate. Sarkar and Moon (116) developed an inventory model with variable backorder and deduced the procedure to reduce setup cost and quality improvement. The production rate is assumed to be constant in the classical supply chain model, however, in many cases, the machine production rate may change (108). Conard and McClamrock's (97) analysis stated that a 10 percent change in processing rate resulting a 50 percent change in machine tool cost. Moreover, the possibility of failure in the production process gradually increases with increasing production rate. As a result, the product quality may deteriorate at some rate. Thus, it is reasonable to consider the production rate as a decision variable not constant. Unit production cost also depends on the production rate and should be treated as one of the decision variables. Giri and Dohi (99) considered a generalized extended EMQ model with variable production rate by assuming stochastic machine breakdown and repair. Chang et al. (95) developed an EMQ model with variable production rate for a two-stage assembly system. Soni and Patel (125) studied an integrated single-supplier single-retailer inventory model with variable production rate and trade-credit policy.

Also, to control and reduce carbon emissions is a global problem. With "United Nations Framework Convention on Climate Change" and "Kyoto Protocol" signed and coming into force, carbon emissions reduction and the cap-and-trade system have become the consensus and inevitable trend. Enterprises are the main bodies of carbon emissions reduction; however the development of modern market economy makes any enterprise be placed in some supply chain systems. Then if we control carbon emissions from the perspective of single enterprise, the spillover effects generated by other enterprises' carbon emissions control in the supply chain will be ignored. Only when we solve the problem of carbon emissions reduction cooperation and coordination among supply chain enterprises effectively can we fundamentally promote carbon emissions reduction work. Otherwise it will be difficult for us to achieve the goal of carbon emissions reduction. With the improvement of cap-and-trade system, the carbon trading price, as the most direct market signal,will regulate the manufacturer's carbon emissions in his production process, thereby affecting the manufacturer's per unit carbon emissions. For example, Baosteel Group, one of the largest steel enterprises in China, has launched two carbon trading projects from 2007 to 2012, aiming to reduce per-unit carbon emissions through energy saving and emissions reduction technologies (127). Then Baosteel Group sold excess carbon credits at the appropriate price in the carbon trading market and profited more than 150 million RMB (128). In addition, with the development of social civilization and low-carbon education, there are more environmentally conscious consumers who prefer low-carbon products. How to meet consumers' low-carbon preference has become a key guarantee to win over in the fierce market competition (129). Therefore, it is of great value and practical significance to study the carbon emissions reduction in the supply chain with considering carbon trading price and consumers' environmental awareness (CEA). The investigated supply chain in this paper consists of single vendor supplies products to multiple buyers to satisfy stochastic and carbon emission dependent demand. At the beginning of production, the manufacturer obtains an initial carbon emissions limit from the government that has established a carbon emissions center. The manufacturer can buy carbon emissions rights from the carbon emissions center if he exceeds his initial carbon emissions limit at the end of production cycle; otherwise, he can sell his unused carbon emissions rights to the carbon emissions center. The manufacturer also can upgrade his production technology to reduce carbon emissions; then he needs to bear the cost of carbon emissions reduction. The consumers have environmental awareness, which means the products' carbon emissions will influence the market demand.

6.2 Literature Review

This paper relates to three major research areas, which are supply chain carbon footprint evaluation and management, decision problem of carbon emissions reduction in supply chain, impacts of CEA on carbon emissions reduction, and supply chain coordination.We review some recent representative works in the literature as follows.

It is a basic work to evaluate and manage supply chain's carbon footprint to control carbon emissions in supply chain. Braithwaite and Knivett (130) proposed a methodology to evaluate supply chain's carbon footprint by introducing the "carbon-to-serve" concept. Cholette and Venkat (131) found that different supply chain configurations can result in vastly different energy and emissions' profiles by using a web-based tool to calculate carbon emissions. Sundarakani et al. (132) examine the carbon footprint across supply chains and use transport methods to construct the analytical model. Lam et al. (133) presented a newmethod for supply chain synthesis and used a demand-driven approach to assess the feasible ways for transferring energy from renewable sources to customers in a given region. Xu and Fan (134) established the carbon footprint calculation models of fixed emissions sources and dynamic emissions sources in the supply chain. Compared with these surveyed papers, ourwork is based on these studies and focuses on the impacts of carbon trading price and CEA on carbon emissions reduction. In the carbon emissions reduction literatures, the decision problem of carbon emissions reduction in supply chain is a hot topic. Benjaafar et al. (135) illustrated how carbon emission concerns could be integrated into operational decision making and analyzed the impact of operational decisions on carbon emissions. Rosiˇc and Jammernegg (136) extended the dual sourcing model based on the newsvendor framework by considering the environmental impact of transport with emission taxes. Song and Leng (137) investigated the classical single-period problem under different carbon emissions policies. Du et al. (138) focused on the impact of "cap-and-trade" mechanism, proposed a game-theoretical analytical model, and derived a unique Nash equilibrium. Jaber et al. (139) presented a supply chain coordination model while accounting for greenhouse gas emissions from manufacturing processes. Xu et al. (140) proposed three differential game models with the emission reduction of the product and the retailer's promotion dependent demand. Li et al. (141) proposed a carbon trading model discussed between enterprises under strict carbon cap. Zhao et al. (142) developed a retailer-driven revenue-sharing contract to coordinate the supply chain with the constraint of product carbon emissions. Lu and Chen (143) studied the supply chain coordination with buyback contract under different carbon emissions policies and found buyback contract can coordinate supply chain. Compared with the surveyed papers in this scope, our work focuses on the coordination mechanism about carbon emissions reduction and order quantity in the supply chain.

CEA affects market demand, which has an impact on the enterprise's carbon emissions. Liu et al. (144) found that asCEA increases, retailers and manufacturers with superior ecofriendly operations will benefit. Zhang et al. (145) analyzed the impact of CEA on order quantities and channel coordination, in which the manufacturer produces the environmental products and the traditional products. Wang and Zhao (146) studied how to determine the optimum order quantity and which situation the supplier should choose to reduce carbon emissions. Xie et al. (147) studied carbon emission reduction and sharing decision-making of supply chain system with the carbon emissions level dependent demand. Chen (148) proposed several important topics for future research based on a brief overview of the current research in the field of low-carbon supply chain management, including the impacts of CEA on carbon emissions reduction. Compared with the literatures in this scope, ourworknot only considers the CEA, but also considers the carbon trading price. Through coordination, supply chain can fully exploit its potential and improve its performance (149). There are many works in the field of supply chain coordination. Heydari and Norouzinasab (150) proposed a coordination mechanism to coordinate both pricing and ordering decisions simultaneously based on quantity discount. Chakraborty and Chatterjee (151) developed the economics of surcharge pricing as a supply chain coordinating mechanism under JIT environment.

The above-mentioned literatures all carry out in-depth studies on the supply chain carbon emissions reduction. They put forward many carbon emissions control methods and models, which are a great contribution to the field of carbon emissions reduction. Meanwhile it is also needed to be pointed out that there are three issues that should be explored in depth: (1) There are few literatures concerning the impacts of carbon trading price on the supply chain carbon emissions reduction. In fact, carbon trading price has important impact on the supply chain carbon emissions reduction and the carbon emissions allocation. (2)There are also few literatures that take both the carbon trading price andCEAinto account. It has become an important problem to study the impacts of carbon trading price on supply chain carbon emissions reduction while taking CEA into account. (3) It is necessary to further study the coordination mechanism in supply chain carbon emissions reduction. Under the constraint of carbon emissions, if the supply chain members try to optimize their own profits to reduce their carbon emissions, carbon emissions reduction of the whole supply chain will often not be the optimal; that is, there is a “marginal double” effect. So it is also needed to explore how to achieve the supply chain carbon emissions reduction coordination.

6.3 Problem definition

In this paper we developed a multi-retailer supply chain model, where single vendor supplies products to multiple buyers to satisfy stochastic and carbon emission dependent demand. The production rate and production cost are considered as a variable quantities. We have considered production cost as a function of production rate, which developed a special ‘U’-shaped function and is used to develop the model (6.1). We have considered a single-setup-multi-delivery (SSMD) policy for delivering products to buyers from supplier. At the buyer’s end, partial backorder is considered for shortages. The lead time demand is considered as stochastic and follows a normal distribution.

Assume that the government has established a carbon emissions trading center and the enterprises buy or sell carbon emissions rights just for normal production; namely, there is no speculation. At the beginning of production cycle, the manufacturer obtains an initial carbon emissions limit from the government. At the end of production cycle, if the manufacturer’s total carbon emissions exceed the initial carbon emissions limit, he needs to buy carbon emissions rights from the carbon emissions trading center at the per unit carbon trading price; if the manufacturer’s total carbon emissions do not exceed the initial carbon emissions limit, he can sell the carbon emissions rights to the carbon emissions trading center at the per-unit carbon trading price.

As the consumers pay more attention to the environmental change, low-carbon products are more and more welcomed by the market. The products’ carbon emissions affect the market demand, which also affect the supply chain members’ market competitiveness. So it is necessary for supply chain members to upgrade their production technology to reduce their carbon emissions. Assume the manufacturer has the ability to reduce his

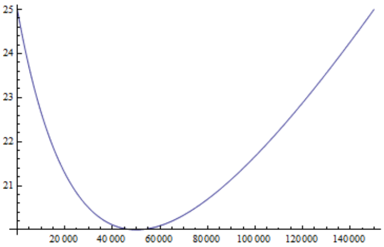

Fig. 6.1 Production Rate Curve

products' carbon emissions. Before the manufacturer decides to upgrade his production technology, the initial carbon emissions per-unit product is e_0. When the manufacturer decides to carry out technological upgrading for carbon emissions reduction, the carbon emissions per-unit product is $e(0 < e \leq e_0)$. The manufacturer needs to bear the cost of carbon emissions reduction when he wants to upgrade his production technology. According to the literatures (127; 129; 140; 146), we can use $\alpha(e_0 - e)^2/2$ as the cost of carbon emissions reduction; here $\alpha(\alpha > 0)$ is the cost coefficient of the manufacturer's carbon emissions reduction.

In this paper we developed a multi-retailer supply chain model, where single vendor supplies products to multiple buyers to satisfy stochastic and carbon emission dependent demand. The production rate and production cost are considered as a variable quantities. We have considered production cost as a function of production rate, which developed a special 'U'-shaped function and is used to develop the model. We have considered a single-setup-multi-delivery (SSMD) policy for delivering products to buyers from supplier. At the buyer's end, partial backorder is considered for shortages. The lead time demand is considered as stochastic and follows a normal distribution. Considering the market demand is affected by consumer's environmental awareness in addition form, the manufacturer considered the technological up-gradation under carbon cap and trade mechanism. Considering all such aspects this paper proposes the methods to determine the optimal order quantity and the optimal level of carbon emissions through model optimization.

6.4 Notations and assumptions

6.4.1 Notations

In this paper we have used following notations to develop the model:

q_i order quantity for buyer i (units)

Sf_i safety factor for buyer i

L_i length of lead time for buyer i (time unit)

n number of buyers

d_i demand of i^{th} buyer per unit time (units)

Co_i ordering cost for i^{th} buyer per order ($ / order)

H_{B_i} holding cost per unit item per unit time for i^{th} buyer ($ / unit / unit time)

C_{SO_i} stock out cost per unit of shortage for i^{th} buyer ($ / unit)

M_{P_i} marginal profit per unit item for i^{th} buyer ($ / unit)

f_i fraction of demand for i^{th} buyer that will be backordered during stockout, $0 \leq \beta \leq 1$.

m number of lots (same for all buyers)delivered in one production cycle (positive integer)

X_i lead time demand for i^{th} buyer

β_i fraction of the demand for buyer i that will be backordered during stock out, $0 \leq \beta_i \leq 1$

P production rate of vendor (unit / unit time)

Q delivery lot size for vendor such that $Q = \sum_{i=1}^{n} q_i$ (units)

$C(P)$ production dependent unit production cost ($ / unit)

CS_v setup cost for vendor ($ / setup)

CH_v holding cost per unit product per unit time ($ / unit / unit time)

CL_v carbon emission limit for vendor (unit Co_2)

E_s carbon emission per lot

C_{CE} cost of per unit carbon emission

C_{PR} per unit carbon trading price ($ / unit Co_2)

e_0 initial carbon emission per unit item production and transportation for vendor (unit Co_2 / unit)

e carbon emission per unit item production and transportation for vendor after technological up-gradation, $0 < e \leq e_0$ (unit Co_2 / unit)

$\alpha(\alpha > 0)$ is the cost coefficient of the manufacturer's carbon emissions reduction

a^+ maximum value of a and 0, i.e. $a^+ = max(a, 0)$

6.4.2 Assumptions

(1) Single setup multiple delivery (SSMD) model with one vendor with one vendor and single type of product supplies to multiple buyers with equal lot size.

(2) Demand d_i for i^{th} buyer is stochastic such that $d_i = d_{1i} + d_{2i}$, where d_{1i} is emission dependent demand such that $d_{1i} = \frac{\xi_i}{e}$ and d_{2i} is random variable which follows (a) normal distribution or (b) other than normal distribution where each d_{2i} 's are independently and identically distributed with mean μ_i and standard deviation (s.d.) s_i. Also $D = \sum_{i=1}^{n} d_i$.

(3) To satisfy the demand of each buyer the vendor supplies a total of Q quantity such that $Q = \sum_{i=}^{n} q_i$.

(4) The vendor manufactures mQ quantity against order q_i quantity of buyer i, i.e., Q quantity of all buyers, but shipment should be in quantity Q over m times. Also shipment procedure follows the relation $q_i = d_i \frac{Q}{D}$ i.e., $\frac{q_i}{d_i} = \frac{Q}{D}$ (= expected cycle time).

(5) The inventory is continuously reviewed by each buyer. According to this policy, an order is placed whenever the level of inventory reaches / decreases to a particular inventory level, called reorder point, denoted as r_i for i^{th} buyer.

(6) Production rate is variable quantity satisfying $P_{min}(P_{min} > D = \sum_{i=1}^{n} d_i$ and P_{max}.

(8) The unit production cost of the vendor is a function of P, having the expression as $C(P) = \frac{a_1}{P} + a_2 P$ as considered by Khouja and Mehrez's [M23], where a_1 and a_2 are constant and must satisfy $P_{min} = \sqrt{\frac{a_1}{a_2}}$, which gives best fit for the function $C(P)$.

(9) Shortage is allowed with partial backorder ratio f_i for i_{th} retailer.

(10) We replace D and d_i's as $D = E(D) = \sum_{i=1}^{n} \frac{\xi}{e} + \sum_{i=1}^{n} \mu_i$ and $d_i = E(d_i) = \frac{\xi}{e} + \mu_i$.

(11)The time horizon is taken as infinite.

6.5 Mathematical modelling of the system

6.5.1 Cost function for vendor

The different cost components for vendor are as follows:

- Setup cost:
  Set up cost per unit time $= \frac{C_s D}{mQ} = \frac{(\sum_{i=1}^{n} \frac{\xi_i}{e} + \sum_{i=1}^{n} \mu_i)}{mQ}$.

- Holding cost:
  The average inventory of the vendor is

$[(mQ(\frac{Q}{P}+(m-1)\frac{Q}{D})-\frac{m^2Q^2}{2P})-(\frac{Q^2}{D}(1+2+3+...+m-1))]\frac{D}{mQ}$
$=\frac{Q}{2}[m(1-\frac{D}{P})-1+\frac{2D}{P}]$.
Therefore, vendor's per unit time holding cost is
$C_{H_v}\frac{Q}{2}[m(1-\frac{D}{P})-1+\frac{2D}{P}]$
$=C_{H_v}\frac{Q}{2}[m(1-(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)\frac{1}{P})-1+2(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)\frac{1}{P}]$

- $*$ Production cost:
  The production cost of the vendor is assumed to be function of production rate P such that, $C(P)=(\frac{a_1}{P}+a_2P)$ () . The production rate that minimizes the production cost $C(P)$ is $P=\sqrt{\frac{a_1}{a_2}}$ and initially we choose a_1 and a_2 such that $P_{min}=\sqrt{\frac{a_1}{a_2}}$.
  Therefore total production cost $=(\frac{a_1}{P}+a_2P)D=(\frac{a_1}{P}+a_2P)(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)$

- $*$ Carbon trading cost:
  Carbon trading cost for vendor $=C_{PR}(eD-C_L)$

- $*$ Technology upgradation cost:
  The technology upgradation cost or carbon emission cost for vendor is $\frac{1}{2}\alpha(e_0-e)$ (**?**)where $\alpha>0$ and α is the cost coefficient of the vendor's carbon emission reduction

Therefore total expected cost for vendor is

$$TC_V(m,Q,P,e)=\frac{C_S(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)}{mQ}+$$
$$C_{H_v}\frac{Q}{2}[m(1-(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)\frac{1}{P})-1+2(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)\frac{1}{P}]$$
$$+(\frac{a_1}{P}+a_2P)(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)+C_{PR}(eD-C_L)+\frac{1}{2}\alpha(e_0-e) \tag{6.1}$$

6.5.2 Cost function for buyers

Before we calculate different cost components, we find $E(X_i-r_i)^+$, where X_i is the lead time demand for i^{th} buyer having a normal distribution with mean and standard deviation (s.d.) respectively $d_iL_i=(\frac{\xi_i}{e}+\mu_i)L_i$ and $\sigma_i\sqrt{L_i}$. Now shortages occur when $X_i>r_i$ for i^{th} buyer. The expected shortage at the end of the cycle for i^{th} buyer is $E(X_i-r_i)^+=\int_{r_i}^{\infty}dF(x_i)=\sigma_i\sqrt{L_i}\psi(Sf_i)$,where $\psi(Sf_i)=\phi(Sf_i)-Sf_i[1-\Phi(Sf_i)]$,

ϕ is standard normal probability distribution function and Φ is the cumulative distribution of a random normal variate.
Now the different cost component of the ith buyer are as follows:

- Ordering cost:
  The expected cycle time for ith buyer is $\frac{q_i}{d_i}$. Therefore the ordering cost for ith buyer $= Co_i\frac{q_i}{d_i} = Co_i\frac{D}{Q} = Co_i\frac{\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i}{Q}$.

- Holding cost:
  In this model the inventory is continuously reviewed by each of the buyers. Therefore the i^{th} buyer places an order q_i when the level of inventory reaches a specified limit r_i, which is called reorder point. So the inventory level of i^{th} buyer just before and after receiving an order q_i is $r_i - d_iL_i$ and $q_i + r_i - d_iL_i$, where $r_i - d_iL_i = r_i - L_i(\frac{\xi_i}{e} - \mu_i)$. Therefore average expected inventory level for i^{th} buyer over the cycle is $\frac{q_i}{2} + r_i - L_i(\frac{\xi_i}{e} - \mu_i)$. Now r_i can be expressed as $L_i(\frac{\xi_i}{e} - \mu_i) + Sf_i\sigma_i\sqrt{L_i}$. Thus the average inventory level for the i^{th} buyer over the cycle is $(\frac{q_i}{2} + Sf_i\sigma_i\sqrt{L_i})$. Again $(1 - f_i)$ be the fraction demand that can not be back ordered. Therefore for i^{th} buyer holding cost for per unit time
  $= H_{B_i}[(\frac{q_i}{2} + Sf_i\sigma_i\sqrt{L_i}) + (1 - f_i)E(X_i - r_i)^+]$
  $= H_{B_i}[(\frac{Q}{2}\frac{\frac{\xi_i}{e}+\mu_i}{\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i} + Sf_i\sigma_i\sqrt{L_i}) + (1 - f_i)\sigma_i\sqrt{L_i}\psi(Sf_i)]$

- Shortage cost:
  As M_{P_i} and C_{SO_i} are the marginal profit and stock out cost respectively, so the shortage cost for i^{th} buyer $= [C_{SO_i} + M_{P_i}(1 - f_i)]\frac{d_i}{q_i}E(X_i - r_i)$
  $= [C_{SO_i} + M_{P_i}(1 - f_i)]\frac{\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i}{Q}\sigma_i\sqrt{L_i}\psi(Sf_i)$.

Therefore total expected cost for i^{th} buyer is

$$TC_{B_i}(Q, m, e, Sf_i, L_i) = Co_i\frac{\sum_{i=1}^{n}\frac{\xi_i}{e} + \sum_{i=1}^{n}\mu_i}{Q} + H_{B_i}[(\frac{Q}{2}\frac{\frac{\xi_i}{e} + \mu_i}{\sum_{i=1}^{n}\frac{\xi_i}{e} + \sum_{i=1}^{n}\mu_i} + Sf_i\sigma_i\sqrt{L_i}) + (1 - f_i)\sigma_i\sqrt{L_i}\psi(Sf_i)] + [C_{SO_i} + M_{P_i}(1 - f_i)]\frac{\sum_{i=1}^{n}\frac{\xi_i}{e} + \sum_{i=1}^{n}\mu_i}{Q}\sigma_i\sqrt{L_i}\psi(Sf_i) \tag{6.2}$$

6.5.3 Integrated supply chain cost

In order to get the centralized optimal decision for both and vendors to minimize the entire supply chain cost, both the supplier and vendors cost function to be considered. Therefore, the joint total expected supply chain cost for both vendor and the buyers is obtained as follows:

$$\begin{aligned} TEC(Q,m,e,P,Sf_i) = {} & \frac{C_S(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)}{mQ} + C_{H_v}\frac{Q}{2}[m(1 \\ & -(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)\frac{1}{P}) - 1 + 2(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i)\frac{1}{P}] + \\ & (\frac{a_1}{P}+a_2P)(\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i) + C_{PR}(eD-C_L) + \frac{1}{2}\alpha(e_0-e) + \\ & \sum_{i=1}^{n}[Co_i\frac{\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i}{Q} + \\ & H_{B_i}[(\frac{Q}{2}\frac{\frac{\xi_i}{e}+\mu_i}{\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i} + Sf_i\sigma_i\sqrt{L_i}) + (1-f_i)\sigma_i\sqrt{L_i}\psi(Sf_i)] + \\ & [C_{SO_i}+M_{P_i}(1-f_i)]\frac{\sum_{i=1}^{n}\frac{\xi_i}{e}+\sum_{i=1}^{n}\mu_i}{Q}\sigma_i\sqrt{L_i}\psi(Sf_i)] \end{aligned} \tag{6.3}$$

6.6 Solution procedure of the model

Now using equation (6.3) we find the optimal system solution i.e., minimum supply chain cost and the optimal values of the decision variables. Here in our model the system becomes an unconstrained minimization problem with five decision variables namely $Q, m, e, P,$ and Sf_i. Now to obtain the minimum $TEC(Q, m, e, P, Sf_i)$, the traditional way is to find the derivative of the objective function $TEC(Q, m, e, P, Sf_i)$ are obtained with respect to all decision variables and equate them with zero. Here m is a discrete decision variable. Now the first order derivative of $TEC(Q, m, e, P, Sf_i)$ with respect to Q, P, Sf_i and e are to be obtained.

As m is a discrete decision variable, for a fixed positive integer value of m the values of $Q, \Phi(Sf_i), P$ and e can be evaluated by equating every individual partial derivative to zero as follows:

$$\frac{\delta TEC(Q,m,e,P,Sf_i)}{\delta Q} = 0 \tag{6.4}$$

$$\frac{\delta TEC(Q,m,e,P,Sf_i)}{\delta P} = 0 \tag{6.5}$$

$$\frac{\delta TEC(Q,m,e,P,Sf_i)}{\delta Sf_i} = 0 \tag{6.6}$$

$$\frac{\delta TEC(Q,m,e,P,Sf_i)}{\delta e}=0 \tag{6.7}$$

But a closed form optimal solution of this model is very difficult to obtain. We apply a spacial type fixed point iteration technique and develop an algorithm for that. Now we apply the following algorithm to find the optimal values of Q, Sf_i, P and e and we use MATLAB code to implement the algorithm.

6.6.1 Solution Algorithm

The algorithm to implement that spacial type of fixed point iteration is as follows:

- $*$ Step 1: Set $m = 1$ and put all the parameter values.
- $*$ Step 2: For all the buyers, $i = 1, 2, 3, ..., n$, assign values of all the parameters and perform the following steps.
- $*$ Step 3: Set $Sf_i = 0$ for each buyer.
- $*$ Step 4: Substitute Sf_i into (6.4) and evaluate Q.
- $*$ Step 5: Use Q to evaluate value of $\Phi(Sf_i)$ for all i from (6.6).
- $*$ Step 6: Now using the value of $\Phi(Sf_i)$, obtain the value of Sf_i using normal distribution table.
- $*$ Step 7: Repeat Step 3 to step 6, until no changes occur in the values of Q and Sf_i and denote those values as Q^* and Sf_i^* respectively.
- $*$ Step 8: Evaluate P^* from (6.5)using the value of Q^*.
- $*$ Step 9: Evaluate e^* from (6.7)using the value of Q^* and P^*.
- $*$ Step 10: Denote the latest update values of Q, Sf_i, P and e as $Q^*_{(m)}, Sf^*_{i(m)}, P^*_{(m)}$ and $e^*_{(m)}$.
- $*$ Step 11: Set $m = m + 1$.
- $*$ Step 12: If $TEC(Q^*_{(m)}, m, e^*_{(m)}, P^*_{(m)}, Sf^*_{i(m)}) \leq TEC(Q^*_{(m-1)}, m-1, e^*_{(m-1)}, P^*_{(m-1)}, Sf^*_{i(m-1)})$, repeat step 2 to step 12. Otherwise, go to step 13.

- $*$ Step 13: Set $TEC(Q^*_{(m)}, m, e^*_{(m)}, P^*_{(m)}, Sf^*_{i(m)}) = TEC(Q^*_{(m-1)}, m-1, e^*_{(m-1)}, P^*_{(m-1)}, Sf^*_{i(m-1)})$, which is the optimal solution and optimal values of the decision variables are $Q^* = Q^*_{(m)}, m^* = m, e^* = e^*_{(m)}, P^* = P^*_{(m)}, Sf^*_i = Sf^*_{i(m)}$.

6.7 Numerical Illustration and conclusion

For numerical illustration, we assumed the following data:

Vendor's setup cost $CS_v = 2000$; variance of mean of lead time demand $Z = [500\ 450\ 400]$; mean of lead time demand $\mu = [150\ 200\ 250]$; vendor's holding cost $CH_v = 12; a = [10000\ 0.01]$; carbon trading price per unit $C_{PR} = 50$; carbon emission limit for vendor $CL_v = 2000$; Cost coefficient of the manufacturer's carbon emission reduction $\alpha = 5000$; initial carbon emission $e_0 = 22$; ordering cost for buyer's per order $C_o = [200\ 225\ 250]$; stock out price $C_{SO} = [15\ 12\ 13]$; marginal profit per unit $M_P = [12\ 11\ 13]; \sigma = [30\ 20\ 35]; Sf = [0\ 0\ 0]; Sf1 = [1\ 1\ 1]$;
$SY = [normpdf(Sf(1)) - Sf(1)*(1-normcdf(Sf(1)))normpdf(Sf(2)) - Sf(2)*(1-normcdf(Sf(2)))normpdf(Sf(3)) - Sf(3)*(1-normcdf(Sf(3)))]$; buyer's holding cost $H_B = [12\ 15\ 17]$; carbon emission per lot $E_s = 280$; cost of per unit carbon emission $C_{CE} = 1.5$; and fraction of demand be back ordered $f = [0.5\ 0.5\ 0.5]$ and lead time for buyer's $L = [5\ 6\ 4]$.

The optimal values of the decision variables are
m = 4, Sf = [0.8471 0.5721 0.5599], Q = 255, P = 833.07, TEC = 656768.53, e = 16.02.

If we change the backorder ratio to zero i.e., f = [0.0 0.0 0.0], then the optimal values of the decision variables are
m = 4, Sf = [1.0676 0.8542 0.8527], Q = 248, P = 838.1, TEC = 657208.9, e = 16.02.

If we change the backorder ratio to 0.8 i.e., f = [0.8 0.8 0.8], then the optimal values of the decision variables are
m = 4, Sf = [0.6376 0.2787 0.2493], Q = 265, P = 825.8, TEC = 656405.2, e = 16.02.

From the above results, we can observe that the value of fraction of demand be back ordered f is much sensitive on the value of Sf, than the other decision variables m, Q, P and TEC.

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List of Publications

Title of paper	Publication details
Product Pricing in a Two-echelon Supply Chain with Stochastic Demand Under Carbon Cap & Trade Regulations	Published in International Journal of Mathematics And its Applications, 10(1)(2022), 67–77
Development of WARKS for Accessing SupplyChain Management	Published in International Journal of Recent Technology and Engineering (IJRTE) ISSN: 2277-3878 (Online), Volume-11 Issue-1, May 2022
Mathematical models for optimization of economic and environmental effects in a two-echelon supply chain management system with stochastic deterioration, seasonal stochastic demand and carbon emission cost	Communicated to Operational Research - An International Journal (ORIJ) , Electronic ISSN: 1866-1505
A multi- retailer supply chain model with stochastic demand, backorder, variable production cost under aspect of carbon cap and trade	To be communicated