Modelling and Analysis of Some Issues in Sustainable Supply Chain Management

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CERTIFICATE FROM THE SUPERVISOR

This is to certify that the thesis entitled **"Modelling and Analysis of Some Issues in Sustainable Supply Chain Management"** submitted by Sri **Sushil Kumar Dey** who got his name registered on 8th November, 2016 (INDEX NO: 181/16/Maths. /25) for the award of Ph. D. (Science) degree of Jadavpur University, is absolutely based upon his own work under the supervision of **Prof. Bibhas Chandra Giri,** Department of Mathematics, Jadavpur University, Kolkata-700032 and that neither this thesis nor any part of it has been submitted for either any degree/diploma or any other academic award anywhere before.

11.02.2022

(Prof. Bibhas Chandra Giri)

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Dedicated to

My Family and Friends

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Chapter 1

Introduction

"The earth is a beautiful place and it will be unfair on the future generations to leave it exhausted and depleted for them."

Blog.mygov.in, 4th sept, 2019

From the beginning, the human race is struggling for existence against different odds. In the field of global business and due to several complexities, the harmful impacts are visible in our environment, our society. The growth of emerging markets in the 21st century is improving global living standards in a way that perhaps did not happen in the previous century. This growth put pressure on our environment, natural resources, climate change, environmental degradation which are going to be some serious threats in near future. Therefore, sustainable development is much needed for our present as well as future generations.

Maintaining the ecological balance has become increasingly clear for the supply chain through sustainable developments. Reusing, recycling, and remanufacturing are some of the best ways an industry can adopt as some prominent activities. Corporate social responsibility (CSR) is another self-regulating business model that helps a company to be socially accountable to its stakeholder, the public, and even itself. All aspects of society including economic, social, and environmental sustainability is, therefore, the backbone of growing industrialization. This thesis aims at sustainable moves of supply chains for the sake of the environment and society without neglecting the economic sustainability of an industry or business organisation.

1.1 Supply chain

A supply chain (SC) is considered as a network of organizations, activities, technologies, resources, and individuals involved in producing and selling a product or service. Thus, through the delivery of raw materials by a supplier, it starts and ends with the fulfillment of customer demand with products or services (see Fig. 1.1).

British logistician Keith Oliver first coined the term supply chain in public, and in 1982, he defined the supply chain management concept as follows:

"Supply chain management (SCM) is the process of planning, implementing, and controlling the operations of the supply chain with the purpose to satisfy customer requirements as efficiently as possible. Supply chain management spans all movement and storage of raw materials, work-in-process inventory, and finished goods from point-of-origin to point-of-consumption" (Oliver et al., 1982).

There is no unique definition of a supply chain. A formal definition of supply chain given by Chopra and Meindl (2001) is as follows:

"A supply chain consists of all stages involved, directly or indirectly, in fulfilling a customer request. The supply chain not only includes the manufacturers and suppliers, but also transporters, warehouses, retailers, and customers themselves".

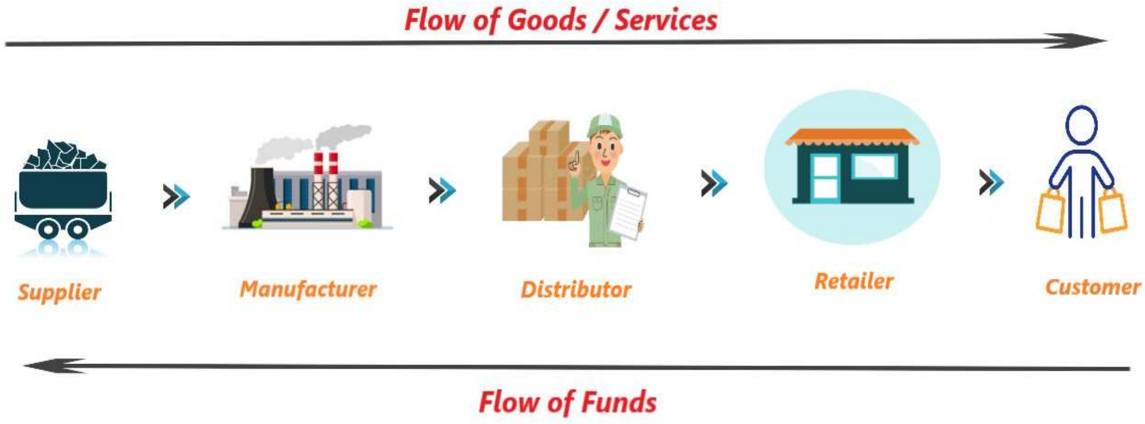

Fig. 1.1: *Supply chain network.*

1.2 Sustainable supply chain

Sustainability is the capacity for the earth's biosphere and human civilization to exist simultaneously. It is also described as the capability of being maintained at a steady level without exhausting natural resources or causing severe ecological damages. A sustainable supply chain is, therefore, the holistic view of a supply chain that serves environment-friendly customers in a better way.

Through sustainability, a company puts effort to consider environmental and human impact of their product's journey through the supply chain. The target remains to minimize environmental harmfulness from several factors like raw material consumption, excessive energy usage, carbon emission, waste production, etc. According to the Brundtland Commission report (1987), sustainable development was mentioned as meeting the need of the present without neglecting the ability of future generations to meet their own needs. The report outlined that the goals of economic and social development must be defined in terms of sustainability, in all countries - developed or developing, market-oriented or centrally planned. Thus, along with environmental consciousness, it targets to create a positive impact on society as a long-run business strategy. Hence sustainability is linked with corporate social responsibility because a socially responsible firm's goal is beyond environmental sustainability and it therefore includes fair trade, good employment practice, proper relation with the customers, moral brand value recognition among the locality, etc. Thus the linkage of sustainability is manifested in Elkington's (1994) 'triple bottom line' or TBL concept, which encircles the planet, profits and the people (see Fig. 1.2). The TBL thought thus ensures that firms should focus on maximizing economic value while ensuring that it does not neglect the environmental and social values for having an assurance of environmental security and proper health standards for living beings. This concept is widely accepted in firms, government and non-governmental organizations (NGOs).

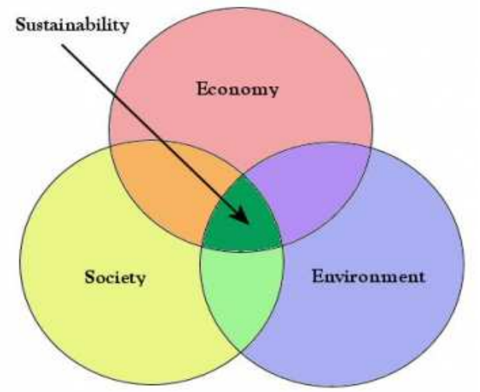

Fig. 1.2: *Dimensions of sustainability.*

1.3 Sustainability in supply chain management

The objective or focus of conventional supply chain management is to enhance sales of goods or services and also reduce cost for a smooth running of all activities. Sustainable supply chain management (SSCM) adds the goal of upholding environmental and social values in the pure economical view of a supply chain (see Fig. 1.3). In recent years, many businesses have focused on the unnecessary use of resources and not generating redundant wastes. That's why sustainability has emerged as a key corporate goal. Companies have started to measure the environmental and social impacts of their goods and services, from beginning to the end of their life cycles. The defined factors that are increased in a sustainable supply chain therefore can be divided into three categories:

Fig. 1.3: *Sustainability in SCM.*

- Improving financial performance of the supply chain *i.e.,* economic sustainability.
- Attracting customers who value environmental sustainability using eco-friendly products.
- Increasing the impacts for a long-run business industry through incorporating social sustainability.

1.3.1 Environmental sustainability

Over the last few decades, exploitation and degradation of our environment have gone up drastically at an alarming rate. In recent years customers, investors, and the government have put increasing pressure on companies to demonstrate greater environmental stewardship and social responsibility. Many industries are therefore prioritizing sustainability voluntarily or may be under governmental pressure. There are many different ways to use the environment sustainably such as- use of renewable energy, recycling of materials, crop rotation, etc. Recycling is one of the most effective ways an industry can adopt, through which used products or wastes are turned into new products. It consumes less energy and natural resources which is a big boost to our environment. Besides all these, recycling reduces water, land, air pollution that is generated by rejection or burning of waste. Recycling is still one of the best ways for green innovation and carbon footprint reduction. Today, a variety of companies have made a business out of recycling that includes local recycling centers, websites like eBay. Paper, plastic, metal, glasses, electronic devices are some very common and available recyclable products. Many apparel brands make clothes from recycled plastic bottles, collect used garments, repair them and then resell them as “upcycled” goods (Netsuite, 2021). Some of the European countries like UK and Germany have already achieved remarkable success through recycling. In Germany, almost 70% of plastic are recycled every year[*]. Companies like Kodak, Canon and HP are implementing recycling strategically and with success.

1.3.2 Economic sustainability

Economic sustainability is the capability of an economy to support a defined level of economic production indefinitely. It is an integral part of sustainability and means that we must use safeguard and sustain resources to create long-term sustainable values by optimal use, recovery, and recycling. In other words, we must conserve finite natural resources today so that future generations too can cater to their needs. Through proper planning and management, the supply chain entities should be encouraged to implement strategies that are economically beneficial. A supply chain needs proper material as well as information flow for economic stability. Channel coordination with contracts, Stackelberg leadership-followership games, etc. can enhance the economic growth of the SC members. The long-term economic sustainability of a supply chain cares about present and future values of natural resources, investments, consumption, markets, and the global economy. So economic sustainability is the pecuniary view of a supply chain or its entities without harming social and environmental aspects.

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[*] http://www.enviropedia.org.uk

1.3.3 Social sustainability

Social sustainability means the aspects of sustainability that relate to people and it's about ensuring that humans have what they need. A conventional supply chain directly or indirectly affects employees, workers, customers, or local communities and that's why it is essential to manage those influences proactively. The first six of the UN global compact's principles focus on the social dimension of sustainability[†]. The lack of social development can hamper business operations and growth while actions to achieve social sustainability may unlock new markets, attract business partners and improve company-community conflicts. Corporate social responsibility, donations for community development, preserving human rights are positive strategies for strengthening social sustainability. For example, using renewable energy sources by firms can reduce draughts as they require less water and energy to maintain. The ultimate goal of social sustainability provides a better quality of life promoting fairly distributed healthcare and supply chain's ethical issues.

Many companies have reported success in improving social sustainability measures like Unilever, the British multinational company that invested significant effort to help emerging economics like Brazil and India wrestle with poverty, water scarcity, and climate change. Walmart redesigned the milk-jug package which helped to use less material and this saved 10 to 20 cents a gallon, compared to old jugs. Starbucks is another example, where coffee growers are helped as social responsibility goals. The company realized that business growth could not sustain without a sustainable move. So they started coffee and farmer equity (CAFE) practices which enhanced sustainable production (Chopra et al., 2013).

1.4 Relevant topics

In this section, we will discuss some of the key areas connected to this study such as closed-loop supply chain, game theory in supply chain, and coordination contract. To manage sustainable supply chains in a better way these areas must be explored.

[†] https://www.unglobalcompact.org

1.4.1 Closed-loop supply chain (CLSC)

Closed-loop supply chains (CLSC) are the supply chain networks that include the return process of used products along with the production of new goods. Total network of a CLSC can be divided into two components: forward supply chain and backward or reverse supply chain (see Fig. 1.4).

Fig. 1.4: *Closed-loop supply chain.*

Forward supply chain

A traditional supply chain is called a forward supply chain when the manufacturer produces new products from fresh raw materials and sells them to customers. It is therefore an open-loop chain and used products do not return to the manufacturer. The forward supply chain becomes divergent in the sense that some raw materials may only be found in a rare few places in the world but end products must get into the homes or places of business of every customer. Customer demand dictates the rate of forwarding logistics, and inventory is kept at each stage to manage variances in that demand.

Backward or reverse supply chain

In a backward or reverse supply chain wastes are collected and then recycled to remanufacture new items. Reverse logistics must be convergent, collecting used products from many points and transporting them back to one or more manufacturing locations.

Members engage in the reverse supply chain for recapturing value by reuse of recycled materials and end-of-life products or for proper disposal following government regulations. The disposal of non-recyclable materials is directly related to waste management and environmental implications. Sometimes reverse supply chain becomes significant not only to remanufacture wastes but also for an incentive behind the plan of sustainability. For instance, as of 2018, 75% of NIKE products were recycled in the reverse chain innovation process. Canon, Adidas and HP are some of the other companies which incorporate the reverse supply chain.

1.4.2 Game theoretic approach in supply chain

Game theory (GT) has become an effective tool in supply chain analysis with multiple agents but conflicting objectives. It deals with decision-making in interactive supply chain optimization problems. Though many economists have worked in the past that can be considered as game-theoretic models, John Von Neumann and Oskar Morgenstern are formally credited as the father of modern game theory (Cachon and Netessine, 2006). GT was then developed by the concepts of Nash equilibrium, cooperative games, games with imperfect information, and many so. Though there are many concepts of game theory but in the context of supply chain management they can be viewed as cooperative or non-cooperative games.

Cooperative game theory deals with how a group of individuals interacts when only the payoffs are known. So it is a game between coalitions of players rather than between individuals. It therefore can be considered as a case of inter supply chain competition under intra supply chain cooperation.

Non-cooperative games deal with how supply chain members deal with one another to achieve their own goals. Each member tries to achieve the equilibrium point when the decision of one entity does not affect another. Such a case called Nash equilibrium was developed by John Nash (1950) and is considered to be the most significant development in game theory, after its invention.

A sequential-move based, simplest possible, non-cooperative dynamic game was introduced by German economist Stackelberg in 1934 and after his name, it is called

Stackelberg game. It is a leader-followership game where the leader moves first and then the follower reacts after him. In most of the literature of supply chain management, manufacturer-Stackelberg game is well known which means the manufacturer first takes its decision. In a duopoly retail market, Stackelberg games can further be classified into two subgames based on duopoly retailers' Cournot or Collusion decisions. For the Cournot game, the duopoly retailers individually take the decision like the Nash game, but in the case of the Collusion game, the retailers jointly act as single player. The later case is also referred to as the sub-supply game. Stackelberg game theoretic strategies are implemented for economic sustainability, especially in different market scenarios.

1.4.3 Supply chain coordination

The supply chain's optimal performance always needs a precise set of actions. Unfortunately, the members are not always focused in the best interest of the supply chain. Rather, they are concerned with the optimization of their own objectives. This self-effort often results in poor performance. However, optimal performance is achievable if the members coordinate with each other and their own objectives align with the supply chain's objective (Cachon, 2003).

Supply chain's optimality depends on the feasible set of strategies. When supply chain members individually make decisions, the model is called the decentralized supply chain model. But when all the players act as a single unit and their optimal desirability aligns with the whole SC, the model is termed the centralized supply chain model. Due to individual decisions in the decentralized model, the members may not always have economically satisfactory outcomes compared to the centralized games. In that situation, the members look for some agreements to enhance their profits and reach win-win situations. These mutual agreements are called coordination contracts.

Supply chain contracts are used in the business relationship between two or more independent participants of the supply chain, as tools for coordination. It diminishes the double-marginalization effects and reduces the risk of uncertainty in a stochastic market scenario. There are many types of contract for business scenarios among which some well-known are- revenue sharing, cost-sharing, buyback, two-part tariff, sales rebate, quality discount contact, etc. Besides all these simple contracts, some composite contracts are also available.

Coordination contracts can be signed between SC members for the fulfillment of one or more of the following objectives: performance improvement in terms of profit maximization or over/under stock cost reduction, facilitation of long-term relationships, and/or risk-sharing among the supply chain partners. Successful implementations of these coordination contracts help the SC members to sustain economically.

1.5 Significance of the study

Supply chain management is an indispensable part of a business's sustainability program. Knowing the levels of social, environmental, and economic impacts of a supply chain, the common goal of almost all industries is to lead our future generations toward a sustainable future. Government pressure, customer awareness, and economical views lead the companies to operate more sustainably than ever before. This doctoral study focuses on sustainable moves of SC under different circumstances like uncertain quality of wastes, stochastic market demand, duopoly retailers with different motives, dual-channel of waste recycling, etc. The significance and scope of the thesis are as follows:

- Closed-loop supply chain with dual-channel waste recycling is analyzed with game-theoretic approaches.
- In case of uncertain supply of waste for recycling, a backup supplier is considered.
- Remanufacturing and closed-loop as well as purely reverse supply chain with sustainability considerations provide the ground of the study.
- Stackelberg games under various scenarios are implemented in maintaining the economic balance of the SC and its entities.
- Social sustainability through corporate social responsibility in CLSC is addressed and coordination contracts for win-win outcomes are investigated.

1.6 Organization of the thesis

The doctoral study aims to develop mathematical models that focus on the development of SC in three different aspects of sustainability namely, environmental, social, and economic. The thesis consists of nine chapters. A brief description of sustainability and its role in supply chain management with several issues and considerations are given in **Chapter 1**. The rest of the thesis is organized as follows:

In **Chapter 2**, a brief review of sustainable development, corporate social responsibility, Stackelberg game, and coordination contracts are presented.

In **Chapter 3**, dual-channel recycling in a closed-loop supply chain (CLSC) which consists of one collector, one recycler, and one manufacturer is considered. In this chapter, Jafari et al.'s (2017) model is extended with a backup supplier considering the uncertainty of the collection of used products. The shortfall quantity of collection is met up by the backup supplier with the estimated fresh raw materials. Under various power structures or interactions of the supply chain entities, different game-theoretic models are developed. It is observed from the numerical study that, depending on the fractional part of the manufacturers requirements of recyclable wastes supplied by the collector, the performance of the supply chain increases compared to that of Jafari et al.'s (2017) model in the absence of the recycler. However, in the presence of the recycler, the whole supply chain's profit surpasses Jafari et al.'s (2017) profit for any amount of used product collection.

Chapter 4 considers a dual-channel closed-loop supply chain for waste recycling in a similar setting to the previous chapter. However, the customer demand is assumed here to be stochastic. Two different cases of recycling are investigated with centralized, decentralized, and fixed markup game strategies. Optimal results for the two game models are obtained through numerical examples. It is seen that ex-ante pricing commitment *i.e.,* fixed markup strategy is beneficial for the whole supply chain as well as the supply chain entities, compared to the decentralized policy. From the numerical study, it is also observed that when the recyclability degree of wastes increases, the expected total profit increases for the whole supply chain. A higher price sensitivity of customer demand leads to lower profit for the chain members.

Chapter 5 demonstrates a socially responsible closed-loop supply chain for waste recycling. To produce the finished product from wastes, two different cases of recycling is considered - either the manufacturer or the recycler does the recycling. The manufacturer makes effort to increase the demand for the finished product due to its corporate social responsibility (CSR). Centralized and manufacturer-Stackelberg game models are developed in each case and, for economic benefits, a joint revenue and cost-sharing contract is implemented. Optimal decisions are obtained analytically and also through a numerical example. It is seen that though the manufacturer bears an extra cost to put effort into increasing the demand, the supply chain members can reach a win-win situation through a suitable revenue and cost-sharing contract. It is further observed that recycling by the recycler is beneficial to the supply chain in comparison to the recycling done by the manufacturer itself. Sensitivity analysis depicts the overall performance of the supply chain with demand sensitive parameters.

In **Chapter 6**, a closed-loop supply chain consisting of a manufacturer, two suppliers and two competitive retailers is explored. One retailer sells manufactured products whereas the other retailer sells remanufactured products and takes up corporate social responsibility. One supplier supplies used products or cores for remanufacturing while the other supplier supplies fresh raw materials for manufacturing new products. The manufacturer sells both new and remanufactured products with different wholesale prices. The chapter analyzes the two competitive retailers' different game strategies when the manufacturer acts as the Stackelberg leader. It is shown that remanufacturing is a good policy to adopt for the whole supply chain, not only for economical benefits but also for environmental sustainability. Optimal decisions of the proposed closed-loop supply chain and its members are also supported by a numerical example. Finally, sensitivity analysis is carried out with respect to key model-parameters.

Chapter 7 studies a three-echelon closed-loop supply chain under sustainability consideration through remanufacturing of waste materials. Depending upon the quality, the collector collects the used products and forwards them to the manufacturer for remanufacturing. The collector offers a reward or incentive to consumers to influence them to return the used items. The shortfall amount of collected used items, if any, is met up by the supplier by supplying fresh raw materials. In three separate cases viz centralized, decentralized, and revenue-sharing contract, optimal incentives for end-customers and optimal profits of supply chain members are determined. The revenue-sharing contract is implemented in two different settings - one including the supplier and the other one excluding the supplier. The win-win outcome for the supply chain members is investigated and a specific range of the sharing parameter for the win-win outcome is obtained. Optimal results are supported by numerical analysis, and the sensitivity of the optimal results concerning key parameters is analyzed.

Chapter 8 investigates the impact of greening and promotional effort-dependent stochastic market demand on the remanufacturer's and the collector's profits when the quality of used products for remanufacturing is uncertain in a reverse supply chain. The proposed model is developed to determine optimal profits of the collector, the remanufacturer, and the whole supply chain. Both the centralized and the decentralized scenarios are considered. To motivate the collector through profit enhancement, the remanufacturer designs a cost-sharing contract. Through numerical examples and sensitivity analysis, the impacts of greenness and promotional effort on optimal profits are investigated. The results show that the remanufacturer gets benefited from greening and promotional effort enhancement. However, a higher value of minimum acceptable quality level decreases the profits of the remanufacturer and the collector. A cost-sharing contract coordinates the supply chain and improves the remanufacturer's and the collector's profits. Besides green innovation, remanufacturing mitigates the harmful effects of waste in the environment. The chapter considers environmental and economic sustainability in a reverse supply chain with a remanufacturer who remanufactures the used products collected by the collector. The quality of used products is uncertain, and customer demand is stochastic, green and promotional effort sensitive. These two types of uncertainty along with green, and promotional effort sensitive customer demand differ the current study from the existing literature.

In **Chapter 9**, an outline of overall conclusion of the works done in this doctoral study is given and some future research scopes are referred, which will target to explore more insights of sustainable supply chains.

Chapter 2

Review of literature

In this chapter, a brief literature review concerning some key issues in sustainable supply chain management that are relevant to this thesis is presented.

2.1 Sustainable development

In the current age, the sustainability consideration has become an emerging priority. Globally the standard of lifestyle is improving in a rapid manner. As a result, pressure is building on environment and natural resources. At least it has been clear that if supply chain does not become sustainable than it was before, world's environment resources will not be sufficient to maintain a healthy growth of our society in near future. World Business Council for Sustainable Development (WBCSD) defined sustainable development as the development which meets the need of the present without compromising the ability of the future generations to meet their own needs (Brundtland, 1987). Kim et al. (2014) defined sustainable supply chain with the view of supply chain profitability maximization considering both environmental and social responsibility. Many industries as well as Governments are giving attention to endure harmful impacts of supply chain with sustainability consideration. Walmart took a sustainable move by reducing and redesigning packaging to use less material.

Until now, different approaches have been proposed for sustainable development. These include green purchasing (Diabat and Govindan, 2011; Tian et al., 2014; Li et al., 2016), green manufacturing (Basiri and Heydari, 2017), design for environment (Kurk and Eagan, 2008; Arnette et al., 2014), pollution control (Aramyan et al., 2011; Sheu and Chen, 2012), remanufacturing (Seitz, 2007; Zhu et al., 2007) and recycling (Chen et al., 2016). Recycling plays a crucial role for better waste management system. An important factor involved in recycling is recyclability degree which implies the percentage of a material that can be used from one unit of the waste. Krikke et al. (2003) analyzed decisions concerning the recyclability of a product. A multi-criteria network model for a sustainable supply chain was developed by Nagurney and Nagurney (2010). Li and Li (2016) developed game theoretic model for sustainable supply chain under competition in product sustainability. Dong et al. (2016) discussed about investment on sustainability of a product under the centralized and decentralized scenarios. Recycling with environmental consideration was discussed by Yu and Solvang (2016). A carbon-sensitive study was considered by Alhaj et al. (2016). Jafari et al. (2017) studied waste recycling in a three-echelon supply chain model. Through different game models, they investigated the optimal decisions in the corresponding scenarios. Ageron et al. (2012) proposed an empirical study for sustainable supply chain management. All three aspects of sustainability (social, economical and environmental) were discussed by Govindan et al. (2014).

Reverse supply chain is another significant network to explore its impacts on sustainability. It pushes the firms not only for remanufacturing but also as an incentive for economically sustainable outcomes of the SC. Heydari et al. (2017) studied a two-stage reverse supply chain where the retailer pays reward to customers to return end-of-life (EOL) products. A competitive collection of waste procurement was presented by Liu et al. (2017). They considered three different dual channels (retailer-manufacturer, manufacturer-third party, and retailer-third party) for collection of used products and showed that manufacturer-retailer dual collecting model is the best option for the manufacturer. Li et al. (2017) examined Government mechanism for increasing the efficiency of collecting channel. Their study demonstrates that both Government and the collector can implement appropriate mechanisms to control or utilize the informal collection channel under different circumstances. A pharmaceutical reverse supply chain with customer incentive was studied by Weraikat et al. (2016). When medicines remain leftover and sold or donated to underdeveloped countries, a proper coordination between the producer and 3PL (third part logistics) is responsible to collect unwanted medicines from customer zones. A brief and systematic review of 198 surveys on sustainability in supply chain between 1995 to 2018 was published by Martins and Pato (2019). Dey and Giri (2022) developed a three-echelon sustainable reverse supply chain with quality uncertainty for recoverable wastes.

2.2 Corporate social responsibility

Bowen and Johnson (1953) first proposed the concept of corporate social responsibility (CSR) and pointed out that business enterprises should not only look for economic interests but also they should pay attention to their behavior. CSR here means nothing but corporate behaviors that display the social and ethical responsibility of a business firm. A good social reputation may improve a companys relation with external factors such as investors, bankers, suppliers and customers. They may also attract the employees or increase current employees motivation and morality as well as their commitment and loyalty to the firm, which in turn may improve financial outcomes. CSR also has internal benefits because the implementation of CSR initiatives can lead to decreased operating costs and increased revenue from grants and incentives. For example, companies which adopt environmental initiatives to reduce waste, reuse materials, recycle, and conserve water and electricity, can frequently obtain grants and incentives for such initiatives and also have benefits, which may help to enhance their financial performance through material efficiency and energy and waste minimization (Branco and Rodrigues, 2006). However, resources are not productive on their own and can only be a source of competitive advantage if they are used by firms to perform their activities. Hence, building good relations with primary stakeholders is susceptible of leading to increased financial returns because it assists firms in developing valuable intangible assets (resources and capabilities), which can be sources of competitive advantage because such assets can differentiate a firm from its competitors. On the other hand, pursuing social issues that are not directly related to the relationship with primary stakeholders may not create such advantages, because participating in social issues is something which can be easily copied by competitors. Thus, one can infer that social responsibility activities can pay off, as long as they are in the interest of the firms primary stakeholders (Taleizadeh et al., 2019). Carroll (1979) described in his CSR pyramid model the reasons behind meeting companys corporate responsibilities. Due to various pressures from the customers as well as governments about the impacts of behavior of a firm, they are forced to implement CSR in a range of selected areas. Thats why many leading brands like Adidas, Wal-mart and Nike have been compelled to incorporate CSR in their supply chains (Amaeshi et al., 2008). As economic responsibility is the strongest desired outcome of a business enterprise, demand plays a vital role in balancing the economic as well as social and environmental considerations of the firm (Elkington, 1998). According to Carroll (2015), the most important attention of the public is to look after environmental sustainability, quality of the product, welfare of the employees, etc. CSR, therefore, has a direct impact on the images of the business partners. It affects customers demand for products (Ma et al., 2017). Jamali (2007) started the core of CSR to resolve and capture the most important concerns of the public. In terms of the impact of CSR on the supply chain, Meng et al. (2012) suggested that bearing CSR could help the enterprises to win much more market share and increase the competitiveness of the supply chain due to CSR consideration. Song et al. (2016) analyzed the influence of CSR consciousness of the supply chain members on the supply chain's decisions and found that a stronger CSR awareness is more the customer surplus resulted in, and a moderate CSR awareness could improve the overall performance of the supply chain. Modak et al. (2014) used consumer surplus to represent CSR and obtained the optimal pricing decisions for decentralized and centralized cases. Dai et al. (2017) analyzed the purchasing decision of the manufacturer and showed that the manufacturer would undertake CSR or not that depends on the pricing difference between the supplier and the customers' willingness to pay. Many studies on supply chain with CSR have taken into account benefit distribution, price discount and cost-sharing strategy. In terms of profit distribution, Guo et al. (2011) proposed Nash equilibrium, Stackelberg game and cooperative games. They pointed out that revenue sharing contract could effectively give Pareto improvement of the supply chain. In terms of cost-sharing, Hsueh and Chang (2008) discussed the coordination of a supply chain with CSR. Panda (2014) explored the coordination of a socially responsible manufacturer-retailer supply chain in two cases - CSR manufacturer and CSR retailer. He found that the CSR manufacturer's pure profit is negative above a threshold value of CSR. Liu, Quan, Xu and Forrest (2019) studied the influence of government subsidy on the decisions of the supply chain with CSR. They exploited a three-echelon Stackelberg supply chain and analyzed the optimal decisions of the supply chain. In a competitive duopoly retail market, the corporate social responsibility was proposed by Dey and Giri (2021).

2.3 Stackelberg game in supply chain

Stackelberg game is used when one firm has greater brand equity or power than other firms and is better known as the leader and the other firms as followers. In Stackelberg game problem, the leader observes reactions of followers and then decides his own best decisions (Zerang et al., 2018). In reality, in most of the cases, the manufacturer has more market power and can influence the decisions of other supply chain members. For instance, giant manufacturers such as General Motors and Toyota act as channel leaders and offer contracts to other members (Sane-Zerang et al., 2020). So, in the manufacturer Stackelberg game model, the manufacturer has strong channel power rather than other members, and he acts as the leader in the supply chain (Jafari et al., 2017; Zerang et al., 2018; Sane-Zerang et al., 2020). In order to describe the autonomy of the supply chain members and enhance their profits in supply chain, Stackelberg games are studied by a vast group of researchers.

The original Stackelberg game model is a sequential quantity choice game in a homogeneous product market. Later, it was extended to both quantity and price choice game. Dastidar (2004) proved that generally quantity Stackelberg games are less competitive than price Stackelberg games. Based on leadership structure, a significant amount of researches have done on Stackelberg games, specially manufacturer Stackelberg games (Chen et al., 2013; Szmerekovsky and Zhang, 2009; Viswanathan and Piplani, 2001). Supply chain with manufacturer as the leader and multiple retailers as follower was proposed by Yang and Zhou (2006) and Yu et al. (2009). Almehdawe and Mantin (2010) studied Vendor Managed Inventory (VMI) with only one manufacturer but multiple retailers. They analyzed both manufacturer dominance and one retailer dominance structures. Based on the numerical analysis, they showed that retailer dominance in general results in higher supply chain efficiency. A different type of supply chain was modeled by Zhao et al. (2014) by implementing two competitive manufacturer but one common retailer for two substitutable products. They analyzed both manufacturer Stackelberg and retailer Stackelberg games. Different Stackelberg games between two manufactures were proposed by Zhao et al. (2017); when one manufacturer uses dual-channel, he/she gets the least profit due to loss from online channel that occurs due to lowest demand resulted from the higher direct price. Joint advertising and pricing decisions was derived by Yue et al. (2013) and Hong et al. (2015). Their main findings were that advertising strongly influences channel members' pricing strategies, used product collection decisions and profits. Li et al. (2017) developed a retailer Stackelberg game for selling a single product with brand differentiation. They showed that the retailer always prefers integrating backward with the manufacturer with higher efficiency of producing and selling the product. For the cooperation between partners in a closed-loop supply chain with remanufacturing, a Stackelberg game framework was considered by Tang et al. (2020).

2.4 Supply chain coordination

Supply chain's optimal decisions always need a feasible set of strategies. Though in a decentralized game the entities have individual goals to optimize their decisions, the game actually results in poor result compared to the centralized game. However, the game can produce a better result when the desires of entities align with the whole supply chain's desire (Cachon, 2003). When self or independent decisions fail to get desired level of outcome, the members need an agreement or a contract to outperform together. Among various contracts available in the supply chain literature, the wholesale price contract, the cost sharing contract, the revenue sharing contract, the quantity discount contract, the sales rebate contract, the buyback contract and the quantity flexibility contracts are mostly used. In a two-echelon supply chain if the wholesale contract fails then a revenue sharing contract can coordinate the supply chain (Xie et al., 2020). Again, sometimes a joint composite contract is beneficial to achieve pareto-improvement (Huang et al., 2020). Chen et al. (2021) used a combined contract of option and cost sharing to coordinate a VMI supply chain. Though there is a vast literature on contract for the forward supply chain, the reverse chain is not that much enriched. Zeng (2013) developed a reverse supply chain with a revenue sharing contract. Govindan and Popiuc (2014) studied reverse logistics for recovering personal computers. They found that profits of two and three-echelon supply chains could be increased by a revenue sharing contract. Hu et al. (2016) studied strategic recycling behaviour of consumers and showed that, with a two-stage price contract, a manufacturer could increase return quantity by offering direct incentive to the collector. They developed five typical contracts in total to coordinate the reverse supply chain (RSC) and found that subsidy contract is helpful to the manufacturer while the cost pulling contract is beneficial to the collector. Hu and Feng (2017) developed a one-supplier, one-buyer supply chain under demand and supply uncertainties, and found that under a revenue sharing contract if the wholesale price remains the same, the RS (revenue sharing) ratio for the supplier will be higher or the wholesale price will be higher when the revenue sharing for the supplier is kept the same. A two-echelon supply chain for deteriorating items with time varying demand was considered by Bai et al. (2017). To coordinate the supply chain, they studied revenue and promotional cost sharing contract and a two-part tariff contract. A multi-echelon supply chain was coordinated under production disruption by Giri and Sarker (2019). They considered both pairwise and spanning revenue sharing contracts. Through a combined contract, a vendor managed inventory was coordinated by Huang et al. (2019). Song and Gao (2018) examined a green supply chain with a revenue sharing contract and proposed that it could effectively improve the greening level of the products together with overall performance of the supply chain. Li et al. (2019) analyzed low carbon strategy under revenue and cost sharing contracts when incentives are given by the retailer to the manufacturer. They noticed that cost sharing and revenue sharing contracts can coordinate the supply chain whereas the corresponding bargaining does not work well. Zeng and Hou (2019) studied the procurement and coordination for a supply chain with stochastic demand in a reverse mobile phone supply chain. Saha et al. (2016) considered a CLSC and collection in the reverse channel with a reward driven return policy. The manufacturer considered discount policy through all three collecting channels. Zheng et al. (2017) analyzed a two-echelon reverse supply chain with incomplete information. The information was hidden through acquisition price, collection effort, wholesale and retail prices. To enhance profit allocation, they considered a two-part tariff contract. In a two-echelon reverse supply chain, consumer reward driven collection was discussed by Heydari et al. (2018). They assumed the manufacturer's remanufacturing capacity as a random variable. The manufacturer shared the capacity risk with the retailer through a revenue sharing contract. Heydari and Ghasemi (2018) extended the previous model by considering uncertain quality of returned items. They also used revenue sharing contract to coordinate the supply chain. The competition between two reverse supply chains was examined by Sadeghi et al. (2019). They considered one supply chain with traditional channel and the other one with dual channel. The willingness of return in each chain was considered as a fraction of self and cross discounts of the competitor. Chen et al. (2019) formulated a two-echelon reverse supply chain where the consumer's environmental awareness is considered through word of mouth effect. Both cooperative and non-cooperative game models were considered when the retailer did the recycling and the manufacturer did the remanufacturing. Collection activity in a supply chain with cap and trade regulation was considered by Kushwaha et al. (2020). From a central remanufacturing facility, they assumed collection capacity from several regions which are separated by quality of products, quantity of products, time of returns and distance of the regions from the centre. Through the buyback contract a supply chain with two competitive manufacturers producing substitutable products was coordinated by Wang et al. (2021). Peng et al. (2021) proposed a two-echelon supply chain and for coordination, they used a spanning revenue-sharing contract.

Chapter 3

Game theoretic analysis of a CLSC with backup supplier under dual channel recycling

3.1 Introduction

Due to rapid environmental degradation, climate change, population growth, and also consumption of natural resources, humanity is under a great threat now-a-days. So, many industries are giving much importance to sustainability for the sake of present and even for future generations. There are various ways available for sustainable development in practice such as green purchasing, reusing, remanufacturing, recycling, etc. (Grimmer and Bingham, 2013; Luchs et al., 2010). Among these approaches, recycling is one of the best ways an industry can adopt. This chapter[*] primarily aims at remanufacturing of the used items to preserve the environment as well as maintain the economical balance. Through these a significant amount of worth values of used products is procured and thus it reduces the harmful impact of wastes like carbon emission and environmental pollution. For example, electonic and electrical devices and compo-

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[*] This chapter is based on the work published in *Computers & Industrial Engineering* , 2019, vol. 129, pp. 179-191.

nents, large household appliances, cooling and freezing appliances, plastic wastes are being recycled by many companies. In UK there is a recycling rate of approximately 60% for iron and steel, most of which comes from scrap vehicles, cooker, fridges and other kitchen appliances and, Germany has the recycling rate approximately 70% for plastic (www.enviropedia.org.uk/Sustainability).

Taking small initiatives like usage of sustainable materials for buildings like recycled bricks and timber or even usage of eco-friendly things can make all the differences. Some industries like Hewlett Packard and Canon undertake the reverse logistics to obtain more profits. In this regard, Lalbakhsh (2012) studied the impact of recycling in sustainable development for developing countries. He discussed the use of different recycling processes such as rain recycling, green space recycling, urban space recycling, garbage recycling and energy recycling. In the process of recycling, used materials are collected from the consumers. In closed-loop supply chain (CLSC), there are various ways by which used items can be returned back from the end customers. Manufacturer himself/herself can collect the used products, the retailer or even a third party may be engaged for the collection. Dual channel for collection can also be implemented in supply chain. Sometimes it is seen that dual channel recycling outperforms single recycling channel (Huang et al., 2013).

However, due to highly competitive business market, it is not always possible for a supply chain entity to get optimal profit margin from the business by his/her own effort. Rather, it is beneficial to go for some agreement or decision making alternatives like leadership-followership power structure. In reality, this is very much rational. Many industries like Adidas and Dell cooperate with their respective raw material suppliers; Coca-Cola company ties up with a third party recycler (Giri et al., 2018). Intel always tries to improve its operations and minimize its impact on the environment. Texas Instruments makes significant investments to efficiently use, reuse, or recycle materials across its operations, and reduces its potential environmental impact by sourcing materials responsibly, as well as appropriately managing waste handling and disposal (www.forbes.com). However, the waste material collection is not always possible from the end customers (Velis, 2014). There are various reasons behind it, e.g., environmental disasters, delay in delivery, lack of human power, etc.

Several relevant questions that may occur regarding recycling of the wastes are as follows:

- What will be the manufacturer's decision when the collector fails to collect desired amount of used products?
- Does a third party recycling really have influences on recycling channel?
- What is the impact of a backup supplier when there is potential shortage at the recycler?
- How much is the effect of recyclability degree of waste in a reverse supply channel?

In order to find the answers of the above questions, we revisit Jafari's (Jafari et al., 2017) model by including a backup supplier that has the ability to supply fresh raw materials to meet any potential shortages at the recycler. We investigate the model by using different game theoretic approaches. The chapter thus concerns the economic as well as environmental aspects of the supply chain. Our aim is also to study the impact of backup supplier in the above mentioned closed-loop supply chain. From the study it can be observed that depending on the fractional part of the manufacturer's requirements of recyclable wastes supplied by the collector, the performance of the supply chain increases compared to that of Jafari et al.'s (2017) model in the absence of the recycler. However, in the presence of the recycler, the whole supply chain's profit surpasses Jafari et al.'s (2017) profit for any amount of used product collection.

3.2 Problem description and assumption

The proposed closed-loop supply chain consists of one manufacturer, one collector, one recycler, and one backup supplier. The manufacturer can get the recycled materials from the recycler to produce the finished product. He is also capable of recycling the wastes. Therefore, he can purchase the recyclable wastes directly from the collector to recycle and then produce the finished product. So, a dual channel is considered to receive recyclable waste and recycled material from the collector and the recycler, respectively (see Fig. 3.1). The manufacturer always tries to satisfy the customer's demand with the help of the collector or the recycler due to low purchasing cost of used item and environmental sustainability.

However, if the collector or the recycler fails to satisfy the manufacturer's demand, then the manufacturer purchases the estimated amount of fresh raw materials from the supplier at a high price to make up the shortfall. To make our model feasible, we assume the following:

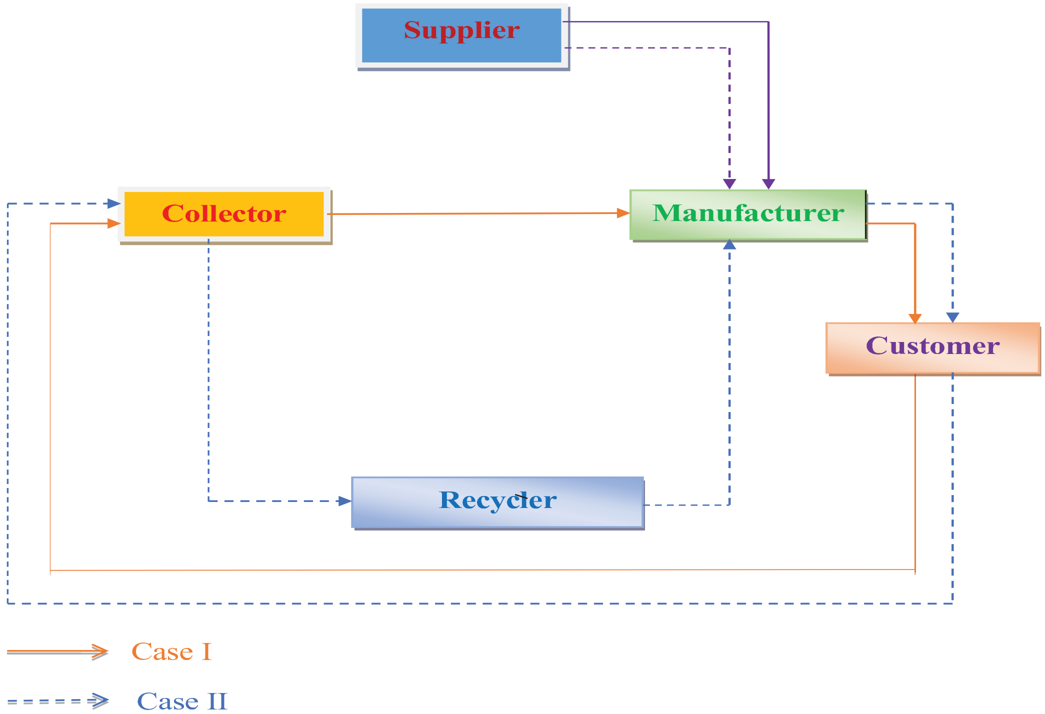


Fig. 3.1: *Material flow diagram.*

(i) $C_r < C_m$ *i.e.*, recycling by the recycler is less expensive than recycling by the manufacturer.

(ii) $\tau_m \geq \tau_c$ and $\tau_m \geq \tau_s$ *i.e.*, manufacturer's profit margin is more than other entities' profit margins.

(iii) We consider the manufacturer as the overall leader and other players as his followers. Since the decision power of the leader is higher than others, the manufacturer expects that his profit would be greater than those of the others. In this setting, the manufacturer accepts to contribute to the business if the proportion of his profit to each follower's profit is more than or equal to a constant value greater than 1, *i.e.,* when the manufacturer acts as the leader, and the collector, supplier, and recycler act as followers, the manufacturer contributes to the business if $\frac{\Pi_m}{\Pi_c} \geq \tau_m$, $\frac{\Pi_m}{\Pi_s} \geq \tau_m$ and $\frac{\Pi_m}{\Pi_r} \geq \tau_m$ where $\tau_m \geq 1$. Similar arguments hold for collector-led and supplier-led games.

(iv) The customer's demand for the finished product is a linear function of the price set by the manufacturer. We take, $D = a - \beta P_m$ where $a, \beta > 0$ and $P_m < \frac{a}{\beta}$. Here a denotes the market's total potential demand but the actual demand is D, and β represents the price sensitivity for customer demand.

The actual demand changes inversely with the manufacturer's retail price P_m, *i.e.,* consumers prefer inexpensive products. This type of demand function is very common in supply chain literature where the customer demand depends on retail price and demand decreases with the increment of retail price (Chen et al., 2013).

3.3 Notation

We use the following notations for parameters, retail prices, demands, and profit functions throughout the chapter:

Parameters		
C_c	:	unit collection cost of recyclable wastes to the collector
C_r	:	unit recycling cost of recyclable wastes to the recycler
C_m	:	unit recycling cost of recyclable wastes to the manufacturer
C_p	:	unit production cost of the finished product to the manufacturer.
θ $(0 < \theta < 1)$	:	recyclability degree of waste denoting the portion of waste that can be recovered and turned into new product
a	:	maximum possible demand faced by the manufacturer for the

finished product

γ (> 1) : quantity of recycled materials required to produce one unit of the finished product

$\frac{\gamma}{\theta}$: quantity of recyclable waste required to produce one unit of the finished product

β (> 0) : price sensitivity of the customer's demand

μ ($0 < \mu \leq 1$) : fractional part of the manufacturer's requirement of recyclable wastes supplied by the collector

λ ($0 < \lambda < 1$) : fractional part of the manufacturer's requirement of recycled materials supplied by the recycler

Prices:

P_d : price charged by the collector to the manufacturer for one unit of recyclable waste

P_c : price charged by the collector to the recycler for one unit of recyclable waste

P_r : price charged by the recycler to the manufacturer for one unit of recycled material

P_m : price charged by the manufacturer to the customers for one unit of finished product

P_s : price charged by the supplier to the manufacturer for one unit of fresh raw material

Demands:

D : customer demand of the finished product at the manufacturer

D_m : manufacturer's demand of recyclable wastes/recycled materials

D_c : manufacturer's demand of recyclable wastes meet up completely or partially by the collector

D_r : manufacturer's demand of recycled materials meet up completely

		or partially by the recycler
D_s	:	manufacturer's demand of fresh raw materials meet up by the backup supplier
Profits:		
Π_c	:	collector's profit
Π_r	:	recycler's profit
Π_s	:	supplier's profit
Π_m	:	manufacturer's profit
Π_{cr}	:	profit obtained from the co-ordination established between the collector and the recycler

3.4 Model formulation and analysis

We develop the proposed model under the following two cases:

Case I: The manufacturer collects the recyclable wastes from the collector, recycles by himself and then produces the finished product. If there is any shortfall of recyclable waste, he purchases the estimated amount of fresh raw materials from the backup supplier to make up the shortfall.

Case II: The manufacturer gets the recycled materials from the recycler to produce the finished product. If there is any shortfall of recycled material, he purchases the estimated amount of fresh raw materials from the backup supplier to make up the shortfall.

3.4.1 Case I: The manufacturer gets recyclable wastes from the collector

Here, we consider the situation where the collector may or may not satisfy the manufacturer's demand of recyclable wastes. In case of any shortfall of recyclable waste, the manufacturer purchases the required amount of fresh raw materials from the backup supplier to make up the shortfall. Therefore, we have, in this case

$$\begin{aligned} D_m &= \frac{\gamma D}{\theta} \\ D_c &= \frac{\gamma D}{\theta}\mu, \qquad 0 < \mu \leq 1 \\ D_s &= \gamma D(1-\mu) \end{aligned}$$

When $\mu = 1$, the manufacturer's demand of recyclable waste is completely meet up by the collector. This situation is investigated by Jafari et al. (2017).

The profit functions of the collector, the supplier and the manufacturer are given by

$$\Pi_c = (P_d - C_c)D_c \tag{3.1}$$

$$\Pi_s = P_s D_s \tag{3.2}$$

$$\text{and} \quad \Pi_m = (P_m - C_p)D - (P_d + C_m)D_c - P_s D_s \tag{3.3}$$

respectively, where the subscripts c, s and m stand for the collector, the supplier and the manufacturer, respectively.

3.4.1.1 Stackelberg game

In this game setting, the manufacturer acts as the overall leader, and the collector and the supplier act as the followers. The manufacturer first fix the price of the finished product. Then, the followers decide their individual prices. Based on the higher decision power of the followers, we further consider three gaming structures - collector-led game, supplier-led game and Nash game.

(A1) Collector-led game

In this game model, the decision power of the collector is considered to be higher than the supplier. So, the collector sets his price first and then the supplier makes his decision. The model is formulated as follows:

$$\begin{cases} L_1\text{: } \max\ \Pi_m = (P_m - C_p)D - (P_d + C_m)D_c - P_s D_s \\ L_2\text{: } \max\ \Pi_c = (P_d - C_c)D_c \\ L_3\text{: } \max\ \Pi_s = P_s D_s \end{cases} \tag{3.4}$$

Using $\dfrac{\Pi_m}{\Pi_c} \geq \tau_m$ we get

$$P_d \leq \frac{(P_m - C_p)\theta - C_m\gamma\mu - P_s\gamma(1-\mu)\theta + C_c\tau_m\gamma\mu}{\gamma\mu(\tau_m + 1)}$$

Since Π_c is linearly increasing in P_d, the optimal value of P_d is the highest of its possible values. Therefore, we have

$$P_d(P_s, P_m) = \frac{(P_m - C_p)\theta - C_m\gamma\mu - P_s\gamma(1-\mu)\theta + C_c\tau_m\gamma\mu}{\gamma\mu(\tau_m + 1)} \tag{3.5}$$

For the collector-led game, using the relation $\dfrac{\Pi_c}{\Pi_s} \geq \tau_c$, we have

$$\frac{(P_d - C_c)D_c}{P_s D_s} \geq \tau_c$$

Now replacing the values of D_s and D_c, we get

$$(P_d - C_c)\mu \geq P_s\tau_c\theta(1-\mu)$$

So, using equation (3.5), we have

$$\begin{aligned}(P_m - C_p)\theta - (C_m + C_c)\gamma\mu &\geq \gamma\theta\tau_c P_s(1-\mu)(\tau_m + 1) + P_s\gamma\theta(1-\mu) \\ &= P_s\theta\gamma(1-\mu)(\tau_c + \tau_m\tau_c + 1)\end{aligned}$$

which implies

$$P_s \leq \frac{(P_m - C_p)\theta - (C_m + C_c)\gamma\mu}{\theta\gamma(1-\mu)(1 + \tau_m\tau_c + \tau_c)}$$

Since Π_s is linearly increasing in P_s, the optimal price decided by the supplier is

$$P_s(P_m) = \frac{(P_m - C_p)\theta - (C_m + C_c)\gamma\mu}{\theta\gamma(1-\mu)(1 + \tau_m\tau_c + \tau_c)} \tag{3.6}$$

Substituting equation (3.6) in equation (3.5), we get

$$P_d(P_m) = \frac{(P_m - C_p)\theta\tau_c - C_m\gamma\mu\tau_c + C_c\gamma\mu(1 + \tau_m\tau_c)}{\gamma\mu(1 + \tau_m\tau_c + \tau_c)} \tag{3.7}$$

Using equations (3.6) and (3.7), we see that

$$\frac{\partial^2\Pi_m}{\partial P_m^2} = -2\beta\left(\frac{\tau_m\tau_c}{1 + \tau_m\tau_c + \tau_c}\right) < 0$$

since $\tau_m,\ \tau_c,\ \beta > 0$. This implies that Π_m is concave with respect to P_m. Hence, optimizing Π_m with respect to P_m, we derive the optimal value of P_m as

$$P_m^* = \frac{(a+\beta C_p)\theta + (C_m + C_c)\beta\gamma\mu}{2\beta\theta} \tag{3.8}$$

We now obtain the optimal prices P_s^* and P_d^* from equations (3.6) and (3.7), respectively.

(A2) Supplier-led game

In this game model, the decision power of the supplier is considered to be higher than the collector. Here, the supplier sets his price first and then the collector makes his decision. The model can be easily formulated as given below:

$$\begin{cases} L_1\text{: max } \Pi_m(P_m) & = & (P_m - C_p)D - (P_d + C_m)D_c - P_sD_s \\ L_2\text{: max } \Pi_s(P_s) & = & P_sD_s \\ L_3\text{: max } \Pi_c(P_d) & = & (P_d - C_c)D_c \end{cases} \tag{3.9}$$

Using $\dfrac{\Pi_m}{\Pi_s} \geq \tau_m$, we obtain after some calculations

$$P_s \leq \frac{(P_m - C_p)\theta - (P_d + C_m)\gamma\mu}{\theta\gamma(1-\mu)(\tau_m + 1)}$$

Since Π_s is linearly increasing in P_s, the optimal value of P_s is given by

$$P_s(P_d, P_m) = \frac{(P_m - C_p)\theta - (P_d + C_m)\gamma\mu}{\theta\gamma(1-\mu)(\tau_m + 1)} \tag{3.10}$$

From the supplier-led game, using the inequality $\dfrac{\Pi_s}{\Pi_c} \geq \tau_s$, we have

$$\frac{P_sD_s}{(P_d - C_c)D_c} \geq \tau_s$$

Now, using D_s and D_c, we get

$$P_s\gamma(1-\mu)\theta \geq (P_d - C_c)\tau_s\gamma\mu$$

So, from equation (3.10), we have

$$(P_m - C_p)\theta - C_m\gamma\mu + C_c\gamma\mu\tau_s(\tau_m + 1) \geq P_d\gamma\mu(\tau_m\tau_s + \tau_s + 1)$$

which implies that

$$P_d \leq \frac{(P_m - C_p)\theta - C_m\gamma\mu + C_c\gamma\mu\tau_s(\tau_m + 1)}{\gamma\mu(1 + \tau_m\tau_s + \tau_s)}$$

Again the collector's profit Π_c is linearly increasing in P_d. Therefore, the optimal price of the collector can be obtained as

$$P_d(P_m) = \frac{(P_m - C_p)\theta - C_m\gamma\mu + C_c\gamma\mu\tau_s(\tau_m + 1)}{\gamma\mu(1 + \tau_m\tau_s + \tau_s)} \tag{3.11}$$

Substituting equation (3.11) in (3.10), we get

$$P_s(P_m) = \frac{(P_m - C_p)\theta\tau_s - (C_m + C_c)\gamma\mu\tau_s}{\theta\gamma(1 - \mu)(1 + \tau_m\tau_s + \tau_s)} \tag{3.12}$$

Using equations (3.11) and (3.12) while optimizing Π_m with respect to P_m, we get the optimal value of P_m as

$$P_m^* = \frac{(a + \beta C_p)\theta + (C_m + C_c)\beta\gamma\mu}{2\beta\theta} \tag{3.13}$$

Putting the value of P_m^* in equations (3.11) and (3.12), we can obtain P_d^* and P_s^*, the optimal prices of the collector and the supplier, respectively.

Now, we use the symbols CL, SL and N to correspond to the collector-led game, supplier-led game, and Nash game, respectively.

Proposition 3.1 *When the manufacturer meets his requirements from the collector and the supplier, the following relationships among the profits of the entities hold:*

(i) $\Pi_c^{SL} \leq \Pi_c^{N} \leq \Pi_c^{CL}$

(ii) $\Pi_s^{CL} \leq \Pi_s^{N} \leq \Pi_s^{SL}$

(iii) $\Pi_m^{N} \leq \Pi_m^{CL} \leq \Pi_m^{SL}$, *for* $\tau_s \geq \tau_c$

(iv) $\Pi_m^{N} \leq \Pi_m^{SL} \leq \Pi_m^{CL}$, *for* $\tau_s \leq \tau_c$

Proof: *(i)* We have

$$\begin{aligned} \Pi_c &= (P_d - C_c)D_c \\ \Pi_c^{SL} &= (P_d - C_c)D_c = \frac{[(P_m - C_p)\theta - (C_m + C_c)\gamma\mu]}{\gamma\mu(1 + \tau_s + \tau_m\tau_s)} \\ \Pi_c^{CL} &= \frac{\tau_c[(P_m - C_p)\theta - (C_m + C_c)\gamma\mu]}{\gamma\mu(1 + \tau_c + \tau_m\tau_c)} \end{aligned}$$

Therefore,

$$\frac{\Pi_c^{CL}}{\Pi_c^{SL}} = \frac{\tau_c + \tau_s(\tau_m\tau_c + \tau_c)}{1 + \tau_m\tau_c + \tau_c} \ \geq 1, \quad \text{since} \ \ \tau_m, \tau_c \ \geq 1$$

Hence, $\Pi_c^{CL} \geq \Pi_c^{SL}$. (a)

In Nash-game model, we have

$$\Pi_c^{N} = \frac{(P_m - C_p)\theta - (C_m + C_c)\gamma\mu}{\gamma\mu(2 + \tau_m)}$$

Therefore,

$$\frac{\Pi_c^{CL}}{\Pi_c^{N}} = \frac{2\tau_c + \tau_m\tau_c}{1 + \tau_m\tau_c + \tau_c} \ \geq 1, \quad \text{since} \ \ \tau_c \ \geq 1$$

So, $\Pi_c^{CL} \geq \Pi_c^{N}$. (b)

Therefore, from the inequalities (a) and (b), we have $\Pi_c^{SL} \leq \Pi_c^{N} \leq \Pi_c^{CL}$.

The proof of part (ii) is similar to part (i). Hence it is omitted.

Proof of (iii):

Substituting the value of P_m in the profit functions of the manufacturer in different game models, we have

$$\begin{aligned}
\Pi_m^{N} &= \Big(\frac{\tau_m}{\tau_m + 2}\Big)(a - \beta P_m)\frac{(P_m - C_p)\theta - (C_m + C_c)\gamma\mu}{\theta} \\
\Pi_m^{CL} &= \Big(\frac{\tau_m\tau_c}{1 + \tau_m\tau_c + \tau_c}\Big)(a - \beta P_m)\frac{(P_m - C_p)\theta - (C_m + C_c)\gamma\mu}{\theta} \\
\Pi_m^{SL} &= \Big(\frac{\tau_m\tau_s}{1 + \tau_m\tau_s + \tau_s}\Big)(a - \beta P_m)\frac{(P_m - C_p)\theta - (C_m + C_c)\gamma\mu}{\theta}
\end{aligned}$$

Therefore, for $\tau_c = \tau_s$, we have $\Pi_m^{CL} = \Pi_m^{SL}$ (c)

Again

$$\begin{aligned}
\frac{\tau_m}{\tau_m + 2} &= \frac{\tau_m\tau_c}{\tau_m\tau_c + \tau_c + \tau_c} \\
&\leq \frac{\tau_m\tau_c}{1 + \tau_m\tau_c + \tau_c}, \quad \text{since} \quad \tau_c \geq \ 1
\end{aligned}$$

Therefore, we have $\Pi_m^{N} \leq \Pi_m^{CL}$ (d)

and also $\Pi_m^{SL} > \Pi_m^{CL}$ for $\tau_s > \tau_c$, (e)

$$\text{since} \ \frac{\tau_m\tau_s}{1 + \tau_m\tau_s + \tau_s} \geq \frac{\tau_m\tau_c}{1 + \tau_m\tau_c + \tau_c}$$

Hence from the relations given by $(c), (d)$ and (e) we get, $\Pi_m^N \leq \Pi_m^{CL} \leq \Pi_m^{SL}$.

Proof of part (iv) is similar to part (iii). Hence it is omitted.

From (i) and (ii) we observe that, among the collector and the supplier, the collector gains higher profit when he acts as the leader and the supplier as the follower. Similarly, the supplier gains higher profit than the collector in the supplier-led game model. As the manufacturer acts as the overall leader, his profit is always greater than the other players. Also, the manufacturer earns more profit in supplier-led and collector-led models rather than the Nash game model.

(A3) Nash game

The Nash game is considered when the collector and the supplier have similar decision powers and they set their prices independently and simultaneously. We formulate the model as given below:

$$\begin{cases} L_1\text{: max } \Pi_m(P_m) & = & (P_m - C_p)D - (P_d + C_m)D_c - P_sD_s \\ L_2\text{: max } \Pi_c(P_d) & = & (P_d - C_c)D_c \\ L_2\text{: max } \Pi_s(P_s) & = & P_sD_s \end{cases} \tag{3.14}$$

Since Π_c is linearly increasing in P_d, in Nash game model, using the inequalities $\dfrac{\Pi_m}{\Pi_c} \geq \tau_m$ and $\dfrac{\Pi_m}{\Pi_s} \geq \tau_m$, we get

$$\begin{aligned} P_d &\leq \frac{(P_m - C_p)\theta - C_m\gamma\mu - P_s\theta\gamma(1-\mu) + C_c\tau_m\gamma\mu}{\gamma\mu(\tau_m + 1)} \\ P_s &\leq \frac{(P_m - C_p) - (P_d + C_m)\dfrac{\gamma\mu}{\theta}}{(\tau_m + 1)\gamma(1-\mu)} \end{aligned}$$

Now, we see that Π_c and Π_s both are linearly increasing in P_d and P_s, respectively. So, the optimal prices of the collector and the supplier are

$$\begin{aligned} P_d &= \frac{(P_m - C_p)\theta - C_m\gamma\mu - P_s\theta\gamma(1-\mu) + C_c\tau_m\gamma\mu}{\gamma\mu(\tau_m + 1)} \\ P_s &= \frac{(P_m - C_p) - (P_d + C_m)\dfrac{\gamma\mu}{\theta}}{(\tau_m + 1)\gamma(1-\mu)} \end{aligned}$$

respectively, which give

$$(P_m - C_p)\theta = P_s\theta\gamma(1-\mu) + C_m\gamma\mu - \tau_m\gamma\mu C_c + P_d\gamma\mu(\tau_m + 1) \tag{3.15}$$

and

$$(P_m - C_p) = (P_d + C_m)\frac{\gamma\mu}{\theta} + P_s\gamma(1-\mu)(\tau_m + 1) \tag{3.16}$$

From equations (3.15) and (3.16), after simplification, we get

$$P_s = \frac{\mu(P_d - C_c)}{\theta(1-\mu)} \tag{3.17}$$

Using equations (3.16) and (3.17) we get,

$$P_d = \frac{(P_m - C_p)\theta - C_m\gamma\mu + C_c\gamma\mu(\tau_m + 1)}{\gamma\mu(\tau_m + 2)} \tag{3.18}$$

Using equation (3.18), we get the value of P_s as

$$P_s = \frac{(P_m - C_p)\theta - (C_m + C_c)\gamma\mu}{\theta\gamma(1-\mu)(\tau_m + 2)} \tag{3.19}$$

Now, using the values of P_d and P_s, we optimize Π_m with respect to P_m and get the optimal value of P_m as

$$P_m^* = \frac{(a + \beta C_p)\theta + (C_m + C_c)\beta\gamma\mu}{2\beta\theta} \tag{3.20}$$

Then the optimal values P_d^* and P_s^* can be obtained from equations (3.18) and (3.19), respectively. The optimal profits of the collector, the supplier and the manufacturer can be obtained from equation (3.14).

Proposition 3.2 *The collector and the supplier share equal profit for Nash-game model.*

Proof: For the Nash game model, using P_d the collector and the supplier's profits are

$$\begin{aligned}\Pi_c^N &= (P_d - C_c)D_c \\ \Pi_s^N &= P_s D_s\end{aligned}$$

Now, using P_d and P_s we get

$$\begin{aligned}(P_d - C_c)D_c &= (P_d - C_c)D\frac{\gamma\mu}{\theta} \\ &= \frac{D((P_m - C_p)\theta - (C_m + C_c)\gamma\mu)}{\theta(\tau_m + 2)} \\ P_s D_s &= \frac{D((P_m - C_p)\theta - (C_m + C_c)\gamma\mu)}{\theta(\tau_m + 2)}\end{aligned}$$

3.4.1.2 Centralized game

The centralized case is defined when all the players act jointly as one player, *i.e.,* there is no chance of preference of leadership or followership. The total profit of the system depends on the finished product's final price set by the manufacturer who sells the product to the end customers. In this case, the total profit is given by

$$\Pi_{cms}(P_m) = (P_m - C_p)(a - \beta P_m) - \frac{\gamma\mu}{\theta}(C_m + C_c)(a - \beta P_m)$$

It is easy to see that Π_{cms} is strictly concave in P_m as $\dfrac{\partial^2 \Pi_m}{\partial P_m^2} = -2\beta < 0,$ since $\beta > 0$. Therefore, optimizing Π_{cms} with respect to P_m, we get

$$P_m^* = \frac{(a + \beta C_p)\theta + (C_m + C_c)\beta\gamma\mu}{2\beta\theta}$$

3.4.2 Case II: The manufacturer gets recycled materials from the recycler

Here, we consider the situation where the recycler may or may not satisfy the manufacturer's demand of recycled materials. In case of any shortfall of recycled material, the manufacturer purchases the required amount of fresh raw materials from the backup supplier to make up the shortfall. Therefore, in this case, we have

$$\begin{aligned} D_m &= \gamma D \\ D_r &= \frac{\gamma D}{\theta}\lambda \\ D_s &= \gamma D(1 - \lambda) \end{aligned}$$

The profits of the collector, recycler, supplier, and manufacturer are given respectively by

$$\Pi_c = (P_c - C_c)D_r \tag{3.21}$$

$$\Pi_r = D_r P_r \theta - (P_c + C_r)D_r \tag{3.22}$$

$$\Pi_s = P_s D_s \tag{3.23}$$

$$\Pi_m = (P_m - C_p)D - D_r P_r \theta - P_s D_s \tag{3.24}$$

3.4.2.1 Stackelberg game

In this case, the manufacturer buys the recycled materials from the recycler, whereas the recycler collects the recyclable materials from the collector. The backup supplier supplies fresh raw materials to satisfy the manufacturer's unsatisfied demand. So, four players in total are involved in this situation. Here also, we consider the manufacturer as the overall leader and the other players as the followers. However, it is cumbersome to consider the gaming structures like supplier-led game, collector-led game, etc. as discussed in Subsection 3.4.1.1. Instead, we consider Nash game and sub-centralized game.

(B1) Nash game

Here, the decision powers of the collector, the recycler and the supplier are the same and they set their prices independently and simultaneously. The model can be formulated as given below:

$$\begin{cases} L_1\colon \max\ \Pi_m(P_m) = (P_m - C_p)D - D_r\theta P_r - P_sD_s \\ L_2\colon \max\ \Pi_r(P_r) = P_rD_r\theta - (P_c + C_r)D_r \\ L_2\colon \max\ \Pi_s(P_s) = P_sD_s \\ L_2\colon \max\ \Pi_c(P_c) = (P_c - C_c)D_r \end{cases} \tag{3.25}$$

Since Π_c is linearly increasing in P_c, the optimal value of P_c is the highest of its possible values. Using the relation $\frac{\Pi_m}{\Pi_c} \geq \tau_m$, we find the optimal price of the collector as

$$P_c = \frac{(P_m - C_p)\theta - \gamma\lambda P_r\theta - P_s\gamma(1-\lambda)\theta + C_c\gamma\lambda\tau_m}{\gamma\lambda\tau_m}$$

Proceeding similarly, we find the optimal prices set by the supplier and the recycler as

$$P_s = \frac{(P_m - C_p) - \gamma\lambda P_r}{\gamma(1-\lambda)(\tau_m + 1)}$$

and

$$P_r = \frac{(P_m - C_p)\theta + \gamma\lambda(P_r + C_r)(\tau_m + 1)}{\theta\gamma\lambda(\tau_m + 2)}$$

respectively. Now, substituting P_s and P_r in Π_m and then optimizing, we get the optimal value of P_m as

$$P_m^* = \frac{(a + \beta C_p)\theta + (C_r + C_c)\beta\gamma\lambda}{2\beta\theta} \tag{3.26}$$

Using this optimal value P_m^*, we can find P_c^*, P_s^* and P_r^*, the optimal prices of the collector, the supplier and the recycler, respectively from the above derivations. Then the profits of all the players involved in the supply chain can be obtained from (3.25).

Proposition 3.3 *When the recycling cost of the recycler is less compared to the manufacturer, customer demand will increase in Case II compared to Case I for the Nash game model.*

Proof: In Nash game of Case I, the optimal retail price

$$(P_m^*)_I = \frac{(a+\beta C_p)\theta + (C_m + C_c)\beta\gamma\mu}{2\beta\theta}$$

and in Case II

$$(P_m^*)_{II} = \frac{(a+\beta C_p)\theta + (C_r + C_c)\beta\gamma\lambda}{2\beta\theta}$$

Now if $C_r < C_m$ then $(P_m^*)_{II} < (P_m^*)_I$ which implies that $a - \beta(P_m^*)_I < a - \beta(P_m^*)_{II}$ i.e., $D_I < D_{II}$. Hence the result.

Note: When $\lambda = 1$ *i.e.,* when the recycler supplies all the recycled materials required by the manufacturer, there is no contribution of the backup supplier; or, we can say that the manufacturer does not need any extra amount of fresh raw materials from the backup supplier. In this situation, P_c reduces to

$$P_c = \frac{(P_m - C_p)\theta - \gamma\theta P_r + C_c\gamma\tau_m}{\gamma\tau_m}$$

Now, since Π_r is linearly increasing in P_r, the condition $\dfrac{\Pi_m}{\Pi_r} \geq \tau_m$ gives the optimal value of P_r as

$$P_r^* = \frac{2\theta(P_m - C_p) + (C_r + C_c)\gamma\tau_m}{\theta\gamma(\tau_m + 2)}$$

Then we have,

$$P_c^* = \frac{(P_m - C_p)\theta + \gamma C_c(\tau_m + 1) - \gamma C_r}{\gamma(\tau_m + 2)}$$

Substituting the values of P_c and P_r in the profit function of the manufacturer and proceeding in a similar fashion, we obtain the optimal value of P_m as

$$P_m^* = \frac{(a+\beta C_p)\theta + (C_r + C_c)\beta\gamma\lambda}{2\beta\theta}$$

These prices and the corresponding profits of the collector, the recycler and the manufacturer are same as derived by Jafari et al. (2017).

(B2) Sub-centralized game

Here, we assume that the collector and the recycler jointly act as one player, and we define this situation as sub-centralized one. Without any loss of generality, we ignore the collector's price P_c as it is an internal decision between the collector and the recycler.

The game model can be formulated as given below:

$$\begin{cases} L_1\text{: } \max\ \Pi_m(P_m) &= (P_m - C_p)D - (P_d + C_m)D_c - P_sD_s \\ L_2\text{: } \max\ \Pi_{cr}(P_r) &= P_rD_r\theta - (C_c + C_r)D_r \\ L_2\text{: } \max\ \Pi_s(P_s) &= P_sD_s \end{cases} \tag{3.27}$$

Proceeding similarly as in the previous game models, the optimal prices of the recycler, the supplier and the manufacturer can be obtained respectively as

$$P_r^* = \frac{2\theta(P_m - C_p) + \gamma\lambda(C_r + C_c)(\tau_m + 1)}{\theta\gamma\lambda(\tau_m + 3)} \tag{3.28}$$

$$P_s^* = \frac{(P_m - C_p)\theta - \gamma\lambda(C_r + C_c)}{\theta\gamma(1 - \lambda)(\tau_m + 3)} \tag{3.29}$$

$$P_m^* = \frac{(a + \beta C_p)\theta + (C_r + C_c)\beta\gamma\lambda}{2\beta\theta} \tag{3.30}$$

3.4.2.2 Centralized game

The total profit, in this case, is the sum of the profits of the collector, the recycler, the supplier and the manufacturer, which is given by

$$\Pi_{crsm} = (P_m - C_p)(a - \beta P_m) - \frac{\gamma\lambda}{\theta}(C_r + C_c)(a - \beta P_m) \tag{3.31}$$

Optimizing Π_{crsm} with respect to P_m, we get $P_m^* = \dfrac{(a + \beta C_p)\theta + (C_r + C_c)\beta\gamma\lambda}{2\beta\theta}$

3.5 Numerical example

To illustrate the developed model numerically, we now consider the parameter-values C_p=5, $\theta = 0.7$, $\gamma = 1.3$, a =1000, $\beta = 1.3$, $C_c = 20$, $C_r = 10$, $C_m = 65$, $\mu = 0.5$, $\lambda = 0.5$, $\tau_m = 1.5$, $\tau_c = 1.2$ and $\tau_s = 1.3$ in appropriate units. For this data set, we obtain the optimal price and profit for each player in the developed game models in Cases I and II as shown in Tables 3.1 and 3.2, respectively.

Table 3.1: Optimal results under different game models in Case I.

Optimal results	Collector-led game	Supplier-led game	Nash game	Centralized game
P_d^*	130.703	134.241	125.431	-
P_s^*	131.789	161.248	150.616	-
P_m^*	426.58	426.58	426.58	426.58
Π_m^*	68684.7	70031.5	65414.0	-
Π_c^*	45789.8	35913.6	43609.3	-
Π_s^*	38158.2	46687.7	43609.3	-
Π_{cms}^*	-	-	-	1,52,633

Table 3.2: Optimal results under different game models in Case II.

Optimal results	Nash game	Sub-centralized game	Centralized game
P_c^*	108.113	-	-
P_r^*	294.609	294.609	-
P_s^*	125.876	125.876	-
P_m^*	401.044	401.044	401.044
Π_m^*	58743.3	58743.2	-
Π_c^*	39162.2	-	-
Π_s^*	39162.2	39162.2	-
Π_r^*	39162.2	-	-
Π_{cr}^*	-	78324.4	-
Π_{crsm}^*	-	-	1,76,230.0

From the results given in Tables 3.1 and 3.2, the following observations are made:

(i) The manufacturer earns higher profit in Case I for all games except sub-centralized game. So, rather than getting recycled materials from the recycler, the manufacturer should try to get recyclable wastes from the collector as much as possible.

(ii) The collector's profit in Case I is more than that in Case II for Nash game model. So, he should also give his maximum effort in collection of recyclable wastes for higher profit when all the players have same decision powers.

(iii) We also observe that, for Nash game model, the manufacturer gets higher profit in Case I compared to Case II. This is quite obvious because, in Nash game model, all the players have similar decision powers and they set their decisions simultaneously.

In the special cases of $\mu = 1$ and $\lambda = 1$ (Jafari et al., 2017), the optimal profits of the closed-loop supply chain are obtained as 119499 and 163147, respectively. This indicates that the profit of the proposed closed-loop supply chain improves compared to that of Jafari et al. (2017). When $\lambda = 1$, the manufacturer's profit in Nash game is obtained as 69920 which is higher than that of our proposed model in Case II. Due to insufficient stock of the recycler, the manufacturer has to buy some fresh raw materials from the supplier at a higher price, which results in decrease of its profit.

3.5.1 Comparison of our model with Jafari et al.'s (2017) model

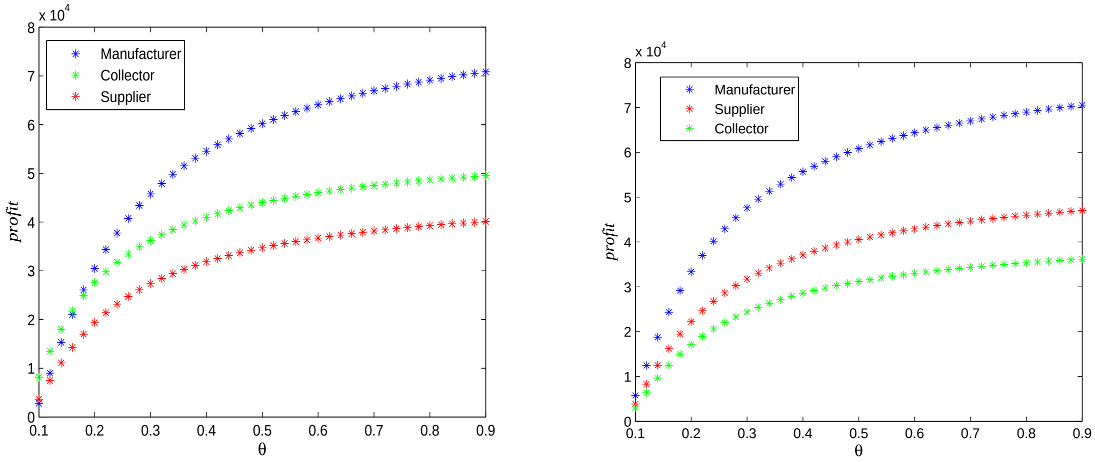
In this section, we discuss the optimal results of the model when there is uncertainty of full recovery of used items for recycling. We keep the same parameter-values as taken by Jafari et al. (2017) and obtain the optimal results for different values of μ and λ as shown in Tables 3.3 and 3.4. In absence of the backup supplier, *i.e.,* when $\mu = 1$ and $\lambda = 1$, Jafari et al. (2017) found that the total profits of the supply chain are 119499.0 and 163147.0, respectively. In our model, for Case I, we see that the supply chain's total profit is greater than that of Jafari et al.'s (2017) model when μ lies within the interval $(0.35, 0.40)$. So, in this case, our model outperforms Jafari et al. (2017) model due to inclusion of the backup supplier, or due to consideration of uncertainty of full recovery of the used products. However, as μ takes value ≥ 0.4, the total profit gradually decreases because of higher competition among the supply chain entities, which corresponds to higher retail price for the manufacturer resulting in lower customer demand for the finished product. For Case II, due to the presence of the recycler, the whole supply chain's profit increases as a result of low recycling price of the recycler and this helps the manufacturer to set lower retail price (401.04) compared to those of Case I (426.58) and Jafari et al.'s (2017) model (414.97). In this case, for any value of λ, the total profit of the supply chain is greater than Jafari et al.'s (2017) model. So, from the whole supply chain's point of view, it is beneficial to include a recycler and a backup supplier in dual channel recycling.

Table 3.3: Manufacturer's profit and total profit of the supply chain for different values of μ under various game models in Case I.

	Manufacturer's profit			
μ	Collector-led game	Supplier-led game	Nash game	Supply chain's profit
0.30	75159.0	76633.0	71580.0	167020.0
0.35	73513.0	74954.5	70012.5	163362.0
0.40	71885.0	73295.0	68462.0	159745.0
0.45	70276.0	71654.0	66929.5	156169.0
0.50	68685.0	70031.5	65414.0	152633.0
0.55	67112.0	68428.0	63920.0	149137.0
0.60	65557.0	66842.0	62435.0	145682.0
0.65	64020.0	65276.0	60972.0	142267.0
0.70	62502.0	63727.5	59526.0	138893.0
0.75	61002.0	62198.0	58097.0	135560.0
0.80	59520.0	60687.0	56686.0	132267.0
0.85	58056.0	59195.0	55292.0	129014.0
0.90	56611.0	57721.0	53915.0	125802.0
0.95	55183.5	56266.0	52556.0	122634.0

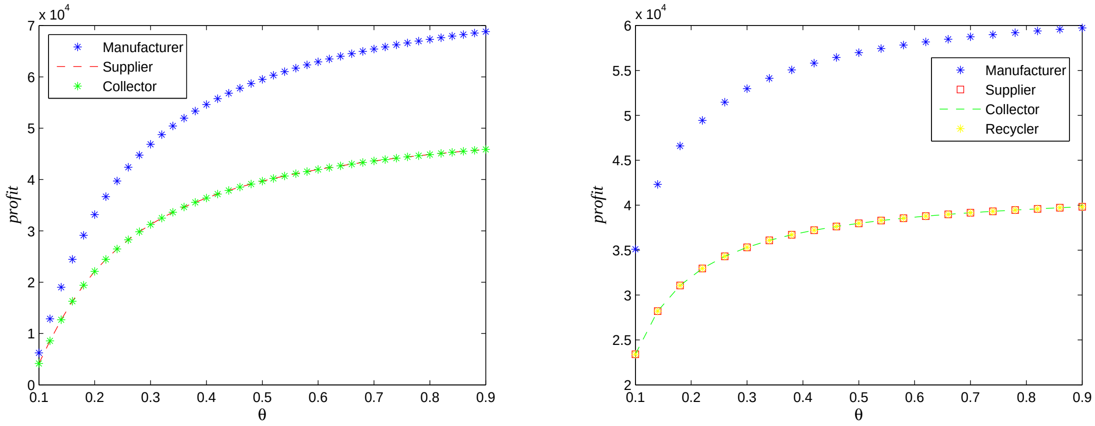
3.5.2 Sensitivity analysis

We now examine the sensitivity of the parameters θ, γ and β on the optimal results of different game models under two cases (Case I and Case II). In both the cases, we see that the optimal retail price P_m^* varies inversely with θ (from equations (3.8), (3.13), (3.20), (3.26) and (3.30)). Therefore, when θ increases, the optimal retail price decreases and it corresponds to higher customer demand. So, with the increment of θ, the gradient or slope of the profit curve increases for all the members (see Fig.



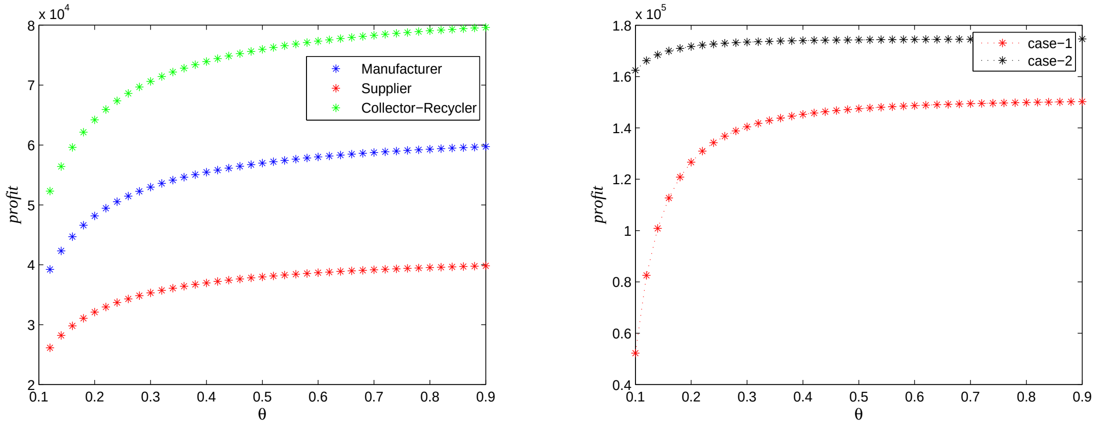
(a) *θ vs. profit (collector-led game)*

(b) *θ vs. profit (supplier-led game)*



(c) *θ vs. profit (Nash game in Case I)*

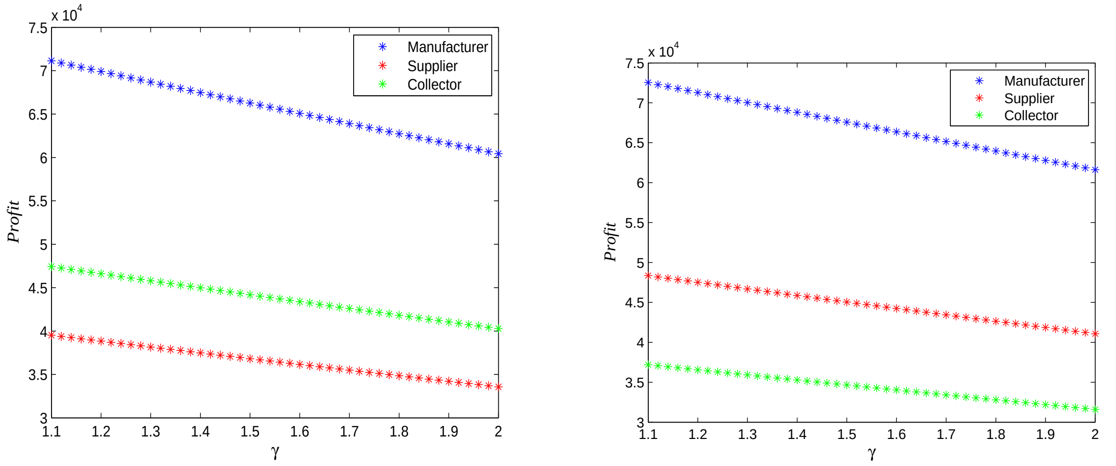
(d) *θ vs. profit (Nash game in Case II)*



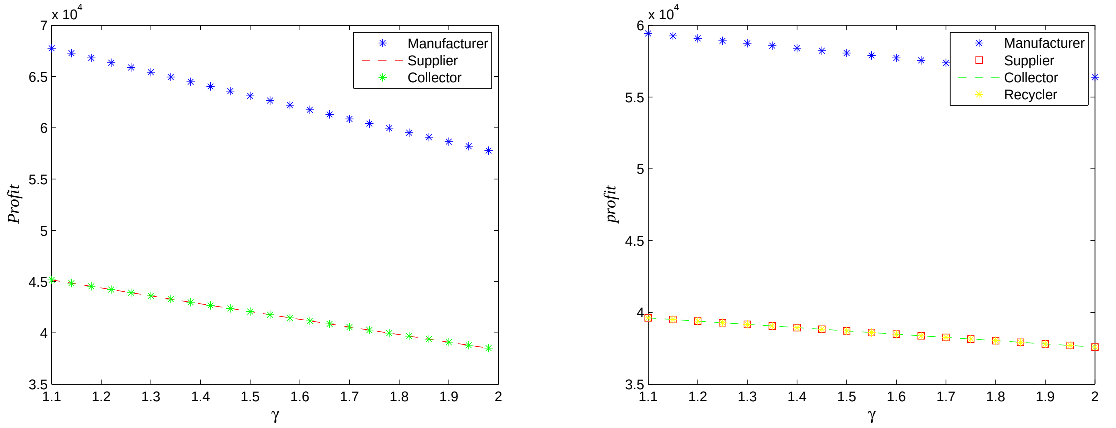
(e) *θ vs. profit (sub-centralized game)*

(f) *θ vs. profit (centralized game)*

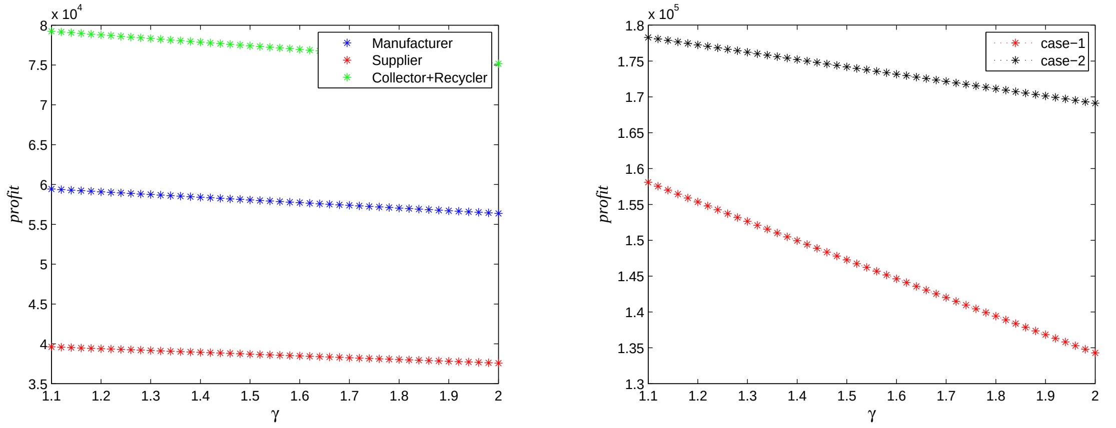
Fig. 3.2: *θ vs. profit under various games.*



(a) *γ vs. profit (collector-led game)* **(b)** *γ vs. profit (supplier-led game)*

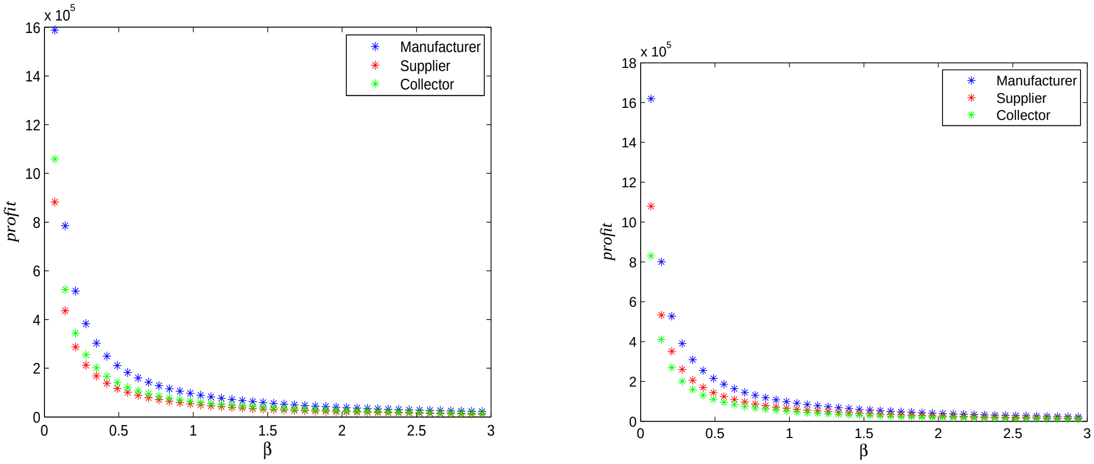


(c) *γ vs. profit (Nash game in Case I)* **(d)** *γ vs. profit (Nash game in Case II)*

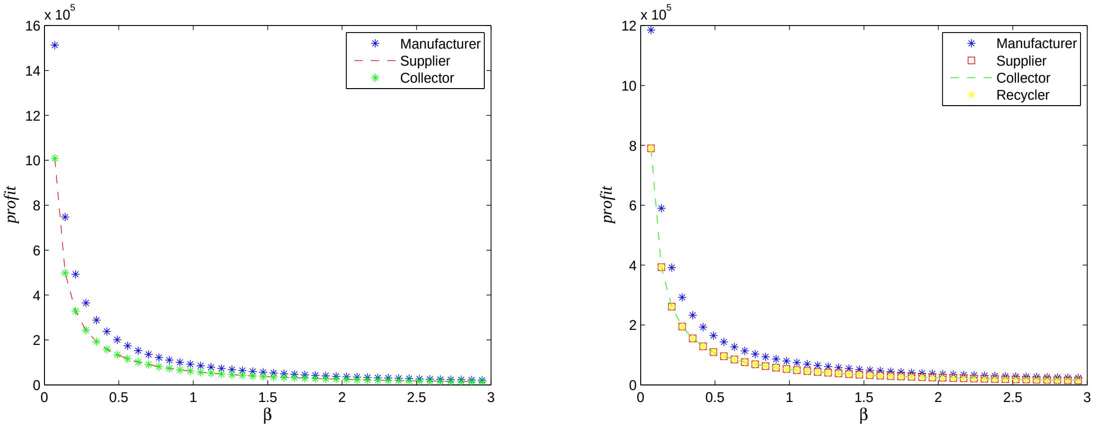


(e) *γ vs. profit (sub-centralized game)* **(f)** *γ vs. profit (centralized game)*

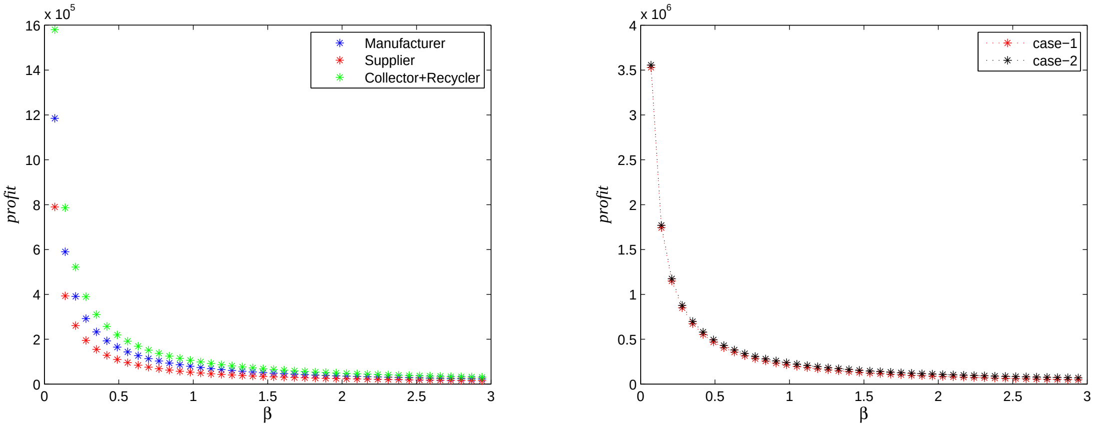
Fig. 3.3: *γ vs. profit under various games.*



(a) *β vs. profit (collector-led game)* **(b)** *β vs. profit (supplier-led game)*



(c) *β vs. profit (Nash game in Case I)* **(d)** *β vs. profit (Nash game in Case II)*



(e) *β vs. profit (sub-centralized game)* **(f)** *β vs. profit (centralized game)*

Fig. 3.4: *β vs. profit under various games.*

Table 3.4: Manufacturer's profit and total profit of the supply chain for different values of λ under various game models in Case II.

	Manufacturer's profit		
λ	Sub-centralized game	Nash game	Supply chain's profit
0.30	60535.0	60535.0	181604.0
0.35	60084.0	60084.0	180253.0
0.40	59636.0	59636.0	178907.0
0.45	59189.0	59189.0	177566.0
0.50	58743.0	58743.0	176230.0
0.55	58300.0	58300.0	174899.0
0.60	57858.0	57858.0	173573.0
0.65	57417.5	57417.5	172253.0
0.70	56979.0	56979.0	170937.0
0.75	56542.0	56542.0	169626.0
0.80	56107.0	56107.0	168321.0
0.85	55673.0	55673.0	167020.0
0.90	55241.5	55241.5	165725.0
0.95	54811.0	54811.0	164434.0

3.2). That means, the more recyclability degree of waste, the higher profits for all the members. In Nash game model in Case I, the supplier and the collector have equal decision power and they share equal profit. However, in collector-led game model, the collector earns more profit whereas in supplier-led game model, the supplier earns more profit. This is possible because of their dominant powers in decision making as leader.

From Fig. 3.3, we see that a higher value of γ leads to lower profit for all members in different game models. As γ increases, the quantity of the recycled materials needed to meet the customer's demand increases. Also the collector has to collect more wastes leading to higher cost. As a result, the profit of each member gradually decreases.

Fig. 3.4 reflects the effect of price sensitivity of customer's demand on the optimal results. As β increases, customer demand decreases resulting in lower profit for each player in different game models.

3.6 Conclusion

In this chapter, we have studied environmental sustainability through collecting and recycling of recyclable wastes from the end customers. Considering uncertainty of waste recovery from end customers, we have extended Jafari et al.'s (2017) paper by including a back up supplier that has the ability to supply fresh raw materials to meet any potential shortages of recyclable product. Here we have considered two cases: (i) The manufacturer collects the recyclable wastes from the collector and if there is any shortfall of wastes, then it is meet up by required amount of fresh raw materials from the backup supplier (Case I). (ii) The manufacturer gets the recycled materials from the recycler and if there is any shortfall of recycled materials, then it is meet up by the required amount of fresh raw materials from the backup supplier (Case II). To study the problem, centralized and manufacturer Stackelberg games have been considered. Manufacturer's Stackelberg game has been further studied under collector-led, supplier-led, and Nash game structures for Case I and sub-centralized, and Nash game structures in Case II. The main results of the study are as follows:

(i) When the manufacturer acts as the Stackelberg leader, various interactions among other players in each case do not have any effect on the manufacturer's pricing decision as well as the total profit of the supply chain.

(ii) Higher recyclability degree of waste leads to higher profit for each player.

(iii) Higher price sensitivity of customer's demand leads to lower profit for each player.

(iv) In the sub-centralized case, the total profit of the collector and the recycler is greater than that of the manufacturer. So, it is a good way to make the recyclable wastes usable collaboratively for environmental sustainability.

(v) The manufacturer's profit in Nash game in Case I is higher than that of Case II.

(vi) In the absence of recycler, depending on the fractional part of the manufacturer's requirements of recyclable wastes supplied by the collector, the performance of the supply chain increases compared to that of Jafari et al.'s (2017) model. But, in the presence of the recycler, the whole supply chain's profit surpasses Jafari et al.'s (2017) profit for any amount of used product collection.

In regard to the questions prescribed in the introduction section, we have the following answers derived from this study:

(i) The manufacturer prefers Case I for his individual profit improvement but from the point of view of whole supply chain, he prefers Case II where the recycling is done by the third party recycler.

(ii) When the recycling cost is low for the third party recycler, the optimal retail price for the manufacturer is lower than that of the other case *i.e.,* Case I (see Proposition 3.3). So then the total customer demand increases and the supply chain's profit also improves.

(iii) In the absence of the backup supplier, when the collector is able to collect the whole amount of required wastes, the total profit of the supply chain reduces in both the cases (see Subsections 3.4.2.1 and 3.5.1). Again, if the collector fails to collect the required amount of waste then the production cannot fulfill the customer demand and it results in lower profits of the supply chain as well as the supply chain entities.

(iv) Optimal price of the manufacturer varies inversely with the recyclability degree of waste. So, higher recyclability degree of waste results in higher customer demand and higher profit of the supply chain simultaneously.

Several future research works related to this topic can be done. One can modify the model to consider different coordination contracts between the players to obtain higher profits. Also, other demand pattern especially stochastic demand of the customer can be assumed for studying similar type of problems.

Chapter 4

Game theoretic models for a CLSC with stochastic demand and backup supplier under dual channel recycling

4.1 Introduction

In the previous chapter, a CLSC for dual-channel recycling and remanufacturing in a deterministic scenario is analyzed. This chapter[*] explores the stochastic environment of customer demand in that setting. In more realistic situation of stochastic customer demand, the process of recycling and remanufacturing remains a critical issue for the supply chain management. Besides environmental aspects, the economic sustainability of the supply chain entities need to be attained accordingly. Through remanufacturing, reusing, recycling as sustainable remedies, many of today's industries are trying to recover values from wastes and reduce the usage of fresh materials. For instance, plastic is the third highest manufacturing sector in the United States where over million of workers are working but for the conscience of environmental sustainability they

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[*] This chapter is based on the work published in *Decision Making: Applications in Management and Engineering*, 2020, vol. 3, issue 1, pp. 108-125.

have installed about $30,000$ recycling drop-off points nationwide and plastic film recycling is continuing to grow (www.nytimes.com). In case of waste recycling, some European countries like Sweden and Germany achieved great results of success even though recycling rate is lower than most of the other countries. Texas instruments make significant investments to reuse or recycle materials across its operations and reduces its potential environmental impact by sourcing materials and managing waste handling and disposal.

But handling the issue of uncertainty is a key decision for a supply chain decision maker. The parties involved in a supply chain therefore need to share the risk of uncertainty with different pricing policies or by implementing several contracts. A fixed markup over the wholesale prices is one of the simple but often used pricing policy. According to Liu et al. (2006) retail fixed markup (RFM) simply exists as an "agreement" more than a formal written code. Markup can also be defined as the difference between the wholesale price and the retail price. Through this policy a pareto-improving solution is desirable specially in a stochastic environment scenario.

In this chapter, for a multi-echelon closed-loop supply chain, we investigate the optimal decisions for pricing and corresponding profit for each player using game theoretic approach. Dual channel recycling (recycling by the manufacturer and recycling by the third party *i.e.,* recycler) is adopted in this chapter in two different models. The customer demand is assumed to be price dependent and stochastic. Optimal results for the two game models are obtained through numerical examples using two possible distribution of the market demand. It is studied therefore whether, ex-ante pricing commitment *i.e.,* fixed markup strategy is beneficial for the whole supply chain or the supply chain entities or not, compared to the decentralized policy. Numerical study and sensitivity analysis with respect to the key parameters are done.

The chapter is organised as follows: Notations and problem description are provided in the next section. Two different game models and their analytical results are discussed in Section 4.3. In Section 4.4, numerical demonstration of the two models along with sensitivity analysis with respect to some key parameters are given. Finally, the chapter is concluded with future research directions in Section 4.5.

4.2 Notation and problem description

In this chapter, for a multi-echelon closed-loop supply chain with price dependent stochastic customer demand, we investigate the optimal decisions for pricing and corresponding profit for each player using game theoretic approach. Dual channel recycling (recycling by the manufacturer and recycling by the third party *i.e.,* recycler) is adopted in this chapter in two different models. The notations that we have used in this chapter are as follows:

C_c : unit collection cost of recyclable wastes to the collector
C_r : unit recycling cost of recyclable wastes to the recycler
C_m : unit recycling cost of recyclable wastes to the manufacturer
C_c : unit collection cost of recyclable wastes to the collector
C_r : unit recycling cost of recyclable wastes to the recycler
C_s : unit procurement cost of recycled waste to the back-up supplier
C_m : unit recycling cost of recyclable wastes to the manufacturer
C_p : unit production cost of the finished product to the manufacturer
u : per unit shortage penalty cost of the manufacturer
v : per unit salvage value of the manufacturer
ϵ : random part of the demand
$\theta(0 < \theta < 1)$: recyclability degree of waste
$\gamma(> 1)$: quantity of recycled materials required to produce one unit of the finished product
q : quantity of finished items produced by the manufacturer
$\frac{\gamma}{\theta}$: quantity of recyclable waste required to produce one unit of the finished product
a : maximum possible demand faced by the manufacturer for the finished product
$b(> 0)$: price sensitivity of the customer's demand

λ $(0 < \lambda < 1)$	:	fractional part of the manufacturer's requirement of recycled materials supplied by the recycler
z	:	stocking factor for the stochastic demand
P_d	:	wholesale price charged by the collector to the manufacturer for one unit of recyclable waste
P_c	:	wholesale price charged by the collector to the recycler for one unit of recyclable waste
P_r	:	wholesale price charged by the recycler to the manufacturer for one unit of recycled material
P_s	:	wholesale price charged by the supplier to the manufacturer for one unit of fresh raw material
P	:	retail price charged by the manufacturer to the customers for one unit of finished product
D	:	customer demand of the finished product at the manufacturer
D_c	:	quantity of raw materials supplied by the collector to the recycler
D_r	:	quantity of recycled waste supplied by the recycler to the manufacturer
D_s	:	quantity of fresh raw materials supplied by the backup supplier to the manufacturer
Π_c	:	collector's profit
Π_r	:	recycler's profit
Π_s	:	supplier's profit
Π_m	:	manufacturer's profit
Π_{cr}	:	profit obtained from the co-ordination established between the collector and the recycler

The proposed closed-loop supply chain consists of one manufacturer, one collector, one recycler and one backup supplier. The manufacturer gets the recycled materials from the recycler and recyclable wastes from the collector. A dual channel is considered to receive recyclable wastes and recycled materials from the collector and the recycler, respectively (see Fig. 3.1). When the collector or the recycler fails to satisfy the manufacturer's need ($q\gamma$ units), the manufacturer takes help of a backup supplier. The manufacturer then buys fresh raw materials from the backup supplier at a high price to make up the shortfall. We assume that the customer's demand D is linear, price-dependent and random in nature. We take $D = a - bP + \epsilon$ where $a, b > 0$, $P < \frac{a}{b}$ and ϵ is the random part of the customer demand. Unit shortage penalty cost and salvage value are also incurred in the model setting as well.

4.3 Model development and analysis

Under the problem scenario mentioned above, we develop two models depending upon two different situations :

Model I: In this model, the collector collects the recyclable wastes from the end customers and then supplies to the manufacturer. The manufacturer first recycles the waste materials and then produces finished goods for the end customers. Any shortfall of wastes is meet up by a backup supplier by supplying fresh raw materials.

Model II: This model includes a recycler. The collector collects the wastes but the recycling is done by the recycler. The recycler recycles the wastes and then sends to the manufacturer for production of finished goods. Any shortfall of recycled material is meet up by a backup supplier by supplying fresh raw materials.

4.3.1 Model I: The manufacturer gets recyclable wastes from the collector

Here, we assume that the collector may or may not satisfy the manufacturer's demand of recyclable wastes. Natural disasters, communication problems or unavailability of resources may be the reasons behind this. We assume that the manufacturer estimates an amount of $q\gamma$ units of raw materials to produce q units of finished product. Let us suppose that the collector can supply $\frac{q\gamma\lambda}{\theta}$ units of wastes where $0 < \lambda \leq 1$ and θ $(0 < \theta < 1)$ is the recyclability degree of waste. So we have in this case

$$\begin{aligned} D_c &= \frac{q\gamma\lambda}{\theta}, \quad 0 < \lambda \leq 1 \quad \text{and} \\ D_s &= q\gamma(1-\lambda) \end{aligned}$$

When $\lambda = 1$, the manufacturer's demand for recyclable wastes is completely meet up by the collector and hence there is no need of any action from the back-up supplier. In this model, we develop three game theoretic approaches, viz. centralized game, decentralized game and fixed-markup game.

4.3.1.1 Centralized game

Since the market demand is stochastic, so sometimes the estimated inventory of the manufacturer may be less than the market demand or sometimes there may be some left over inventory in hand. We assume that the manufacturer sells the left over inventory in a secondary market with a salvage value v per unit. Unit shortage penalty cost is u. Then the expected profits of the manufacturer, the collector and the supplier are given by

$$\begin{aligned} \Pi_m &= E\Big[P \times min(q, D) - u(D-q)^+ + v(q-D)^+ - P_s D_s \\ &\quad -(P_d + C_m)D_c - qC_p\Big] \quad (4.1) \\ \Pi_c &= E\Big[(P_d - C_c)D_c\Big] \quad (4.2) \\ \Pi_s &= E\Big[(P_s - C_s)D_s\Big] \quad (4.3) \end{aligned}$$

respectively, where $X^+ = \max\ (X, 0)$ and the subscripts c, s and m stand for the collector, the supplier and the manufacturer, respectively. We replace $z = q - y(P)$, where z is the stocking factor on which the shortage or overage depends. Stocking factor is also sometimes called safety stock factor. Our objective is to find the optimal selling price, stocking factor rather than selling price and the stocking quantity. The expected total profit in the centralized game is given by

$$\begin{aligned}
\Pi &= \Pi_m + \Pi_c + \Pi_s \\
&= E\Big[P \times min(q,D) - u(D-q)^+ \ + \ v(q-D)^+ - P_s D_s \\
&\quad -(P_d + C_m)D_c + (P_d - C_c)D_c + (P_s - C_s)D_s\Big] \\
&= (P - C_p)\Big[y(p) + \mu\Big] - (C_p - v)\ \phi(z) - (P + u - C_p)\psi(z) \\
&\quad -(C_m + C_c)\Big[z + y(P)\Big]\frac{\gamma\lambda}{\theta}
\end{aligned}$$

where $\phi(z) = \int_0^z (z-t)f(t)dt$ and $\psi(z) = \int_z^\infty (t-z)f(t)dt$.

Now our problem is to maximize Π with respect to z and P *i.e.,* $\underset{z;P}{Maximize}\ \Pi$. We consider the first and second order partial derivatives of Π with respect to z and P. When z is fixed, we get the optimal value of P as

$$P^*(z) = \frac{a + bC_p + \mu + (C_m + C_c)\frac{b\gamma\lambda}{\theta} - \psi(z)}{2b}$$

and optimal value of z for a fixed P is

$$z^*(P) = F^{-1}\left[1 - \frac{(C_p - v) + (C_m + C_c)\frac{\gamma\lambda}{\theta}}{P + u - v}\right]$$

Corollary : *The profit function* Π *is concave in* z *for a given value of* P *and concave in* P *for a given value of* z.

Proof: The expected total profit in the centralized Model I is

$$\begin{aligned}
\Pi &= (P - C_p)\Big[y(p) + \mu\Big] - (C_p - v)\ \phi(z) - (P + u - C_p)\psi(z) \\
&\quad -(C_m + C_c)\Big[z + y(P)\Big]\frac{\gamma\lambda}{\theta}
\end{aligned}$$

Taking first and second order partial derivatives of Π with respect to z and P we get

$$\begin{aligned}
\frac{\partial \Pi}{\partial z} &= -(C_p - v) + (P + u - v)\Big[1 - F(z)\Big] - (C_m + C_c)\frac{\gamma\lambda}{\theta} \\
\frac{\partial^2 \Pi}{\partial z^2} &= -(P + u - v)f(z) < 0, \text{ since } v < P \\
\frac{\partial \Pi}{\partial p} &= (P - C_p)(-b) + (a - bP + \mu) - (C_m + C_c)\frac{\gamma\lambda}{\theta}(-b) - \psi(z) \\
\frac{\partial^2 \Pi}{\partial p^2} &= -2b < 0, \text{ since } b > 0.
\end{aligned}$$

Now we see that the second order partial derivatives $\dfrac{\partial^2 \Pi}{\partial z^2}$ and $\dfrac{\partial^2 \Pi}{\partial p^2}$ are negative. So, the profit function Π is concave in z and P respectively, and hence the result.

Proposition 4.1 *(i) The optimal retail price increases with the stocking factor and (ii) the optimal stocking factor of the manufacturer is also an increasing function of the retail price.*

Proof: (i) We have the optimal retail price

$$P^*(z) = \frac{a + bC_p + \mu + (C_m + C_c)\frac{b\gamma\lambda}{\theta} - \psi(z)}{2b}$$

Then,

$$\begin{aligned} \frac{dP^*}{dz} &= -\Big(\frac{1}{2b}\Big)\frac{d}{dz}\psi(z) \\ &= \Big(\frac{1}{2b}\Big)\int_z^\infty f(t)dt > 0, \text{ since } f(t) \geq 0 \text{ for all } t. \end{aligned}$$

(ii) For the optimal stocking factor z^* we have

$$F(z^*) = 1 - \frac{(C_p - v) + (C_m + C_c)\frac{\gamma\lambda}{\theta}}{P + u - v}$$

Differentiating partially with respect to p, we get

$$\begin{aligned} f(z^*)\frac{dz}{dP} &= \frac{(C_p - v) + (C_m + C_c)\frac{\gamma\lambda}{\theta}}{(P + u - v)^2}, \text{ which implies} \\ \frac{dz}{dP} &= \frac{1}{f(z^*)}\,\frac{(C_p - v) + (C_m + C_c)\frac{\gamma\lambda}{\theta}}{(P + u - v)^2} > 0, \text{ since } f(z) \geq 0. \end{aligned}$$

Proposition 4.2 *Under linear additive demand function, the supply chain's centralized solution is to set quantities z^*, P^* and to order $a - bP^* + z^*$ such that -*

(i) if $F(\cdot)$ is an arbitrary distribution, then the entire support must be searched to find z^*, and

(ii) if $F(\cdot)$ satisfies $2r(z)^2 + \frac{dr(z)}{dz} > 0$ where $r(z) = \frac{f(z)}{1 - F(z)}$ is the hazard rate, then z^* is the largest z satisfying the first order condition.

Proof: We have,

$$\frac{d\Pi}{dz} = -(C_p - v) + (P + u - v)[1 - F(z)] - (C_m + C_c)\frac{\gamma\lambda}{\theta}$$

Let, $R(z) = \dfrac{d\Pi}{dz}$. So,

$$\begin{aligned} \frac{dR(z)}{dz} &= \frac{d}{dz}\Big[\frac{d\Pi}{dz}\Big] \\ &= \frac{d}{dz}\big[-(C_p - v) - (C_m + C_c)\frac{\gamma\lambda}{\theta}\big] + \frac{d}{dz}\big[(P + u - v)(1 - F(z))\big] \end{aligned}$$

$$\begin{aligned} \text{where, } P(z) &= \frac{a + bC_p + \mu + \frac{b\gamma\lambda}{\theta} - \psi(z)}{2b} \\ &= P^0 - \frac{\psi(z)}{2b}, \text{ where } P^0 = \frac{a + bC_p + \mu}{2b} + (C_m + C_c)\frac{\gamma\lambda}{2\theta} \end{aligned}$$

$$\begin{aligned} \text{Now, } \frac{dR(z)}{dz} &= \frac{d}{dz}\big[(P^0 - \frac{\psi(z)}{2b} + u - v)(1 - F(z))\big] \\ &= \frac{1}{2b}\big[1 - F(z)\big]^2 - \big(P^0 + u - v - \frac{\psi(z)}{2b}\big)f(z) \\ &= \frac{f(z)}{2b}\big[2b(P^0 + u - v) - \psi(z) - \frac{1 - F(z)}{r(z)}\big], \\ \text{where } r(z) &= \frac{f(z)}{1 - F(z)}, \text{ the hazard rate.} \end{aligned}$$

$$\begin{aligned} \text{Again }, \frac{d^2R(z)}{dz^2} &= \frac{d}{dz}\Big(\frac{dR(z)}{dz}\Big) \\ &= \frac{dR(z)/dz}{f(z)}\,\frac{df(z)}{dz} - \frac{f(z)}{2b}\left[(1 - F(z)) + \frac{f(z)}{r(z)} + \frac{(1 - F(z))\big[\frac{dR(z)}{dz}\big]}{[r(z)]^2}\right] \end{aligned}$$

$$\text{Therefore, } \frac{d^2R(z)}{dz^2}\Big|_{\frac{dR(z)}{dz}=0} = \frac{-f(z)\big[1 - F(z)\big]}{2b[r(z)]^2}\Big[2[r(z)]^2 + \frac{dr(z)}{dz}\Big]$$

Now if $F(\cdot)$ be a probability distribution function which satisfies the condition

$$2r(z)^2 + \frac{dr(z)}{dz} > 0$$

then it follows that $R(z)$ is either monotone or unimodal implying that

$$\begin{aligned}
R(z) &= \frac{d\Pi\big[z, P(z)\big]}{dz} \text{ has at most two roots. Again,} \\
\underset{\lim z\to\infty}{R(z)} &= -(C_p - v) - (C_m + C_c)\frac{\gamma\lambda}{\theta} < 0
\end{aligned}$$

So, if $R(z)$ has only one root then it gives the maximum value of $\Pi(z, P)$ and if it has two roots then the larger of them corresponds to the maximum value of $\Pi(z, P)$.

4.3.1.2 Decentralized game

Here our objective is to maximize separately the expected profits of the manufacturer, the supplier and the collector, which are as follows:

$$\begin{aligned}
\Pi_m &= E\Big[P \times min(q, D) - u(D-q)^+ + v(q-D)^+ - P_sD_s - (P_d + C_m)D_c - qC_p\Big] \\
\Pi_c &= E\Big[(P_d - C_c)D_c\Big] \\
\Pi_s &= E\Big[(P_s - C_s)D_s\Big]
\end{aligned}$$

Now, we suppose that the profit margins for the players in this game are same *i.e.,* $P_d = \frac{P+C_c}{2}$ and $P_s = \frac{P+C_s}{2}$. Using these relations, we derive the optimal values of P and z as

$$\begin{aligned}
P^*(z) &= \frac{a + \mu + bC_p - \psi(z) + b\big[(P_d + C_m)\frac{\gamma\lambda}{\theta} + P_s\gamma(1-\lambda)\big]}{2b} \\
z^*(P) &= F^{-1}\left[1 - \frac{(C_p - v) + (P_d + C_m)\frac{\gamma\lambda}{\theta} + P_s\gamma(1-\lambda)}{P + u - v}\right]
\end{aligned}$$

4.3.1.3 Fixed markup strategic game

In the fixed markup strategic game, we assume that the supplier's wholesale price is $P_s = (1-\alpha_1)P$ where $0 < \alpha_1 < 1$ and the collector's wholesale price is $P_d = (1-\alpha_2)P$ where $0 < \alpha_2 < 1$ and that $0 < \alpha_1 \leqslant \alpha_2 < 1$. Using these relations in the profit functions

$$\begin{aligned}\Pi_m &= E\Big[P \times min(q, D) - u(D-q)^+ + v(q-D)^+ - P_s D_s - (P_d + C_m)D_c - qC_p\Big]\\ \Pi_c &= E\Big[(P_d - C_c)D_c\Big]\\ \Pi_s &= E\Big[(P_s - C_s)D_s\Big]\end{aligned}$$

we obtain the optimal price of the manufacturer as

$$P^* = \frac{a + \mu + bC_p - \psi(z) + b[C_m\frac{\gamma\lambda}{\theta} - C_s\gamma(1-\lambda)] - (z-a)[(1-\alpha_2)\frac{\gamma\lambda}{\theta} + (1-\alpha_1)\gamma(1-\lambda)]}{2b - 2b(1-\alpha_2)\frac{\gamma\lambda}{\theta} - 2b(1-\alpha_1)\gamma(1-\lambda)}$$

and optimal stocking factor as

$$z^* = F^{-1}\Bigg[1 - \frac{(C_p - v) + ((1-\alpha_2)P + C_m)\frac{\gamma\lambda}{\theta} + ((1-\alpha_1)P - C_s)\gamma(1-\lambda)}{P + u - v}\Bigg]$$

The optimal wholesale prices of the supplier and the collector are given by the relations $P_s^* = (1-\alpha_1)P^*$ and $P_d^* = (1-\alpha_2)P^*$.

4.3.2 Model II : The manufacturer gets recycled materials from the recycler

Here, we consider the situation where the collector supplies recyclable wastes to the recycler for recycling. However, the recycler may or may not satisfy the manufacturer's demand of recycled materials. In case of any shortfall of recycled materials, the manufacturer purchases the required amount of fresh raw materials from the backup supplier. Therefore, in this case we have,

$$\begin{aligned}D_c &= \frac{q\gamma}{\theta}\lambda, \quad 0 < \lambda \leq 1\\ D_r &= q\gamma\lambda\\ D_s &= q\gamma(1-\lambda)\end{aligned}$$

4.3.2.1 Centralized game

Like the previous model, here we assume that the manufacturer needs a total q units of finished product to satisfy customer demand. If the market demand exceeds the order quantity, shortage occurs and the shortage penalty cost of the manufacturer is then $u(D-q)$. On the other hand, if the market demand is less than the total quantity q, the leftover inventory is sold in a secondary market at a lower cost v. Then the total revenue from the leftover inventory is $v(q-D)$. Thus the expected profits of the manufacturer, the collector, the recycler and the supplier are given by

$$\begin{aligned}
\Pi_m &= E\Big[P\times min(q,D)-u(D-q)^{+}+v(q-D)^{+}-P_sD_s-(P_d+C_m)D_c \\
&\quad -qC_p\Big] && (4.4)\\
\Pi_c &= E\Big[(P_c-C_c)D_c\Big] && (4.5)\\
\Pi_r &= E\Big[P_rD_r-(P_c+C_r)D_c\Big] && (4.6)\\
\Pi_s &= E\Big[(P_s-C_s)D_s\Big] && (4.7)
\end{aligned}$$

where $X^{+}=$ max $(X,0)$ and the subscripts c, s, r and m stand for the collector, the supplier, the recycler and the manufacturer, respectively. The expected total profit in the centralized game is

$$\begin{aligned}
\Pi &= \Pi_m+\Pi_c+\Pi_r+\Pi_s\\
&= E\Big[P\times min(q,D)-u(D-q)^{+}+v(q-D)^{+}-P_sD_s-(P_d+C_m)D_c\\
&\quad +(P_d-C_c)D_c+(P_s-C_s)D_s\Big]\\
&= (P-C_p)[y(P)+\mu]-(C_p-v)\ \phi(z)-(p+u-C_p)\ \psi(z)\\
&\quad -(C_r+C_c)[z+y(P)]\frac{\gamma\lambda}{\theta}
\end{aligned}$$

where

$$\phi(z)=\int_0^z (z-t)f(t)dt \ \text{ and } \ \psi(z)=\int_z^\infty (t-z)f(t)dt.$$

When z is fixed, we derive the optimal value of P as

$$P^{*}(z)=\frac{a+bC_p+\mu+(C_r+C_c)\frac{b\gamma\lambda}{\theta}-\psi(z)}{2b}$$

and for a fixed P, the optimal value of z as

$$z^*(P) = F^{-1}\left[1 - \frac{(C_p - v) + (C_r + C_c)\frac{\gamma\lambda}{\theta} - \psi(z)}{P + u - v}\right]$$

Corollary : *The profit function* Π *is concave in* z *for a given value of* P *and concave in* P *for a given value of* z.

Proof: The expected total profit in the centralized Model II is

$$\begin{aligned}\Pi &= (P - C_p)[y(P) + \mu] - (C_p - v)\ \phi(z) - (P + u - C_p)\ \psi(z) \\ &\quad -(C_r + C_c)[z + y(P)]\frac{\gamma\lambda}{\theta}\end{aligned}$$

Therefore, the first and second order partial derivatives of Π with respect to P and z are given by

$$\begin{aligned}\frac{\partial\Pi}{\partial z} &= -(C_p - v) + (P + u - v)[1 - F(z)] - (C_r + C_c)\frac{\gamma\lambda}{\theta} \\ \frac{\partial^2\Pi}{\partial z^2} &= -(P + u - v)f(z) < 0,\ \text{since } v < P \ \text{ and } f(z) \geq 0. \\ \frac{\partial\Pi}{\partial p} &= (P - C_p)(-b) + (a - bP + \mu) - (C_r + C_c)\frac{\gamma\lambda}{\theta}(-b) - \psi(z) \\ \frac{\partial^2\Pi}{\partial p^2} &= -2b < 0,\ \text{since } b > 0. \text{ Hence it is proved.}\end{aligned}$$

4.3.2.2 Decentralized game

The decentralized game is considered when all the members in the supply chain have similar decision powers and they are not interested for a collaborative business all together. There may be some mutual agreements between a pair of members but they will never collaborate all together like a centralized model. So our problem is now to maximize separately the expected profits of the manufacturer, the collector and the supplier, which are

$$\begin{aligned}\Pi_m &= E\Big[P \times min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s - (P_d + C_m)D_c - qC_p\Big] \\ \Pi_c &= E\Big[(P_d - C_c)D_c\Big] \\ \Pi_s &= E\Big[(P_s - C_s)D_s\Big]\end{aligned}$$

Similar to the previous model, we now suppose that the profit margins for the players in this game are the same *i.e.,*

$$P_d = \frac{P + C_c}{2} \text{ and } P_s = \frac{P + C_s}{2}.$$

Then the optimal values of P and z are given by

$$\begin{aligned} P^*(z) &= \frac{a + \mu + bC_p - \psi(z) + b[(P_d + C_m)\frac{\gamma\lambda}{\theta} + P_s\gamma(1-\lambda)]}{2b} \text{ and} \\ z^*(P) &= F^{-1}\left[1 - \frac{(C_p - v) + (P_d + C_m)\frac{\gamma\lambda}{\theta} + P_s\gamma(1-\lambda)}{P + u - v}\right] \end{aligned}$$

Proposition 4.3 *The joint profit for all the members in the supply chain in Model II is greater than that of Model I if $C_m > C_r$.*

Proof: In Model I, the expected total profit of the supply chain is

$$\begin{aligned} \Pi_D^I &= \Pi_m + \Pi_s + \Pi_c \\ &= (P - C_p)\Big[y(P) + \mu\Big] - (C_p - v)\ \phi(z) - (P + u - C_p)\psi(z) \\ &\quad - \Big[(C_m + C_c)\frac{\gamma\lambda}{\theta} + C_s\gamma(1-\lambda)\Big]\Big[z + y(P)\Big] \end{aligned}$$

and the expected total profit of the supply chain in Model II is

$$\begin{aligned} \Pi_D^{II} &= \Pi_m + \Pi_s + \Pi_c + \Pi_r \\ &= (P - C_p)\Big[y(P) + \mu\Big] - (C_p - v)\ \phi(z) - (P + u - C_p)\ \psi(z) \\ &\quad - \Big[(C_r + C_c)\frac{\gamma\lambda}{\theta} + C_s\gamma(1-\lambda)\Big]\Big[z + y(p)\Big] \end{aligned}$$

Therefore, whenever $C_m > C_r$, $\Pi_D^{II} > \Pi_D^I$. Hence the result.

4.3.2.3 Fixed markup strategic game

In the fixed markup strategic game also, each of the players wants to maximize its own profit individually. Each downstream player wants to fix his wholesale price greater than that of the preceding upstream member. Hence, we assume that, the collector's wholesale price $P_c = (1 - \alpha_3)P_r$, the recycler's wholesale price $P_r = (1 - \alpha_4)P$, and the

supplier's wholesale price $P_s = (1-\alpha_5)P$, and that $0 < \alpha_5 \leqslant \alpha_4 \leqslant \alpha_3 < 1$.
Using the above relations in the profit functions

$$\begin{aligned}
\Pi_m &= E\Big[P \times min(q,D) - u(D-q)^+ + v(q-D)^+ - P_sD_s - P_rD_r - qC_p\Big] \\
\Pi_c &= E\Big[(P_c - C_c)D_c\Big] \\
\Pi_r &= E\Big[P_rD_r - (P_c + C_r)D_c\Big] \\
\Pi_s &= E\Big[(P_s - C_s)D_s\Big]
\end{aligned}$$

we get the optimal price of the manufacturer as

$$P^*(z) = \frac{a + \mu + bC_p - \psi(z) - (a+z)\Big[(1-\alpha_4)\gamma\lambda + (1-\alpha_5)\gamma(1-\lambda)\Big]}{2b\Big[1 - (1-\alpha_4)\gamma\lambda - (1-\alpha_5)\gamma(1-\lambda)\Big]}$$

and the optimal value of the stocking factor z as

$$z^*(P) = F^{-1}\left[1 - \frac{(C_p - v) + P[(1-\alpha_4)\gamma\lambda + (1-\alpha_5)\gamma(1-\lambda)]}{P + u - C_p}\right].$$

4.4 Numerical examples

In this section, we illustrate the developed models numerically. For the random customer demand, two examples with different distributions are assumed.

4.4.1 Example 1 for Model I

In this example, we set the parameter-values for Model I. We assume that the random demand follows: (i) exponential distribution *i.e.,* $f(\alpha, x)$=$\alpha e^{-\alpha x}$, $x > 0$ with $\alpha = 0.02$, mean $\mu = 50$; and (ii) uniform distribution *i.e.,* $f(z)$=$\frac{1}{100}$, $0 \leqslant z \leqslant 100$, with mean $\mu = 50$. We consider the other parameter-values as follows: C_p=5, $\theta = 0.7$, $\gamma = 1.3$, a =1000, $b = 1.3$, $C_c = 15$, $C_m = 65$, $C_s = 100$, $\mu = 10$, $\alpha_1 = 0.65$, $\alpha_2 = 0.65$, $\lambda = 0.6$, $u = 3$, $v = 4$ in appropriate units. For this set of data, we obtain the optimal price, optimal stocking factor and profit for each player in different games. The optimal

Table 4.1: Optimal results in Model I for exponential distribution.

Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	471.103	498.87	479.14
z^*	59.81	15.10	15.96
Π_m^*	-	45336.70	48973.6
Π_c^*	-	63836.90	66882.3
Π_s^*	-	18989.40	13837.7
Expected total profit	1,33,691.0	1,28,163.0	1,29,693.6

Table 4.2: Optimal results in Model I for uniform distribution.

Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	475.19	501.98	482.24
z^*	70.024	26.18	27.42
Π_m^*	-	46103.2	49776.7
Π_c^*	-	65495.7	68630.8
Π_s^*	-	19555.7	14325.2
Expected total profit	1,37,255.0	1,31,154.6	1,32,732.7

results for exponential and uniform demand distributions are shown in Tables 4.1 and 4.2, respectively.

Tables 4.1 and 4.2 show that the expected total profits for all the gaming approaches are higher in case of uniform demand distribution compared to the respective models in exponential demand distribution. The optimal retail price of the product is lower in case of fixed markup strategy which results in higher customer demand and higher profit. The optimal profits of the manufacturer and the collector are higher in case of the fixed markup strategy than those in decentralized policy.

4.4.2 Example 2 for Model II

Here also we consider two types of demand distribution as given below :
(i) exponential distribution *i.e.,* $f(\alpha, x)$=$\alpha e^{-\alpha x}$, $x > 0$ with $\alpha = 0.02$, mean $\mu = 50$; (ii) uniform distribution *i.e.,* $f(z)$=$\frac{1}{100}$, $0 \leqslant z \leqslant 100$, with same mean $\mu = 50$. We consider the parameter-values as follows: C_p=5, $\theta = 0.7$, $\gamma = 1.3$, a =1000, $b = 1.3$, $C_c = 15$, $C_m = 65$, $C_r = 10$, $C_s = 100$, $\mu = 10$, $\alpha_3 = 0.45$, $\alpha_4 = 0.4$, $\alpha_5 = 0.35$, $\lambda = 0.6$, $u = 3$, $v = 4$ in appropriate units. For this set of data, we obtain the optimal price, optimal stocking factor and expected profit of each player in different gaming

approaches, as shown in Tables 4.3 and 4.4.

Tables 4.3 and 4.4 depict the optimal results for each player as well as for the whole supply chain in Model II. Optimal retail prices are lower in this model compared to those in Model I which corresponds to higher demand. Here also optimal values of the profits are greater for the uniform distribution, and for both the distributions, the expected total profit of the supply chain is improved in the markup policy, compared to the decentralized game. The expected total profits of the supply chain for the two decentralized cases in model II are higher than those of the respective cases in Model I due to the lower recycling cost of the recycler (Proposition 4.3).

Table 4.3: Optimal results in Model II for exponential distribution.

Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	442.75	465.98	402.32
z^*	84.90	8.24	8.15
Π_m^*	-	26788.0	27448.9
Π_c^*	-	67415.1	70915.1
Π_s^*	-	38296.1	40743.6
Π_r^*	-	20463.4	14526.3
Expected total profit	1,62,928.0	1,52,964.6	1,53,633.9

Table 4.4: Optimal results in Model II for uniform distribution.

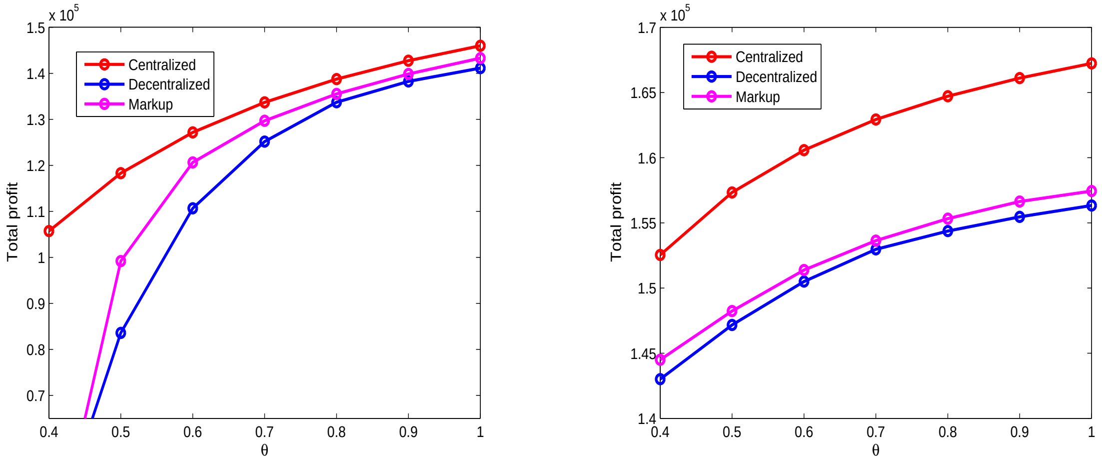
Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	445.639	467.762	403.72
z^*	81.80	15.21	15.04
Π_m^*	-	27042.9	27663.9
Π_c^*	-	68464.8	65064.7
Π_s^*	-	38928.0	41584.4
Π_r^*	-	20808.0	14469.9
Expected total profit	1,66,501.0	1,55,244.0	1,56,534.0

4.4.3 Sensitivity analysis

Now, for the exponential distribution, we examine the sensitivity of the key parameters θ, b, and γ on the optimal prices as well as the expected profit of the supply chain under different strategies of both the game models.

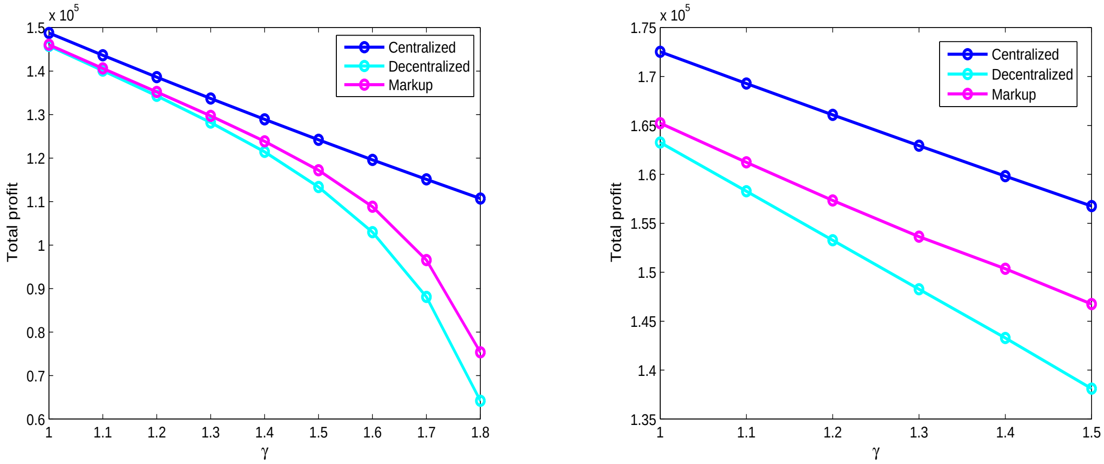
4.4.3.1 Sensitivity with respect to θ

As the value of θ increases, in Model I, the supply chain's expected total profit increases for the centralized, decentralized and markup strategic game models. This happens because higher recyclability degree results in higher quality value of the used wastes, and this reduces the recycling cost and also the usage of total wastes. We see that the profit of the manufacturer increases as θ increases but the collector and the backup supplier's optimal profits are obtained for their respective specific values of θ.



(a) *θ vs. total profits (Model I)* **(b)** *θ vs. total profits (Model II)*

Fig. 4.1: *Sensitivity with respect to θ.*



(a) *γ vs. total profits (Model I)* **(b)** *γ vs. total profits (Model II)*

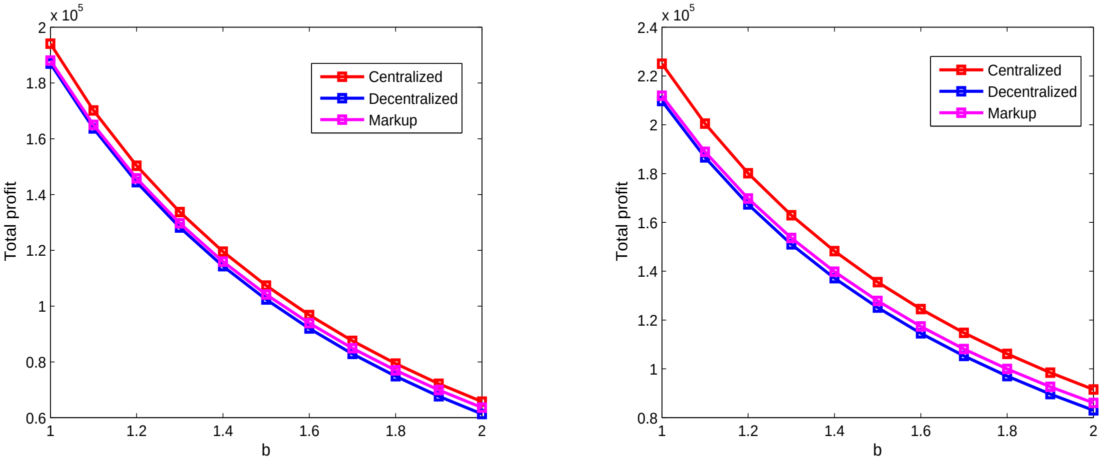
Fig. 4.2: *Sensitivity with respect to γ.*

In Model II also, the expected total profit increases with θ. The expected total profit is maximum in the centralized model, which is the benchmark case. For the markup strategy, the expected total profit is higher compared to that of the decentralized gaming strategy (see Fig. 4.1).

4.4.3.2 Sensitivity with respect to γ

As the value of γ increases, the manufacturer requires more recyclable wastes to produce q units of finished product. Hence the production cost increases for the manufacturer and that leads to lower profit. However, the collector and the backup supplier attain higher profits for increasing γ, as they will have to supply more raw materials.

If the value of γ increases, the amount of recycled materials to be supplied by the recycler to the manufacturer increases. So, in that case, the expected profit decreases in all the three types of gaming approach. Because of ex-ante price markup commitment, the expected total profit in case of markup policy is higher compared to that of the decentralized policy (see Fig. 4.2).



(a) *b vs. total profits (Model I)* **(b)** *b vs. total profits (Model II)*

Fig. 4.3: *Sensitivity with respect to b.*

4.4.3.3 Sensitivity with respect to b

For higher values of b (price sensitivity of customer demand), the customer demand is lower. As a result, the profits of all individual entities decrease for higher values of b. In the markup policy, the supply chain's optimal expected profit becomes higher than that in the decentralized game (see Fig. 4.3).

Similar observation is made for the game Model II. The expected total profit of the supply chain as well as individual profits decrease with the values of b.

4.5 Conclusion

In this chapter, we have studied a closed-loop supply chain scenario where recycling is the main concern for environmental sustainability. A manufacturer performs recycling using two different channel of recycling, directly by his own and also by the help of another recycler. For two different game models, depending on different ways of recycling, we have analyzed the optimal pricing strategy of all the supply chain members. For stochastic demand, it is not always easy to get closed form solution of the model. So, we have obtained the optimal solutions numerically for two types of demand distribution - uniform and exponential. From the sensitivity analysis, we have the following observations:

(i) Ex-ante markup strategy is beneficial (compared to decentralized model) for the supply chain entities, specially the manufacturer. However, profit is not always pareto-improving in case of stochastic demand scenario (here specially for the backup supplier in Model I and recycler in Model II), which supports the result of Liu et al. (2010).

(i) For higher value of θ, the supply chain will gain higher profit. The individual entities will also gain higher profit for a particular range of θ.

(iii) When the recycler recycles the wastes at a cost lower than the manufacturer, the expected total profit of the supply chain becomes higher.

(iv) A higher price sensitivity of customer's demand decreases customer demand, and hence it leads to lower profit for the manufacturer.

Several future studies can be done using different contract policies among the chain members. One can also assume multiplicative form of stochastic demand. Instead of fixed markup, the entities can go for variable markup policy also.

Chapter 5

Corporate social responsibility in a CLSC with dual channel waste recycling

5.1 Introduction

Social issues like environmental degradation, food safety, labor conflicts, etc. are becoming more and more prominent in today's supply chain management. Pressures are also accumulating for environmental and socially responsible supply chain practices due to sustainability consideration. In addition to Government's concerns at various levels, the issue of consumer awareness of corporate social responsibility (CSR) is also becoming stronger than ever before. Various business organisations are performing environmental as well social responsibility through recycling. With the growing trend of globalization, CSR is receiving notable attention from academic and business communities. This chapter* focuses on social responsibility in a closed-loop supply chain without neglecting economic concern of the supply chain members. It has an important influence on the coordinated development of economy and society. More and more consumers now-a-days try to purchase sustainable products keeping public welfare in

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*This chapter is based on the work published in *International Journal of Systems Science: Operations & Logistics*, 2021, DOI: 10.1080/23302674.2021.2005844 mind. Sometimes Government pushes directly the business firms for socially desirable outcomes which can take a variety of forms. Providing subsidy is one of the growing topics in recent years. Although many supply chains incorporate social responsibility, the allocation of social responsibility has not emerged as the main focus. Also, it is a critical issue for supply chain members to collaboratively manage CSR. When the question of economic sustainability of the supply chain members arises, all may not be able to reach up to the optimal mark, decided by their own.

In this chapter, a dual channel closed-loop supply chain for recycling is considered where the manufacturer as well as other members are socially responsible. The manufacturer considers sustainability through product recycling and remanufacturing. Therefore, the manufacturer exerts effort for quality improvement or marketing of the finished product as the manufacturer itself sells the produced items. The manufacturer, in this way, attracts the attention of customers leading to enhance the market demand. The primary objectives of this study are to investigate

(i) the optimal decisions of the supply chain entities under integrated, decentralized and contract policies,

(ii) the significance of effort on consumer demand to bear social sustainability pressure,

(iii) the economic satisfaction of all the supply chain entities, and

(iv) the impact of recyclability degree of waste on optimal behaviors of the supply chain entities.

This chapter considers recycling of wastes in two different settings. Either the recycler or the manufacturer itself recycles the collected used products. Thus, recycling work seems flexible in between the recycler and the manufacturer. In this way, the optimality of the supply chain is examined in two completely different cases. Besides, the manufacturer's decisions on social responsibility through corporate social responsibility is analysed. To mitigate the economical issues between the supply chain members, a joint revenue and cost sharing contract is implemented. Thus the manufacturer's quality improvement effort for remanufactured products and the gain of consumer surplus are considered together in the proposed closed-loop supply chain.

The rest of the chapter is organised as follows: In the next section, assumptions and notations are given. In Section 5.3, models are developed and analyzed for two separate cases. In Section 5.4, a numerical example is given and also sensitivity analysis is carried out with respect to key model-parameters. Finally, the chapter is concluded in Section 5.5 with managerial insights and some future research directions.

5.2 Notation and assumption

The proposed dual-channel closed-loop supply chain consists of one collector, one recycler and one manufacturer for production of a single product. Wastes are collected by the collector and then supplied to the recycler for recycling. It is assumed that the manufacturer is also capable of recycling. Therefore, we study two different cases - presence of the recycler and absence of the recycler. In the absence of the recycler, the manufacturer itself does the recycling and then produces the finished product. To develop the proposed model, we use the following notations throughout the chapter:

C_c	:	unit collection cost of recyclable wastes to the collector
C_r	:	unit recycling cost of recyclable wastes to the recycler
C_m	:	unit recycling cost of recyclable wastes to the manufacturer
C_p	:	unit production cost of the finished product to the manufacturer
$\theta(0 < \theta < 1)$	:	recyclability degree of waste
k	:	coefficient of the manufacturer's effort cost
a	:	maximum possible customer demand for the finished product
$\alpha(> 0)$	:	effort sensitivity of the customer's demand
e	:	manufacturer's effort level behind corporate social responsibility
$\beta(> 0)$	:	price sensitivity of the customer's demand
$\gamma(> 1)$	:	quantity of recycled materials required to produce one unit of the finished product
$\frac{\gamma}{\theta}$	:	quantity of recyclable waste required to produce one unit of the finished product
P_d	:	wholesale price charged by the collector to the manufacturer for one

		unit of recyclable waste
P_c	:	wholesale price charged by the collector to the recycler for one unit of recyclable waste
P_r	:	wholesale price charged by the recycler to the manufacturer for one unit of recycled waste
P_m	:	retail price charged by the manufacturer to the customers for one unit of finished product
R	:	customer demand of the finished product at the manufacturer
D_m	:	manufacturer's demand of recyclable wastes/recycled materials
D_r	:	quantity of recycled waste supplied by the recycler to the manufacturer

To develop the proposed model, we make the following assumptions:

(i) Customer demand is linear in retail price (P_m) and effort level (e), and it is of the form $R = a - \beta P_m + \alpha e$ where $\alpha (> 0)$ and β (> 0) are the price and effort sensitivity parameters, respectively. Again, the inequality $0 < P_m < \frac{a+\alpha e}{\beta}$ ensures that the demand is always positive. Promotional or quality improvement effort and retail price dependent linear demand is well known in supply chain literature (Ma et al., 2013; Giri et al., 2017).

(ii) $C_r < C_m$ *i.e.,* recycling by the recycler is less expensive than recycling by the manufacturer; otherwise, there will be no role of the recycler in the supply chain.

(iii) There is no shortage of used product for remanufacturing, *i.e.,* the collector collects sufficient amount of used product to fulfill the manufacturer's or the recycler's need for used products.

5.3 Model development and analysis

In this proposed model, we assume that the manufacturer faces intense pressure for CSR and so it gives effort behind it. On the other hand, consumers are willing to pay an extra price for social sustainability. We consider the socially responsible manufacturer's concern for social responsibility. We suppose that the manufacturer earns a fraction $y \in [0,1]$ of total consumer surplus, which is given by

$$y \int_{P_{min}}^{P_{max}} R \, dP_m = y \int_{\frac{a-R+\alpha e}{\beta}}^{\frac{a+\alpha e}{\beta}} (a - \beta P_m + \alpha e) \, dP_m = y \frac{(a - \beta P_m + \alpha e)^2}{2\beta} = \frac{yR^2}{2\beta}$$

The manufacturer earns this consumer surplus along with its pure profit, and it depends on both effort and retail price. $y = 0$ implies that the manufacturer only maximizes its pure profit and $y = 1$ means that the manufacturer is perfectly responsible for social responsibility.

5.3.1 Case I: The manufacturer gets recyclable wastes from the collector

Here, we consider the situation where the collector satisfies the manufacturer's demand of recyclable wastes. The manufacturer recycles the wastes and produces the finished product. The collector needs a total of D_m $(= \frac{\gamma R}{\theta})$ units of recyclable waste to supply to the manufacturer.

5.3.1.1 Decentralized model

In the decentralized scenario, the supply chain entities try to maximize their own profits individually. The collector sets the optimum reward level and the remanufacturer decides its optimal effort level. The entities take their individual optimal decisions sequentially. We suppose that the collector acts as the Stackelberg leader and the manufacturer as the follower. Total profits of the collector and the remanufacturer are given separately in the following:

$$\begin{aligned} \Pi_m &= (P_m - C_p)(a - \beta P_m + \alpha e) - (P_d + C_m)(a - \beta P_m + \alpha e)\frac{\gamma}{\theta} - \frac{ke^2}{2} \\ &\quad + \frac{y}{2\beta}(a - \beta P_m + \alpha e)^2 \qquad (5.1) \\ \Pi_c &= (P_d - C_c)(a - \beta P_m + \alpha e)\frac{\gamma}{\theta} \qquad (5.2) \end{aligned}$$

In equation (5.1), the first term denotes the manufacturer's earn by selling the products to the customers, the second term is the cost of buying the used products from the collector, the third term presents the cost associated with the effort level e and the last term exhibits the fraction of consumer surplus for the socially responsible manufacturer. Equation (5.2) presents the collector's profit for selling the used products to the manufacturer. Optimizing Π_m with respect to P_m and e and then using them in the collector's profit function, we get the collector's optimal wholesale price as

$$P_d^* = \frac{a\theta - \beta\theta C_p + \beta\gamma(C_c - C_m)}{2\beta\gamma} \tag{5.3}$$

Using equation (5.3), we get respectively the optimal retail price and the optimal effort of the manufacturer as

$$\begin{aligned} P_m^* &= \frac{(C_c + C_m)\beta\gamma(\alpha^2 - k\beta) + \Big[a(\alpha^2 + k(-3 + 2y)\beta) + \beta C_p(\alpha^2 - k\beta)\Big]\theta}{2S\beta\theta} \\ e^* &= \frac{\alpha\Big\{\beta\theta C_p + (C_m + C_c)\beta\gamma - a\theta\Big\}}{2S\theta} \end{aligned}$$

where, $S = \alpha^2 + k(-2 + y)\beta$. We have $\frac{\partial^2\Pi_m}{\partial e^2} = \dfrac{(\alpha^2 y - k\beta)}{\beta}$; $\frac{\partial^2\Pi_m}{\partial P_m^2} = \beta(y - 2) < 0$, since $y \in [0, 1]$; $\frac{\partial^2\Pi_c}{\partial P_d^2} = \dfrac{-2\beta\gamma^2}{\theta^2} < 0$. This shows that the collector's profit function is concave in P_d and the manufacturer's profit function is concave in P_m. Also, the manufacturer's profit function is concave in e provided that $k\beta - \alpha^2 y > 0$.

5.3.1.2 Centralized model

In the centralized scenario, all the partners jointly take the decisions, or in other words, all the members act as a single decision maker. The total profit of the system depends on the finished product's final price set for the end customers. In this case, the total profit is

$$\begin{aligned} \Pi_{cm} &= (P_m - C_p)(a - \beta P_m + \alpha e) - \frac{\gamma}{\theta}(C_m + C_c)(a - \beta P_m + \alpha e) - \frac{ke^2}{2} \\ &\quad + \frac{y}{2\beta}(a - \beta P_m + \alpha e)^2 \end{aligned} \tag{5.4}$$

In equation (5.4), the first and the last terms denote the total earnings due to selling the finished products and consumer surplus, respectively whereas the second and the third terms present the costs associated with recycling of the collected used products and quality improvement effort level. The associated Hessian matrix is given by

$$H = \begin{pmatrix} \frac{\partial^2 \Pi_{cm}}{\partial e^2} & \frac{\partial^2 \Pi_{cm}}{\partial e \partial P_m} \\ \frac{\partial^2 \Pi_{cm}}{\partial P_m \partial e} & \frac{\partial^2 \Pi_{cm}}{\partial P_m^2} \end{pmatrix} = \begin{pmatrix} \dfrac{\alpha^2 y - k\beta}{\beta} & \alpha(1-y) \\ \alpha(1-y) & \beta(y-2) \end{pmatrix}$$

Since the first order principal minor $H_{11} = \dfrac{\alpha^2 y - k\beta}{\beta}$ and $|H| = (\alpha^2 y - k\beta)(y-2) - \alpha^2(1-y)^2$, therefore, the profit function Π_{cm} is concave in P_m and e if $H_{11} < 0$ *i.e.,* $k\beta - \alpha^2 y > 0$ and $|H| > 0$ *i.e.,* $(\alpha^2 y - k\beta)(y-2) > \alpha^2(1-y)^2$.

Assuming that the Hessian matrix H is negative definite, the optimal values of P_m and e can be obtained from the first order conditions as

$$P_m^* = \frac{(C_c + C_m)(\alpha^2 - k\beta)\gamma + \Big[ak(y-1) + C_p(\alpha^2 - k\beta)\Big]\theta}{S\theta} \tag{5.5}$$

$$e^* = \frac{\alpha\Big\{(C_m + C_c)\beta\gamma - a\theta + \beta\theta C_p\Big\}}{S\theta} \tag{5.6}$$

Proposition 5.1 *The optimum effort level increases with* θ *provided that* $k\beta(2-y) - \alpha^2 > 0$.

Proof: The optimum effort level of the supply chain is

$$\begin{aligned} e^* &= \frac{\alpha\Big\{(C_m + C_c)\beta\gamma - a\theta + \beta\theta C_p\Big\}}{S\theta}, \quad \text{which implies that} \\ \frac{de^*}{d\theta} &= \frac{\alpha(C_m + C_c)\beta\gamma}{\theta^2\big(k\beta(2-y) - \alpha^2\big)} > 0, \ \text{when } k\beta(2-y) - \alpha^2 > 0, \end{aligned}$$

since the other parameters are all positive. Hence the result.

Proposition 5.2 *The optimum effort level in the centralized model is twice the corresponding value of the decentralized model.*

Proof: The proof follows from the comparison of the results of the centralized and the decentralized models and hence omitted.

Theorem 5.1 *The total profit of the supply chain increases with recyclability degree of waste.*

Proof: The consumer demand R depends on two decision variables θ and e. Now, differentiating P_m^* and e^* with respect to θ, we get

$$\begin{aligned} \frac{dP_m^*}{d\theta} &= -\frac{\gamma(C_m+C_c)(\alpha^2-k\beta)}{\big(\alpha^2-k(2-y)\beta\big)\theta^2} < 0, \text{ since } \gamma,\ \theta > 0 \\ \text{and} \quad \frac{de^*}{d\theta} &= \frac{\alpha(C_m+C_c)\beta\gamma}{\theta^2\big(k\beta(2-y)-\alpha^2\big)} > 0, \text{ by Proposition 5.1} \end{aligned}$$

So, we notice that the optimal price decreases but the optimal effort level increases with θ. This implies that the demand $R = a - \beta P_m + \alpha e$ is an increasing function of θ, the recyclability degree of waste. As the total profit of the supply chain increases with demand, hence the theorem is proved.

5.3.1.3 Revenue and cost sharing contract

Under this contract, we assume that the manufacturer shares a fraction λ $(0 < \lambda < 1)$ of his revenue with the collector, and also the collector agrees to bear a portion $(1-\mu)$ of the cost of effort of the manufacturer where $0 < \mu < 1$. The manufacturer and the collector would like to optimize their individual profits and target to gain more (compared to the decentralized policy) through this collaboration or information sharing between each other. Under this contract, the profit functions of the collector and the manufacturer are given by

$$\begin{aligned} \Pi_m &= (\lambda P_m - C_p)(a-\beta P_m+\alpha e) - (P_d+C_m)(a-\beta P_m+\alpha e)\frac{\gamma}{\theta} - \mu\frac{ke^2}{2} \\ &\quad + \frac{y}{2\beta}(a-\beta P_m+\alpha e)^2 \\ \Pi_c &= (P_d-C_c)(a-\beta P_m+\alpha e)\frac{\gamma}{\theta} + (1-\lambda)P_m(a-\beta P_m+\alpha e) - (1-\mu)\frac{ke^2}{2} \end{aligned}$$

The concavity of the profit functions can be checked in a similar manner as prescribed in subsection 5.3.1.2. Following sequential approach, the optimal values of the decision variables are obtained as

$$\begin{aligned} e^* &= \frac{\alpha\Big\{(C_m + C_c)\beta\gamma - a\theta + \beta\theta C_p\Big\}\lambda\mu}{\theta\Big\{\alpha^2\lambda\big(\lambda(\mu-1) + 2\mu\big) + 2k\beta\mu^2(-1 + y - \lambda)\Big\}} \\ P_d^* &= \frac{\alpha^2\lambda^2\big(- \beta(\gamma C_m + \theta C_p) + a\theta\lambda\big) - a^2\lambda\mu U + k\beta\mu^2 V}{\beta\gamma\Big(2k\beta\mu^2(1 - y + \lambda) + \alpha^2\big(\lambda - (2+\lambda)\mu\big)\Big)} \\ P_m^* &= \frac{\beta\mu(-a^2\lambda + k\beta\mu)\Big((C_m + C_c)\gamma + \theta C_p\Big) + a\theta\Big[k\beta(1 - 2y + 2\lambda)\mu^2 - \alpha^2\lambda(\lambda(-1+\mu) + \mu)\Big]}{\beta\theta W} \end{aligned}$$

where, $U = -2\beta\gamma C_m - 2\beta\theta C_p + \lambda\Big\{\beta\gamma C_c + a\theta(1+\lambda)\Big\}$, $V = \beta\gamma C_m(y-2) - \beta\gamma C_c(y - 2\lambda) + \theta\Big\{C_p - (y-2)\beta + a(y - 2y\lambda + 2\lambda^2)\Big\}$, $W = 2k\beta\mu^2(1 - y + \lambda) + \alpha^2\lambda\big(\lambda - (2+\lambda)\mu\big)$.

Theorem 5.2 *When the effort level* $e > 0$*, the collector earns extra profit from the contract (compared to the decentralized model) provided that the feasibility condition holds.*

Proof: If Π_c^{rcs} and Π_c^d denote the profit functions of the collector in revenue-cost sharing model and decentralized model, respectively then we have

$$\Pi_c^{rcs} > \Pi_c^d \text{ if } (1-\lambda)p_m(a - \beta P_m + \alpha e) > (1-\mu)\frac{ke^2}{2}$$

This implies that

$$\beta p_m^2 - (a + \alpha e)p_m < -\left(\frac{1-\mu}{1-\lambda}\right)\frac{ke^2}{2} < 0$$

since $0 < \mu, \lambda < 1$ and $k > 0$. So, $p_m < \dfrac{a + \alpha e}{\beta}$, which is the feasibility condition.

Theorem 5.3 *The manufacturer's profit is infeasible under the contract (compared to the decentralized model) when it bears the most of the CSR effort cost.*

Proof: We have $\Pi_m^{rcs} > \Pi_m^d$ when

$$\beta p_m^2 - (a + \alpha e)p_m > -\left(\frac{1-\mu}{1-\lambda}\right)\frac{ke^2}{2}$$

which implies that

$$\beta p_m^2 > (a + \alpha e)p_m - \left(\frac{1-\mu}{1-\lambda}\right)\frac{ke^2}{2}$$

When $\mu \to 1$ *i.e.,* the manufacturer itself bears the total cost of CSR effort, $p_m > \dfrac{a+\alpha e}{\beta}$, which is contrary to the feasibility condition. Also we notice that feasibility is indeterminable when $\mu \to 1$ and $\lambda \to 1$ simultaneously.

5.3.2 Case II : The manufacturer gets the recycled materials from the recycler

Here, we consider the situation where the collector sends the collected wastes to the recycler for recycling. Then the recycler satisfies the manufacturer's demand of recycled materials. Therefore, in this case, we have

$$\begin{aligned} D_m &= \gamma R \\ D_r &= \frac{\gamma R}{\theta} \end{aligned}$$

5.3.2.1 Decentralized model

In this case, the profit functions of the collector, the recycler and the manufacturer are given by

$$\Pi_c = (P_c - C_c)(a - \beta P_m + \alpha e)\frac{\gamma}{\theta} \tag{5.7}$$

$$\Pi_r = P_r\gamma(a - \beta P_m + \alpha e) - (P_c + C_r)(a - \beta P_m + \alpha e)\frac{\gamma}{\theta} \tag{5.8}$$

$$\begin{aligned} \Pi_m = {} & (P_m - C_p)(a - \beta P_m + \alpha e) - P_r\gamma(a - \beta P_m + \alpha e) - \frac{ke^2}{2} \\ & + \frac{y}{2\beta}(a - \beta P_m + \alpha e)^2 \end{aligned} \tag{5.9}$$

The collector's profit function is composed of total earning by selling the used products to the recycler minus the total procurement or collection cost. Equation (5.8) denotes the recycler's profit for recycling the used products and then selling to the manufacturer for final production, and the manufacturer's profit function is similar to that of case I. Here we have $\frac{\partial^2\Pi_m}{\partial e^2} = \dfrac{(\alpha^2 y - k\beta)}{\beta}$; $\frac{\partial^2\Pi_m}{\partial P_m^2} = \beta(y-2) < 0$ and $\frac{\partial^2\Pi_m}{\partial e\partial p_m} = \frac{\partial^2\Pi_m}{\partial p_m\partial e} = \alpha(1-y)$. Hence the manufacturer's profit function is concave in P_m and e provided that $k\beta - \alpha^2 y > 0$ and $(\alpha^2 y - k\beta)(y-2) > \alpha^2(1-y)^2$.

In the sequential approach, the collector first decides its optimal wholesale price. Then the recycler recycles the wastes and determines its selling price for the manufacturer. After production, the manufacturer sells the finished product with a retail price P_m. By backward induction method, first differentiating equation (5.9) with respect to P_m and e and then obtaining the corresponding values, we put them in equation (5.8). From equation (5.8), we then get the optimal value of P_r in terms of P_c. From equation (5.7), we get the optimal value of P_c. By substituting this value in backward process, we get the optimal values of P_r and P_m. Optimal decisions are thus obtained as

$$\begin{aligned}
e^* &= \frac{\alpha\Big\{(C_c + C_r)\beta\gamma - a\theta + \beta\theta C_p\Big\}}{4S\theta} \\
P_m^* &= \frac{(C_c + C_r)\beta\gamma(\alpha^2 - k\beta) + \Big[3a\alpha^2 + \beta C_p(\alpha^2 - k\beta) + ak(-7 + 4y)\beta\Big]\theta}{4S\beta\theta} \\
P_r^* &= \frac{(C_c + C_r)\beta\gamma + 3\theta(a - \beta C_p)}{4\beta\gamma\theta} \\
P_c^* &= \frac{(C_c - C_r)\beta\gamma + \theta(a - \beta C_p)}{2\beta\gamma}
\end{aligned}$$

5.3.2.2 Centralized model

Here the collector, the recycler and the manufacturer jointly participate in the business and the total profit of the system is given by

$$\begin{aligned}
\Pi_{crm} &= (P_m - C_p)(a - \beta P_m + \alpha e) - \frac{\gamma}{\theta}(C_r + C_c)(a - \beta P_m + \alpha e) - \frac{ke^2}{2} \\
&\quad + \frac{y}{2\beta}(a - \beta P_m + \alpha e)^2
\end{aligned} \tag{5.10}$$

Similar to Case I, here also the total profit of the supply chain is the sum of the revenue earned by selling the finished product and the fraction of consumer surplus minus the collection cost, recycling cost and effort implementation cost. For optimum of Π_{crm}, the first order necessary conditions give

$$\begin{aligned}
P_m^* &= \frac{(C_c + C_r)(\alpha^2 - k\beta)\gamma + \Big\{ak(-1 + y) + C_p(\alpha^2 - k\beta)\Big\}\theta}{S\theta} \\
e^* &= \frac{\alpha\Big\{\beta\theta C_p - a\theta + (C_r + C_c)\beta\gamma\Big\}}{S\theta}
\end{aligned}$$

From the results of Subsections 5.3.2.1 and 5.3.2.2, we have the following proposition:

Proposition 5.3 *The optimum effort level in the centralized model is greater than that of the decentralized model.*

Proof: Comparing the optimal effort levels in subsections 5.3.2.1 and 5.3.2.2 it can be seen that the effort level in the centralized model is four times the effort level of the decentralized model.

Proposition 5.4 *The optimum effort level in the centralized model in case II is greater than that of case I.*

Proof: The optimum effort level in the centralized model for case I is

$$e_I^* = \frac{\alpha\Big\{(C_m + C_c)\beta\gamma - a\theta + \beta\theta C_p\Big\}}{S\theta}$$

and that for case II is

$$e_{II}^* = \frac{\alpha\Big\{\beta\theta C_p - a\theta + (C_r + C_c)\beta\gamma\Big\}}{S\theta}$$

Therefore, from the feasibility condition of the model we have, $e_{II}^* > e_I^*$.

5.3.2.3 Revenue and cost sharing contract

Suppose that a portion of the revenue of the manufacturer is shared between the recycler and the collector. Let a portion λ be shared out of which $\delta(1-\lambda)$ fraction is shared by the recycler and the remaining fraction $(1-\delta)(1-\lambda)$ is shared by the collector, where $\lambda,\ \delta \in (0,1)$. Like Case I, here also the effort cost of the remanufacturer is shared jointly by the recycler and the collector. Suppose that $(1-\mu)$ fraction of the total effort cost is shared between the recycler and the collector, among which the recycler shares $\phi(1-\mu)$ fraction and the remaining fraction $(1-\phi)(1-\mu)$ is shared by the collector, where $\phi,\ \mu \in (0,1)$.

The profit functions of the supply chain members are given by

$$\begin{aligned}
\Pi_m &= (\lambda P_m - C_p)(a-\beta P_m+\alpha e) - P_r\gamma(a-\beta P_m+\alpha e) - \mu\frac{ke^2}{2} + \frac{y}{2\beta}(a-\beta P_m+\alpha e)^2\\
\Pi_r &= P_r\gamma(a-\beta P_m+\alpha e) - (P_c+C_r)(a-\beta P_m+\alpha e)\frac{\gamma}{\theta} + \delta(1-\lambda)P_m(a-\beta P_m+\alpha e)\\
&\quad -\phi(1-\mu)\frac{ke^2}{2}\\
\Pi_c &= (P_c-C_c)(a-\beta P_m+\alpha e)\frac{\gamma}{\theta} + (1-\delta)(1-\lambda)P_m(a-\beta P_m+\alpha e) - (1-\phi)(1-\mu)\frac{ke^2}{2}
\end{aligned}$$

Proceeding similarly as in the previous case, the optimal decisions can be obtained as

$$\begin{aligned}
e^* &= \frac{\alpha\lambda\Big\{\beta\gamma(C_c+C_r)+\theta E_{11}\Big\}}{2\theta\big(E_{12}\alpha^2\lambda+E_{13}k\beta-2E_{14}\big)}\\
P_m^* &= \frac{\beta\gamma C_c(\alpha^2\lambda-k\beta)+C_r\beta\gamma(\alpha^2\lambda-k\beta)+\theta(F_{11}+aF)}{2\beta\theta\big(F_{14}+k\beta F_{15}\big)}\\
P_r^* &= \frac{k\beta G_{11}+\beta\lambda G_{12}+\lambda^2G_{13}-\lambda^3G_{14}+\lambda^4G_{15}}{2\beta\theta\gamma\big(\alpha^2\lambda G_{16}+k\beta G_{17}\big)}\\
P_c^* &= \frac{\delta\Big\{C_c\beta\gamma K_{11}+\theta\big(aK_{12}+C_p\beta(K_{13}+k\beta K_{14})\big)\Big\}-C_r\beta\gamma(k\beta K_{15}+\alpha^2\lambda K_{16})}{\beta\gamma(\alpha^2\lambda K_{17}+k\beta K_{18})}
\end{aligned}$$

where,

$E_{11}=C_p\beta\big(1+(\delta-1)\lambda\big)-a(\delta+\lambda)+a\lambda(\delta+\lambda-\delta\lambda),\quad E_{12}=2\delta+\lambda-\delta\lambda+(\delta-1)\lambda^2,$

$E_{13}=y\big(1+\delta+(\delta-1)\lambda\big),\quad E_{14}=\delta+\lambda+(\delta-1)\lambda^2,$

$F_{11}=C_p\beta(k\beta-\alpha^2\lambda)\big(1+(\delta-1)\lambda\big),\quad F=a^2\lambda F_{12}+k\beta F_{13},$

$F_{12}=3\delta+\lambda-\delta\lambda+(\delta-1)\lambda^2,\quad F_{13}=-3(\delta+\lambda)+2y\big(1+\delta+(\delta-1)\lambda\big)-\lambda\big(\delta+3(\delta-1)\lambda\big),$

$F_{14}=\alpha^2\lambda\big(2\delta+\lambda-\delta\lambda+(\delta-1)\lambda^2\big),\quad F_{15}=y\big(1+\delta+(\delta-1)\lambda\big)-2\big(\delta+\lambda+(\delta-1)\lambda^2\big),$

$G_{11}=-2(C_c+C_r)k\beta\gamma+\Big\{C_pk(2+y)\beta-C_p(4\alpha^2+ky\beta)\delta+ak(y-2\delta+3y\delta)\Big\}\theta,$

$G_{12}=-(C_c+C_r)k\beta\gamma+C_pk(2+y)\beta-C_p(4\alpha^2+ky\beta)\delta+ak(y-2\delta+3y\delta)\theta,$

$G_{13}=(C_c+C_r)\alpha^2\beta\gamma+\big(-ak(2+y)\beta+a\big(3\alpha^2+k(y-2)\beta\big)\delta+C_p\beta\big(2k\beta(\delta-1)+\alpha^2(2\delta-1)\big)\big),$

$G_{14}=\Big\{C_p\alpha^2\beta+a(\alpha^2+2k\beta)\Big\}(\delta-1)\theta,\quad G_{15}=a\alpha^2(\delta-1)\theta,\quad G_{16}=2\delta+\lambda-\delta\lambda+(\delta-1)\lambda^2,$

$G_{17}=y\big(1+\delta+(\delta-1)\lambda\big)-2\big(\delta+\lambda+(\delta-1)\lambda^2\big)$

$K_{11}=-\alpha^2\lambda+k\beta(1-y+\lambda),\quad K_{12}=-a^2\delta\lambda+k\beta\big(\delta+\lambda-(\delta-1)(\lambda-2)\lambda^2+y\big(-1+(\delta-1)(\lambda-2)\lambda\big)\big),$

$K_{13}=a^2\lambda\Big(-(\lambda-1)^2+\delta\big(2+(\lambda-2)\lambda\big)\Big),\quad K_{14}=(\lambda-1)^2+\delta(-2+y+\lambda-\lambda^2),$

$K_{15}=y-\delta+\big(-2+y(\delta-1)+\delta\big)\lambda-2(\delta-1)\lambda^2,\quad K_{16}=\delta+\lambda-\delta\lambda+(\delta-1)\lambda^2,$

$K_{17}=2\delta+\lambda-\delta\lambda+(\delta-1)\lambda^2,\quad K_{18}=y\Big\{1+\delta+(\delta-1)\lambda\Big\}-2\Big\{\delta+\lambda+(\delta-1)\lambda^2\Big\}$

Theorem 5.4 *The collector will not accept the contract if* $\lambda \to 1$ *or* $\delta \to 1$ *i.e., the revenue sharing proportion tends to zero.*

Proof: We have, $\Pi_c^{rcs} > \Pi_c^d$ when

$$(a - \beta p_m + \alpha e)p_m > \frac{(1-\mu)(1-\phi)}{(1-\lambda)(1-\delta)}\frac{ke^2}{2}$$

Now, if $\lambda \to 1$ or, $\delta \to 1$ *i.e.,* the manufacturer or the recycler shares no revenue with the collector, we notice that the right hand side of the inequality becomes undefined. Therefore, it is not a feasible decision for the collector to agree with the contract. However, nothing can be concluded when $\phi \to 1$ or $\mu \to 1$.

5.3.2.4 Sub-supply game

In a sub-supply game, two or more than two members (but not all) of a supply chain jointly act as a single player. The purpose behind this is to negotiate the double marginalization effect. In this model, we assume that the collector and the recycler jointly act as a single player and so p_c is eliminated as it becomes an internal decision between them. The joint total profit of the collector and the recycler is, therefore,

$$\begin{aligned} \Pi_{cr} &= \Pi_c + \Pi_r \\ &= P_r\gamma(a - \beta P_m + \alpha e) - (C_c + C_r)\frac{\gamma(a - \beta P_m + \alpha e)}{\theta} \end{aligned} \tag{5.11}$$

The first term of the above equation is the revenue earned due to selling of recycled items to the manufacturer and the second term is the sum of the collection cost and the recycling cost of used items. The manufacturer's profit which remains the same as in the previous cases, is given by

$$\begin{aligned} \Pi_m &= (P_m - C_p)(a - \beta P_m + \alpha e) - P_r\gamma(a - \beta P_m + \alpha e) - \frac{ke^2}{2} \\ &\quad + \frac{y}{2\beta}(a - \beta P_m + \alpha e)^2 \end{aligned} \tag{5.12}$$

The optimal decisions are obtained following backward substitution approach. With the optimal retail price and effort level of the manufacturer (in terms of p_r), the joint decision maker's optimal retail price reduces to

$$P_r^* = \frac{(C_c + C_r)\beta\gamma + \theta(a - \beta C_p)}{2\beta\gamma\theta}$$

Now, using P_r^*, the optimal retail price and the effort level of the manufacturer are obtained as

$$\begin{aligned} e^* &= \frac{\alpha\Big\{(C_c + C_r)\beta\gamma - a\theta + \beta\theta C_p\Big\}}{2S\theta} \\ P_m^* &= \frac{(C_c + C_r)\beta\gamma(\alpha^2 - k\beta) + \Big[a\big\{\alpha^2 + k(-3 + 2y)\beta\big\} + \beta C_p(\alpha^2 - k\beta)\Big]\theta}{2S\beta\theta} \end{aligned}$$

5.4 Numerical illustration

In this section, we illustrate the developed models in both case I and case II numerically.

5.4.1 Numerical example

To analyze the developed models numerically, we consider the parameter-values as $C_p = 60$, $\theta = 0.7$, $\gamma = 1.3$, a =1000, $\beta = 1.3$, $C_c = 20$, y= 0.7, $k = 15$, $C_m = 120$, $\alpha = 1.1$, $\lambda = 0.98$ and $\mu = 0.75$ in appropriate units. These parameter-values satisfy the concavity condition $k\beta - \alpha^2 y > 0$ for the manufacturer's profit function, given in equation (5.1). For this data set, we obtain the optimal price and profit for each player in the developed game models in Case I as shown in Table 5.1.

Table 5.1: Optimal results under different game models in Case I.

Optimal results	Centralized game	Decentralized game	Revenue and cost sharing contract
e^*	26.61	13.31	17.37
P_m^*	428.86	599.05	445.93
P_d^*	-	140.95	72.57
Π_m^*	-	26490.5	31689.0
Π_c^*	-	52980.9	62131.0
Π_{cm}^*	105962.0	79471.4	93820.0

We now use the superscripts c, rcs and d to denote the centralized, revenue-cost sharing contract and decentralized models, respectively. From the numerical results depicted in Table 5.1, we have the following observations:

(i) $e^{*(c)} > e^{*(rcs)} > e^{*(d)}$ *i.e.,* in the centralized model, the optimum effort level is the highest compared to the other two models. This happens because collaborative joint decision in any supply chain is inclined to optimal benchmark decisions. Between the other two models, the optimal effort is more in case of joint contract, compared to the centralized model.

(ii) $p_m^{*(c)} < p_m^{*(rcs)} < p_m^{*(d)}$ *i.e.,* the optimal retail price is the lowest in the centralized model and the highest in the decentralized model.

(iii) $\Pi_m^{*(rcs)} > \Pi_m^{*(d)}$ and $\Pi_c^{*(rcs)} > \Pi_c^{*(d)}$ *i.e.,* through the revenue and cost sharing contract, the manufacturer and the collector both get more profits than their corresponding decentralized profits.

(iv) $\Pi_{cm}^{*(c)} > \Pi_{cm}^{*(rcs)} > \Pi_{cm}^{*(d)}$ which means that the total supply chain profit is highest in the centralized model but between the other two models, the total supply chain profit under contract is more compared to the decentralized model.

For numerical experiment in Case II, we keep the parameter-values same as Case I except the cost and revenue sharing contract parameters. We take $\lambda = 0.96,\ \mu = 0.92, \delta = 0.92, \phi = 0.92$. Optimal results are given in Table 5.2. It is observed that, similar to case I, the optimum effort level in the centralized model is maximum among the four models. The optimum retail price for the final product follows the inequality $p_m^{*(c)} < p_m^{*(ss)} < p_m^{*(rcs)} < p_m^{*(d)}$, where the superscript ss denotes the sub-supply game. Due to lower retail price and higher effort level, the total demand of the final product is the highest in the centralized model and the lowest in the decentralized model. Each supply chain entity gains more profit under the contract rather than the profit of the decentralized model. In the special situation (sub-supply game) when the collector and the recycler behave as a single decision maker, the profit of the supply chain increases compared to the decentralized model or the model with revenue-cost sharing contract.

Table 5.2: Optimal results under different game models in Case II.

Optimal results	Centralized game	Decentralized game	Revenue and cost sharing contract	Sub-supply game
e^*	30.46	6.52	7.89	14.56
P_m^*	376.03	678.21	608.53	532.82
P_r^*	-	468.02	422.84	337.32
P_c^*	-	153.65	151.92	-
Π_m^*	-	27378.0	29378.65	36556.32
Π_r^*	-	18247.0	20393.20	-
Π_c^*	-	49461.0	51798.0	-
Π_{cr}^*	-	-	-	74390.0
Π_{crm}^*	117246.0	95086.0	101569.85	110946.0

5.4.2 Sensitivity analysis

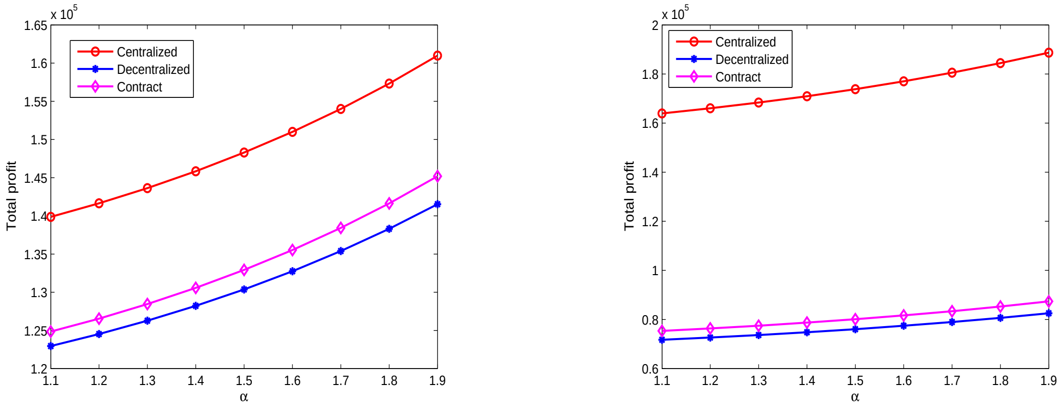
In this section, we examine the sensitivity of the key parameters α, β, θ and y on the total profit of the supply chain for different strategies in both the cases.

5.4.2.1 Sensitivity with respect to α

With increasing values of α, the demand increases. As a result, the manufacturer earns higher profit. Since the manufacturer needs more wastes to fulfill customer demand, the collector or the recycler also automatically benefits by supplying more recyclable or recycled wastes. Fig. 5.1 depicts the sensitivity analysis with respect to α in both the cases. From Fig. 5.1 we see that, under the contract, the supply chain's total profit is higher than that in the decentralized policy.

5.4.2.2 Sensitivity with respect to β

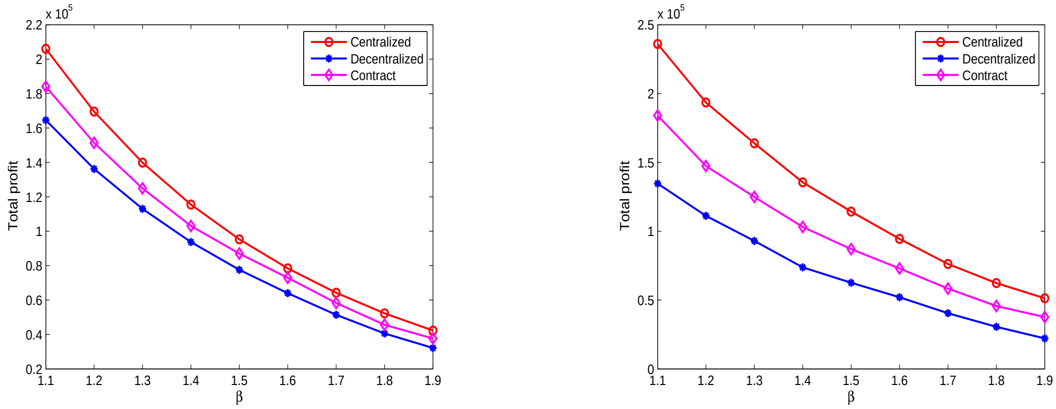
Since the demand is inversely proportional to β, the profits of the supply chain members as well as the whole supply chain decrease with higher value of β. From Fig. 5.2(a), it is clear that, though the total profit is marginally higher for the benchmark case, still it decreases more rapidly when $\beta > 1.5$. Among the decentralized and contract policies, supply chain's total profit is greater for the contract policy when $\beta \in (1.1, 1.4)$ and for other values



(a) α *vs. total profits (Case I)* **(b)** α *vs. total profits (Case II)*

Fig. 5.1: *Sensitivity with respect to* α*.*

of β (> 1.4), the decentralized policy gives better profit. However, Fig. 5.2(b) shows that contract policy is beneficial for the whole supply chain when $\beta > 1.2$.

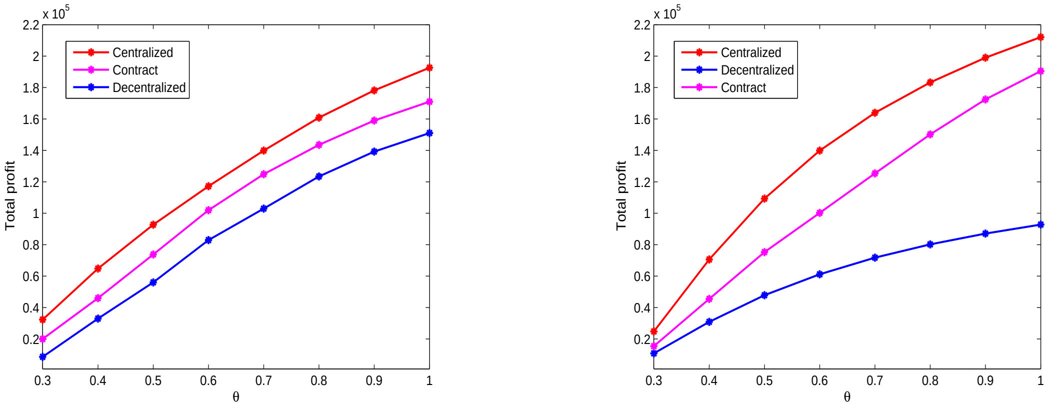


(a) β *vs. total profits (Case I)* **(b)** β *vs. total profits (Case II)*

Fig. 5.2: *Sensitivity with respect to* β*.*

5.4.2.3 Sensitivity with respect to θ

Analytical result shows that the parameter θ has direct impact on consumer demand which increases with recyclability degree θ. So, if θ increases, the manufacturer and all other entities are benefitted from recyclable wastes. From one unit of recyclable waste, the recycler or the manufacturer procures more recycled product and this results in lower price. From Fig. 5.3, we see that the total profit of the supply chain increases with θ and the total profit of the supply chain enhances under a revenue and cost sharing contract.

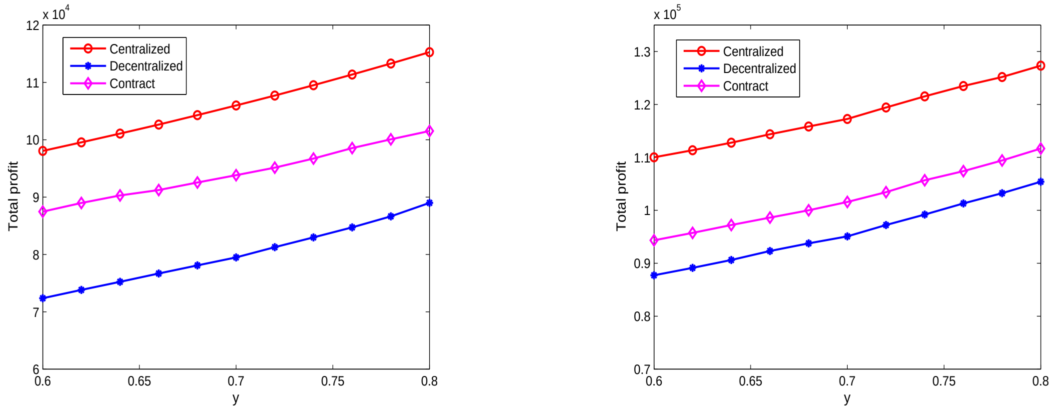


(a) *θ vs. total profits (Case I)* **(b)** *θ vs. total profits (Case II)*

Fig. 5.3: *Sensitivity with respect to θ.*

5.4.2.4 Sensitivity with respect to y

Socially responsible manufacturer's total profit is directly proportional to the fraction of consumer surplus y. From the numerical results, it is clear that the manufacturer's retail price decreases with y and the effort level increases when customer demand for the product is more. Profits of the manufacturer and the collector increase for higher values of y. Fig. 5.4 further depicts that the supply chain members can reach to a win-win situation for a higher value of y.



(a) *y vs. total profits (Case I)* **(b)** *y vs. total profits (Case II)*

Fig. 5.4: *Sensitivity with respect to y.*

5.5 Conclusion

In this chapter, we have studied a closed-loop supply chain to address recycling of wastes together with social responsibility concern. The manufacturer gives effort for social sustainability and the other members (the collector and the recycler) cooperate with him through cost sharing. From our study, the following meaningful insights are obtained:

(i) A properly designed joint revenue and cost sharing contract helps the members of the supply chain for better decision making and achieving a win-win situation. In the decentralized model, due to double marginalization effect, the individual profits of the supply chain entities and the overall total profit of the supply chain are lower compared to those of the centralized model. A suitable contract can partially mitigate this issue.

(ii) The Stackelberg leader in general should get higher profit than other members. In our study, the collector being the leader, gets more profit in the decentralized model as well as in contract policy. But for the sub-supply game, the joint total profit becomes higher than each of their individual profits.

(iii) Since the demand of the supply chain is price and effort dependent, a higher effort level and a lower retail price always give better profit for the whole supply chain. Consumer surplus *i.e.,* extra profit of the manufacturer thus increases with demand.

(iv) The total profit of the supply chain increases with recyclability degree of waste because the consumer demand increases with θ. The individual entities also gain higher profit for a particular range of θ.

(v) The fraction of consumer surplus is directly related to individual profit as well as total profit of the supply chain. The socially concerned manufacturer gets higher profit for increasing value of y.

Several future studies can be done considering different forms of deterministic or stochastic demand. Social sustainability can be considered through incentive such as government subsidy. In a completely stochastic scenario where the collection of used products depends on quality of the product of reward level, the problem become more complex and realistic too. Besides social responsibility, other actions like social work donation can be proposed. Inter supply chain competition or the competitions among multi-retailers can be further investigated. Government subsidy for social responsibility and carbon footprint consideration may also be considered as future research avenues.

Chapter 6

Analyzing a closed-loop sustainable SC with duopolistic retailers under different game structures

6.1 Introduction

Sustainability has become an emerging issue now-a-days for a chaotic environment surrounding us. As the population is growing and human lifestyle is becoming richer, the consumption of natural resources is gradually increasing, and therefore, people are in no mood except thinking about sustainability in a parallel way. Brundtland (1987) defined sustainable development as "the development that meets the needs of the present without compromising the ability of future generations to meet their own need". It includes environmental, social and economic stability or balance on our planet. Over the last two decades, many of the enterprises have taken social responsibility into account along with their general business policies, which has been an emerging way for sustainability consideration. In this chapter* a socially responsible supply chain is considered with monopoly as well as duopoly retailers. Through corporate social responsibility (CSR) consideration, the firms not only can draw the attention of customers but also generate scope for a better and positive response for well business policy in

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the future. Supports for a charity, social ethics, safety standards and protection of workers in industry are considered social responsibilities for any business firm (Ageron et al., 2012; Liu et al., 2019). Two opposite perspectives to CSR have emerged- business view and societal view. Through a business view, a large enterprise makes a contribution to the society by making profit which is related to wages, taxes, employment, investment etc. However, in societal view, stakeholder's welfare and social responsibilities are prioritized. Thus CSR covers the relationship between the society and the enterprises. CSR is therefore an integral part of a firm's strategy. Though there are some arguments like moral, ethical and specially the economic argument but, through CSR implementation, a company creates a greater market advantage in the competitive market scenario. It is increasingly relevant in today's competitive market and so the organisations who fail to maximize the adoption of CSR strategy may be left behind. According to a global survey result in 2002, it was noted that about 94% organizations trust the implementation of CSR strategy to produce real business benefits. Thus, with business goals, the companies are looking for social and environmental issues although only 11% of them have successfully set up the CSR strategies in their organizations (Panda et al., 2017).

Remanufacturing provides a golden opportunity towards reaching a sustainable future (Copani and Behnam, 2020; Reimann et al., 2019) and it is also a potential solution that facilitates sustainable business practices (Atasu et al., 2008). Many enterprises like Kodak, Hewlett-Packard, etc. have already participated in product recovery and remanufacturing (Qiang et al., 2013). Though recycling and remanufacturing both have environmental merits, the re-marketing or reselling of a remanufactured product is always challenging (Long et al., 2019). Tolio et al. (2017) highlighted the main challenges and opportunities that are faced in a demanufacturing as well as remanufacturing system. From customer perspective, the forward and the backward supply chains are not of equal importance. Customer's initial response for remanufactured product may not be positive because of a variety of reasons. The main reason may be the uncertainty about the quality of the remanufactured product (Ovchinnikov, 2011). To resolve this issue, various power tools are required and that causes an expense for the socially responsible retailer or the manufacturer. Moreover, understanding the market demand for manufactured and remanufactured products is a critical issue. The challenge remains when the market consists of different competitive retailers selling the same product.

In a supply chain with duopolistic retailers, the main concern is the manufacturer's decision especially when he/she acts as the Stackelberg leader (Yang and Zhou, 2006). In a Stackelberg game, one firm (or member) has greater power than other firms (or members). When one of the members acts as the Stackelberg leader, others have to follow the leader. The Stackelberg leader's decision making power is higher than the other players. Thus, at the time of decision making, the Stackelberg leader first takes into account the decisions of the followers and then makes its own decisions. It is also important to check whether the manufacturer's decision is affected by the competitive retailers; specifically, whether the superiority in gaining profit remains despite the retailers' competitive behavior (Lau and Lau, 2003).

Considering sustainability through remanufacturing and competition in the retail market, a closed-loop supply chain model is developed in this chapter. Two competitive retailers are involved in the supply chain. Corporate social responsibility is considered by one of the retailers. The main objectives of this study are as follows:

(i) To investigate whether remanufacturing is a good policy under competition between two retailers or not.

(ii) To investigate the manufacturer's economic sustainability through overall performance because besides environmental driven strategy by remanufacturing, economic stability remains a vital factor for the manufacturer.

(iii) To determine socially responsible retailer's optimal decisions and investigate whether social responsibility consideration turns out to be a profitable policy or not. In fact, social responsibility consideration by the retailer is a big move especially in the decentralized scenario when each entity takes decisions individually.

(iv) To determine the optimal decisions of the two suppliers who supply fresh raw materials and used products or cores for manufacturing and remanufacturing, respectively.

The chapter is organised as follows: In the next section, problem description and notations are given. In Section 6.3, models are developed from centralized, decentralized and sub-supply chain perspectives. Model analysis and a special case with no competitive retailers are discussed there. In Section 6.4, the validity of the developed models is checked with the help of a numerical example and also sensitivity analysis is carried out with respect to key model-parameters. Finally, the chapter is concluded in Section 6.5 with managerial insights and some future research directions.

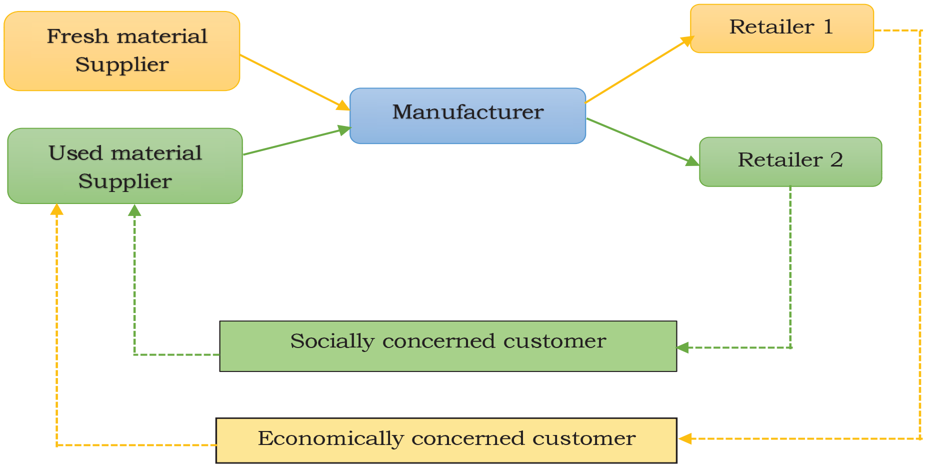


Fig. 6.1: *Structure of the proposed CLSC model.*

In the present chapter, a three-echelon closed-loop supply chain is considered with one manufacturer, two suppliers and two competitive retailers. Corporate social responsibility is taken by one of the retailers (see Fig. 6.1). Several game strategies are implemented between the manufacturer and the duopoly retailers.

6.2 Notation and problem description

We consider a closed-loop supply chain with one manufacturer, two suppliers and two competitive retailers for trading a single product. For two different kinds of customer in the existing market, the manufacturer produces both fresh product and remanufactured product and then sells them with different wholesale prices through two different retailers. Among them one takes social responsibility into account but the other one focuses only on economic benefits. Since the retailers are competitive in nature, the retail price of fresh product has great impact on demand of the remanufactured product and conversely the retail price of remanufactured product has similar impact on the demand of fresh product. On the other hand, for production of the final product, the manufacturer depends on two different kinds of supplier. The collection of used products or cores and fresh virgin materials are two completely different activities in manufacturer's perspective. Therefore, in general, two different players or entities are involved in those separate activities, especially when a third party collects the used materials for remanufacturing. Other reasons may be due to separate collection hubs in different places, delay in delivery for raw materials, communication gap or disagreement among the members in the supply chain. That's why, one supplier supplies used products or cores and the other one supplies fresh virgin materials. The following notations are used throughout the chapter:

c_1 : unit collection cost of fresh raw material supplier
c_2 : unit collection cost of used product supplier
s_1 : unit wholesale price of fresh raw material supplier
s_2 : unit wholesale price of used product supplier
w_1 : manufacturer's unit wholesale price for manufactured product
w_2 : manufacturer's unit wholesale price for remanufactured product
p_1 : retailer-1's unit retail price
p_2 : retailer-2's unit retail price
a : maximum possible total market demand of the product
b_1 : price sensitivity of retailer-1's demand
b_2 : price sensitivity of retailer-2's demand
θ : price sensitivity due to competition of the retailers
y : retailer-2's social responsibility factor
α : sensitivity of retailer-2's social responsibility factor
β : unit social responsibility investment cost for retailer-2
D_1 : customer demand for fresh product
D_2 : customer demand for remanufactured product

From now onwards, we use the terms retailer-1 ($r1$) and retailer-2 ($r2$) to denote the fresh product selling retailer and the remanufactured product selling retailer, respectively. Some basic assumptions made for development of the proposed model are as follows:

(i) Supplier's unit wholesale price for fresh raw material is greater than that of used product *i.e.,* $s_1 > s_2$. Manufacturer's unit re-manufacturing cost is less than unit manufacturing cost (Hong et al., 2017; Savaskan et al., 2004).

(ii) Unit cost of core is less than that of fresh raw material *i.e.,* $c_2 < c_1$; otherwise, remanufacturing will not be an optimal strategy from the manufacturer's point of view.

(iii) Market demand is sensitive to both price and social responsibility (Reimann et al., 2019). The demand at the retailer-1 is $D_1 = (a - b_1p_1 + \theta p_2)$ and the demand at the retailer-2 is $D_2 = (a - b_2p_2 + \theta p_1 + \alpha y)$.

6.3 Model development and analysis

In this section, the model is developed upon different game structures of the manufacturer and the duopoly retailers. Several cases of socially responsible as well as non-responsible retailer's decisions are considered and two special cases for single retailer are discussed. Analytical results with respect to the key decision variables are also derived.

6.3.1 Centralized model: The benchmark case

In the centralized scenario, the manufacturer, two suppliers and two competitive retailers act as a single decision maker without worrying about their own profits. The joint profit of the supply chain in this case is given by

$$\begin{aligned}\Pi &= (p_1 - c_1)D_1 + (p_2 - c_2)D_2 - \frac{\beta y^2}{2}\\ &= (p_1 - c_1)(a - b_1p_1 + \theta p_2) + (p_2 - c_2)(a - b_2p_2 + \theta p_1 + \alpha y) - \frac{\beta y^2}{2}\end{aligned}$$

where the first term denotes the profit by selling new products, second term represents the profit for selling the remanufactured products and last term is the CSR implementation cost. The total profit of the supply chain thus depends on the retailer's optimal retail prices and social responsibility factor.

Proposition 6.1 *The total profit* Π *of the supply chain is jointly concave (in strict sense) in* y, p_1 *and* p_2 *if* $b_1b_2 > \theta^2$, $2\beta b_2 > \alpha^2$ *and* $b_1\alpha^2 - 2b_1b_2\beta + 2\beta\theta^2 < 0$.

Proof: We have

$$\begin{aligned}
\frac{\partial \Pi}{\partial y} &= -\beta y + \alpha(p_2 - c_2) \\
\frac{\partial \Pi}{\partial p_1} &= a + \theta p_2 - 2b_1 p_1 + \theta(p_2 - c_2) \\
\frac{\partial \Pi}{\partial p_2} &= a + \theta(p_1 - c_1) + \theta p_1 + \alpha y - 2b_2 p_2
\end{aligned}$$

And the second order partial derivatives are $\frac{\partial^2 \Pi}{\partial y^2} = -\beta;\quad \frac{\partial^2 \Pi}{\partial p_1^2} = -2b_1;\quad \frac{\partial^2 \Pi}{\partial p_2^2} = -2b_2.$

The Hessian matrix is given by $H = \begin{pmatrix} \frac{\partial^2 \Pi}{\partial y^2} & \frac{\partial^2 \Pi}{\partial p_1 \partial y} & \frac{\partial^2 \Pi}{\partial p_2 \partial y} \\ \frac{\partial^2 \Pi}{\partial y \partial p_1} & \frac{\partial^2 \Pi}{\partial p_1^2} & \frac{\partial^2 \Pi}{\partial p_2 \partial p_1} \\ \frac{\partial^2 \Pi}{\partial y \partial p_2} & \frac{\partial^2 \Pi}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi}{\partial p_2^2} \end{pmatrix} = \begin{pmatrix} -\beta & 0 & \alpha \\ 0 & -2b_1 & 2\theta \\ \alpha & 2\theta & -2b_2 \end{pmatrix}$

Now the principal minors of order 1 $\left(i.e., \frac{\partial^2 \Pi}{\partial y^2}, \frac{\partial^2 \Pi}{\partial p_1^2}, \frac{\partial^2 \Pi}{\partial p_2^2}\right)$ are all negative. In order the profit function to be concave, the second order principal minors have to be positive and the third order principal minor must be negative. So, the conditions $b_1 b_2 > \theta^2$, $2\beta b_2 > \alpha^2$ and $b_1\alpha^2 - 2b_1 b_2 \beta + 2\beta\theta^2 < 0$ must hold for the concavity of the profit function Π. Hence the proposition is proved.

The first of the above three conditions for concavity of the profit function Π implies that $b_1 b_2 > \theta^2$ *i.e.,* the product of the direct retail price sensitive parameters b_1 and b_2 is greater than the product of the price sensitivities due to competition. This suggests that the market demand is more price sensitive than the competition among manufactured and remanufactured products of two competitive retailers. In particular case, if $b_1 = b_2$ $(= b$, say$)$ then $b^2 > \theta^2$ *i.e.,* $b > \theta$ (since $b, \theta > 0$) and this also implies the same. The second condition $(2\beta b_2 > \alpha^2)$ implies that, for social responsibility consideration of the retailer, the product of price sensitivity for the CSR implementation cost and price sensitivity for remanufactured product is greater than half of the square of CSR sensitivity parameter of the demand. The third condition is $b_1\alpha^2 - 2b_1 b_2 \beta + 2\beta\theta^2 < 0$ *i.e.,* $b_1\alpha^2 + 2\beta\theta^2 < 2b_1 b_2 \beta$ which can be deduced from the first two conditions. Therefore, it can be concluded that optimal profit of the supply chain can be achieved when the market demand is more retail price sensitive, and, for the socially responsible retailer's CSR consideration, the condition $2\beta b_2 > \alpha^2$ should hold.

Theorem 6.1 *The optimal social responsibility level and retail prices are given by*

$$\begin{aligned}
y^* &= \frac{\alpha\Big\{a(b_1+\theta)+c_2(\theta^2-b_1b_2)\Big\}}{b_1(2\beta b_2-\alpha^2)-2\beta\theta^2} \\
p_1^* &= \frac{1}{2}\left[c_1+\frac{c_2\alpha^2\theta+a\Big\{\alpha^2-2\beta(b_2+\theta)\Big\}}{b_1(\alpha^2-2b_2\beta)+2\beta\theta^2}\right] \\
p_2^* &= \frac{b_1c_2\alpha^2-b_1(a+b_2c_2)\beta+\beta\theta(c_2\theta-a)}{b_1(\alpha^2-2b_2\beta)+2\beta\theta^2}
\end{aligned}$$

Proof: Using the first order optimality conditions and solving the corresponding three equations (for three decision variables) simultaneously, we can easily get the above optimal decisions y^*, p_1^* and p_2^*, which correspond to the maximum profit of the whole supply chain.

6.3.2 Decentralized model

In the decentralized model, each entity determines his/her own individual optimal decisions. We use the subscripts $m, r1, r2, s1$ and $s2$ to denote the manufacturer, retailer-1, retailer-2, supplier-1 and supplier-2, respectively. The individual profit functions of the supply chain entities are given by

$$\begin{aligned}
\Pi_m &= (w_1-s_1)(a-b_1p_1+\theta p_2)+(w_2-s_2)(a-b_2p_2+\theta p_1+\alpha y) \\
\Pi_{r1} &= (p_1-w_1)(a-b_1p_1+\theta p_2) \\
\Pi_{r2} &= (p_2-w_2)(a-b_2p_2+\theta p_1+\alpha y)-\frac{\beta y^2}{2} \\
\Pi_{s1} &= (s_1-c_1)(a-b_1p_1+\theta p_2) \\
\Pi_{s2} &= (s_2-c_2)(a-b_2p_2+\theta p_1+\alpha y)
\end{aligned}$$

In the decentralized situation, we consider a manufacturer-Stackelberg game in which the manufacturer is the leader and retailers as well as suppliers are the followers. The sequence of events is as follows : First, according to retailers' order quantities, the manufacturer orders for raw materials from the two suppliers with purchasing costs s_1 per unit and s_2 per unit, respectively. Knowing the supplier's fixed selling prices, the manufacturer decides the wholesale prices w_1 per unit and w_2 per unit. Then, knowing the manufacturer's decisions, the retailers finalize their retail prices p_1, p_2 and CSR level y.

Proposition 6.2 *The retailer-2's profit function* Π_{r2} *is jointly concave in* y *and* p_2 *if* $2\beta b_2 > \alpha^2$ *whereas retailer-1's profit function* Π_{r1} *is strictly concave in* p_1*.*

Proof: We have

$$\begin{aligned}\frac{\partial \Pi_{r2}}{\partial y} &= -\beta y + \alpha(p_2 - w_2)\\ \frac{\partial \Pi_{r2}}{\partial p_2} &= a + \theta p_1 + \alpha y - 2b_2 p_2 + b_2 w_2\end{aligned}$$

And the second order partial derivatives are $\dfrac{\partial^2 \Pi_{r2}}{\partial y^2} = -\beta < 0$; $\dfrac{\partial^2 \Pi_{r2}}{\partial p_2^2} = -2b_2 < 0$ and $\dfrac{\partial^2 \Pi_{r1}}{\partial p_1^2} = -2b_1 < 0$. The Hessian matrix corresponding to the profit function of the retailer-2 is

$$H = \begin{pmatrix} \frac{\partial^2 \Pi_{r2}}{\partial y^2} & \frac{\partial^2 \Pi_{r2}}{\partial p_2 \partial y} \\ \frac{\partial^2 \Pi_{r2}}{\partial y \partial p_2} & \frac{\partial^2 \Pi_{r2}}{\partial p_2^2} \end{pmatrix} = \begin{pmatrix} -\beta & \alpha \\ \alpha & -2b_2 \end{pmatrix}$$

This implies that the retailer-2's profit function Π_{r2} is concave in y and p_2 provided that $|H| > 0$ *i.e.,* $2\beta b_2 > \alpha^2$ whereas the retailer-1's profit function Π_{r1} is strictly concave in p_1. Hence the result.

Thus, in the manufacturer-Stackelberg game, when the retailers play a Cournot game, the corporate social responsible retailer gets the optimum profit provided that the condition $2\beta b_2 > \alpha^2$ holds. Therefore, when the two retailers have similar decision powers but the manufacturer is the leader in decision making, the optimality conditions for best profits of the retailers are given in the proposition above. The socially responsible retailer gets maximum profit by selling the remanufactured products when $2\beta b_2 > \alpha^2$, which is same as the second case of Proposition 6.1. Similarly the other retailer who sells newly manufactured products gets best profit when he/she fixes the retail price p_1^* and the manufacturer settles the optimal wholesale prices w_1^* and w_2^* given in the following theorem.

Theorem 6.2 *When the manufacturer acts as the Stackelberg leader and the duopoly retailers play Cournot game, the optimal wholesale prices of the manufacturer, retail prices of the* *retailers and CSR level y are given by*

$$\begin{aligned}
w_1^* &= \frac{4b_1b_2^2(a+b_1s_1)\beta(\alpha^2-2b_2\beta)^2+A\theta+B\theta^2+C\theta^3+D\theta^4}{(b_1(\alpha^2-2b_2\beta)+\beta\theta^2)+(8b_1b_2^2\beta(\alpha^2-2b_2\beta)+(\alpha^2-4b_2\beta)^2\theta^2)}\\
w_2^* &= \frac{4b_1b_2(a+b_2s_2)\beta(-\alpha^2+2b_2\beta)+E\theta-F\theta^2-G\theta^3}{8b_1b_2^2\beta(-\alpha^2+2b_2\beta)-(\alpha^2-4b_2\beta)^2\theta^2}\\
p_1^* &= \frac{2b_1b_2^2(3a+b_1s_1)\beta(\alpha^2-2b_2\beta)^2+I\theta+J\theta^2+K\theta^3+L\theta^4}{(b_1(\alpha^2-2b_2\beta)+\beta\theta^2)(8b_1b_2^2\beta(\alpha^2-2b_2\beta)+((\alpha^2-4b_2\beta)^2)\theta^2)}\\
p_2^* &= \frac{4b_1^2b_2\beta(\alpha^2-2b_2\beta)((a+b_2s_2)\alpha^2-b_2(3a+b+2s_2)\beta)-M\theta+N\theta^2+P\theta^3+Q\theta^4+R\theta^5}{(b_1(\alpha^2-2b_2\beta)+\beta\theta^2)(8b_1b_2^2\beta(\alpha^2-2b_2\beta)+((\alpha^2-4b_2\beta)^2)\theta^2)}\\
y^* &= \frac{-S}{(b_1(\alpha^2-2b_2\beta)+\beta\theta^2)(8b_1b_2^2\beta(\alpha^2-2b_2\beta)+((\alpha^2-4b_2\beta)^2)\theta^2)}
\end{aligned}$$

where,

$A = 2b_1b_2\beta(\alpha^2-2b_2\beta)((a+b_2s_2)\alpha^2-4ab_2\beta);$

$B = (\alpha^2-2b_2\beta)(b_1s_1\alpha^4-b_2(a+5b_1s_1\alpha^2)\beta+4b_2^2(a+3b_1s_1)\beta^2);$

$C = b_2\beta(-s_2\alpha^4+3(a+b_2s_2)\alpha^2\beta-8ab_2\beta^2);$

$D = s_1\beta(\alpha^4-5b_2\alpha^2\beta+8b_2^2\beta^2);$

$E = (\alpha^2-2b_2\beta)\{(a-b_1s_1)\alpha^2-4ab_2\beta\};$

$F = \beta\{(a-3b_2s_2)\alpha^2+8b_2^2s_2\beta\};$

$G = s_1(\alpha^2)\beta;$ $I = b_1b_2\beta(\alpha^2-2b_2\beta)(3(a+b_2s_2)\alpha^2-2b_2(5a+b_2s_2)\beta);$

$J = (\alpha^2-2b_2\beta)(b_1s_1\alpha^4-b_2(2a+3b_1s_1)\alpha^2\beta+2b_2^2(4a+3b_1s_1)\beta^2);$

$K = b_2\beta(-2s_2\alpha^4+(5a+7b_2s_2)\alpha^2\beta-4b_2(3a+b_2s_2)\beta^2);$

$L = s_1\beta(\alpha^4-3b_2\alpha^2\beta+4b_2^2\beta^2);$

$M = b_1(\alpha^2-2b_2\beta)((a-b_1s_1)\alpha^4+b_2(-5a+b_1s_1)\alpha^2\beta+2b_2^2(5a+b_1s_1)\beta^2);$

$N = b_1b_2\beta(-3s_2\alpha^4+4(2a+3b_2s_2)\alpha^2\beta-2b_2(9a+5b_2s_2)\beta^2);$

$P = \beta((-a+b_1s_1)\alpha^4+b_2(7a+b_1s_1)\alpha^2\beta-6b_2^2(2a+b_1s_1)\beta^2);$

$Q = b_2(\beta^2)(-s_2\alpha^2+2(a+b_2s_2)\beta);$ $R = 2b_2s_1\beta^3;$

$S = \alpha((b_1b_2-\theta^2)(-b_1(\alpha^2-2b_2\beta)(4b_2^2s_2\beta+s_1\alpha^2\theta-2b_2s_1\beta\theta)+\beta\theta^2(2b_2s_2\alpha^2-6b_2^2s_2\beta-s_1\alpha^2\theta+2b_2s_1\beta\theta))+a(4b_1^2b_2^2\beta(\alpha^2-2b_2\beta)+\beta\theta^3(4b_2^2\beta+\alpha^2\theta-b_2(\alpha^2+2\beta\theta))+b_1\theta(-4b_2^3\beta^2+\alpha^4\theta+10b_2^2\beta^2\theta+b_2(\alpha^4-6\alpha^2\beta\theta)))).$

Proof: Due to complexity, here it is not possible to compare analytically the profits of individual players. However, through a numerical example, their respective profits can be compared, based on their decision powers.

6.3.3 Sub-supply chain

When two or more members of a supply chain recognize their inter-dependence and agree to act in union to maximize their total profit, it is termed as sub-supply chain. It is a intermediate stage between Centralized and Nasg game. In this case, the manufacturer chooses to merge with either the supplier or the retailer and then acts as a single entity. In other words, the manufacturer chooses to open its own raw material manufacturing facility or retailing facility. For instance, if LENOVO decides to manufacture the motherboard of laptop by itself then it is an example of manufacturer-supplier merger. Similarly, when mobile manufacturers use their exclusive showrooms to sell their products, it can be termed as manufacturer-retailer merger. The composite coordinating strategy to take decisions lead to greater profits than individual total profits in the decentralized supply chain. The supply chain literature is mainly enriched with manufacturer-retailer, retailer-retailer or supplier-manufacturer sub-supply games (Giri et al., 2017; Guo et al., 2011; Yang and Zhou, 2006).

In this Subsection, we focus on the situation where the manufacturer either ties up with retailer-1 or retailer-2 for a better business policy with economical benefit. The motivation behind this consideration is that when the manufacturer ties up with the suppliers, only the supplier's wholesale price s_1 or s_2 will be negotiated, which is considered to be fixed. All the decision variables remain the same. On the other hand, in the manufacturer-retailer sub-supply game, the decision variable w_1 or w_2 will be eliminated from the decision making analysis. The joint decision between the manufacturer and the retailer therefore focuses only on retail price and CSR level. Here the joint total profit is the sum of their individual profits. We denote the models as m-r1 and m-r2 when the manufacturer collaborates with the retailer-1 and the retailer-2, respectively. The manufacturer's wholesale prices w_1 and w_2 are eliminated in m-r1 and m-r2, respectively as they are internal decisions of the sub-supply chain.

6.3.3.1 m-r1 model

In this subsection, we assume that the manufacturer and the retailer-1 jointly venture in the optimal decision making. Hence the wholesale price w_1 vanishes due to negotiation between the manufacturer and the retailer-1. The social responsibility is considered by the retailer-2 independently. Therefore, in this case, the profit functions are given by

$$\begin{aligned}
\Pi_{m-r1} &= (p_1 - s_1)(a - b_1p_1 + \theta p_2) + (w_2 - s_2)(a - b_2p_2 + \theta p_1 + \alpha y) \\
\Pi_{r2} &= (p_2 - w_2)(a - b_2p_2 + \theta p_1 + \alpha y) - \frac{\beta y^2}{2} \\
\Pi_{s1} &= (s_1 - c_1)(a - b_1p_1 + \theta p_2) \\
\Pi_{s2} &= (s_2 - c_2)(a - b_2p_2 + \theta p_1 + \alpha y)
\end{aligned}$$

The manufacturer and the retailer-1 jointly decide their optimal retail price for the final product and the wholesale price for the retailer-2. However, the retailer-2 independently optimizes his/her retail price (p_2) and CSR level (y). The optimal decisions are given by

$$\begin{aligned}
y^* &= \frac{\alpha\Big\{a(b_1 + \theta) + s_2(\theta^2 - b_1b_2)\Big\}}{b_1(2\beta b_2 - \alpha^2) - 2\beta\theta^2} \\
w_2^* &= \frac{2b_1b_2\beta(a + b_2s_2)(2b_2\beta - \alpha^2) + U\theta + \beta\big\{-a\alpha^2 + b_2s_2(\alpha^2 - 4b_2\beta)\big\}\theta^2 - s_1\alpha^2\beta\theta^3}{4b_1b_2^2\beta(-\alpha^2 + 2b_2\beta) - (\alpha^4 - 4b_2\alpha^2\beta + 8b_2^2\beta^2)\theta^2} \\
p_1^* &= \frac{-2b_2^2\beta(a + b_1s_1)(\alpha^2 - 2b_2\beta) + V\theta - s_1(\alpha^4 - 3b_2\alpha^2\beta + 4b_2^2\beta^2)\theta^2}{4b_1b_2^2\beta(2b_2\beta - \alpha^2) - (\alpha^4 - 4b_2\alpha^2\beta + 8b_2^2\beta^2)\theta^2} \\
p_2^* &= s_2 + \frac{\beta\Big\{a(b_1 + \theta) + s_2(-b_1b_2 + \theta^2)\Big\}}{2b_1b_2\beta - b_1\alpha^2 - 2\beta\theta^2}
\end{aligned}$$

where $U = (\alpha^2 - 2\beta b_2)\big\{(a - b_1s_1)\alpha^2 - 2ab_2\beta\big\}$ and $V = b_2\beta\big\{-(a + b_2s_2)\alpha^2 + 4ab_2\beta\big\}$.

Here w_2^* and p_1^* represent the optimal wholesale price and retail price jointly set by the manufacturer and the retailer-1. The retailer-2 independently decides his optimal retail price p_2^* and CSR level y^*.

6.3.3.2 m-r2 model

Similar to the subsection above, here we consider that the manufacturer and the retailer-2 collaborate with each other for optimal decisions. The manufacturer produces remanufactured products and shares the cost of social responsibility with the retailer-2. The profit functions in this case are given by

$$\begin{aligned}
\Pi_{m-r2} &= (p_2 - s_2)(a - b_2p_2 + \theta p_1 + \alpha y) + (w_1 - s_1)(a - b_1p_1 + \theta p_2) - \frac{\beta y^2}{2} \\
\Pi_{r1} &= (p_1 - w_1)(a - b_1p_1 + \theta p_2) \\
\Pi_{s1} &= (s_1 - c_1)(a - b_1p_1 + \theta p_2) \\
\Pi_{s2} &= (s_2 - c_2)(a - b_2p_2 + \theta p_1 + \alpha y)
\end{aligned}$$

Optimizing the above profit functions, the optimal wholesale price (w_1^*), retail prices (p_1^* and p_2^*) and the CSR level (y^*) are obtained as

$$\begin{aligned}
w_1^* &= \frac{1}{2}\left(s_1 + \frac{s_2\alpha^2\theta + a\{\alpha^2 - 2\beta(b_2+\theta)\}}{b_1(\alpha^2 - 2b_2\beta) + 2\beta\theta^2}\right) \\
p_1^* &= \frac{1}{4}\left(\frac{a}{b_1} + s_1 + \frac{s_2\theta}{b_1} + \frac{2\Big\{s_2\alpha^2\theta + a\{\alpha^2 - 2\beta(b_2+\theta)\}\Big\}}{b_1(\alpha^2 - 2b_2\beta) + 2\beta\theta^2}\right) \\
p_2^* &= s_2 + \frac{\beta\Big\{a(b_1+\theta) + s_2(-b_1b_2 + \theta^2)\Big\}}{2b_1b_2\beta - b_1\alpha^2 - 2\beta\theta^2} \\
y^* &= \frac{\alpha\Big\{a(b_1+\theta) + s_2(\theta^2 - b_1b_2)\Big\}}{b_1(2\beta b_2 - \alpha^2) - 2\beta\theta^2}
\end{aligned}$$

Here, the manufacturer and the retailer-2 jointly decide the optimal wholesale price w_1^*, retail price p_2^* for remanufactured product and optimal CSR level y^*. The retailer-1 individually decides the optimal retail price p_1^* for the manufactured product.

6.3.4 Special case: Single retailer instead of two competitive retailers

When there is no competitive retailer in the market, two special cases may arise depending on the nature of single retailer who may or may not take the corporate social responsibility. Based on these two cases, we now develop the centralized, decentralized, and sub-supply game models.

6.3.4.1 Case I: Socially responsible single retailer

In this situation, when the retailer wishes to take responsibility for social welfare, he/she needs to exert effort which causes an additional cost. However, if the demand depends on CSR effort then the retailer is always interested to take up CSR activity. Assuming that the retailer-1 and the supplier-1 are absent, the profit functions of the supplier, the manufacturer and the socially responsible retailer can be derived directly from the decentralized model

(Subsection 6.3.2) as

$$\Pi_{s2} = (s_2 - c_2)(a - b_2p_2 + \alpha y) \tag{6.1}$$

$$\Pi_{m} = (w_2 - s_2)(a - b_2p_2 + \alpha y) \tag{6.2}$$

$$\Pi_{r2} = (p_2 - w_2)(a - b_2p_2 + \alpha y) - \frac{\beta y^2}{2} \tag{6.3}$$

Now, like the previous section, we discuss the centralized game, the decentralized game and sub-supply game in the following.

(A1) Centralized game

In the centralized game, all the three members assist each other and jointly target to achieve the optimum economic outcome. The wholesale prices (s_2 and w_2) of the supplier-2 and the manufacturer vanish due to mutual negotiation. The total profit of the supply chain is given by

$$\Pi(y, p_2) = (p_2 - c_2)(a - b_2p_2 + \alpha y) - \frac{\beta y^2}{2}$$

where the first term denotes the profit by selling the remanufactured products and the second term represents the CSR implementation cost.

Proposition 6.3 *The total profit function* Π *is concave in* p_2 *and* y *if* $2\beta b_2 > \alpha^2$.

Proof: We have $\frac{\partial \Pi}{\partial y} = \alpha(p_2 - c_2) - \beta y$; $\frac{\partial^2 \Pi}{\partial y^2} = -\beta$; $\frac{\partial \Pi}{\partial p_2} = a + \alpha y + b_2c_2 - 2b_2p_2$; and $\frac{\partial^2 \Pi}{\partial p_2^2} = -2b_2$. The Hessian matrix is given by

$$H = \begin{pmatrix} \frac{\partial^2 \Pi}{\partial y^2} & \frac{\partial^2 \Pi}{\partial p_2 \partial y} \\ \frac{\partial^2 \Pi}{\partial y \partial p_2} & \frac{\partial^2 \Pi}{\partial p_2^2} \end{pmatrix} = \begin{pmatrix} -\beta & \alpha \\ \alpha & -2b_2 \end{pmatrix}$$

Since both $\frac{\partial^2 \Pi}{\partial y^2}$ and $\frac{\partial^2 \Pi}{\partial p_2^2}$ are negative, the profit function Π is concave in both p_2 and y if the second order principal minor is positive *i.e.,* $2\beta b_2 > \alpha^2$. Hence the proposition is proved.

In case of a single retailer in the market and specially when he/she is socially responsible, the joint total profit of the whole supply chain depends only on retail price p_2 and CSR level y. Therefore, when the optimality condition given in the above proposition (same as Propositions 6.1 and 6.2) holds, the profit of the supply chain is maximum when the decision maker takes the optimal decisions y^* and p_2^* given in the theorem below. Therefore, remanufacturing and retailing of those products are optimal from the whole supply chain's perspective when the single decision maker, in this case, takes the optimal decisions given in the following theorem:

Theorem 6.3 *The optimal retail price and CSR level* y *are given by*

$$\begin{aligned} y^* &= \frac{\alpha(a-b_2c_2)}{2b_2\beta-\alpha^2} \\ p_2^* &= \frac{a\beta-\alpha^2c_2+\beta b_2c_2}{2b_2\beta-\alpha^2} \end{aligned}$$

Proof: The proof is straightforward and, therefore, it is omitted.

Thus, in case of monopoly retail market, the centralized policy attains the highest profit when $2\beta b_2 > \alpha^2$, and the optimal retail price and CSR level for the socially responsible retailer are obtained as given in Theorem 6.3.

(A2) Decentralized game

In the decentralized game, the profit functions of the supplier, the manufacturer and the retailer are given in equations (6.1), (6.2) and (6.3), respectively. Optimizing the retailer's profit function with respect to p_2 and y, we get

$$\begin{aligned} y &= \frac{\alpha(a-b_2w_2)}{2b_2\beta-\alpha^2} \\ p_2 &= \frac{a\beta-\alpha^2w_2+\beta b_2w_2}{2b_2\beta-\alpha^2} \end{aligned}$$

Now, substituting these values in the manufacturer's profit function and then optimizing with respect to w_2, the optimal wholesale price of the manufacturer is obtained as

$$w_2^* = \frac{a+b_2s_2}{2b_2}$$

Hence, for given wholesale price of the supplier, the optimal retail price and CSR level are given by

$$\begin{aligned} y^* &= \frac{\alpha(a-b_2s_2)}{4b_2\beta-2\alpha^2} \\ p_2^* &= \frac{(a+b_2s_2)\alpha^2-b_2(3a+b_2s_2)\beta}{2b_2(\alpha^2-2b_2\beta)} \end{aligned}$$

So, when the manufacturer is the Stackelberg leader and there is a single retailer in the market, the manufacturer decides his optimal wholesale price as w_2^* and the retailer decides his optimal CSR level and retail price as y^* and p_2^*, respectively.

(A3) Sub-supply game

In this situation, the manufacturer and the retailer-2 jointly derive the optimal decisions. The total profit function of the manufacturer and the retailer, and the profit function of the supplier are given by

$$\begin{aligned}\Pi_{m-r2} &= (p_2 - s_2)(a - b_2p_2 + \alpha y) - \frac{\beta y^2}{2}\\ \Pi_{s2} &= (s_2 - c_2)(a - b_2p_2 + \alpha y)\end{aligned}$$

It is easy to show that the joint profit function Π_{m-r2} is strictly concave in p_2 and y. The optimal CSR level and the retail price are obtained as

$$\begin{aligned}y^* &= \frac{\alpha(a - b_2s_2)}{2b_2\beta - \alpha^2}\\ p_2^* &= s_2 + \frac{\beta(a - b_2s_2)}{2b_2\beta - \alpha^2}\end{aligned}$$

6.3.4.2 Case II: Socially non-responsible single retailer

When the retailer does not want to take the social responsibility, he/she need not to bear any extra cost related to CSR. Assuming that the retailer-2 and the supplier-2 are absent, the profit functions of the manufacturer, the retailer and the fresh material supplier in this case can be obtained from the decentralized model (Subsection 6.3.2) as

$$\Pi_{s1} = (s_1 - c_1)(a - b_1p_1) \tag{6.4}$$

$$\Pi_m = (w_1 - s_1)(a - b_1p_1) \tag{6.5}$$

$$\Pi_{r1} = (p_1 - w_1)(a - b_1p_1) \tag{6.6}$$

Like Case I, here also we develop the centralized, the decentralized and the sub-supply game models but, unlike the previous case, the demand is assumed to be dependent on the retail price only *i.e.,* $D = (a - b_1p_1)$, since the retailer is not socially responsible.

(B1) Centralized game

The profit function in the centralized game is given by

$$\Pi(p_1) = (p_1 - c_1)(a - b_1p_1)$$

The only decision variable in this case is the retail price p_1 and the profit function $\Pi(p_1)$ is strictly concave in p_1 since $\frac{\partial^2\Pi(p_1)}{\partial p_1^2} = -2b_1 < 0$. The optimal retail price for the centralized profit is obtained as

$$p_1^* = \frac{a + b_1 c_1}{2b_1}$$

(B2) Decentralized game

Similar to the previous case, optimal values of the manufacturer's wholesale price and the retailer's retail price are obtained as

$$\begin{aligned} p_1^* &= \frac{3a + b_1 s_1}{4b_1} \\ w_1^* &= \frac{a + b_1 s_1}{2b_1} \end{aligned}$$

Comparing the decentralized models developed in Subsections 6.3.4.1 and 6.3.4.2, it is clear that the manufacturer's wholesale prices w_2^* and w_1^* are sensitive to suppliers' wholesale prices s_2 and s_1. Also, wholesale prices w_2^* and w_1^* increase whenever s_2 or s_1 increases in the respective case.

(B3) Sub-supply game

When the manufacturer and the retailer jointly take part in the decision making, their profit function and the supplier's profit function are given by

$$\begin{aligned} \Pi_{m-r1} &= (p_1 - s_1)(a - b_1 p_1) \\ \Pi_{s1} &= (s_1 - c_1)(a - b_1 p_1) \end{aligned}$$

Clearly the joint profit function Π_{m-r1} is strictly concave in p_1 and the optimal retail price of the new product is

$$p_1^* = \frac{a + b_1 s_1}{2b_1}$$

From Subsections 6.3.4.1 and 6.3.4.2, it is clear that retail prices increase whenever the suppliers' wholesale prices increase. Thus the collection cost of cores for remanufacturing or procuring cost for fresh raw materials has direct impact on the supply chain even though the players agree to take decisions collaboratively.

6.4 Numerical analysis

In this section, we demonstrate the developed models through a numerical example. For suitably chosen parameter-values, optimal decisions as well as optimal profits of the supply chain and its members are obtained.

6.4.1 Numerical example

We consider the following parameter-values: $a = 40$, $\theta = 0.7$, $\beta = 1.5$, $b_1 = 1.2$, $b_2 = 1.3$, $\alpha = 1.2$, $c_1 = 2$, $c_2 = 0.5$, $s_1 = 4$ and $s_2 = 2$ in appropriate units. These parameter-values are so chosen that the concavity criteria given in Propositions 6.1, 6.2 and 6.3 are in tact, and the assumptions remain valid for all the game models. Tables 6.1 and 6.2 present the optimal results of the developed game models.

Table 6.1: Optimal prices, CSR levels and supply chain profits of different game models.

Model	y	p_1^*	p_2^*	w_1^*	w_2^*	Π
Centralized	60.67	62.20	76.84	-	-	2673.39
Decentralized	17.56	57.54	56.01	46.27	34.06	1902.76
m-r1 game	15.0	45.27	50.24	-	31.48	1933.82
m-r2 game	59.80	70.48	76.76	62.86	-	2589.58

Table 6.2: Optimal profits under different competitive game models.

Model	Π_m	Π_{m-r1}	Π_{m-r2}	Π_{r1}	Π_{r2}	Π_{s1}	Π_{s2}
Decentralized	1344.3	-	-	114.43	395.17	20.31	28.53
m-r1 game	-	1579.13	-	-	288.62	41.68	24.39
m-r2 game	-	-	2440.19	69.77	-	18.3	61.32

Observations:

The following observations can be made from the numerical results presented in Tables 6.1 and 6.2:

(i) In the decentralized game, we find that $w_2^* = 34.06$ and $w_1^* = 46.27$. Therefore, the manufacturer sells the newly manufactured products at a higher price compared to the remanufactured products. This result supports the feasibility as well as optimality conditions from the manufacturer's point of view because the manufacturer's unit remanufacturing cost is less than unit manufacturing cost. As a result, the retailer would sell the remanufactured product at a lower rate to the customers than the newly manufactured product. Many electronic equipments like computers, printer cartridges etc. are therefore sold at a lower rate through online as well as offline retail channel. From this result, the socially responsible retailer will be influenced for social activities.

(ii) $\Pi_{r2} > \Pi_{r1}$ *i.e.,* the socially responsible retailer's profit is greater than that of the socially non-responsible retailer. This is due to the effect of the manufacturer's wholesale price.

(iii) Though the socially responsible retailer's retail price is less than that of the other retailer, the socially responsible retailer gets more profit from the decentralized game model.

(iv) In the decentralized scenario, the supplier who supplies reusable materials, earns higher profit compared to fresh material supplier *i.e.*, $\Pi_{s2} > \Pi_{s1}$.

(v) In the sub-supply game, for both m-r1 and m-r2 game models, the coordinated profits Π_{m-r1} and Π_{m-r2} are greater than the sum of individual profits in the decentralized game. This indicates that the concept of sub-supply chain is a good choice to adopt for the supply chain members from economical point of view.

(vi) In m-r2 game model, the total profit of the supply chain is more than that in m-r1 game model because the CSR factor y increases in the former case significantly and also wholesale price (w_1) of the non-CSR retailer increases.

Now, in Tables 6.3 and 6.4, we demonstrate the optimal results of the special cases.

Table 6.3: Optimal results of different game models with a single socially responsible retailer.

Model	y^*	p_2^*	w_2^*	Π_m	Π_{r2}	Π_{m-r2}	Π_{s2}	Π
Centralized	18.88	24.60	-	-	-	-	-	456.61
Decentralized	9.12	27.79	16.38	213.23	106.61	-	14.82	334.66
m-r2 game	18.24	24.80	-	-	-	426.45	29.64	456.09

Observations:

For better illustration, we use the superscripts '*cent*' and '*decent*' to designate the centralized and the decentralized models, respectively. The major findings from the optimal results are given below:

Table 6.4: Optimal results of different game models with a single socially non-responsible retailer.

Model	p_1^*	w_1^*	Π_m	Π_{r1}	Π_{m-r1}	Π_{s1}	Π
Centralized	17.67	-	-	-	-	-	294.53
Decentralized	26.0	18.67	129.06	64.53	-	17.6	211.2
m-r1 game	18.67	-	-	-	258.13	35.2	293.33

(i) $y^{*\ cent} > y^{*\ m-r2} > y^{*\ decent}$ *i.e.,* the optimal CSR level is higher than that in the centralized model and also it is better in the manufacturer-retailer sub-supply game model than the decentralized model.

(ii) The retail price follows the sequence $p_1^{*\ cent} < p_1^{*\ m-r1} < p_1^{*\ decent}$ and this leads to $\Pi^{*\ cent} > \Pi^{*\ m-r1} > \Pi^{*\ decent}$ *i.e.,* in the centralized game, the highest profit is obtained. Among the other two game models, the sub-supply game strategy is better than the decentralized policy from the whole supply chain's point of view.

(iii) Similar results are observed for the models with socially responsible retailer instead of socially non-responsible retailer, *i.e.,* $p_2^{*\ cent} < p_2^{*\ m-r1} < p_2^{*\ decent}$ and $\Pi^{*\ cent} > \Pi^{*\ m-r2} > \Pi^{*\ decent}$.

(iv) In case of CSR retailer, the total profit of the supply chain increases in the centralized, the decentralized and sub-supply games. Among the two retailers, the CSR retailer's individual profit is better than the non-CSR retailer. It is therefore a good policy for the retailer to adopt corporate social responsibility.

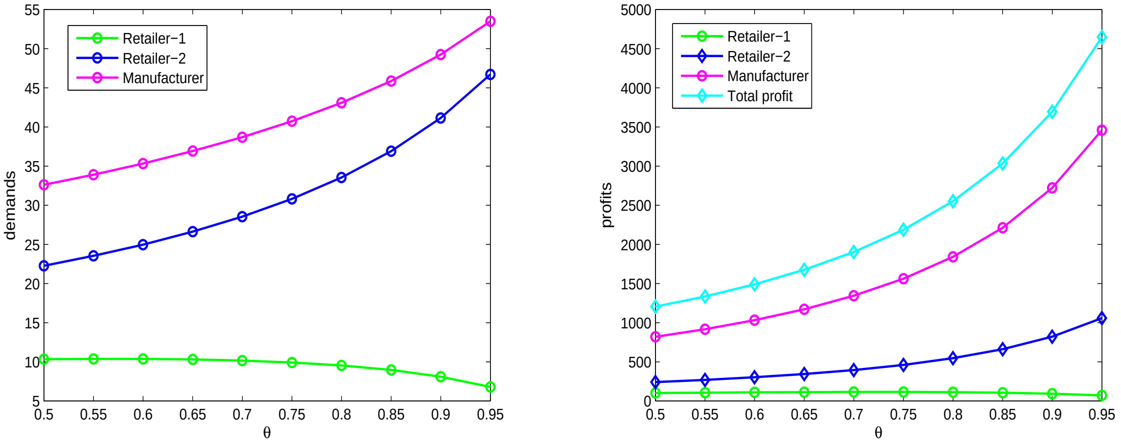
(v) Joint profits in the sub-supply games are greater than the sum of individual profits of the manufacturer and the retailers.

6.4.2 Sensitivity analysis

In this section, we perform the sensitivity analysis with respect to model-parameters θ, α, and β. Change in demands, individual profits and total profits of the supply chain are discussed with respect to these parameters.

6.4.2.1 Sensitivity with respect to θ

Since one retailer considers CSR and the other does not consider it, price sensitivity parameter θ has different impacts on different retailers' demands. From the numerical study it is observed that, as the value of θ increases, the demand of the CSR responsible retailer's remanufactured product increases but the demand of the non-CSR retailer decreases. A higher value of θ indicates a higher retail price of the product, which implies a lower product demand. The overall total demand of the supply chain however increases (see Fig. 6.2(a)). As a result, the profit of the CSR retailer increases with θ and it also results in more profit for the whole supply chain (see Fig. 6.2(b)). For the non-CSR retailer, the profit increases with θ at a very low rate up to the value of $\theta = 0.75$ and thereafter the profit decreases when the competition level becomes higher. The manufacturer earns a higher profit with a higher value of θ.



(a) *θ vs. demands*

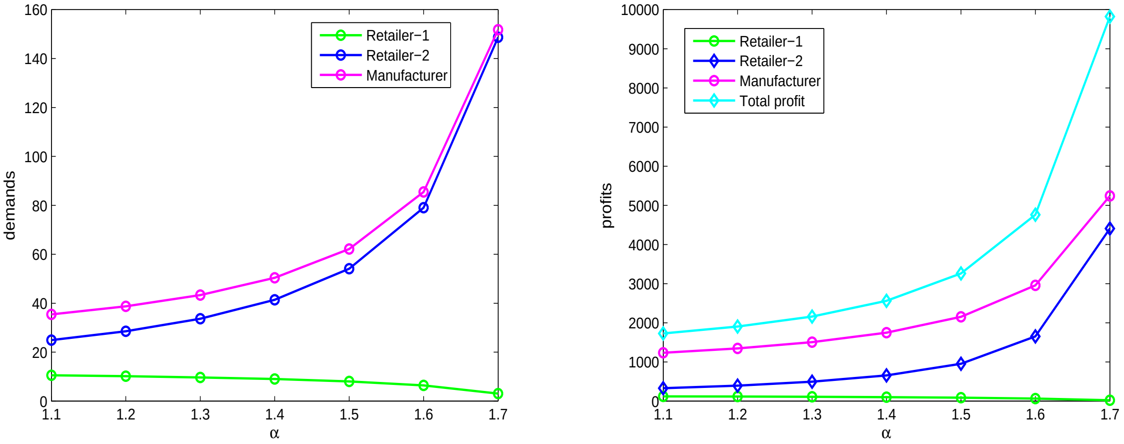
(b) *θ vs. individual and total profits*

Fig. 6.2: *Sensitivity analysis with respect to the parameter θ.*

6.4.2.2 Sensitivity with respect to α

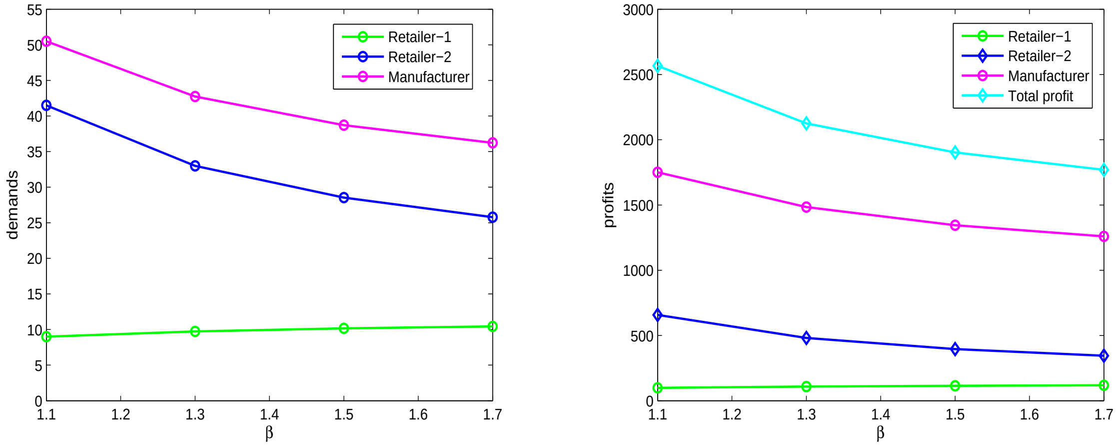
Since the demand of the remanufactured product is positively related to retailer's CSR sensitivity factor α, therefore when α increases, the demand of the remanufactured product increases. Due to retailer's socially responsible move, customers will be willing to buy remanufactured products. The demand of the competitive retailer's fresh product would then decrease. Nevertheless the total demand of the supply chain improves with the value of α

(see Fig. 6.3(a)). That is why, the socially responsible retailer gets more profit for a higher value of α. On the other hand, the profit of the non-CSR retailer decreases as α increases. Since the overall demand increases, the manufacturer is benefited alongside the CSR retailer. Consequently, the whole supply chain's profit enhances for higher values of α (see Fig. 6.3(b)).



(a) *α vs. demands* **(b)** *α vs. individual and total profits*

Fig. 6.3: *Sensitivity analysis with respect to the parameter α.*



(a) *β vs. demands* **(b)** *β vs. individual and total profits*

Fig. 6.4: *Sensitivity analysis with respect to the parameter β.*

6.4.2.3 Sensitivity with respect to β

The parameter β represents the CSR implementation cost sensitivity of the CSR retailer. The CSR implementation cost is inversely proportional to the production of the remanufactured product. So the socially responsible retailer's cost increases with increasing value of β. As β increases, the retail price sensitive customers' demand increases but the increment is very low (see Fig. 6.4(a)). The overall total demand of the supply chain however decreases with a higher value of β. The CSR responsible retailer's profit decreases and the non-CSR retailer's profit increases as β gradually increases. The manufacturer's profit as well as the whole supply chain's profit therefore decrease with the sensitivity parameter β (see Fig. 6.4(b)).

6.5 Managerial insights and conclusion

To run a business firm successively, the managers must have to take several efficient and effective management policies. The present study considers corporate social responsibility in management policies. So the manager's concern remains both for economic and social responsibilities. The manufacturer produces both manufactured and remanufactured products and sell them through two competitive retailers. One of the retailers put a CSR level for the remanufactured product and displays social responsibility. Since the remanufactured product has lower price compared to the new product, the manager should effectively manage the remanufacturing process for economic and environmental sustainability. Also, CSR plays a significant role to attract customers. Thats why, the manager's social responsibility can stimulate the economic stability of the business firm. For example, Starbucks company has joined with the US refugee agency to sculp up the company's support and effort to reach refugee candidates to hire 10000 refugees by 2022. Sub-supply game strategy turns to an economically beneficial strategy for the manufacturing firm. The manager therefore should try to set the manufacturing company's own retail hub or to bring the manufacturer and the retailer under a mutual agreement. Sometimes it has good control over production and pricing of the retail products. Also, it is seen that socially responsible retailer earns higher profit compared to the other retailer in the case of their individual decisions. So, social responsibility consideration is a good economical strategy to be taken by the manager.

In this chapter, a sustainable closed-loop supply chain is investigated where the market demand consists of two types of customer - environmentally concerned customers and economically concerned customers. Therefore, both new and remanufactured products are in demand in the market. The supply chain consists of two competitive retailers. One of them sells remanufactured products and is socially responsible while the other one sells new products only and does not take into account social responsibility. The manufacturer produces both new and remanufactured products. Under the two competitive retailer's price and corporate social responsibility effort dependent demand, three different game models viz. centralized, decentralized and sub-supply chain game models are developed. Some significant results we find from our study are as follows:

(i) Because of lower purchasing cost of used product, the manufacturer has lower wholesale price for the socially responsible retailer. This results in higher profit for the socially responsible retailer compared to the other retailer.

(ii) The sub-supply chain concept is good from the economical point of view because in both $m-r1$ and $m-r2$ game models, total profits Π_{m-r1} and Π_{m-r2} are greater than the sum of their profits in the decentralized game.

In the special cases where there is only a single retailer instead of two competitive retailers, we have the following major findings:

(i) The socially responsible retailer's individual profit is greater than that of the non-responsible retailer.

(ii) The centralized game always provides the highest profit in any situation; sub-supply game model provides a better strategy compared to the decentralized model from the point of view of the whole supply chain's overall performance. The reason behind this is the elimination of double marginalization effect through negotiation between the manufacturer and retailers.

Many leading companies like HP, Dell, Ford and Starbucks adopt sustainable moves along with their economic business strategies. HP, one of the best electronic brands, actively participates in the procurement of used electronic equipments. It takes back the used parts for a monetary incentive policy. HP collects cores to resale through remanufacturing in 74 countries and territories worldwide. In closed-loop ink cartridge recycling process, recycled plastics from HP ink cartridges are combined with recycled plastic bottles to create new and original ink cartridges. As a socially responsible move, HP engages with Governments to help improve national and international legislation governing the movement of electronic waste such as Basel Convention on the control of hazardous waste and their disposal (www8.hp.com).

In summary, it can be suggested that social responsibility consideration in our study results in economic and social sustainability from the individual as well as whole supply chain's points of view. The presence or absence of retailer competition for trade has a fruitful impact on the collection and remanufacturing of cores through the bearing of social responsibility effort cost. There are still certain limitations that can be considered in the future study like consideration of different collection centres for used products. Besides this, the customer incentive especially quality based customer incentive for returned items would be an interesting problem to deal with. The frontier of this study still can be enlarged in different directions, especially in stochastic environment for the demand of the product or consideration of stochastic remanufacturing capacity. Multi-retailer or multi-supplier consideration in the supply chain would be another research direction especially when n-tier supplier's competitive nature is presented there. The management task is therefore to assemble a group of resources and then to take optimal decisions from all the resources. An unknown or variable collection capacity would be another way to analyze the present model in future.

Chapter 7

Coordination of a sustainable reverse supply chain with revenue sharing contract

7.1 Introduction

In this 21^{st} century, human race is now in a great danger due to environmental degradation, climate change and shortages of natural resources. All these have led Governments and policy makers to rethink about the coming threat in near future. Many researchers and scientists are trying to advance the frontier of the supply chain taking sustainability as a major decision. Customer's growing concern for environmental issues has encouraged many industries to adopt sustainability as a fruitful remedy. Reverse and closed-loop supply chains are being adopted to achieve sustainability goals (Eskandarpour et al., 2015). This chapter* is based on a reverse supply chain, which completely focus on environmental sustainability through remanufacturing. Customers are now-a-days resisting products which lack health safety and humanistic care. For this reason, many companies are implementing recycling and remanufacturing with a noble goal towards a cleaner world. Recently a remarkable initiative had been taken by 2020 Tokyo Olympic and Paraolympic game organisers. Their target

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*This chapter is based on the work published in *Journal of Industrial and Management Optimization*, 2022, vol. 18, issue 1, pp. 487-510.

was to make all the medals from electronic wastes including old smart phones and laptops (www.usatoday.com). Previously in 2016 Rio Olympic game, about 30% of silver and bronze medals came from recycled materials (www.bbc.com).

However, this recycling process is not smooth to apply, specially when the quality of wastes or used products become a great factor. In a totally uncertain environment, quality of waste raises concern about economic stability of the supply chain entities. Customers also, on the other hand, need a strong encouragement to take part in the collection of wastes voluntarily. Many researchers tried monetary incentives in this regard (Govindan and Popiuc, 2014; Saha et al., 2016) but the decision on how a collector should give incentive to end-customers is a critical point. Heydari and Ghasemi (2018) considered a minimum acceptable quality level of wastes for waste collection index. In a competitive market, it is actually a real challenge that who should take the initiative and how should it be implemented. Therefore, managing chain competition is one of the biggest issues among the supply chain entities (Taleizadeh and Sadeghi, 2019). Besides competition and technical issues, one of the significant challenges is to design products and supply chain in which each task could be done economically. The economic interests should be understood for all the parties involved, and activities should be performed accordingly. For instance,

- Does a company or industry gain or loss and affect economically by selling remanufactured products along with the new ones?
- Who should be responsible for waste or used product collection (manufacturer, collector or third party)?
- Does it require any decision change if the used product has a lower quality?

The answers to these questions are really relevant up to which a supply chain would be sustainable. So economic satisfaction of each member of a supply chain comes into front when sustainability is concerned. Decentralized decisions are not always profitable and so a mutual agreement or contract plays the crucial role. In this chapter, we consider a three-echelon supply chain for remanufacturing. Depending on quality (which is uncertain) of wastes, the manufacturer produces finished product from wastes. The collector may not satisfy the manufacturer's demand of wastes. In that case, the supplier meets up the shortfall amount by supplying fresh raw materials. Then the following relevant questions may arise:

(i) How will the manufacturer manage the collector's and the supplier's responses?

(ii) What will be the manufacturer's decision? Does a contract really help to improve the profitability of supply chain entities?

(iii) Does uncertainty of waste quality really has any impact on economic sustainability of the supply chain entities?

To find the answers of the above questions, the proposed closed-loop supply chain (CLSC) model is developed. The possibility of material shortage in remanufacturing is considered and so a backup supplier is introduced (see Fig. 7.1). Also, the customer demand for the finished product is assumed to be price dependent. In most of the previous studies on reverse supply chain, it is assumed as constant. The novelty of the study is as follows: The chapter considers uncertainty of quality of waste and, therefore, uncertainty of waste collection for remanufacturing. It analyses the manufacturer's decisions regarding sustainability without compromising economical stability of all entities. In the previous studies of the reward driven reverse supply chain, the revenue was shared mainly between two members. However, in our model, it is considered in two different cases. In one case, the revenue is shared between the manufacturer and the collector, and in the other case, the revenue is shared with both the collector and the supplier. To the best of our knowledge, this type of model setting is not considered in any of the previous studies.

The rest of the chapter is organised as follows: In the next section, some basic assumptions and notations are given. In Section 7.3, the proposed model is developed and analyzed for two separate cases of revenue sharing policy. In Section 7.4, a numerical example is given and sensitivity analysis is carried out with respect to key model-parameters. Finally, the chapter is concluded in Section 7.5 with managerial insights and some future research directions.

7.2 Notation and assumption

The notations which are used in this chapter are as follows:

Notations	:	**Description**
c_i	:	inspection cost of the collector
c_r	:	refinement cost of the manufacturer

c_s : procurement cost of raw material for the supplier

c_p : production cost of the manufacturer

Q : available total wastes in the market

w_c : wholesale price of the collector

w_s : wholesale price of the supplier

L : minimum acceptable quality level of wastes

a : maximum possible market demand of the finished product

b : price sensitivity for customer demand

$t(\geq 0)$: binary variable representing acceptance (t=1 if $L \leq v < 1$) or rejection (t=0 if $0 < v < L$) of an item at the collector

r^{max} : maximum reward level at which all consumers wish to return their used products

Decision variables :

p : retail price of the manufacturer

r : reward (price) given to the consumers by the collector for items with minimum allowed quality level L

Stochastic variables :

v : quality of returned items with probability density function $g(\cdot)$ and cumulative distribution function $G(\cdot)$

Functions :

$E(\Pi_c)$: collector's expected total profit

$E(\Pi_s)$: supplier's expected profit

$E(\Pi_m)$: manufacturer's expected total profit

$E(\Pi)$: supply chain's expected total profit

Here we consider a three-echelon supply chain with one manufacturer, one collector and one supplier. Customer demand is price dependent in nature. The collector collects the used products and sends to the manufacturer for remanufacturing. The collector offers a reward to consumers for bringing back the used products. Since used products may have different quality levels, the collector maintains a minimum quality level L to accept the used products from consumers. The collector inspects all the collected items with per unit inspection cost c_i. After inspection, those items which satisfy quality level L are forwarded to the manufacturer with wholesale price w_c per unit and other wastes are rejected. After receiving wastes from the collector, the manufacturer refines the materials with per unit refinement cost c_r and then produces with per unit production cost c_p.

When the collector fails to supply the desired quantity of used products, the supplier fulfills the shortfall amount by supplying fresh raw materials with a wholesale price w_s per unit. In this way, the manufacturer's demand is satisfied and the manufacturer then fulfills the customer demand D. The previous researches considered that the manufacturer's demand is met up by collected wastes only. However, in practice, it is not always possible to get 100% collection of wastes to fulfill the entire demand of the manufacturer because of various reasons such as low quality of wastes, natural disasters, delay in delivery, and so on. To deal with this situation, we consider a general case where customer satisfaction is prioritized through production from accepted wastes and fresh raw materials, keeping in mind both environmental and economical issues. To develop the proposed model, we make the following assumptions:

(i) The customer demand (D) is price dependent. We take $D = a - bp$, where $a(> 0)$ is the maximum possible demand, $b(> 0)$ is price sensitivity parameter and p is the manufacturer's retail price so that D is always positive *i.e.,* $a > bp$. In the supply chain literature, this form of demand function is very common where demand specifically decreases with the retail price (Hua et al., 2010; Chen et al., 2017).

(ii) Consumers' willingness to bring back the used items is a function of the reward level r. We define it as

$$W = f(r) = \begin{cases} \frac{r}{r^{max}} : 0 < r < r^{max} \\ 1 \quad : r \geq r^{max} \end{cases} \tag{7.1}$$

where r is the collector's reward level and r^{max} is the maximum reward level for consumers. Since the quality of used products varies from one consumer to other, it is very critical to separate each product with exact quality level. To resolve this issue, a fixed amount of monetary incentive is assumed for each returned item, which fulfills a desired quality level set up by the collector. Similar consideration for consumers' willingness is assumed by Heydari and Ghasemi (2018).

(iii) The acceptance or rejection of a returned item by the collector depends on its quality level. We define variable 't' for this purpose. If the returned product has the minimum allowed quality level L, it takes the value 1; otherwise, it takes the value 0.

$$t = \begin{cases} 0 : 0 < v < L \\ 1 : L \leq v < 1 \end{cases} \tag{7.2}$$

Therefore, the expected amount of used products sent by the collector to the manufacturer can be calculated as $E(T) = WQE(t) = \int_L^1 WQg(v)dv$.

(iv) The manufacturer's production is flawless i.e., one unit of final product is produced from one unit of raw material.

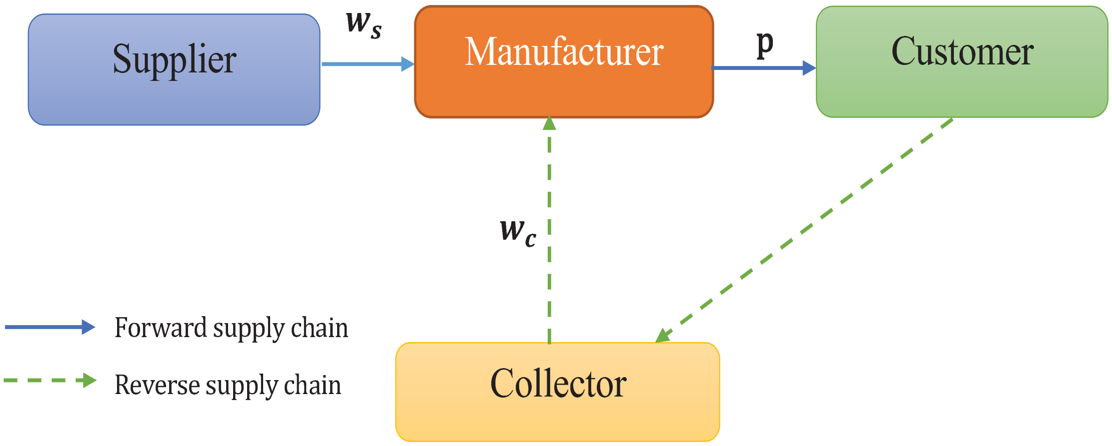


Fig. 7.1: *Diagram of the supply chain model.*

7.3 Model development and analysis

In this section, the proposed model is developed and analyzed with game-theoretic approaches. Centralized policy, decentralized policy, and revenue-sharing contracts are implemented to investigate the environmental and economic sustainability of the supply chain.

7.3.1 Decentralized model

In the decentralized scenario, all the entities try to optimize their own profits. Here each of the collector and the manufacturer individually tries to optimize its own profit. The supplier also optimizes his profit through a feasible wholesale price. From now onwards, we use the superscripts d, c and RS_i ; $i = 1, 2$ to denote decentralized, centralized and revenue sharing contract, and the subscripts c, s and m to denote the collector, the supplier and the manufacturer, respectively.

7.3.1.1 Collector's profit.

During the collection process, the collector inspects all the wastes with unit inspection cost c_i. The wastes which fail to satisfy collector's desired quality level L, are rejected. So, the collector's profit per unit of returned product is given by

$$\Pi_c^d = \begin{cases} -c_i : 0 < v < L \\ w_c - r - c_i : L \leq v < 1 \end{cases}$$

Therefore, the collector's expected total profit is given by

$$\begin{aligned} E(\Pi_c^d) &= \int_L^1 WQ(w_c - r - c_i)g(v)dv - \int_0^L c_i WQ g(v)dv \\ &= \int_L^1 (a - bp)\frac{r}{r^{max}}(w_c - r - c_i)g(v)dv - \int_0^L c_i(a - bp)\frac{r}{r^{max}}g(v)dv, \end{aligned}$$

where the first term represents the expected revenue of the collector for selling the reusable products to the manufacturer and the second term denotes the expected inspection cost of the wastes. The available amount of wastes can be expressed as $Q = \theta D$ where $0 \leq \theta \leq 1$. However, for modelling simplicity, we assume that $Q = D = a - bp$.

Now, to check the concavity of the profit function, we differentiate $E(\Pi_c^d)$ with respect to r twice and get

$$\begin{aligned} \frac{dE(\Pi_c^d)}{dr} &= \int_L^1 (a - bp)\frac{1}{r^{max}}(w_c - 2r - c_i)g(v)dv - \int_0^L (a - bp)\frac{c_i}{r^{max}}g(v)dv \\ \frac{d^2E(\Pi_c^d)}{dr^2} &= -2\int_L^1 \frac{(a - bp)}{r^{max}}g(v)dv < 0. \end{aligned}$$

So, the collector's expected profit is strictly concave in reward value r and the optimal reward is given by

$$\begin{aligned} r^{d*} &= \frac{\int_L^1 \frac{(a-bp)}{r^{max}}(w_c - c_i)g(v)dv - \int_0^L (a-bp)\frac{c_i}{r^{max}}g(v)dv}{2\int_L^1 \frac{(a-bp)}{r^{max}}g(v)dv} \\ &= \frac{w_c I - c_i}{2I} \quad \text{where} \quad I = \int_L^1 g(v)dv. \end{aligned} \tag{7.3}$$

Proposition 7.1 *The optimal reward r^{d*} for consumers decreases with the minimum acceptable quality level L. For fixed quality level L, the collector would pay more reward to consumers.*

Proof: In the decentralized model, the optimal reward given by the collector to the end-customer is

$$\begin{aligned} r^{d*} &= \frac{w_c I - c_i}{2I} \quad \text{where} \quad I = \int_L^1 g(v)dv \\ &= \frac{w_c}{2} - \frac{c_i}{2I} \end{aligned}$$

The first order derivative of r^{d*} with respect to L gives

$$\frac{dr^{d*}}{dL} = \frac{c_i}{2I^2}\frac{dI}{dL} = \frac{c_i}{2I^2}(-1)g(l) = -\frac{c_i}{2I^2}g(l) < 0,$$

since $g(l), c_i$ and I^2 are all > 0. Thus, when L increases, the consumers get lesser reward because then the collector fails to collect sufficient amount of wastes for remanufacturing. Quite naturally the collector gets lesser profit when L increases.

On the other hand, the collector supplies a fraction $\int_L^1 \frac{r}{r^{max}}g(v)dv$ of the demand D to the manufacturer. Let, $f = \int_L^1 \frac{r}{r^{max}}g(v)dv$. Then we have

$$\frac{df}{dr} = \int_L^1 \frac{1}{r^{max}}g(v)dv > 0.$$

This implies that f increases with r for a fixed quality level L and, therefore, the collector would pay more reward to consumers for returning the used products.

7.3.1.2 Manufacturer's profit

We suppose that the collector supplies a fraction $\int_L^1 \frac{r}{r^{max}}g(v)dv$ of the total demand D with per unit wholesale price w_c and the remaining fraction $\{1 - \int_L^1 \frac{r}{r^{max}}g(v)dv\}$ is meet up by the supplier with per unit wholesale price w_s. Then the manufacturer's expected profit is given by

$$\begin{aligned} E(\Pi_m^d) &= (p - c_p - c_r)(a - bp) - w_c(a - bp)\int_L^1 \frac{r}{r^{max}} g(v)dv \\ &\quad -w_s(a - bp)\left[1 - \int_L^1 \frac{r}{r^{max}} g(v)dv\right] \end{aligned}$$

where the first term denotes the sales revenue, and the second and the third terms represent respectively the used product cost paid to the collector and raw material cost paid to the supplier. Now,

$$\begin{aligned} \frac{dE(\Pi_m^d)}{dp} &= a + b(c_p + c_r) - 2bp + w_c b\int_L^1 \frac{r}{r^{max}} g(v)dv + w_s b\left\{1 - \int_L^1 \frac{r}{r^{max}} g(v)dv\right\} \\ \frac{d^2E(\Pi_m^d)}{dp^2} &= -2b \end{aligned}$$

So, the manufacturer's expected profit is concave in p and the optimal retail price is given by

$$p^*(r) = \frac{a + b(c_p + c_r) + bw_c \int_L^1 \frac{r}{r^{max}} g(v)dv + bw_s\left\{1 - \int_L^1 \frac{r}{r^{max}} g(v)dv\right\}}{2b}$$

Using the collector's optimal decision of reward level, the manufacturer decides his retail price as

$$p^{d*} = \frac{2\{a + b(c_p + c_r)\}r^{max} + bw_c(w_c I - c_i) + bw_s(2r^{max} + c_i - w_c I)}{4br^{max}}$$

Proposition 7.2 *The manufacturer gets lesser profit for higher value of L.*

Proof: The optimal retail price of the manufacturer is given by

$$p^{d*} = \frac{2\{a + b(c_p + c_r)\}r^{max} + bw_c(w_c I - c_i) + bw_s(2r^{max} + c_i - w_c I)}{4br^{max}}$$

Differentiating p^{d*} with respect to L, we get

$$\frac{dp^{d*}}{dL} = \frac{b(w_c^2 - w_c w_s)}{4br^{max}} \frac{dI}{dL} = -\frac{(w_c^2 - w_c w_s)}{4r^{max}} g(l) > 0,$$

since $w_c < w_s$ and $w_c, w_s > 0$. Therefore, the optimal retail price increases with L and consequently the demand decreases as L increases. Hence, the manufacturer gets lesser profit for higher value of L.

Proposition 7.3 *The manufacturer's expected profit function $E(\Pi_m^d)$ is an increasing function of r.*

Proof: Differentiating the manufacturer's expected profit function of the decentralized model with respect to r, we get

$$\begin{aligned}
\frac{dE(\Pi_m^d)}{dr} &= -w_c(a-bp)\int_L^1 \frac{1}{r^{max}}g(v)dv - w_s(a-bp)\left[0-\int_L^1 \frac{1}{r^{max}}g(v)dv\right] \\
&= w_s(a-bp)\int_L^1 \frac{1}{r^{max}}g(v)dv - w_c(a-bp)\int_L^1 \frac{1}{r^{max}}g(v)dv \\
&= (w_s-w_c)\int_L^1 \frac{1}{r^{max}}g(v)dv > 0, \quad \text{since } w_s > w_c.
\end{aligned}$$

Therefore, the manufacturers expected profit is an increasing function of r.

The manufacturer's optimal retail price in the decentralized model is obtained as

$$p^*(r) = \frac{a+b(c_p+c_r)+bw_c\int_L^1 \frac{r}{r^{max}}g(v)dv + bw_s\Big\{1-\int_L^1 \frac{r}{r^{max}}g(v)dv\Big\}}{2b}$$

Differentiating p^{d*} with respect to r, we get

$$\begin{aligned}
\frac{dp^*}{dr} &= \frac{bw_c\int_L^1 \frac{1}{r^{max}}g(v)dv + bw_s\Big\{-\int_L^1 \frac{1}{r^{max}}g(v)dv\Big\}}{2b} \\
&= \frac{(w_c-w_s)I}{2r^{max}} < 0, \quad \text{since } w_c < w_s.
\end{aligned}$$

This implies that demand increases with r and consequently, the total collection of wastes i.e. $\int_L^1 \frac{r}{r^{max}}g(v)dv$ also increases. As the collector supplies more wastes to the manufacturer, the manufacturer needs lesser amount of raw materials from the supplier. Therefore, the manufacturer earns more profit as $w_c < w_s$.

7.3.1.3 Supplier's profit

According to our assumption, the amount of raw materials supplied by the supplier is $\{1-\int_L^1 \frac{r}{r^{max}}g(v)dv\}$ fraction of the total demand D. So, the supplier's expected profit is

$$E(\Pi_s^d) = (w_s-c_s)(a-bp)\left\{1-\int_L^1 \frac{r}{r^{max}}g(v)dv\right\}$$

Since the supplier is not directly related to remanufacturing initiated by the collector and the manufacturer, the supplier can't take decisions of his own. However, the manufacturer agrees to pay more than the collector's wholesale price at the time of need. In this situation, depending upon optimal decisions (p^{d*}, r^{d*}) of the collector and the manufacturer, the supplier decides his wholesale price.

7.3.2 Centralized model

Unlike the decentralized model, here the collector, the manufacturer and the supplier jointly determine optimal decisions. It is needless to say that, the centralized strategy usually eliminates the double marginalization effect between any two supply chain entities by negotiation with a benchmark profit of the whole supply chain in target. Therefore, the expected total profit of the supply chain in the case is given by

$$\begin{aligned} E(\Pi) &= \Pi_m + \Pi_c + \Pi_s \\ &= (p - c_p - c_r)(a - bp) - c_s(a - bp)(1 - \int_L^1 \frac{r}{r^{max}} g(v)dv) - (r + c_i) \\ &\quad \times (a - bp)\int_L^1 \frac{r}{r^{max}} g(v)dv - c_i(a - bp)\int_0^L \frac{r}{r^{max}} g(v)dv \end{aligned}$$

where the first term represents the sales revenue, the second term is the expected procurement cost for fresh raw materials, the third term denotes the expected total collection cost of used products and the last term is the expected inspection cost for the disposed items which are not eligible for remanufacturing.

Proposition 7.4 *The expected total profit $E(\Pi)$ of the whole supply chain is jointly concave in p and r provided that $4I(a - bp)r^{max} > b(2rI - c_sI + c_i)^2$.*

Proof: The first and the second order partial derivatives of $E(\Pi)$ with respect to p and r give

$$\begin{aligned} \frac{\partial E(\Pi)}{\partial p} &= a + b(c_p + c_r) - 2bp + b\int_L^1 \frac{r^2}{r^{max}} g(v)dv \\ &\quad + bc_s\{1 - \int_L^1 \frac{r}{r^{max}} g(v)dv\} + bc_i \frac{r}{r^{max}} \\ \frac{\partial^2 E(\Pi)}{\partial p^2} &= -2b < 0 \\ \frac{\partial E(\Pi)}{\partial r} &= c_s(a - bp)\int_L^1 \frac{g(v)}{r^{max}} r dv - c_i(a - bp)\frac{1}{r^{max}} - 2r(a - bp)\int_L^1 \frac{g(v)}{r^{max}} dv \\ \frac{\partial^2 E(\Pi)}{\partial r^2} &= -2(a - bp)\int_L^1 \frac{g(v)}{r^{max}} dv < 0, \quad \text{since } a > bp \text{ and } \int_L^1 \frac{g(v)}{r^{max}} dv > 0. \end{aligned}$$

So, the corresponding Hessian matrix is given by

$$H = \begin{pmatrix} \frac{\partial^2 E(\Pi)}{\partial p^2} & \frac{\partial^2 E(\Pi)}{\partial p \partial r} \\ \frac{\partial^2 E(\Pi)}{\partial r \partial p} & \frac{\partial^2 E(\Pi)}{\partial r^2} \end{pmatrix}$$

$$= \begin{pmatrix} -2b & b(2r - c_s) \int_L^1 \frac{g(v)}{r^{max}} dv + \frac{bc_i}{r^{max}} \\ b(2r - c_s) \int_L^1 \frac{g(v)}{r^{max}} dv + \frac{bc_i}{r^{max}} & -2(a - bp) \int_L^1 \frac{g(v)}{r^{max}} dv \end{pmatrix}$$

From the Hessian matrix H, we have the first order principal minor $H_{11} = -2b < 0$, second order principal minor $H_{22} = -2(a - bp) \int_L^1 \frac{g(v)}{r^{max}} dv < 0$, and determinant of H is

$$|H| = 4b(a - bp) \int_L^1 \frac{g(v)}{r^{max}} dv - \left\{ 2br \int_L^1 \frac{g(v)}{r^{max}} dv - bc_s \int_L^1 \frac{g(v)}{r^{max}} dv + \frac{bc_i}{r^{max}} \right\}^2$$

So, H is concave in p and r if $|H| > 0$ *i.e.,*

$$4b(a - bp) \int_L^1 \frac{g(v)}{r^{max}} dv - \left\{ 2br \int_L^1 \frac{g(v)}{r^{max}} dv - bc_s \int_L^1 \frac{g(v)}{r^{max}} dv + \frac{bc_i}{r^{max}} \right\}^2 > 0$$

which implies that $4I(a - bp)r^{max} > b(2rI - c_s I + c_i)^2$.

Proposition 7.5 *The optimal reward and retail price in the centralized strategy are given by*

$$\begin{aligned} r^{c*} &= \frac{c_s I - c_i}{2I} \quad \textit{where} \quad I = \int_L^1 g(v) dv \\ p^{c*} &= \frac{4Ir^{max}\left\{a + b(c_p + c_r)\right\} + b(c_s I - c_i)^2 + 2bc_i(c_s I - c_i) + 2Ibc_s(2r^{max} + c_i - c_s I)}{8bIr^{max}} \end{aligned} \tag{7.4}$$

Proof: The proof is omitted as the results given by (7.4) can be easily derived.

The values of r^{c*} and p^{c*} correspond to the optimal reward and retail price, respectively for the whole supply chain. Due to lower retail price, the demand is enhanced in this case. Also, the absence of double-marginalization effect drives the supply chain's total profit to increase.

7.3.3 Revenue sharing contract

We assume that the manufacturer signs a revenue sharing contract with the collector and the supplier in two different ways. In one case, the manufacturer shares revenue with the collector only and, in other case, the revenue is shared with both the collector and the supplier (see Fig. 7.2).

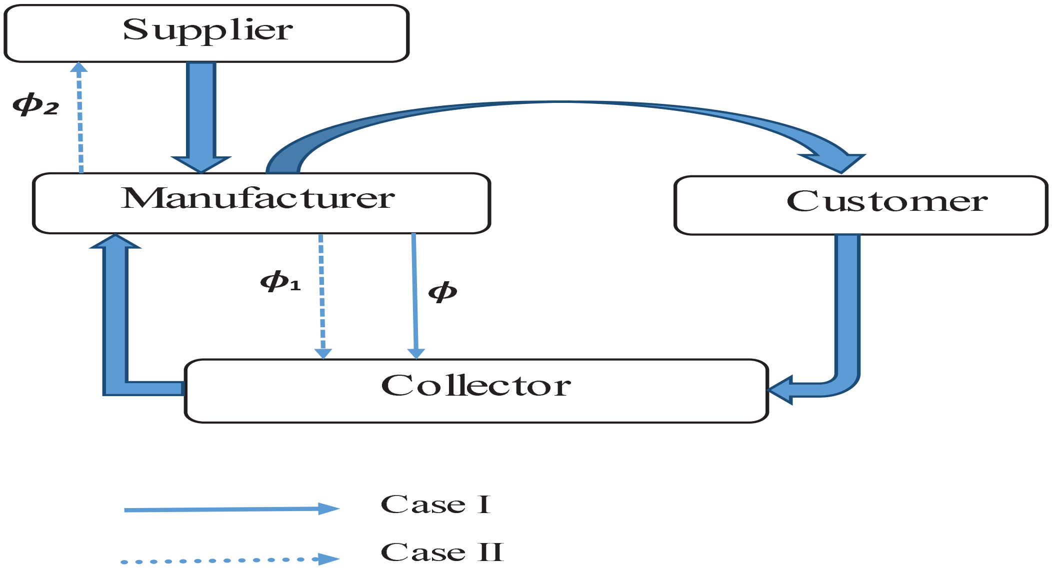


Fig. 7.2: *Proposed supply chain model under revenue sharing contract.*

7.3.3.1 Revenue sharing with the collector only

In this case, the manufacturer shares a fraction $(1-\phi)$ of his revenue with the collector but does not share with the supplier. So, the profits of the manufacturer, the collector and the supplier are given by

$$\begin{aligned} E(\Pi_m^{RS_1}) &= (\phi p - c_p - c_r)(a-bp) - w_c^{RS_1}(a-bp)\int_L^1 \frac{r}{r^{max}} g(v)dv \\ &\quad -w_s(a-bp)\Big\{1-\int_L^1 \frac{r}{r^{max}} g(v)dv\Big\} \\ E(\Pi_c^{RS_1}) &= \int_L^1 (a-bp)\frac{r}{r^{max}}(w_c^{RS_1} - r - c_i)g(v)dv - \int_0^L c_i(a-bp)\frac{r}{r^{max}} g(v)dv \\ &\quad +(1-\phi)p(a-bp) \\ E(\Pi_s^{RS_1}) &= (w_s - c_s)(a-bp)\Big\{1-\int_L^1 \frac{r}{r^{max}} g(v)dv\Big\} \end{aligned}$$

where the term $(1-\phi)p(a-bp)$ in the collector's profit is the revenue shared by the manufacturer. Also, ϕp and $w_c^{RS_1}$ are new retail price and the wholesale price of the manufacturer and the collector, respectively and the rests are same as the decentralized model. Now,

$$\begin{aligned}\frac{dE(\Pi_c^{RS_1})}{dr} &= \int_L^1 \frac{(a-bp)}{r^{max}}(w_c^{RS_1} - 2r - c_i)g(v)dv - \int_0^L (a-bp)\frac{c_i}{r^{max}}g(v)dv \ \ \text{and} \\ \frac{d^2E(\Pi_c^{RS_1})}{dr^2} &= -2\frac{(a-bp)}{r^{max}}I < 0.\end{aligned}$$

Hence, the optimal reward is obtained as

$$\begin{aligned}r^{RS_1*} &= \frac{\int_L^1 \frac{(a-bp)}{r^{max}}(w_c^{RS_1} - c_i)g(v)dv - \int_0^L (a-bp)\frac{c_i}{r^{max}}g(v)dv}{2\int_L^1 \frac{(a-bp)}{r^{max}}g(v)dv} \\ &= \frac{w_c^{RS_1}I - c_i}{2I} \ \ \text{where} \ \ I = \int_L^1 g(v)dv\end{aligned}$$

Taking the first and the second order derivatives of $E(\Pi_m^{RS_1})$ with respect to p, we get

$$\begin{aligned}\frac{dE(\Pi_m^{RS_1})}{dp} &= \phi a + b(c_p + c_r) - 2b\phi p + bw_c^{RS_1}\int_L^1 \frac{r}{r^{max}}g(v)dv \\ &\quad + bw_s\Big\{1 - \int_L^1 \frac{r}{r^{max}}g(v)dv\Big\} \\ \frac{d^2E(\Pi_m^{RS_1})}{dp^2} &= -2b\phi < 0\end{aligned}$$

This shows that, the manufacturer's profit is concave in p and the optimal retail price is

$$p^{RS_1}(r) = \frac{\phi a + b(c_p + c_r) + bw_c^{RS_1}\int_L^1 \frac{r}{r^{max}}g(v)dv + bw_s\{1 - \int_L^1 \frac{r}{r^{max}}g(v)dv\}}{2b\phi}.$$

When the optimal reward r^{RS_1*} is obtained, the manufacturer's optimal retail price is

$$p^{RS_1*} = \frac{2r^{max}\{\phi a + b(c_p + c_r)\} + bw_c^{RS_1}(w_c^{RS_1} - c_i) + bw_s(2r^{max} - w_c^{RS_1}I + c_i)}{4b\phi r^{max}}.$$

Now, when the manufacturer shares a fraction $(1-\phi)$ of his total revenue with the collector, the collector agrees to settle his wholesale price as $w_c^{RS_1} < w_c^d$. In this situation, the supply chain is coordinated provided that the retail price and the reward are same as those of the centralized model. Now, equating the reward r^{RS_1*} with the value r^{c^*}, we get

$$\frac{c_sI - c_i}{2I} = \frac{\int_L^1 (w_c^{RS_1} - c_i)g(v)dv - \int_0^L (a-bp)c_ig(v)dv}{2\int_L^1 g(v)dv}$$

and this implies that $w_c^{RS_1} = c_s$. Similarly, equating the retail price p^{RS_1*} with p^{c^*}, we get

$$\phi = \frac{b(c_p + c_r) + bw_c^{RS_1}I\frac{d}{r^{max}} + bw_s(1 - \frac{rI}{r^{max}})}{b(c_p + c_r) + \frac{br^2I}{r^{max}} + bc_i\frac{r}{r^{max}} + bc_s(1 - \frac{rI}{r^{max}})}$$

Substituting the optimal value of the reward r as $r = r^{RS_1^*} = r^{c^*} = \frac{c_sI-c_i}{2I}$ and $w_c^{RS_1} = c_s$, ϕ takes the following simplified expression

$$\phi = \frac{4Ir^{max}(c_p + c_r + w_s) + 2I(c_sI - c_i)(c_s - w_s)}{4Ir^{max}(c_p + c_r + c_s) - (c_sI - c_i)^2} \tag{7.5}$$

Under revenue sharing contract, the manufacturer and the collector both try to improve their profits. The manufacturer settles his portion of share and the collector determines his wholesale price accordingly. The manufacturer will earn profit higher than that of the decentralized model *i.e.,* $\Pi_m^{RS_1} \geq \Pi_m^d$ which gives

$$(\phi p^{RS_1} - c_p - c_r)(a - bp^{RS_1}) - w_c^{RS_1}(a - bp^{RS_1}) \int_L^1 \frac{r^{RS_1}}{r^{max}} g(v)dv - w_s(a - bp^{RS_1})$$
$$\left\{1 - \int_L^1 \frac{r^{RS_1}}{r^{max}} g(v)dv\right\} \geq \ (p^d - c_p - c_r)(a - bp^d) - w_c(a - bp^d) \int_L^1 \frac{r^d}{r^{max}} g(v)dv$$
$$- w_s(a - bp^d)\left\{1 - \int_L^1 \frac{r^d}{r^{max}} g(v)dv\right\}$$

Since the manufacturer does not share any revenue with the supplier, the supplier's wholesale price remains the same as that of the decentralized model. When the reward amount and the retail price are known, using the above inequality, we have

$$\phi \geq \ \frac{1}{p^{RS_1}(a - bp^{RS_1})}\Bigg[p^d(a - bp^d) - {w_c}^d(a - bp^d) \int_L^1 \frac{r^d}{r^{max}} g(v)dv - w_s(a - bp^d)$$
$$\left\{1 - \int_L^1 \frac{r^d}{r^{max}} g(v)dv\right\} + w_c^{RS_1}(a - bp^{RS_1}) \int_L^1 \frac{r^{RS_1}}{r^{max}} g(v)dv + w_s(a - bp^{RS_1})\Big\{1-$$
$$\int_L^1 \frac{r^{RS_1}}{r^{max}} g(v)dv\Big\}\Bigg]$$

Similarly, if the collector's profit is considered, then the inequality $\Pi_c^{RS_1} \geq \Pi_c^d$ gives

$$\phi \leq \ 1 - \frac{1}{p^{RS_1}(a - bp^{RS_1})}\Bigg[\int_L^1 (a - bp^d)\frac{r^d}{r^{max}}({w_c}^d - r^d - c_i)g(v)dv - \int_0^L c_i$$
$$(a - bp^d)\frac{r^d}{r^{max}} g(v)dv + \int_0^L c_i(a - bp^{RS_1})\frac{r^{RS_1}}{r^{max}} g(v)dv - \int_L^1 (a - bp^{RS_1})\frac{r^{RS_1}}{r^{max}}$$
$$(w_c^{RS_1} - r^{RS_1} - c_i)g(v)dv\Bigg]$$

Now, from the above two inequalities, we obtain the feasible range of the revenue sharing parameter ϕ as $\phi_L \leq \phi \leq \phi_R$ where,

$$\phi_L = \frac{1}{p^{RS_1}(a-bp^{RS_1})}\Bigg[p^d(a-bp^d) - {w_c}^d(a-bp^d)\int_L^1 \frac{r^d}{r^{max}}g(v)dv - w_s(a-bp^d)$$
$$\Big\{1-\int_L^1 \frac{r^d}{r^{max}}g(v)dv\Big\} + w_c^{RS_1}(a-bp^{RS_1})\int_L^1 \frac{r^{RS_1}}{r^{max}}g(v)dv + w_s(a-bp^{RS_1})\Big\{1-$$
$$\int_L^1 \frac{r^{RS_1}}{r^{max}}g(v)dv\Big\}\Bigg], \quad \text{and,}$$

$$\phi_R = 1-\frac{1}{p^{RS_1}(a-bp^{RS_1})}\Bigg[\int_L^1 (a-bp^d)\frac{r^d}{r^{max}}({w_c}^d - r^d - c_i)g(v)dv - \int_0^L c_i$$
$$(a-bp^d)\frac{r^d}{r^{max}}g(v)dv + \int_0^L c_i(a-bp^{RS_1})\frac{r^{RS_1}}{r^{max}}g(v)dv - \int_L^1 (a-bp^{RS_1})\frac{r^{RS_1}}{r^{max}}$$
$$(w_c^{RS_1} - r^{RS_1} - c_i)g(v)dv\Bigg]$$

7.3.3.2 Revenue sharing with both the collector and the supplier

Suppose the manufacturer shares a fraction ϕ_1 of revenue with the collector and another fraction ϕ_2 with the backup supplier satisfying the condition $\phi_1+\phi_2 < 1$ where $0 < \phi_1, \phi_2 < 1$. The manufacturer signs the revenue sharing contract with both the members to get reduced wholesale prices from them. We suppose that, under the contract, the wholesale prices of the collector and the supplier are $w_c^{RS_2}$ and $w_s^{RS_2}$, respectively. Then the profits of the manufacturer, the collector and the backup supplier are given by

$$\begin{aligned}
E(\Pi_m^{RS_2}) &= \Big(\{1-(\phi_1+\phi_2)\}p - c_p - c_r\Big)(a-bp) - w_c^{RS_2}(a-bp) \\
&\quad \int_L^1 \frac{r}{r^{max}}g(v)dv - w_s^{RS_2}(a-bp)\{1-\int_L^1 \frac{r}{r^{max}}g(v)dv\} \\
E(\Pi_c^{RS_2}) &= \int_L^1 (a-bp)\frac{r}{r^{max}}(w_c^{RS_2} - r - c_i)g(v)dv - \int_0^L c_i(a-bp)\frac{r}{r^{max}} \\
&\quad g(v)dv + \phi_1 p(a-bp) \\
E(\Pi_s^{RS_2}) &= (w_s^{RS_2} - c_s)\Big(1-\int_L^1 \frac{r}{r^{max}}g(v)dv\Big)(a-bp) + \phi_2 p(a-bp)
\end{aligned}$$

Differentiating $E(\Pi_m^{RS_2})$ with respect to p and equating to zero, we get the optimal retail price as

$$p^{RS_2}(r) = \frac{\{1-(\phi_1+\phi_2)\}a + b(c_p+c_r) + bw_c^{RS_2}\int_L^1 \frac{r}{r^{max}}g(v)dv + bw_s^{RS_2}\Big(1-\int_L^1 \frac{r}{r^{max}}g(v)dv\Big)}{2b\Big\{1-(\phi_1+\phi_2)\Big\}}$$

Similarly, the optimal reward can be obtained as

$$\begin{aligned} r^{RS_2^*} &= \frac{\int_L^1 (w_c^{RS_2}-c_i)g(v)dv - \int_0^L c_i g(v)dv}{2\int_L^1 g(v)dv} \\ &= \frac{w_c^{RS_2}I - c_i}{2I} \quad \text{where} \quad I = \int_L^1 g(v)dv. \end{aligned}$$

When optimal reward $r^{RS_2^*}$ is obtained, the manufacturer sets his retail price as

$$p^{RS_2^*} = \frac{2r^{max}\Big[\{1-(\phi_1+\phi_2)\}a + b(c_p+c_r+w_s^{RS_2})\Big] - bw_c^{RS_2}IM + bc_iM}{4br^{max}\Big\{1-(\phi_1+\phi_2)\Big\}}$$

where $M = (w_s^{RS_2} - w_c^{RS_2})$. When the manufacturer shares a fraction ϕ_1 of his revenue with the collector and a fraction ϕ_2 with the supplier, he seeks lower wholesale prices from the collector and the supplier, *i.e.,* $w_c^{RS_2} < w_c^d$ and $w_s^{RS_2} < w_s^d$. For supply chain coordination, we equate these decision variables with the centralized ones. The optimal rewards are then also equal and so when $r^{RS_2} = r^c$, we get

$$\frac{c_sI - c_i}{2I} = \frac{\int_L^1 (w_c^{RS_2}-c_i)g(v)dv - \int_0^L c_i g(v)dv}{2\int_L^1 g(v)dv}$$

which implies that $w_c^{RS_2} = c_s$. Now, comparing the optimal price with that of the centralized one, we get

$$\begin{aligned} &\frac{\{1-(\phi_1+\phi_2)\}a + b(c_p+c_r) + bw_c^{RS_2}\int_L^1 \frac{r}{r^{max}}g(v)dv + bw_s^{RS_2}(1-\int_L^1 \frac{r}{r^{max}}g(v)dv)}{2b\{1-(\phi_1+\phi_2)\}} \\ &= \frac{4Ir^{max}\{a+b(c_p+c_r)\} + b(c_sI-c_i)^2 + 2bc_i(c_sI-c_i) + 2Ibc_s(2r^{max}+c_i-c_sI)}{8bIr^{max}} \end{aligned}$$

Substituting the optimal reward of the collector, the total fraction of shared revenue is obtained as

$$\begin{aligned} &\phi_1+\phi_2 \\ &= \frac{4Ir^{max}(c_s - {w_s}^{RS_2}) + 2I^2w_c^{RS_2}({w_s}^{RS_2} - {w_c}^{RS_2}) - 2c_i({w_s}^{RS_2} - {w_c}^{RS_2}) - T^2}{4Ir^{max}(c_p+c_r+c_s) - T^2} \end{aligned} \tag{7.6}$$

where $T = (c_sI - c_i)$.

Theorem 7.1 *There exists a win-win situation when* $p(\phi_1+\phi_2) = (w_c^d - w_c^{RS_2})A + (w_s^d - w_s^{RS_2})(1-A)$*, where* p *is the optimal retail price of the manufacturer in the coordinated case and* $A = \int_L^1 \frac{r}{r^{max}} g(v)dv$.

Proof: The collector, the manufacturer and the supplier will reach to the win-win situation when their profits under revenue-sharing contract are better than their corresponding profits in the decentralized case. So, $E(\Pi_m^{RS_2}) \geq E(\Pi_m^d)$, $E(\Pi_c^{RS_2}) \geq E(\Pi_c^d)$ and $E(\Pi_s^{RS_2}) \geq E(\Pi_s^d)$. Now, $E(\Pi_c^{RS_2}) \geq E(\Pi_c^d)$ gives $p\phi_1 \geq (w_c^d - w_c^{RS_2})A$, and $E(\Pi_s^{RS_2}) \geq E(\Pi_s^d)$ gives $p\phi_2 \geq (w_s^d - w_s^{RS_2})(1-A)$. Again, comparing profits of the manufacturer in the two cases (presented in Subsections 7.3.3.1 and 7.3.3.2), we get $(\phi_1+\phi_2)p \leq (w_c^d - w_c^{RS_2})A + (w_s^d - w_s^{RS_2})(1-A)$. Combining these three inequalities above, we conclude that $p(\phi_1+\phi_2) = (w_c^d - w_c^{RS_2})A + (w_s^d - w_s^{RS_2})(1-A)$. Hence the result.

It is to be noted that when the marginal gain in the manufacturer's profit due to revenue sharing equals to the total shared revenue between the collector and the supplier for particular values of ϕ_1 and ϕ_2, the supply chain members achieve win-win outcome. Hence, the collector, the manufacturer as well as the supplier are benefitted by the revenue sharing contract and they gain more than their individual decentralized profits.

Theorem 7.2 *When* $(1-\phi) = \phi_1+\phi_2$ *in both the cases, (i)* $w_c^{RS} = \frac{2Ir^{max}+c_i}{I^2}$ *provided that* $w_c^{RS_1} = w_c^{RS_2}$ *and (ii)* $w_s = \frac{(w_c^{RS_1}+w_c^{RS_2})I^2 - c_i}{I^2}$ *provided that* $w_s = w_s^{RS_2}$.

Proof: When $\phi_1+\phi_2 = (1-\phi)$, we have from equations (7.5) and (7.6),

$$\begin{aligned} 4Ir^{max}(w_s - w_s^{RS_2}) + 2I^2(w_c^{RS_2}w_s^{RS_2} - {w_c^{RS_2}}^2 + {w_c^{RS_1}}^2 - w_c^{RS_1}w_s) \\ = 2c_i(w_s^{RS_2} - w_c^{RS_2} - w_s + w_c^{RS_1}) \end{aligned} \tag{7.7}$$

As it is not possible to get the explicit form of the wholesale price under revenue sharing contract in both the cases, we consider the following two circumstances:

(i) $w_c^{RS_1} = w_c^{RS_2}$ *i.e., the collector's wholesale prices in both cases are the same.*

In this case, $w_c^{RS_1} = w_c^{RS_2} = w_c^{RS}$ (say). The equation (7.7) then reduces to $4Ir^{max}(w_s - w_s^{RS_2}) + 2I^2(w_c^{RS}w_s^{RS} - w_c^{RS}w_s) = 2c_i(w_s^{RS_2} - w_s)$ which implies that $(4Ir^{max} + 2c_i -$

$2I^2 w_c^{RS})(w_s^{RS} - w_s) = 0$. If $w_s \neq w_s^{RS}$, then we have $4Ir^{max} + 2c_i - 2I^2 w_c^{RS} = 0$ giving $w_c^{RS} = \frac{2Ir^{max}+c_i}{I^2}$.

(ii) $w_s^{RS_2} = w_s$ *i.e., the supplier's wholesale prices in both the cases are the same.*
In this case, the equation (7.7) reduces to $2I^2(w_c^{RS_2} w_s - {w_c^{RS_2}}^2 + {w_c^{RS_1}}^2 - w_c^{RS_1} w_s) = 2c_i(w_c^{RS_1} - w_c^{RS_2})$. Simplifying we get $(w_c^{RS_1} + w_c^{RS_2} - w_s)I^2 - c_i = 0$ which gives $w_s = \frac{(w_c^{RS_1}+w_c^{RS_2})I^2 - c_i}{I^2}$.

From the economic point of view, it can be stated that, when the condition given in Theorem 7.1 holds, the players will be individually benefited; they all will gain compared to their decentralized profits. From Theorem 7.2 where particular situations are considered, it is also observed that the win-win situation can be achieved for the prescribed wholesale prices of the collector and the supplier.

7.4 Numerical analysis

7.4.1 Numerical example

To demonstrate the proposed model numerically, we consider the following numerical example. The parameter-values are taken as $L = 0.25$, $r^{max} = 100$, $a = 1000$, $b = 1.3$, $c_i = 5$, $c_r = 20$, $c_p = 30$, $c_s = 60$, $w_c = 100$, $w_s = 110$ in appropriate units. For these parameter values, the expected profit function of the centralized model is found to be concave (see Fig. 7.3).

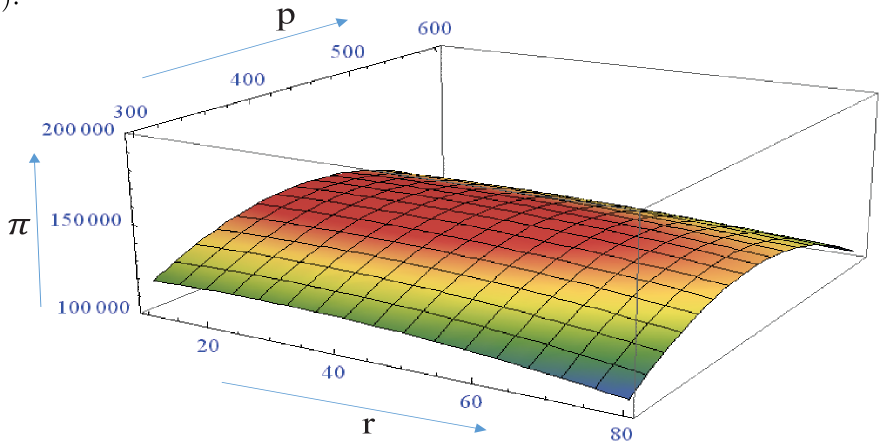


Fig. 7.3: *Concavity of* $E(\Pi)$ *with respect to* p *and* r.

We take $w_s > w_c$ so that the manufacturer is convinced to buy used products from the collector, not to buy raw materials directly from the supplier. Also $w_s > c_s$. We assume that the quality of acceptable used products follows a uniform distribution over $[0, 1]$. Table 7.1 depicts the optimal results of different models.

Table 7.1: Optimal results.

Model	r^*	p^*	$1-\phi$	$\phi_1+\phi_2$	$E(\Pi_c)$	$E(\Pi_m)$	$E(\Pi_s)$	$E(\Pi)$
Centralized	26.67	437	-	-	-	-	-	143535
Decentralized	46.67	463	-	-	6505	122018	10355	138878
RS-I	26.67	437	.01	-	7662	125518	10355	143535
RS-II	26.67	437	-	0.03	-	124536	-	143535

Table 7.1 shows that the optimal reward level is higher in the decentralized model whenever $w_c > c_s$ (from equations (7.3) and (7.4)), because the collector maximizes his collection with a better reward to draw more attention of consumers when he deals with a better wholesale price. The optimal retail price, on the other hand, is lower in the centralized policy. As demand increases with a lower retail price, the overall profit of the supply chain increases. The supplier, on the other hand, supplies the required shortage materials with a higher wholesale price compared to the collector. When the manufacturer makes a revenue sharing contract with the collector only, it is notable that the manufacturer and the collector both improve their profits and the supply chain is perfectly coordinated. For a particular wholesale price $w_c^{rs1} = 90$, the value of parameter ϕ is obtained as 0.989. So, when the manufacturer shares $(1-\phi)$ or 1% of his revenue with the collector, both the collector and the manufacturer achieve win-win outcome. However, the supplier's profit remains the same as that of the decentralized model.

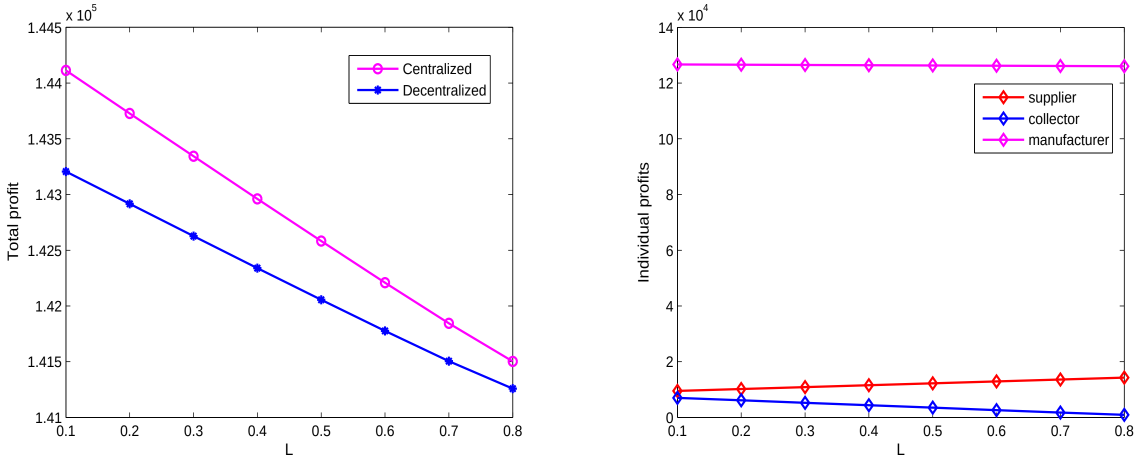
In the second case, when the manufacturer shares revenue with both the collector and the supplier, the wholesale prices of the collector and the supplier are reduced compared to those values of the decentralized model. Hence, we have $w_c^{rs2} < w_c^d$ and $w_s^{rs2} < w_s^d$. For a particular setting of the wholesale prices $w_c^{rs2} = 90$ and $w_s^{rs2} = 95$, the total fraction of revenue share $(\phi_1 + \phi_2)$ is obtained as 0.03. Because of decreased wholesale prices of the collector and the supplier, the manufacturer will be keen to share revenue with both of them. From Table 7.1 it is seen that, the manufacturer shares 3% of his revenue in total with the collector and the supplier. However, the manufacturer's profit in this situation is lower than the first case but still the manufacturer reaches to better profit margin compared to the decentralized game. The exact portions of share between the collector and the supplier i.e. ϕ_1 and ϕ_2 are dependent on one another when the players seek a win-win outcome. If one of ϕ_1 and ϕ_2 is known then the other can be obtained, and the individual profits of the collector and the supplier can also be determined.

7.4.2 Sensitivity analysis

In this section, we carry out the sensitivity analysis for some key parameters of our model.

7.4.2.1 Sensitivity with respect to L

The quality of waste is directly proportional to recyclability degree of waste i.e. the amount of waste that can be reused for further production. In our model, the collector decides the minimum acceptable quality level considering the manufacturer's demand of waste materials.



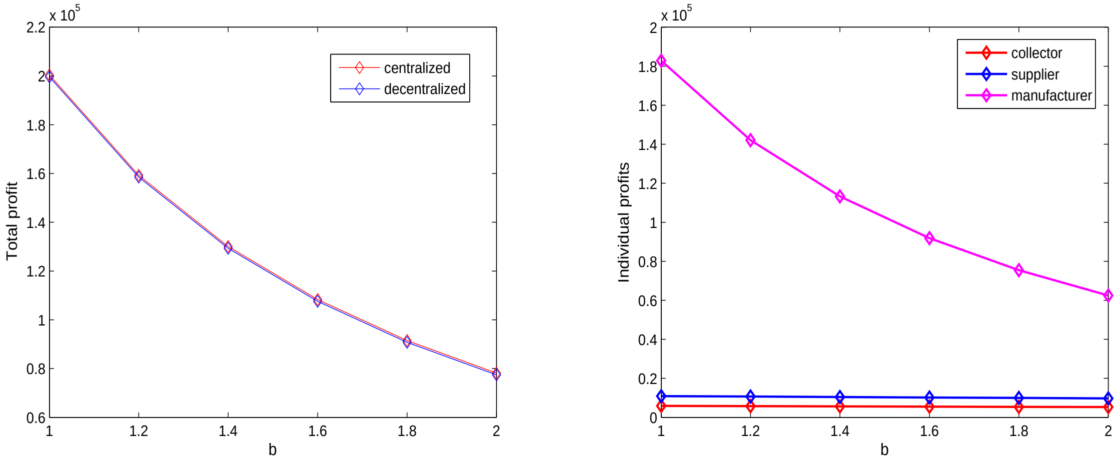
(a) *L vs total profit of the SC.* **(b)** *L vs individual profits.*

Fig. 7.4: *L vs. profits of the SC and its entities.*

For limited amount of collected wastes, as the value of L increases, the collector's profit decreases. Then the manufacturer has to purchase more raw materials from the supplier at a higher rate and the supplier in return gets more profit (see Fig. 7.4(b)). The supplier's wholesale price being higher, the manufacturer pays more for raw materials and this results in lower profit with increasing value of L. The optimal value of the reward (r^*) decreases as L increases and so the profit of the supply chain decreases in both the centralized and the decentralized models (see Fig. 7.4(a)).

7.4.2.2 Sensitivity with respect to b

From Fig. 7.5(a), it is observed that the total profit of the supply chain gradually decreases with high price sensitivity coefficient b. The profits of the collector and the manufacturer decrease for lower demand and insufficient wastes. Though the supplier is not explicitly responsible for the customer demand, his profit reduces due to lower production of the manufacturer and higher sensitivity of price (see Fig. 7.5(b)).

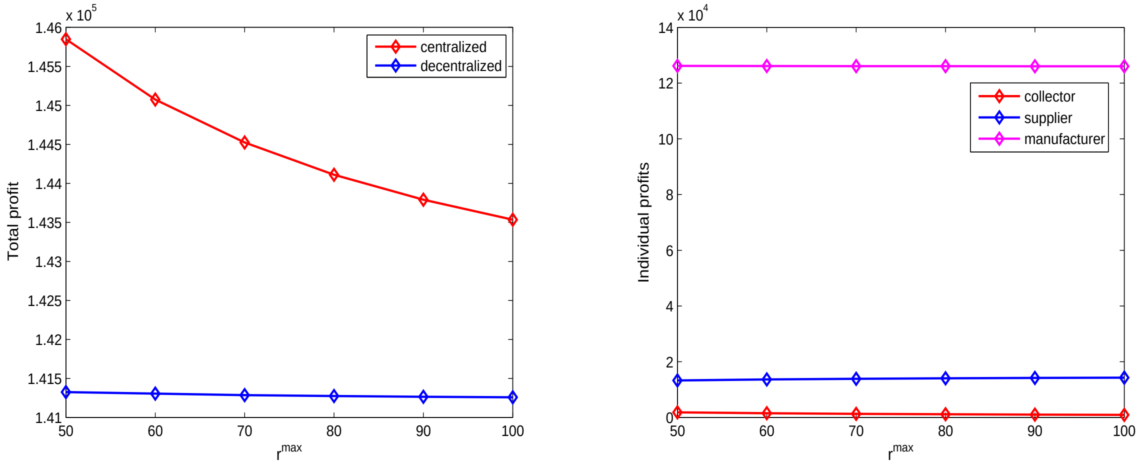


(a) *b vs total profit of the SC.* **(b)** *b vs individual profits.*

Fig. 7.5: *b vs. profits of the SC and its entities.*

7.4.2.3 Sensitivity with respect to r^{max}

From Fig. 7.6(a) it can be observed that, in the decentralized model, the total profit of the supply chain decreases slowly but, in the centralized model, it decreases more rapidly as the optimal reward r^* increases. When the collector gives more reward to consumers for returning wastes, his profit decreases. Then the manufacturer's profit also decreases but the supplier benefits from the supply chain (see Fig. 7.6(b)).



(a) r^{max} *vs total profit of the SC.* **(b)** r^{max} *vs individual profits.*

Fig. 7.6: r^{max} *vs. profits of the SC and its entities.*

7.5 Managerial implications and conclusion

To run a business successfully, a well managed supply chain by a competent manager is very much essential. Through several important decisions, a supply chain need to be well-managed for optimal outcomes of each individual associated with the business firm. In this study, a three-echelon supply chain model is analyzed for collection of used products, and remanufacturing of used products to meet customer demand. Stochastic quality of used product is considered in the proposed model. Under a certain acceptable quality level for used products, the collector offers reward to consumers. The supplier plays a vital role when sufficient used products are not available to the manufacturer. The manufacturer has to purchase fresh raw materials from the supplier at a high rate than the collector in the time of need. In three different cases, namely, centralized, decentralized and revenue sharing contract, the proposed model is analyzed and optimal results are obtained. The revenue sharing contract is implemented for two different situations and a special case is discussed for equal sharing of revenue. Through a numerical example with uniform distribution of uncertain quality of used products, optimal decisions of each individual are determined. Some managerial insights derived from this study are as follows:

(i) Through the revenue sharing contract, the manufacturer, the collector and the supplier can benefit for certain values of the sharing parameters ϕ, ϕ_1 and ϕ_2. So the decision maker must come up with a revenue-sharing strategy which is very effective for the remanufacturing firm.

(ii) The manufacturer can share a higher percentage of revenue in the case when he shares with both the collector and the supplier. But his individual profit will be more in the case when he shares revenue only with the collector. So, the manufacturer should encourage the collector to collect sufficient amount of used products for remanufacturing so that supply of raw materials from the supplier reduces to nil or negligible.

(iii) The supply chain as well as its entities are highly sensitive to minimum acceptable quality level (L) of the used product. A higher value of L results in higher profit of the supplier, although it does not give optimal result to the manufacturer and the collector. Therefore, the whole supply chain's profit will decrease with an increase in the value of L. Therefore, the manager should look into the acceptable quality level which is very sensitive to the supply chain's overall performance.

(iv) Price sensitivity coefficient and the maximum reward value have great impact on the profits of the whole supply chain and its members. Therefore, critical decisions for collection of used products as well as the demand of final remanufactured product would be a great concern for the supply chain as a whole.

(v) For sustainable consideration through used product remanufacturing, the manufacturer should play the key role in improving the total profit of the supply chain. More collection of used products means that the manufacturer has to consume less fresh raw materials from the supplier. So it is advisable that the manufacturer should make attempt to collect used products as much as possible to gain economic benefit.

There are several ways in which the frontier of the proposed model can be advanced. Quality dependent return rate will be a new direction where the stochastic behaviour as well as customer's attention will be drawn with fruitful sustainability consideration. In particular, quality improvement strategy is another avenue where the manufacturer or the respective recycler may be economically more sustainable.

Chapter 8

Reverse supply chain coordination under effort and green sensitive stochastic demand and uncertain quality of returned products

8.1 Introduction

Over the last two decades, many business firms have started to consider recycling or remanufacturing as a remedial step to protect natural resources and environment. Many industries are giving priority to sustainability for the sake of our present and future generations. In the previous chapter a sustainable reverse supply chain is modelled when the quality of returned items is stochastic. Now, in this chapter* a broader but more realistic view of a sustainable supply chain is considered. Reverse supply chain and logistics with green and effort sensitive but stochastic demand is under consideration for remanufacturing. Remanufacturing is one of the best ways an industry can adopt to restore the used products to their original state. Environmental regulation such as Waste Electrical and Electronic Equipment (WEEE) directive in Europe offers important motivations to implement reverse logistics (Heydari et al., 2018). Under this regulation, remanufacturers have to abide and implement remanufacturing

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*This chapter is based on the work submitted to *Kybernetes* (Revised), 2022 of used products focusing on limitations of natural resources. There is no doubt that remanufacturing is beneficial from environmental perspective but it is sometimes critical for the remanufacturing industry. Marketing of remanufactured products, especially when there are competitive retailers is always challenging. Customers' initial response to a remanufactured product remains debatable (Dey and Giri, 2021). A key challenge is drastic competition on labor costs. That's why, German company Bosch uses a two-way strategy: increasing automation in the German plant while the labor-based works are transferred to the plants in Ukraine and Slovakia (Tolio et al., 2017).

The quality of end-of-life (EOL) products usually remains uncertain and this makes the reverse logistics more complicated than the forward logistics. The fluctuation in quality level affects collection of used items, recycling rate, and above all it raises the supply chain members' concern for economic sustainability. Sometimes consumers need strong encouragement to send back their used products. Many researchers used monetary incentives for consumers who take part in the return of used products (Govindan and Popiuc, 2014; Saha et al., 2016; Heydari et al., 2018). However, determining the amount of incentive is critical especially when quality is a key factor behind used products collection. Heydari and Ghasemi (2018) considered a minimum quality level of used products for eligibility of remanufacturing.

In the competitive business market, environmental sustainability is a big challenge. Many of the industries are giving promotional efforts and also producing environment-friendly products to draw the attention of consumers. Promotional effort plays a significant role in the context of conventional supply chains and their outcomes. Free servicing up to a certain time period, replacement of damaged products, and maintaining a good buyer-customer relationship are some forms of promotional effort. For a long-run business advertisement, display of eco-friendly green products is very effective. Again, in a reverse supply chain, the greening level of the remanufactured product, promotional effort, and remanufacturing are interrelated. As the greening level of products increases, the demand increases in a green-conscious market. As a result, more green products are sold and then the collector becomes more interested to collect used products. The remanufacturer tries to enhance the green invention level for marketing of remanufactured products. The overall profit of the supply chain thus increases for remanufacturing green products. So, greening initiatives have successful implementation scopes for a supply chain in several dimensions (Toktaş-Palut, 2021).

However, the implementations may not be optimal from a supply chain member's economical perspective. Supply chain members often join together under a mutual agreement through a suitable contract so that their objectives are aligned with the supply chain's objective. Especially when self-decisions of the members are not feasible in comparison to their desired outcomes, they would agree for suitable contracts. When remanufacturing is considered for environmental sustainability, greenness and promotional effort for the product are considered for economic sustainability, and the market demand is uncertain, the supply chain problem becomes complex. Several questions for this type of setting may arise for optimal decisions:

(i) How much is the impact of greening and promotional effort level on the remanufacturer's decision?

(ii) If a third party collects the used products then what will be the collector's decision for customer reward value?

(iii) Which contract will be suitable for enhancing profits of supply chain entities as well as the whole supply chain compared to the decentralized scenario?

To find the answers, we consider a two-echelon reverse supply chain with one remanufacturer and one collector. The collector collects the used products from consumers and sends them to the remanufacturer for remanufacturing. The used product's quality being uncertain, the collector collects only those items which fulfill a minimum quality level. The remanufacturer refurbishes and makes them as green products for the sake of environmental sustainability. A marketing effort is also done by the remanufacturer to reach more customers when the market demand is uncertain but sensitive to the green level. Since the study focuses on challenges arising from promotional effort and greening investment by the remanufacturer, the risk of uncertainties is shared with the collector through a cost-sharing contract. Ghosh and Shah (2015) assumed a green supply chain with a cost-sharing contract. Unlike their model, the remanufacturer's sustainable moves for green product and promotional effort in a stochastic environment are taken into account in the model. Therefore, uncertain quality of used products and promotional effort- and green-sensitive stochastic customer demand in a sustainable supply chain are the main features of this chapter.

The rest of the chapter is arranged as follows: In the next section, the problem is described and also notations are given. In Sections 8.3, the proposed model is developed and then analyzed for the implementation of centralized game, decentralized game, and coordination contracts. In Section 8.4, a numerical example is given and sensitivity analysis is carried out with respect to key model-parameters. Managerial implications are discussed in Section 8.5. Finally, the chapter is concluded in Section 8.6 with some future research scopes.

8.2 Notation and problem description

We use the following notations throughout the chapter:

Parameters	:	**Description**
c_c	:	unit inspection cost of the collector
c_m	:	unit remanufacturing cost
w_c	:	unit purchasing cost paid by the remanufacturer to the collector
L	:	minimum acceptable quality level
v	:	unit salvage value for items at the secondary market
s	:	unit shortage penalty cost
Q	:	total amount of existing end-of-life (EOL) products in the market
d^{max}	:	maximum reward level for the consumers
a	:	potential market demand
t	:	binary variable representing acceptance ($t = 1$ if $L \leq \phi < 1$) or rejection ($t = 0$ if $0 < \phi < L$) of an item at the collector's site
w	:	a real number between 0 and 1 representing the consumer's willingness to return EOL item as a function of the reward
Decision variables	:	
d	:	reward per unit offered to consumers by the collector for items with the minimum allowed quality level L

θ	:	green level innovation at the remanufacturer's site
e	:	promotional effort level of the remanufacturer
Random variables	:	
ϕ	:	quality of returned items, a random variable with probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$
ϵ	:	stochastic component of the demand with p.d.f. $g(\cdot)$ and cumulative distribution function $G(\cdot)$
Profit functions	:	
$E(\Pi_c)$	:	collector's expected total profit
$E(\Pi_m)$	:	remanufacturer's expected total profit
$E(\Pi)$	:	supply chain's expected total profit

Here we consider a reverse supply chain with one remanufacturer and one collector. Customer demand is stochastic but promotional effort and green-level sensitive. The collector offers a reward to consumers for bringing back the used products (Saha et al., 2016). The used products may have different quality levels but the collector maintains a fixed quality level to accept the used products from consumers. Since the market demand is stochastic, the remanufacturer has to bear the goodwill loss cost for any shortage. For the proposed model, we make the following assumptions:

(i) Demand X is stochastic but dependent on promotional effort and green level of the product. We take $X = a + be + c\theta + \epsilon$ where $a, b, c > 0$ and ϵ is the stochastic component of the demand.

(ii) Consumer's willingness to bring back the used products is assumed as a function of reward as

$$w = f(d) = \begin{cases} \frac{d}{d^{max}} & : 0 < d < d^{max} \\ 1 & : d \geq d^{max} \end{cases} \tag{8.1}$$

(iii) Acceptance or rejection of returned products depends on their quality level. We define a variable 't' for this purpose. If the returned product has the minimum allowed quality level L, the variable t will take the value 1; otherwise, it will take the value of 0. Thus we have

$$t = \begin{cases} 0 : 0 < \phi < L \\ 1 : L \leq \phi < 1 \end{cases} \tag{8.2}$$

Therefore, the expected amount of EOL products sent to the remanufacturer is

$$E(T) = wQE(t) = \int_L^1 wQf(t)dt$$

(iv) The remanufacturer gets salvage value for overage production or he has to pay a penalty cost for each shortage unit.

8.3 Model development and analysis

In this section, a reverse supply chain with uncertainty in demand and quality of returned items is considered in decentralized, centralized and revenue-sharing contract scenarios.

8.3.1 Decentralized model

In the decentralized model, all the supply chain entities try to maximize their profits individually. Here, the collector tries to optimize his decision to set optimum reward level and the remanufacturer tries to optimize promotional effort and green level of the product individually. In the following, the expected profits of the collector and the remanufacturer are given separately. We use the superscript D to denote the decentralized case.

8.3.1.1 Collector's profit

The collector's profit per unit of the returned product is given by

$$\Pi_c^D = \begin{cases} -c_c : 0 < \phi < L \\ w_c - d - c_c : L \leq \phi < 1 \end{cases} \tag{8.3}$$

The above indicates that the collector bears a loss of c_c per unit for used products that fail to maintain the quality level L but earns a profit of $(w_c - d - c_c)$ per unit for used products that are accepted for remanufacturing. Therefore, the collector's expected total profit is given by

$$\begin{aligned} E(\Pi_c^D) &= \int_L^1 wQ(w_c - d - c_c)f(t)dt - \int_0^L c_c wQf(t)dt \\ &= \int_L^1 Q\frac{d}{d^{max}}(w_c - d)f(t)dt - c_c Q\frac{d}{d^{max}} \end{aligned}$$

where the first term is the profit associated with collection and supply to the remanufacturer, and the second term indicates the total inspection cost for used products.

Proposition 8.1 *The collector's expected profit function is concave in d and the optimal value of the reward is given by*

$$d^* = \frac{\int_L^1 w_c f(t)dt - c_c}{2\int_L^1 f(t)dt} \tag{8.4}$$

Proof: We have

$$\begin{aligned} \frac{\partial E(\Pi_c^D)}{\partial d} &= \int_L^1 \frac{Q}{d^{max}}(w_c - 2d)f(t)dt - c_c\frac{Q}{d^{max}} \\ \frac{\partial^2 E(\Pi_c^D)}{\partial d^2} &= -2\int_L^1 \frac{Q}{d^{max}}f(t)dt < 0 \end{aligned}$$

So, the collector's expected profit function is strictly concave in d, and it implies that the collector's optimal reward value exists uniquely. Therefore, from the first-order optimality condition, we can obtain the optimal value of d as given in equation (8.4).

It is obvious from equation (8.4) that if per unit inspection cost increases then the optimum reward level decreases. Then the collector will not be benefited economically.

8.3.1.2 Remanufacturer's profit

The remanufacturer's expected profit is given by

$$\begin{aligned} \Pi_m^D &= (p - w_c) \times min[E(T), X] + v[E(T) - X]^+ - s[X - E(T)]^+ \\ &\quad - c_m E(T) - Ie^2 - K\theta^2 \end{aligned}$$

where the first term denotes the remanufacturer's expected profit for remanufacturing, the second term is the expected salvage value for the overage production, the third term implies the expected shortage penalty cost, the fourth term is the total expected remanufacturing cost, and the fifth and the sixth terms indicate promotional effort cost and green innovation cost, respectively where

$$[E(T)-X]^+ = \begin{cases} E(T)-X \text{ ; when } E(T) > X \\ 0 \qquad\quad \text{ ; otherwise} \end{cases} \tag{8.5}$$

So, the expected total profit of the remanufacturer is

$$\begin{aligned} E(\Pi_m^D) &= (p-w_c-c_m+s)E(T)-(p-w_c+s-v)\int_0^{E(T)}\{E(T)-X\}g(u)du \\ &\quad -s(a+be+c\theta+\overline{X})-Ie^2-K\theta^2 \end{aligned}$$

where $\overline{X}$ is the mean customer demand. Now,

$$\begin{aligned} \frac{\partial E(\Pi_m^D)}{\partial e} &= b(p-w_c+s-v)\int_0^{E(T)} g(u)du - bs - 2Ie \\ \frac{\partial E(\Pi_m^D)}{\partial \theta} &= c(p-w_c+s-v)\int_0^{E(T)} g(u)du - cs - 2k\theta \end{aligned}$$

and $\dfrac{\partial^2 E(\Pi_m^D)}{\partial e^2} = -2I < 0,\quad \dfrac{\partial^2 E(\Pi_m^D)}{\partial \theta^2} = -2K < 0,\quad \dfrac{\partial^2 E(\Pi_m^D)}{\partial e\partial\theta} = 0,\quad \dfrac{\partial^2 E(\Pi_m^D)}{\partial\theta\partial e} = 0.$

Therefore, the Hessian matrix is given by,

$$H_2 = \begin{pmatrix} \frac{\partial^2 E(\Pi_m^D)}{\partial e^2} & \frac{\partial^2 E(\Pi_m^D)}{\partial e\partial\theta} \\ \frac{\partial^2 E(\Pi_m^D)}{\partial\theta\partial e} & \frac{\partial^2 E(\Pi_m^D)}{\partial\theta^2} \end{pmatrix} = \begin{pmatrix} -2I & 0 \\ 0 & -2K \end{pmatrix} \text{ and } |H_2| = 4IK > 0.$$

Hence the remanufacturer's expected total profit is strictly concave in θ and e. Using the optimal reward level of the collector, the remanufacturer will optimize his effort and green level.

Proposition 8.2 *The remanufacturer's optimal effort and greening level decrease with I and K, respectively.*

Proof: For optimality of $E(\Pi_m^D)$, we have

$$\begin{aligned} \frac{\partial E(\Pi_m^D)}{\partial e} &= b(p-w_c+s-v)\int_0^{E(T)} g(u)du - bs - 2Ie \\ &= 0 \end{aligned}$$

which gives the optimal effort

$$e^* = \frac{b(p-w_c+s-v)\int_0^{E(T)} g(u)du - bs}{2I}$$

Again,

$$\begin{aligned}\frac{\partial E(\Pi_m^D)}{\partial \theta} &= c(p - w_c + s - v)\int_0^{E(T)} g(u)du - cs - 2k\theta \\ &= 0\end{aligned}$$

gives the optimum green level

$$\theta^* = \frac{c(p - w_c + s - v)\int_0^{E(T)} g(u)du - cs}{2K}$$

From the above, we see that the optimal effort is inversely proportional to I *i.e.,* the effort level decreases with I. Optimal profit of the remanufacturer, therefore, decreases with the sensitivity parameter I. Similar result holds for the green sensitive parameter.

8.3.2 Centralized model: The benchmark case

In the centralized model, the collector and the remanufacturer jointly participate in the business. So, the expected total profit of the supply chain is given by

$$\begin{aligned}E(\Pi) &= p \times min[E(T), X] + v[E(T) - X]^+ - s[X - E(T)]^+ - c_m E(T) \\ &\quad - c_c Q\frac{d}{d^{max}} - dE(T) - Ie^2 - K\theta^2 \\ &= (p + s - c_m - d)E(T) - (p + s - v)\int_0^{E(T)} \{E(T) - X\}g(u)du \\ &\quad - s(a + be + c\theta + \overline{X}) - Ie^2 - K\theta^2 - c_c Q\frac{d}{d^{max}}\end{aligned}$$

where the notation $[\cdot]^+$ carries the same meaning as defined in equation (8.5). Since the decisions in the centralized model is controlled by a single decision-maker, the term $d \times E(T)$ denotes the expected total reward for consumers' return of used products and all other terms are same as already specified in subsection 8.3.1.2.

Proposition 8.3 *The expected total profit of the whole supply chain is jointly concave in d, e, and θ.*

Proof: We have

$$\begin{aligned}\frac{\partial E(\Pi)}{\partial d} &= A(p+s-c_m-2d)-c_c\frac{Q}{d^{max}}\\ \frac{\partial E(\Pi)}{\partial e} &= b(p+s-v)\int_0^{E(T)} g(u)du-bs-2Ie\\ \frac{\partial E(\Pi)}{\partial \theta} &= c(p+s-v)\int_0^{E(T)} g(u)du-cs-2k\theta\end{aligned}$$

and the second order derivatives are $\frac{\partial^2 E(\Pi)}{\partial d^2}=-2A$, $\frac{\partial^2 E(\Pi)}{\partial e^2}=-2I$, $\frac{\partial^2 E(\Pi)}{\partial \theta^2}=-2K$ where, $A=\int_L^1 \frac{Q}{d^{max}}f(t)dt$. The corresponding Hessian matrix is

$$H_3=\begin{pmatrix}\frac{\partial^2E(\Pi)}{\partial d^2} & \frac{\partial^2E(\Pi)}{\partial e\partial d} & \frac{\partial^2E(\Pi)}{\partial \theta\partial d}\\ \frac{\partial^2E(\Pi)}{\partial d\partial e} & \frac{\partial^2E(\Pi)}{\partial e^2} & \frac{\partial^2E(\Pi)}{\partial \theta\partial e}\\ \frac{\partial^2E(\Pi)}{\partial d\partial \theta} & \frac{\partial^2E(\Pi)}{\partial \theta\partial e} & \frac{\partial^2E(\Pi)}{\partial \theta^2}\end{pmatrix}=\begin{pmatrix}-2A & 0 & 0\\ b(p+s-v)Ag(E(T))d(E(T)) & -2I & 0\\ c(p+s-v)Ag(E(T))d(E(T)) & 0 & -2K\end{pmatrix}$$

The principal minors of the Hessian matrix are

$$|H_1|=-2A<0;\ |H_2|=det\begin{pmatrix}-2A & 0\\ b(p+s-v)Ag(E(T))d(E(T)) & -2I\end{pmatrix}=4\text{AI}>0$$

and $|H_3|=$ -8AIK <0. This shows that the expected total profit $E(\Pi)$ is strictly concave in d, e, and θ. Therefore, there exists a unique optimal solution that maximizes the profit function $E(\Pi)$. The corresponding optimal values d^*, e^*, and θ^* are obtained from the first-order optimality conditions. We obtain these results numerically as closed-form solution cannot be obtained in this case due to complexity.

The results of Proposition 8.2 also hold for the centralized model. In this case, the optimal effort and the optimal greening level are

$$\begin{aligned}e^* &= \frac{b(p+s-v)\int_0^{E(T)} g(u)du-bs}{2I}\\ \theta^* &= \frac{c(p+s-v)\int_0^{E(T)} g(u)du-cs}{2k}\end{aligned}$$

8.3.3 Coordination contract

In this subsection, we try to find a mutual settlement via a suitable contract which leads to enhance the profits of the supply chain members compared to the decentralized case. We first consider the popularly known revenue-sharing contract. We use superscript/subscript rs to denote this contract.

8.3.3.1 *Revenue sharing contract*

Under the revenue-sharing contract, we suppose that the remanufacturer shares $(1-\beta)$ percent of revenue earned by selling the remanufactured products, and in turn, the collector agrees to sell the collected EOL products at a lower wholesale price w_{rs} to the remanufacturer. The expected profit functions of the remanufacturer and the collector under this contract are given respectively by

$$\begin{aligned} E(\Pi_m^{rs}) &= (\beta p - w_{rs} - c_m + s)E(T) - (\beta p - w_{rs} + s - v)\int_0^{E(T)} \{E(T) - X\}g(u)du \\ &\quad -s(a + be + c\theta + \overline{X}) - Ie^2 - K\theta^2 \end{aligned}$$

$$\begin{aligned} E(\Pi_c^{rs}) &= \int_L^1 Q\frac{d}{d^{max}}(w_{rs} - d)f(t)dt + p(1-\beta)E(T) - c_c Q\frac{d}{d^{max}} - p(1-\beta) \\ &\quad \times \int_0^{E(T)} \{E(T) - X\}g(u)du \end{aligned}$$

Proposition 8.4 *Channel coordination cannot be achieved through the revenue sharing contract.*

Proof: For channel coordination, the optimal decisions of the centralized game and the decisions under the revenue sharing contract are the same. So, equating the optimal effort levels of these two cases (*i.e.,* taking $e_c^* = e_{rs}^*$), we have

$$\frac{b(p+s-v)\int_0^{E(T)} g(u)du - bs}{2I} = \frac{b(\beta p + s - v - w_{rs})\int_0^{E(T)} g(u)du - bs}{2I}$$

which implies that

$$(p + s - v) = \beta p - w_{rs} + s - v, \quad \text{since} \;\; d_c^* = d_{rs}^*$$

i.e., optimal reward is also the same in both the cases, which implies that $w_{rs} = \beta p - p = p(\beta - 1) < 0$, since $0 < \beta < 1$. This implies that channel coordination is not possible for revenue sharing contract. Hence the result.

We now try to coordinate the supply chain with a cost-sharing contract where the collector shares a portion of the remanufacturer's cost due to effort and greening of the product. We use the superscript/subscript cs to denote the cost-sharing contract.

8.3.3.2 Cost-sharing contract

Since the remanufacturer bears the costs due to effort and greening of the product, he would try to convince the collector to share a portion of the costs. In turn, the collector would try to increase his selling price of collected EOL items in order to accommodate the remanufacturer's request. The expected profit functions of the remanufacturer and the collector under this contract are given respectively by

$$\begin{aligned} E(\Pi_m^{cs}) &= (p - w_{cs} - c_m + s)E(T) - (p - w_{cs} + s - v)\int_0^{E(T)} \{E(T) - X\}g(u)du \\ &\quad -s(a + be + c\theta + \overline{X}) - \gamma I e^2 - \lambda K\theta^2 \\ E(\Pi_c^{cs}) &= \int_L^1 Q\frac{d}{d^{max}}(w_{cs} - d)f(t)dt - c_c Q\frac{d}{d^{max}} - (1-\gamma)Ie^2 - (1-\lambda)K\theta^2 \end{aligned}$$

where, $0 < \gamma,\ \lambda < 1$. The first-order condition for the collector's optimal profit is given by $\frac{\partial E(\Pi_c^{cs})}{\partial d} = 0$ which gives

$$\begin{aligned} d^* &= \frac{\int_L^1 \frac{Q}{d^{max}} w_{cs} f(t)dt - c_c\frac{Q}{d^{max}}}{2\int_L^1 \frac{Q}{d^{max}} f(t)dt} \\ &= \frac{Aw_{cs} - c_r\frac{Q}{d^{max}}}{2A},\ \text{where } A = \int_L^1 \frac{Q}{d^{max}} f(t)dt \qquad (8.6) \end{aligned}$$

The optimal decision for the consumer reward in the centralized model is obtained from $\frac{\partial E(\Pi)}{\partial d} = 0$ which gives the optimal value

$$d^{**} = \frac{A\left\{(p + s - c_m) - (p + s - v)\left[\int_0^{E(T)} g(u)du - (a + be + c\theta)g(E(T))\right]\right\} - c_c\frac{Q}{d^{max}}}{2A} \qquad (8.7)$$

Now, from equations (8.6) and (8.7), we get the wholesale price

$$\begin{aligned} w_{cs} &= (p + s - c_m) - (p + s - v)\int_0^{E(T)} g(u)du \qquad (8.8) \\ &\quad +(p + s - v)(a + be + c\theta)g\{E(T)\} \end{aligned}$$

Similarly, equating the optimal effort and green level of the centralized model and those of the model with cost-sharing contract, we have the following relations :

$$w_{cs} = (1-\gamma)(p+s-v) \tag{8.9}$$

$$w_{cs} = (1-\lambda)(p+s-v) \tag{8.10}$$

From equations (8.9) and (8.10), we get $\gamma = \lambda$ *i.e.*, both the fractions of cost-share are the same.

Proposition 8.5 *The expected amount of returned products increases with the increment of effort level e and green level θ.*

Proof: From equations (8.8) and (8.10), we get

$$\begin{aligned} w_{cs} &= (p+s-c_m) - (p+s-v)\int_0^{E(T)} g(u)du + (p+s-v)(a+be+c\theta)g\{E(T)\} \\ &= (1-\lambda)(p+s-v) \end{aligned}$$

from which it is deduced that

$$E(T) = G^{-1}\left\{\frac{p+s-c_m}{p+s-v} + (a+be+c\theta)g\{E(T)\} + \lambda - 1\right\}$$

Since G is monotonically increasing, G^{-1} is also a monotonically increasing function. So, with increasing values of θ and e, the expected amount of returned EOL products increases.

Proposition 8.6 *The remanufacturer's optimal effort level increases with the collector's reward for the used products.*

Proof: Optimizing the remanufacturer's profit with respect to the effort level e, we obtain the optimum effort level as

$$e^* = \frac{b(p-w_{cs}+s-v)\int_0^{E(T)} g(u)du - bs}{2\gamma I} \tag{8.11}$$

Differentiating equation (8.11) with respect to d, we get

$$\frac{\partial e^*}{\partial d} = \frac{b(p-w_{cs}+s-v)\Big(\int_L^1 \frac{Q}{d^{max}} f(t)dt\Big) g\{E(T)\}}{2\gamma I}$$

which is positive since $p > w_{cs}$ and $s > v$. Hence the optimal effort is an increasing function of the consumer reward d.

8.3.3.3 Win-win outcome through cost-sharing contract

It can be shown that, for a finite range of values of the cost-sharing parameter λ, both the remanufacturer and the collector reach a win-win situation. We deduce this range analytically using the condition that profits under the contract for the collector and the remanufacturer will be greater than those of the decentralized model. So, using the relation $E(\Pi_m^{cs}) \geq E(\Pi_m^D)$, we get

$$\lambda \leq \frac{(p - w_{cs} - c_m + s)E(T) - (p - w_{cs} + s - v)\int_0^{E(T)}\{E(T) - X\}g(u)du - s(a + be + c\theta + \overline{X}) - E(\Pi_m^d)}{Ie^2 + K\theta^2}$$

Using equation (8.10) in the above relation, we get

$$\lambda \leq \frac{(c_m - v)E(T) + s(a + be + c\theta + \overline{X}) + E(\Pi_m^d)}{(p + s - v)E(T) - (p + s - v)\int_0^{E(T)}\{E(T) - X\}g(u)du - Ie^2 - K\theta^2} \tag{8.12}$$

Again, from the relation $E(\Pi_c^{cs}) \geq E(\Pi_c^D)$, we have

$$\lambda \geq 1 - \frac{\int_L^1 Q\frac{d}{d^{max}}(w_{cs} - d)f(t)dt - c_c Q\frac{d}{d^{max}} - E(\Pi_c^d)}{Ie^2 + K\theta^2}$$

Using equation (8.10) the above inequality reduces to

$$\lambda \geq \frac{Ie^2 + K\theta^2 - (p + s - v - d)\int_L^1 Q\frac{d}{d^{max}}f(t)dt + c_c Q\frac{d}{d^{max}} + E(\Pi_c^d)}{Ie^2 + K\theta^2 - (p + s - v)\int_L^1 Q\frac{d}{d^{max}}f(t)dt} \tag{8.13}$$

Now, from equations (8.12) and (8.13), we get a specified range of λ, which is given by

$$\begin{aligned} &\frac{Ie^2 + K\theta^2 - (p + s - v - d)\int_L^1 Q\frac{d}{d^{max}}f(t)dt + c_c Q\frac{d}{d^{max}} + E(\Pi_c^d)}{Ie^2 + K\theta^2 - (p + s - v)\int_L^1 Q\frac{d}{d^{max}}f(t)dt} \leq \lambda \\ &\leq \frac{(c_m - v)E(T) + s(a + be + c\theta + \overline{X}) + E(\Pi_m^d)}{(p + s - v)E(T) - (p + s - v)\int_0^{E(T)}\{E(T) - X\}g(u)du - Ie^2 - K\theta^2} \end{aligned}$$

For each value of λ in the above range, both the players are benefited in terms of profits compared to the decentralized scenario.

8.4 Numerical analysis

In this section, we demonstrate the proposed models through a numerical example. Assuming suitable parameter-values, the optimal decisions as well as optimal profits of the supply chain and its members are obtained for each model.

8.4.1 Numerical example

To demonstrate the proposed model numerically, we consider the following parameter-values: $p = 600,\ w_c = 160,\ v = 5,\ s = 30,\ I = 2,\ K = 2,\ L = 0.3,\ d^{max} = 150,\ a = 200,\ b = 1.3,\ c = 1.2,\ c_m = 210,\ c_c = 20,\ Q = 1000$ in appropriate units. The parameters are so chosen that the model assumptions remain valid and optimality conditions become feasible.

We assume that the customer demand D is stochastic and it varies within 0 to 2000 that means $D \in [D_{min}, D_{max}]$ where $D_{min} = 0$ and $D_{max} = 2000$. Therefore, the expected customer demand for the product, $\overline{X} = 1000$. Also, the quality (ϕ) of the returned product takes a value within the interval $[0, 1]$ and so it follows a uniform distribution in the interval $[0, 1]$ *i.e.,* $\phi \sim U[0, 1]$. Because the quality of returned products varies from consumer to consumer, it is equally possible to take any value between 0 and 1. In the following table, we now show the optimal results for the centralized model, the decentralized model, and the coordination model with a cost-sharing contract.

Table 8.1: Optimal results under different models.

Optimal results	Centralized	Decentralized	Cost sharing	Profit increment(%)
d	143.98	95.71	143.98	-
e	58.49	19.65	58.49	-
θ	53.99	18.14	53.99	-
λ_{min}	-	-	0.563	-
λ_{max}	-	-	0.623	-
$E(\Pi_c)$	-	42752.4	47631.7	11.41
$E(\Pi_m)$	-	52652.5	66757.3	26.79
$E(\Pi)$	114389.0	95404.9	114389.0	19.90

Observations:

The major findings from the numerical study are given below:

(i) The optimal promotional effort level and the optimal greening level are higher in the centralized model compared to those of the decentralized model. This happens because the double-marginalization effect is ignored in the centralized model through mutual negotiation.

(ii) In the decentralized model, the remanufacturer's expected profit is higher than that of the collector because the collector's optimal collection of used products is dependent on customer reward d. Since d is lower in the decentralized model compared to the centralized model, the collector's collection level reduces. On the other hand, the remanufacturer's promotional effort and greening level enhance the consumer demand, which result in better expected profit.

(iii) When the cost-sharing parameter λ increases, the remanufacturer's cost increases and, therefore, the collector's share reduces. Again from equation (8.13), the collector's wholesale price w_{cs} decreases with λ, which means that the collector's wholesale price is more when he shares more of the remanufacturer's promotional effort and green innovation cost. Thus both the players tend to reach to a Pareto-optimal situation through the cost-sharing contract. Numerical results show that when the cost-sharing parameter λ lies within the interval (0.563, 0.623), both the players achieve a win-win outcome. For $\lambda = 0.602$, the supply chain is perfectly coordinated and the total expected profit is equal to that of the benchmark case (see Fig. 8.1).

(iv) Promotional effort and greening level both increase in the cost-sharing model in comparison to the decentralized model. Thus, increase in the remanufacturer's expected profit through the cost-sharing contract is more than that of the collector when compared with the decentralized model. This is because the remanufacturer's profit is directly proportional to the demand of green remanufactured product, and the demand increases due to promotional effort and greening level investment.

(v) The whole supply chain is economically benefited due to the cost-sharing contract. It is seen that when the supply chain is perfectly coordinated, about 20% of the total expected profit is enhanced compared to the profit of the decentralized model.

8.4.2 Sensitivity analysis

We now perform the sensitivity analysis to investigate the effects of key model-parameters on the optimal results. We change the value of one parameter at a time and keep the other parameter-values unchanged. The results are shown in Tables 8.2-8.4.

From Table 8.2, it is seen that, with the increasing value of the minimum acceptable quality of return products, the expected profits of the collector and the remanufacturer decrease.

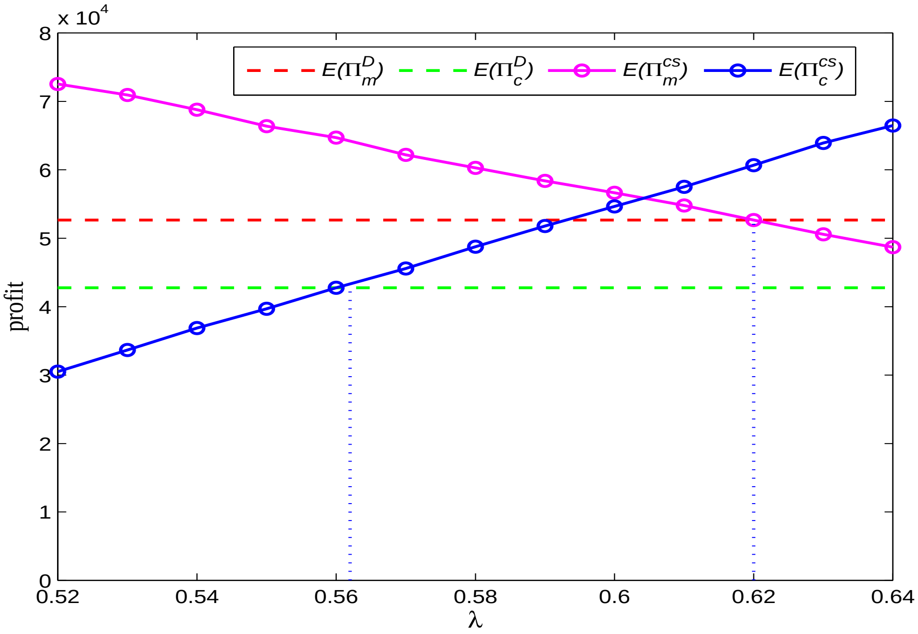


Fig. 8.1: *Win-win outcome through cost sharing contract.*

It is also observed that the expected profit decreases more drastically for the collector than the remanufacturer. This is because the collector's total collection quantity reduces when he is strict on quality level for the acceptance of used products. The optimal reward level also decreases with L because the collector finds it difficult to collect the used products for a higher value of L. The expected profits of both the remanufacturer and the collector also decrease with L in the cost-sharing contract.

Table 8.2: Sensitivity analysis for different values of L.

Quality level(L)	$E(\Pi_m^{cs})$	$E(\Pi_c^{cs})$	$E(\Pi_m^D)$	$E(\Pi_c^D)$	d^D
0.10	84020.7	67574.3	74231.8	58674.1	98.89
0.15	76252.8	57397.9	69180.7	54684.3	98.24
0.20	71534.2	54045.0	63900.4	50700.0	97.50
0.25	68905.4	51868.5	58391.0	46722.2	96.67
0.30	66757.3	47631.7	52652.5	42752.4	95.71
0.35	49804.4	40234.2	46684.9	38792.3	94.62
0.40	42652.5	37834.2	40488.2	34844.4	93.33
0.45	36397.9	32180.7	34062.4	30912.1	91.82
0.50	29905.4	28062.2	27407.4	27000.0	90.00

The expected profit of the remanufacturer in the decentralized model decreases with the increments in promotional effort and greening cost sensitivity parameters I and K. From Table 8.3, it is clear that the optimal effort decreases with a higher rate in the centralized and the decentralized models when I increases.

Table 8.4 depicts similar results for optimal greening level when greening cost sensitivity parameter K increases. But the impact of greening cost sensitivity on optimum effort level or the impact of promotional effort cost sensitivity on optimum greening level is less compared to the impacts on total expected profits and other optimal decisions. The remanufacturer's expected profit is therefore highly sensitive to the promotional effort and greening cost parameters. The total expected profit of the supply chain decreases gradually with I and K.

Table 8.3: Sensitivity analysis for the parameter I.

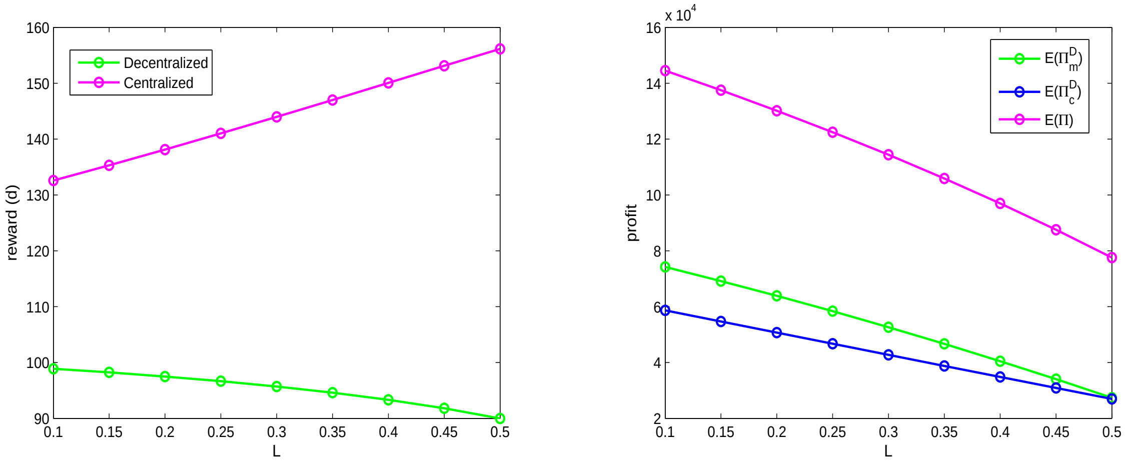
Effort sensitivity(I)	$E(\Pi^C)$	$E(\Pi_m^D)$	e^C	e^D	θ^C
1.0	121684	53424.5	124.73	39.29	57.56
1.2	119147	53167.2	101.69	32.74	56.32
1.4	117401	52983.4	85.84	28.07	55.47
1.6	116126	52845.5	74.27	24.56	54.84
1.8	115154	52738.3	65.44	21.83	54.37
2.0	114389	52652.5	58.49	19.65	54.00
2.2	113770	52582.4	52.88	17.86	53.69
2.4	113260	52523.9	48.24	16.37	53.44

Table 8.4: Sensitivity analysis for the parameter K.

Green sensitivity(K)	$E(\Pi^C)$	$E(\Pi_m^D)$	e^C	θ^D	θ^C
1.0	120544	53310.3	61.76	36.27	114.02
1.2	118417	53091.0	60.63	30.22	93.28
1.4	116945	52934.4	59.85	25.91	78.92
1.6	115866	52817.0	59.27	22.67	68.39
1.8	115040	52725.6	58.84	20.15	60.35
2.0	114389	52652.5	58.49	18.14	54.00
2.2	113861	52592.7	58.21	16.49	48.85
2.4	113425	52542.9	57.98	15.11	44.60

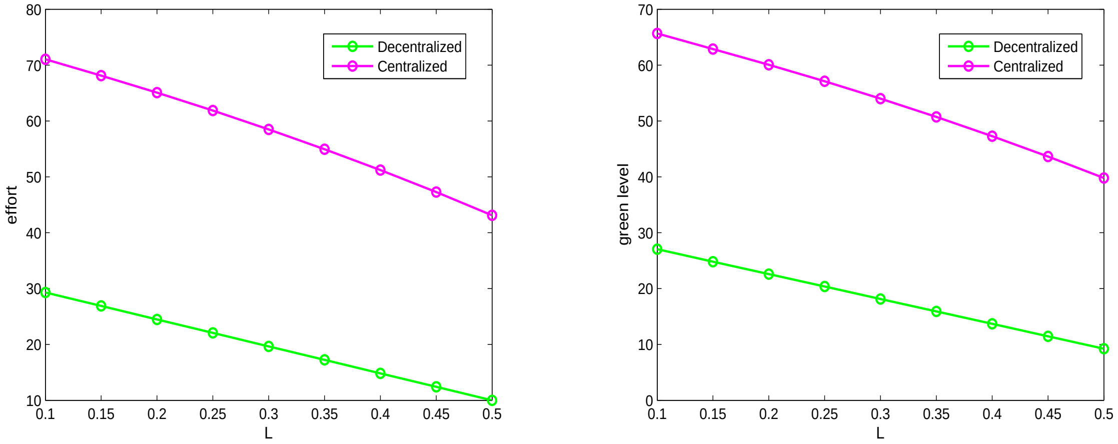
8.5 Managerial insights

A business firm runs successfully by several efficient and effective decisions made by its managers. The present study considers remanufacturing through the collection of used products from consumers with the help of a collector. The uncertainty of quality level remains an integral part of reusing the used products. Inspection of the collected items and effort to produce environment-friendly green products are vital decisions to the managers.



(a) *L vs customer reward amount* **(b)** *L vs profits*

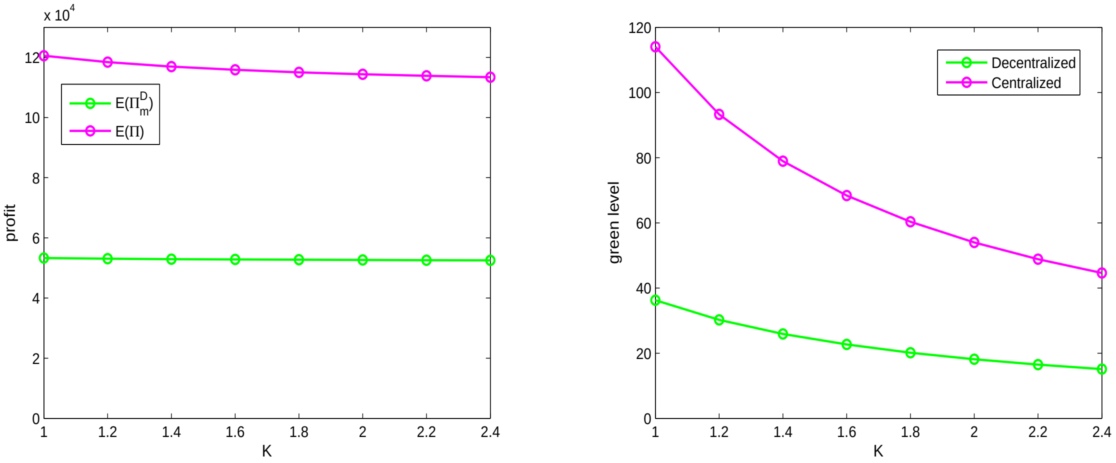
Fig. 8.2: *Changes in rewards and profits with respect to L.*



(a) *L vs. efforts* **(b)** *L vs. green levels*

Fig. 8.3: *Changes in efforts and green levels with respect to L.*

The uncertainty in consumer demand for remanufactured products makes the model more complex. A remanufacturing supply chain with greenness of product and promotional effort consideration compels the decision-maker in vital thinking. Therefore, the remanufacturing firm manager needs to decide carefully as the optimal decisions are highly sensitive to greening and promotional effort levels. Some important managerial insights of our study are given below:



(a) *K vs. profits*

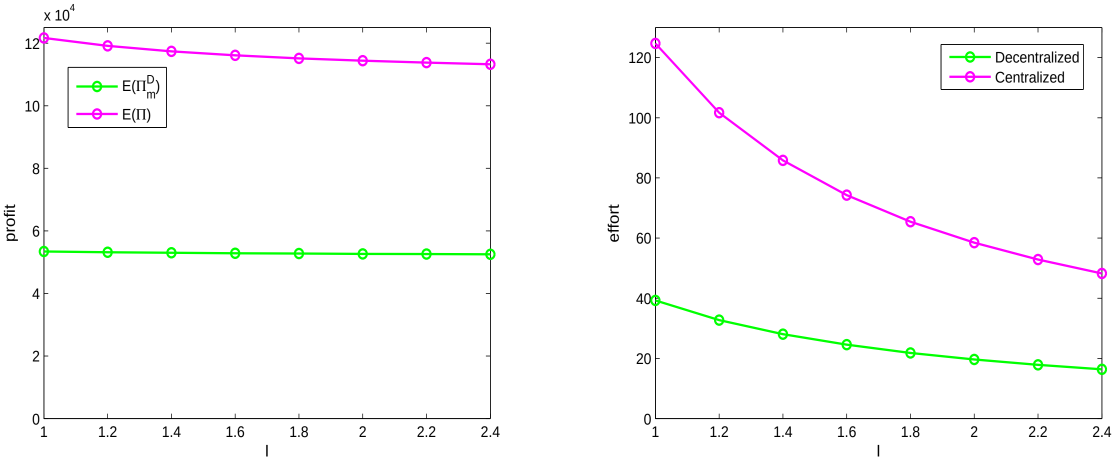
(b) *K vs. green levels*

Fig. 8.4: *Changes in profits and green levels with respect to K.*

The collector collects the used products (which have a minimum quality level) by giving rewards to consumers according to the quality level. In the decentralized policy, the collector's reward decreases with L because the collector fails to procure a sufficient amount of used products or cores for remanufacturing, and also inspection cost increases with L (see Fig. 8.2(a)). Therefore, the expected total profit of the remanufacturer decreases. The expected total profit of the centralized system also has the same characteristics (see Fig. 8.2(b)).

The managers need to focus on promotional effort and green investment to keep a balance between the consumer's demand and the profitability of business firms. From the numerical study, it is found that both promotional effort and greening level decrease with L (see Fig. 8.3) because the amount of used products collection decreases with L. Therefore, the remanufacturer's production uncertainty becomes high, which enhances the shortage penalty cost of unfulfilled demand.

The remanufacturer exerts promotional effort and pays greening level innovation cost for the demand of the final product. But if both promotional effort and greening cost sensitivity parameters increase, optimal effort and greening level decrease (see Fig. 8.4). Therefore, the expected profits of the remanufacturer and the whole supply chain decrease, and the economic stability is hampered (see Fig. 8.5).



(a) *I vs. profits* **(b)** *I vs. efforts*

Fig. 8.5: *Changes in profits and effort levels with respect to I.*

8.6 Conclusion

In the present study, a two-echelon reverse supply chain is investigated in the presence of uncertain market demand and uncertain quality of returned products. To drive out the business complexities due to market competition, demand uncertainty and double marginalization effect, a cost-sharing contract is prescribed for individual profit improvements of both the members. Supply chain entities agree to share the risk of both greening and effort cost investment. From the numerical study, it is clear that, for a range of values of the cost-sharing parameter, both the members can achieve a win-win situation, and channel coordination is possible for a certain value of the parameter within the range. The chapter presents the following important results of the proposed reverse supply chain:

(i) Due to an uncertain environment when the consumer demand is sensitive to promotional effort and green level of the product, a cost-sharing contract can help each supply chain member to reach Pareto optimal solution through individual profit improvement.

Perfect coordination between the collector and the remanufacturer is achieved for a particular value of cost-sharing parameter λ. Thus the supply chain, as well as its members, reaches economic stability even in a stochastic environment of customer demand.

(ii) A higher value of the minimum acceptable quality level leads to lower profit for the remanufacturer. This happens because the collector fails to collect a significant amount of used items from consumers and it raises the uncertainty of the customer demand. That is why, through green product innovation, the supply chain mitigates the harmful effect of wastes in the environment.

(iii) For a fixed quality level L, the reward for the customer increases but the optimal reward decreases with L because the collector fails to collect a sufficient amount of used products in that case. Therefore, the collector wants to diminish his loss from collection of wastes which have no core value for remanufacturing.

(iv) Optimal effort and greening level are sensitive to the parameters I and K, respectively. For higher values of these sensitivity parameters, the optimal values get diminished. As a result, individual and total profits for the decentralized and the centralized models decrease.

(v) Optimal effort and green level both increase in the centralized as well as coordination contract scenarios. The remanufacturer would be benefited most from the reverse supply chain by increasing demand for green products. So, a greening initiative by a remanufacturing firm has a high positive impact on environmental as well as economic stability.

The main contributions of the present study are, (i) assumption of two uncertainties in the form of customer demand and quality of returned products for remanufacturing, (ii) remanufacturer's promotional effort and green investment for environment-friendly green products and (iii) cost-sharing contract for the coordination of the reverse supply chain. The outcomes of the developed models imply that the quality of returned product, promotional effort and green investment are crucial decisions for a remanufacturing industry.

There are several scopes for future research related to this topic. We have taken a fixed reward for each collected item. In future study, quality dependent reward amount may be assumed. Further, unit remanufacturing cost can be taken as a variable depending on the quality level of returned products. The quality improvement strategy of the returned used products can also be considered in future study.

Chapter 9

Conclusion and future research prospects

In this thesis, we have studied the impact of sustainability issues in successful supply chain management. Environmental, economic, and social sustainability have been considered as remedial business strategies for long run. Rapid globalization, population growth, health safety concerns of the population, labor conflicts, and above all highly competitive business market have urged the business managers to revisit their existing traditional strategies and advance the frontier with sustainability consideration. For environmental sustainability, a supply chain with dual-channel waste recycling has been analyzed in the third chapter. Stackelberg game-theoretic models have been developed for the economic sustainability of SC members. The fourth chapter has included an uncertain environment of customer demand which is more realistic. Markup strategy as an economically sustainable policy has been considered for the supply chain and its members.

Social responsibility consideration has been implemented in the fifth chapter and its impact has been measured. However, the limitations like competitive retail market or demand dependency on social responsibility, quality of return wastes are crucial. Henceforth, we have dealt with the competitive retail channels in the sixth chapter, and two different suppliers for fresh and used materials have been considered. Our results show that remanufacturing and corporate social responsibility are beneficial for the whole supply chain and the individuals. A reverse supply chain has been investigated in the seventh chapter which has focused on economic and environmental sustainability through remanufacturing and coordination contracts, respectively. Under uncertain quality of returned items, the supply chain model has been analyzed for two types of revenue sharing contract, and economic sustainability has been reflected through win-win outcomes. Analysis of reverse supply chain is a leading extension for environmental sustainability. The recovery of wastes for different quality of wastes has been modeled in the eighth chapter. Stochastic quality of returned items along with uncertain market demand which also depends on green and effort level is a significant development for a completely reverse supply chain

The most conventional supply chains focus on developing strategies mainly for economic benefits. But successful initiatives of sustainable development is just started in many developing countries, among which social sustainability consideration is very rare. So, in future research effort, it is of urgency to be implemented its several dimensions.

Again, contrary to forward logistics, reverse logistics is a relatively new area for researchers to explore. Especially the analysis of contract implementation among supply chain members is gaining increasing attention of researchers and practitioners. Furthermore, environmental sustainability under different domains of customer demand can be assumed and SC entities with pure and composite contracts for the target of sustainable development will make interesting scenarios for future investigation.

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3. **Analyzing a closed-loop sustainable supply chain with duopolistic retailers under different game structures,** Sushil Kumar Dey & B. C. Giri, *CIRP Journal of Manufacturing Science and Technology*, 2021, Volume 33, pages 222 - 233.

4. **Corporate social responsibility in a closed-loop supply chain with dual-channel waste recycling,** Sushil Kumar Dey & B. C. Giri, *International Journal of Systems Science: Operations & Logistics*, 2021, (Published online), DOI: 10.1080/23302674.2021.2005844.

5. **Coordination of a sustainable reverse supply Chain with revenue sharing contract,** Sushil Kumar Dey & B. C. Giri, *Journal of Industrial and Management Optimization*, 2022, Volume 18, issue 1, pages 487- 510.

6. **Reverse supply chain coordination under effort and Green sensitive stochastic demand and uncertain Quality of returned products,** B. C. Giri & Sushil Kumar Dey, *Kybernetes*, 2022, (Revision submitted).

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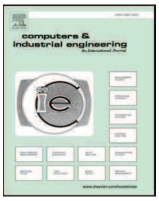

Game theoretic analysis of a closed-loop supply chain with backup supplier under dual channel recycling

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ABSTRACT

In a recent article, Jafari et al. (2017) considered dual-channel recycling in a closed-loop supply chain (CLSC) consisting one collector, one recycler and one manufacturer. In this paper, their model is extended with a backup supplier considering uncertainty of collection of used products. The shortfall quantity of collection is met up by the backup supplier with the estimated fresh raw materials. Under various power structures or interactions of the supply chain entities, different game theoretic models are developed. It is observed from the numerical study that, depending on the fractional part of the manufacturer's requirements of recyclable wastes supplied by the collector, the performance of the supply chain increases compared to that of Jafari et al.'s (2017) model in the absence of the recycler. However, in the presence of the recycler, the whole supply chain's profit surpasses Jafari et al.'s (2017) profit for any amount of used product collection.

1. Introduction

Due to rapid environmental degradation, climate change, population growth and also consumption of natural resources, humanity is under a great threat now-a-days. So, many industries are giving much importance to sustainability for the sake of present and even for future generations. There are various ways available for sustainable development in practice such as green purchasing, reusing, remanufacturing, recycling, etc. (Grimmer & Bingham, 2013; Luchs, Naylor, Irwin, & Raghunathan, 2010; Salimifard & Raeesi, 2014). Among these approaches, recycling is one of the best ways an industry can adopt to procure the significant amount of worth values of used products and also to reduce the harmful impact of wastes like carbon emission and environmental pollution. As example, electronic and electrical devices and components, large household appliances, cooling and freezing appliances, plastic wastes are being recycled by many companies (Kilic, Cebeci, & Ayhan, 2015). UK has a recycling rate of approximately 60% for iron and steel, most of which comes from scrap vehicles, cooker, fridges and other kitchen appliances and, Germany has the recycling rate approximately 70% for plastic (www.enviropedia.org.uk/Sustainability/Waste_Recycling.php). Taking small initiatives like usage of sustainable materials for buildings like recycled bricks and timber or even usage of eco-friendly things can make all the differences. Some industries like Hewlett Packard and Canon undertake the reverse logistics to obtain more profits (Savaskan & Van Wassenhove, 2006). In this regard, Lalbakhsh (2012) studied the impact of recycling in sustainable development for developing countries. He discussed the use of different recycling processes such as rain recycling, green space recycling, urban space recycling, garbage recycling and energy recycling. In the process of recycling, used materials are collected from the consumers. In closed-loop supply chain (CLSC), there are various ways by which used items can be returned back from the end customers. Manufacturer himself/herself can collect the used products, the retailer or even a third party may be engaged for the collection. Dual channel for collection can also be implemented in supply chain. Sometimes it is seen that dual channel recycling outperforms single recycling channel (Feng, Govindan, & Li, 2017; Huang, Song, Lee, & Ching, 2013). However, due to highly competitive business market, it is not always possible for a supply chain entity to get optimal profit margin from the business by his/her own effort. Rather, it is beneficial to go for some agreement or decision making alternatives like leadership-followership power structure. In reality, this is very much rational. Many industries like Adidas and Dell cooperate with their respective raw material suppliers; Coca-Cola company ties up with a third party recycler (Giri, Mondal, & Maiti, 2018). Intel always tries to improve its operations and minimize its impact on the environment. Texas Instruments makes significant investments to efficiently use, reuse, or recycle materials across its operations, and reduces its potential environmental impact by sourcing materials responsibly, as well as appropriately managing waste handling and disposal (www.forbes.com). Recently, Hrncir,

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Analyzing a closed-loop sustainable supply chain with duopolistic retailers under different game structures

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ABSTRACT

This article investigates a closed-loop supply chain consisting of a manufacturer, two suppliers and two competitive retailers. One retailer sells manufactured products whereas the other retailer sells remanufactured products and takes up corporate social responsibility (CSR). One supplier supplies used products or cores for remanufacturing while the other supplier supplies fresh raw materials for manufacturing new product. The manufacturer sells both new and remanufactured products with different wholesale prices. The paper analyzes the two competitive retailer's different game strategies when the manufacturer acts as the Stackelberg leader. It is shown that remanufacturing is a good policy to adopt for the whole supply chain, not only for economical benefits but also for environmental sustainability. Optimal decisions of the proposed closed-loop supply chain and its members are also supported by a numerical example. Finally, sensitivity analysis is carried out with respect to key model-parameters.

Introduction

Sustainability has become an emerging issue now-a-days for a chaotic environment surrounding us. As the population is growing and human lifestyle is becoming richer, the consumption of natural resources is gradually increasing, and therefore people are in no mood except thinking about sustainability in a parallel way. Brundtland [8] defined sustainable development as 'the development that meets the needs of the present without compromising the ability of future generations to meet their own needs'. It includes environmental, social and economic stability or balance on our planet. Over the last two decades, many of the enterprises have taken social responsibility into account along with their general business policies, which has been an emerging way for sustainability consideration. Through corporate social responsibility (CSR) consideration, the firms not only can draw the attention of customers but also generate scope for a better and positive response for well business policy in the future. Supports for a charity, social ethics, safety standards and protection of workers in industry are considered social responsibilities for any business firm [2,28]. Two opposite perspectives to CSR have emerged – business view and societal view. Through a business view, a large enterprise makes a contribution to the society by making profit which is related to wages, taxes, employment, investment, etc. However, in societal view, stakeholder's welfare and social responsibilities are prioritized. Thus CSR covers the relationship between the society and the enterprises. CSR is therefore an integral part of a firm's strategy. Though there are some arguments like moral, ethical and specially the economic argument but, through CSR implementation, a company creates a greater market advantage in the competitive market scenario. It is increasingly relevant in today's competitive market and so the organizations who fail to maximize the adoption of CSR strategy may be left behind. According to a global survey result in 2002, it was noted that about 94% organizations trust the implementation of CSR strategy to produce real business benefits. Thus, with business goals, the companies are looking for social and environmental issues although only 11% of them have successfully set up the CSR strategies in their organizations [40].

Remanufacturing provides a golden opportunity towards reaching a sustainable future [11,42] and it is also a potential solution that facilitates sustainable business practices [4]. By remanufacturing, used products or cores are renewed so that they may operate like new products [1]. Many enterprises like Kodak, Hewlett-Packard, etc. have already participated in product recovery and remanufacturing [41]. Though recycling and remanufacturing both have environmental merits, the re-marketing or reselling of a remanufactured product is always challenging [31]. Tolio et al. [48] highlighted the main challenges and opportunities that are faced in a demanufacturing as well as remanufacturing

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COORDINATION OF A SUSTAINABLE REVERSE SUPPLY CHAIN WITH REVENUE SHARING CONTRACT

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Abstract. In this article, a three-echelon closed-loop supply chain is considered under sustainability consideration through remanufacturing of waste materials. Depending upon quality, the collector collects the used products and forwards to the manufacturer for remanufacturing. The collector offers a reward or incentive to consumers to influence them to return the used items. The shortfall amount of collected used items, if any, is meet up by the supplier by supplying fresh raw materials. In three separate cases viz centralized, decentralized and revenue-sharing contract, optimal incentives for end-customers and optimal profits of supply chain members are determined. The revenue-sharing contract is implemented in two different settings - one including the supplier and the other one excluding the supplier. The win-win outcome for the supply chain members is investigated and a specific range of the sharing parameter for win-win outcome is obtained. Optimal results are supported by numerical analysis, and sensitivity of the optimal results with respect to key parameters is analyzed.

1. **Introduction.**

In this 21^{st} century, human race is now in a great danger due to environmental degradation, climate change and shortages of natural resources. All these have led Governments and policy makers to rethink about the coming threat in near future. Many researchers and scientists are trying to advance the frontier of the supply chain taking sustainability as a major decision. Customers' growing concern for environmental issues has encouraged many industries to adopt sustainability as a fruitful remedy. Reverse and closed-loop supply chains are being adopted to achieve sustainability goals [13]. Customers are also resisting products which lack health safety and humanistic care. For this reason, many companies are implementing recycling and remanufacturing with a noble goal towards a cleaner world not only for this generation but also for future generations [12]. For example, many famous brands like Nike, GAP and Adidas are incorporating sustainability in their supply chains. Many electronic equipments, household wastes and plastics are being recycled by many companies [28, 8, 41]. Recently a remarkable initiative has been taken by 2020 Tokyo Olympic and Paraolympic game organisers. Their target is to make all the medals from electronic wastes including old smart phones and laptops (www.usatoday.com). Previously in 2016 Rio Olympic game, about 30% of silver and bronze medals came from recycled materials (www.bbc.com). However,

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Corporate social responsibility in a closed-loop supply chain with dual-channel waste recycling

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ABSTRACT
This paper proposes a socially responsible closed-loop supply chain for waste recycling. To produce the finished product from wastes, two different cases of recycling are considered – either the manufacturer or the recycler does the recycling. The manufacturer puts effort to increase the demand for the finished product due to its corporate social responsibility (CSR). Different game models are developed in each case. Optimal decisions are obtained analytically and also through a numerical example. It is seen that though the manufacturer bears an extra cost to put effort for increasing the demand, the supply chain members can reach a win-win situation through a suitable revenue and cost sharing contract. It is further observed that recycling by the recycler is beneficial to the supply chain in comparison to the recycling done by the manufacturer itself. Sensitivity analysis depicts the overall performance of the supply chain with demand sensitive parameters.

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1. Introduction

Social issues like environmental degradation, food safety, labour conflicts, etc. are becoming more and more prominent in today's supply chain management. Pressures are also accumulating for environmental and socially responsible supply chain practices due to sustainability considerations. In addition to Government's concerns at various levels, the issue of consumer awareness of corporate social responsibility (CSR) is also becoming stronger than ever before. Environment-friendly products are in a growing demand for public welfare. In the present time, consumers are concerned about the products which lack human care and health safety. For example, many leading brands like Nike, GAP, Adidas and McDonalds have been urged to incorporate CSR in their supply chains. Business firms are playing a vital role in considering recycled waste as an instrument for environmental sustainability. For instance, electronic components, plastic wastes and household appliances are being recycled by many prominent companies (Kilic et al., 2015). A significant step towards waste recycling has been adopted by 2020 Tokyo Olympic and Paraolympic game organisers. They have set a target to make all medals from electronic wastes, including old smart phones and laptops. Required recycled materials are to be collected from the Japanese public as well as business and industry. Previously in the 2016 Rio Olympic also, about 30% of the silver and bronze medals were made of recycled materials (www.bbc.com). Various business organisations are performing environmental as well as social responsibility through recycling. With the growing trend of globalisation, CSR is receiving notable attention from academic and business communities. It has an important influence on the coordinated development of economy and society. More and more consumers are trying to purchase sustainable products keeping public welfare in mind. Sometimes Government pushes directly the business firms for socially desirable outcomes which can take a variety of forms. Providing subsidy is one of the growing topics in recent years. Although many supply chains incorporate social responsibility, the allocation of social responsibility has not emerged as the main focus. Also, it is a critical issue for supply chain members to collaboratively manage CSR. When the question of economic sustainability of the supply chain members arises, all may not be able to reach up to the optimal mark, decided by their own.

This study focuses on social responsibility in a closed-loop supply chain without neglecting economic concern of the supply chain members. A dual channel closed-loop supply chain for recycling is considered where the manufacturer as well as other members are socially responsible. The manufacturer considers sustainability through product recycling and remanufacturing. Therefore, the manufacturer exerts effort for quality improvement or

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GAME THEORETIC MODELS FOR A CLOSED-LOOP SUPPLY CHAIN WITH STOCHASTIC DEMAND AND BACKUP SUPPLIER UNDER DUAL CHANNEL RECYCLING

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Abstract: *In this paper, a dual-channel closed-loop supply chain is consi-dered for waste recycling. The manufacturer produces the finished product using recycled and recyclable waste materials as well as fresh raw materials. The recyclable wastes collected by the collector are supplied to the manufacturer directly or indirectly via a third party (recycler). Two different game models are considered for two different cases of recycling: recycling by the manufacturer and recycling by the recycler. If the collector fails to supply the required amount of waste materials, the backup supplier meets up the shortfall by supplying fresh raw materials. The customer demand is assumed to be stochastic. Optimal results for the two game models are obtained through numerical examples. It is seen that ex-ante pricing commitment i.e, fixed markup strategy is beneficial for the whole supply chain as well as the supply chain entities, compared to the decentralized policy. From the numerical study, it is also observed that when the recyclability degree of wastes increases, the expected total profit increases for the whole supply chain. A higher price sensitivity of customer demand leads to lower profit for the chain members.*

Keywords: *Supply chain management, Closed-loop supply chain, Recycling, Markup-strategy.*

1. Introduction

One of the biggest concerns of our society today is degradation of environment. Increasing consumption, richer lifestyle, higher level of logistics and transportation have led to higher carbon emissions and as a consequence, all these are raising important questions about environmental sustainability. Most of the supply chains in today's business scenario are attentive to sustainability, not only for the present age but also for the future generations. Various ways like remanufacturing, reusing, green

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