Abstract

Title: NUMERICAL RADIUS INEQUALITIES OF HILBERT SPACE OPERATORS AND THEIR APPLICATIONS

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Index No: 136/18/Math./26

The numerical range $W(A)$ of a bounded linear operator A on a complex Hilbert space $\mathcal{H}$ is defined as the range of the continuous mapping $x \longmapsto \langle Ax, x\rangle$ on the unit sphere of the Hilbert space, i.e., $W(A) = \{\langle Ax, x\rangle : x \in \mathcal{H}, \|x\| = 1\}$. Clearly $W(A)$ is a bounded subset of the scalar field and its closure contains the spectrum of the operator. The bounds of the numerical range helps in estimating the spectrum of the operator. In this connection the numerical radius $w(A)$, which is defined as the radius of the smallest circle with center at the origin that contains the numerical range $W(A)$, plays a very important role. The main focus of this thesis is to develop stronger lower and upper bounds of the numerical radius using various technique. We obtain improvements and generalizations of the inequalities $w(A) \leq \frac{1}{2}\left(\|A\| + \|A^2\|^{1/2}\right)$ and $\frac{1}{4}\|A^*A + AA^*\| \leq w^2(A) \leq \frac{1}{2}\|A^*A+AA^*\|$. Then we study the numerical radius inequality of the generalized commutator and anti-commutator operators which improves and generalizes the inequality $w(AB \pm BA) \leq 2\sqrt{2}\|B\|w(A)$. Next we present upper bounds for the numerical radius of bounded linear operators which generalize and improve on the well-known upper bound $w^2(A) \leq \frac{1}{2}\|A^*A + AA^*\|$. We obtain an upper bound for the numerical radius of the sum of the product operators which generalizes and improves on the existing ones. We present equivalent conditions for the equality of $w(A) = \frac{\|A\|}{2}$ as well as $w^2(A) = \frac{1}{4}\|A^*A + AA^*\|$ in terms of the geometrical shape of the numerical range of A. Next we develop a number of inequalities using the properties of t-Aluthge transform. We show that the bounds obtained here are better than the existing ones. We also estimate the spectral radius of the sum of the product of n pairs of operators. Then we present upper and lower bounds for the numerical radius of 2×2 operator matrices. Applying the bounds obtained here, to Frobenius companion matrix of a complex monic polynomial $p(z)$ of degree greater than or equal to three, we obtain new bounds for the zeros of $p(z)$.

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