APPLICATION OF PIEZOELECTRIC PATCH IN CRACK DETECTION AND REPAIR IN BEAM TYPE ELEMENTS UNDER STATIC AND DYNAMIC LOADS.

Thesis submitted by

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"Statement of Originality"

I, Goutam Roy, registered on 18.05.2018 do hereby declare that this thesis entitled" **APPLICATION OF PIEZOELECTRIC PATCH IN CRACK DETECTION AND REPAIR IN BEAM TYPE ELEMENTS UNDER STATIC AND DYNAMIC LOADS"** contains literature survey and original research work done by the undersigned candidate as part of Doctoral studies.

All information in this thesis have been obtained and presented in accordance with existing academic rules and ethical conduct. I declare that, as required by these rules and conduct, I have fully cited and referred all materials and results that are not original to this work.

I also declare that I have checked this thesis as per the "Policy on Anti Plagiarism, Jadavpur University, 2019", and the level of similarity as checked by iThenticate software is **3** %.

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Certified by Supervisor:

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CERTIFICATE FROM THE SUPERVISOR

This is to certify that the thesis entitled ***"Application of piezoelectric patch in crack detection and repair in beam type elements under static and dynamic loads"*** *submitted by* ***Shri GOUTAM ROY****, who got his name registered on 18.05.2018 for the award of Ph.D. (Engineering) degree of Jadavpur University is absolutely based upon his own work under my supervision and that neither his thesis nor any part of the thesis has been submitted for any degree/ diploma or any other academic award anywhere before.*

Goutam Pohit 5/4/22

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Acknowledgement

At this juncture of my life, I am at the end of my PhD research work after more than three years of continuous effort at The Jadavpur University. On this occasion, I would like to take this opportunity to deliver my sincere gratitude to all the persons who have continuously supported me in different intervals of my life, without their suggestions and relations my PhD would not have been achievable. In the main, I would like to convey my bottomless heartfelt gratitude and indebtedness to my honourable supervisor Prof. Goutam Pohit, Professor, Department of Mechanical Engineering, Jadavpur University, for his supreme guidance from the 1st day of my research work. I am thankful to the Jadavpur University for providing me the rare opportunity to work as a research scholar in the department of Mechanical Engineering. I also want to convey my sincere gratitude to two of my teachers, Sujit Chakrobortty Sir and Titov Banerjee Sir for their invaluable support towards me. I would like to thank Dr. Brajesh Panigrahi and Dr. Amit Banerjee , former PhD scholars of my supervisor, for their strong help and guidance to complete this research work. I acknowledge that for me, the present research work never would be possible to complete without the continuous support of Dr. Brajesh panigrahi. My special thank goes to one of my colleagues Dr. Bikash Panja, for his suggestions and helps, he had also worked in the same Lab where I have pursed my research work.. I also want to acknowledge the support which I have received from one of my present colleague Mr. Abishek Hazra, he was also a senior of me during M.Tech. I want to express my gratefulness towards the department of Mechanical Engineering of Narula Institute of Technology for the kind co-operation which I have received during my PhD tenure. I am deeply indebted to Prof. M.R Kanjilal, the Principal of Narula Institute of Technology, for her continuous motivation and support,. I want to convey my thanks to all my friends, colleagues who have continuously supported me to pursue my PhD. I would like to convey my thanks to all the persons who have helped me during the experimental works in Machine Lab. Finally, I am grateful to my all family members whose overwhelming support and belief in me helped me to curve my way out to the completion of the thesis work. I am deeply indebted to my father Raghu Nath Roy, my mother Rekha Roy, my aunt Usha Chakrobortty, my elder brother Sreemanta Roy, my younger brothers, my sisters, my in laws and my beloved wife Swarupa Roy. I dedicate this thesis to my supervisor Prof. Goutam pohit , my family, all of my teachers, all of my friends and the almighty Lord Shiva.

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Contents

W	Transverse displacement
Ψ	Rotational displacement (Flexure)
Ø	Rotational displacement (Shear)
U	Total strain energy
E	Young modulus
I	Moment of Inertia (area)
G	Modulus of rigidity
A	Cross sectional area
k	Shear correction factor
K_t	Stiffness of rotational spring
S	Flexibility due to crack
ν	Poisson's ratio
ς	Crack depth ratio
a	Crack depth
h	Beam depth
F (ς)	Crack correction factor

F	Externally applied force
V	Work done by the external load
x_p	Load point
L	Length of Beam
L_1	Length of Sub Beam
e_{31}	Piezoelectric stress constant
C_v	Capacitance
δ, p1, p2	Thickness, left end, right end of patch
c_{11}^E	Young modulus of patch
$k_1^b, k_2^b, k_1^p, k_2^p$	Stiffness parameters
u_{nc}	Mode shape function
$\sigma_{xx}, \varepsilon_{xx}$	Longitudinal stress, strain
ρ_b, ρ_p	Densities
Q	Width (Beam)

Page No.

Catastrophic failure of a structure or a machine element endangers human lives. The main cause behind this sort of failure is the propagation of a crack which leads to failure. At the primary stage of a crack, it may be dormant and can be repaired. So, the detection and repair of cracks is a crucial part of research in the field of mechanics. In this regard, several procedures are already developed. But still, in intricate structures consisting of beam-type elements, it is burdensome to take apart all the elements to facilitate crack detection and repair. The same problem comes in the case of machine elements also. Correspondingly another drawback of the already-developed detection techniques is they are not effective to detect tiny cracks. If the cracks are detected at a very early stage then they can be repaired and hence it is very significant to locate the crack when it has not considerably propagated. For this purpose, smart materials conceivably have the utmost potential. Among the different smart materials, piezoelectric ceramics are the most effective in terms of their performance and cost. Only limited works are so far carried out where piezoelectric patches are employed to act as either sensor or actuator to detect as well as to fix the crack in a beam. But those developed techniques are cumbersome, and not easy to employ. The present effort proposes easy and effective ways to employ piezoelectric patches to detect as well as repair a crack within beam-type elements. It is noted that an open crack in the beam discontinues the slope curve at the location of the crack. It leads a way for the detection of cracks. It is found that a piezoelectric patch can produce a voltage, proportional to this kind of the discontinuity. The first part of the present work shows how a crack potentially is identified in a beam under static load by using the piezoelectric sensor. A finite element model and Semi-analytical of the cracked beam with the attached piezoelectric patch are prepared to build up the procedure and the same is validated by the experimental result already published in a refereed

journal. Subsequently, it is revealed that by applying an external voltage field on a piezoelectric actuator a local moment can be generated near the crack which can nullify the discontinuity which had formed by the crack. It is proposed that this way a crack may be repaired in a beam. In the next part of the research, the same concept is applied to beams under vibration. To detect a crack in a beam under dynamic loading, finite element analysis, and analytical analysis is performed. In addition, an experimental setup is developed consisting of a digital oscilloscope, piezoelectric patch, cracked beam, beam holder, and exciter to validate the theoretically obtained results. At the last, the new crack identification technique has been used to model an instrument to measure the slope at a point on a deformed beam.

INTRODUCTION

1.1 Background and motivation:

Crack within structural elements or in machine parts, causes a change in stiffness near the crack. As a result, the dynamic response as well as the behaviour under the static load of the element gets altered considerably. [Chondros and Dimarogonas (1998)] It will possibly change the distribution of mass and damping behaviour of the structure. Often the cracks propagate fast causing a catastrophic failure that may endanger human life on top of a loss in the economy. Sometimes, catastrophic failures direct the failure to the interconnected parts with it. The structural elements mainly fail due to fatigue, because of unstable structural geometry, and poor material properties or calculations. The failure typically initiates with a crack form at the maximum stressed point. These cracks propagate as the object is repeatedly loaded and unloaded, eventually reaching a critical extent and causing catastrophic failure under regular conditions. Indeed a deep study on fracture mechanics may prevent this kind of failure. [Cotterell (2002)] The primary application of fracture mechanics was to envisage the crack or to find out the reason for the fracture. Another use is to set inspection to check the reliability. Therefore premature identification of cracks is a vital part of the research.

Here in this paragraph, a concise on a few catastrophic failures have been discussed. On 28th Dec of 1879, a catastrophic failure of rail a bridge in Scotland caused the death of 75 people. The collapse was due to the failure of several bridges made of cast iron. On 7th November of 1940, the Tacoma Bridge at v Washington collapsed within a few months after its run. On 1st August 1967 in Minnesota the Mississippi River Bridge collapsed suddenly in the evening busy hours.

The accident caused the death of thirteen people and 145 people were injured. Thereafter the collapse the department of Highway suggested the state for examining the 700 other similar bridges in the U.S. On 4th April of 2013 at Thane in Mumbai, a building catastrophically collapsed. It was called a horrible failure of a building which had caused the killing of 74 people of which 18 numbers of children died. On 24th April of 2013 in Dhaka, a high-rise commercial shop collapsed suddenly. The investigation on the dead finished on the 13th of May which stated that 1129 people died. On 29th June of 1995 in Seoul, a five-story building catastrophically collapsed which killed 502 numbers of people. The study of this failure of the building in Seoul revealed that at first in April month cracks were found on the ceiling because of the presence of AC on the weakened roof of the defectively designed structure. On 29th June in the morning, several new cracks formed on the ceiling radically and subsequently the whole fifth floor collapsed suddenly. The above reviews show the potency of a crack to cause catastrophic failure which results in loss of lives.

Therefore, it is a major concern for the researchers who are working in the field of structural health monitoring to inspect such cracks. Most of the structural parts are prepared of metals mainly steel which has a high chance to get fractured in at cold environment while welding, due to fatigue or on account of corrosion [Mann (2011)]. Some of the manufacturing processes like flame cutting, punching, and welding aggravate the chances of the formation of cracks particularly if the element is having high depth or thickness. There is a further chance of the failures of structural elements while operating under the excess load than the prescribed capacity. The mode of fracture in the aforesaid elements is in general ductile in type. In elements made of concrete or cast iron, the failure is brittle. Engineers are supposed to appreciate the mechanisms behind the cracks to lessen the possibility of such failure**.** In addition, the Designer has to lay enormous effort and time to keep an eye on the complex structures comprising beam elements. One consistent approach is the inspection of damage while the structure is in run or online. In several contemporary researches the structural elements are investigated under static as well as dynamic loading. As pointed out earlier that a crack reduces the stiffness near its vicinity under static load as well as dynamic load. The monitoring of structure through both the above stated ways are persuasive since in both cases the inspection perhaps accomplishes in the running condition. The research on health monitoring of structures is divided in two aspects. In one way the variation or changes in different properties of the structure e.g. change in stiffness, deflection of beam, natural frequency etc. These properties are evaluated to identify a specific location of crack. This approach is called direct problem. In another approach the location of crack is detected by studying the changes in different parameters as well as the response of the structure. This is called inverse problem. In direct problem the center of attention is on the formulation of crack and its effect on different parameters.

Another part of research in this field is repairing of the cracked beam. There are a few conventional techniques which can effectively repair a beam made of RC or metal. [Ahmad et al. (2013)] By injecting epoxy resin in to the crack of a RC beam to restore the strength. With the advent of electronic sensors, amplifiers as well as smart materials (which is capable of sensing or producing mechanical forces) the research scope in detection and repair of crack have become bigger. The uses of smart materials in the field of crack detection and repair is a very recent phenomenon. Since the smart materials are controlled by or their outputs are electrical voltage due to that the incorporation of smart material in this field may be quite effective. One of the effective smart materials is piezoelectric material because it is readily available in markets and it is easy to control. One more fact is a single patch made of piezoelectric material can be used for both purposes i.e. detection and repair. It is mentioned earlier that a crack induces a local change in stiffness which alters the response of the element. But the inspection or study of those changes is not easy.

This becomes easy when a patch made of piezoelectric material is employed to inspect. Piezoelectric material catches the changes in form of generated electrical charges in it which results in voltage output. Subsequently by applying external voltage, the piezoelectric patch can be deformed or strained as required leading to control/monitor damages efficiently.

Beam type elements are the major parts of any structure. The possible behavior of intricate structures under static or dynamic loading can be foretold by studying beams. It is manifested that the theories of beam are competent to analyze diverse structural as well as machine parts, such as, crane, wings of airplane, blades of turbine, arms of manipulator, and chassis of vehicles. A small number of industrially used elements are given away in Fig. 1.1. The response of such things can be effortlessly estimated by using the established theories of beam such as Euler theory, Timoshenko beam theory. Hence these items can be treated appropriately as beam like elements.

1.2 Literature review

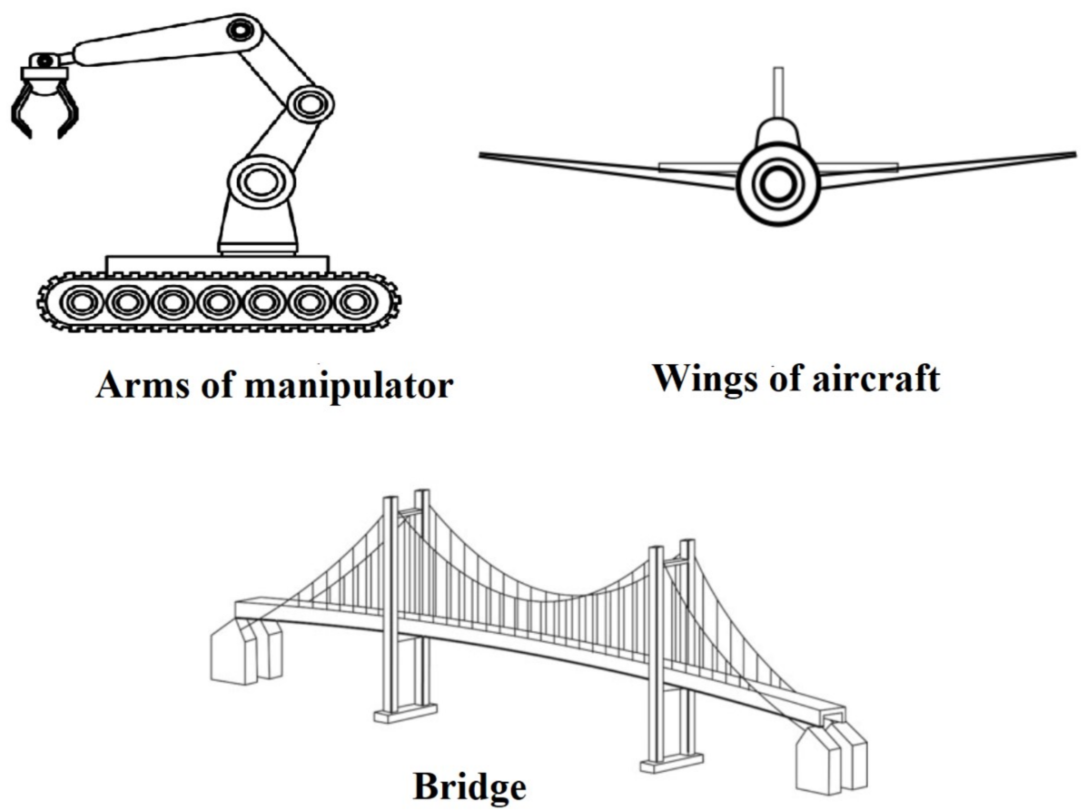


Figure 1.1. Few examples of Beam type structural elements

In recent times, the uses of piezoelectric materials in different mechanical aspects like controlling, actuating and monitoring different elements have gained huge importance. Undoubtedly it should be the focus of researchers because piezoelectric materials make it much easier and more reliable. At the beginning of this literature review, a brief discussion is presented on the different uses of piezoelectric materials like energy harvest action, controlling and monitoring. The latter part is divided into three sections as per the present work viz. formulation of crack, crack detection and repair by piezoelectric patch in static/dynamic loading conditions.

1.2.1 Uses of piezoelectric material

Piezoelectric material gets polarized when strained and deforms when an external voltage is applied. Duan et al. (2010), in their review paper, have elaborated that some materials in nature show piezoelectricity, e.g. some ceramics (Lead Zirocondate Titanate/PZT), some polymers (Polyvinylidene Fluoride/ PVDF) and also the composites made of PZT with PVDF. These are used as either Sensors or Actuators for different Engineering purposes. Among these Ceramic is cheap and more easily fabricated than polymers.

Energy harvestation: Several research works have been constituted based on energy harvestation by using piezoelectric material. Changki et al. (2010) proposed a work on the energy-generating performance of a system under pressure with a piezoelectric transducer. In his work, harvestation of energy is performed by a simply supported piezoelectric plate, which is kept under fluctuating pressure and the piezoelectric plate converts the energy into electrical voltage. Li et al. (2016) presented that the capacity to generate power of piezoelectric material depends on the amplitude of vibration and modes of fixation. Here a commercial piezo plate is used in different boundary conditions to generate power. Fakhzan and Asan (2011) presented a model of a cantilever beam attached with a layer of piezoelectric material (Unimorph) following Euler–Bernoulli beam theory. Najini et al. (2017) proposed a system based on technical simulation to generate electricity by piezoelectric patch from road traffic. Safaei et al. (2018) have discussed in their review paper that a range of materials can convert vibrational energy into voltage, for example, electromagnetic material and piezoelectric material.

. Piezoelectric material, for its capability of electromechanical transformation and considerable power density in comparison to electromagnetic transducers, has been extensively used to transform mechanical energy into electrical voltage. Mitcheson et al. (2008) explained that piezoelectric transduction is appropriate for the energy harvestation in reciprocating motions. Ottman et al. (2002) narrated the prospect of harvesting electrical energy from vibrating piezoelectric elements. A mechanically excited piezoelectric element is different from that of a usual power source because it has a capacitive impedance rather than inductive and can be steered under vibration with different amplitudes. Xu et al. (2016) presented that piezoelectric harvesters (PEHs) are small architectures which are elegantly designed to capture vibration and to convert it into electrical voltage. The PEH idea comprises the unimorph cantilever beam, bimorph PEH and clamped edge PEH. As a final point, approaches on PEHs for aerospace purposes are considered. Turkmen and Celik (2018) intended to retrieve the energy that transfers to the ground while walking by using piezoelectric materials. For this purpose, a sole has been modelled with the integrated piezoelectric harvester. Li et al. (2014) reviewed the present state of the study on PEH devices, under low frequency (0–100 Hz) and the procedures which are developed to advance the outputs of the PEHs. Kim et al. (2019) explained that by the concepts of the gating effect of piezoelectric material, the collected charges on the surface due to turbo elasticity can act as gate voltage to control the carrier in transistor devices. The piezoelectric effect is observed by Maurya et al. (2015) in crystalline substances that have inversion symmetric points. This causes polarization of electric charges proportional to the vibration (direct effect). This direct effect is utilized for energy harvest action. The harvesting process very much depends on the combined effect of loading and the property of piezoelectric materials. Another capability of the piezoelectric material is to survive under repetitive load. Eghbali et al. (2020) performed a study to estimate the effective geometrical, and electrical parameters and properties of the materials for higher power extraction and optimized those to turn up the optimum results. It is presented that by using the auxetic model a significant enhancement in the power harvestation is possible. The method of harnessing the ambient energy forms into serviceable electrical voltage is known as harvesting [Nechibvute et al.(2012)]. This harvesting increases the opportunity for self-power-driven systems that are omnipresent and autonomous. Among the usual energy, sources vibrations is a striking source of energy mostly because it is widely accessible and appropriate for the transducers made of piezoelectric materials, which are capable of converting mechanical energy into electrical voltage. [Khazaee et al.(2020)] This paper accords a finite element articulation to estimate power harvestation by piezoelectric materials in a wider range. The proposed shear FEM model is not merely is appropriate for thick harvesters made of composites but also takes in the flaws of the earlier methods. The model prepared for finite element analysis is established by experimental as well as analytical consequences. Mallouli and Chouchane (2020) presented a mathematical model of a bimorph harvester made of composite macro fibre. The amalgamation rule is used to resolve the homogenized and corresponding properties of the structures. Song et al.(2015) proposed a piezoelectric energy harvester induced by vortex (VIPEH), enthused by the cylindrical vortex vibration (VIV) in a fluid. This harvester incorporates a PZT in cantilever boundary condition in a cylindrical shape. The ability of energy harvesting of the VIPEH is studied theoretically as well as experimentally. An Euler–Bernoulli-based model for the energy harvester is established. Speciale et al. (2020) presented a piezoelectric harvester using a buckling mechanism. The harvester is made of two major parts: a mechanical bistable structure and a cantilever beam made of PZT. The model is developed by analytical methods as well as FEA. A prototype is prepared and tested under a mechanical excitation of low frequency. Yang et al. (2018) described a complete and significant review of up-to-date research energy harvesting by PZT. Researchers are always focused on whether a harvester can produce enough electrical energy under varying mechanical loads. This present paper concentrated on different methods which lead to adequate power output. A range of procedures, nonlinear approaches, optimization, and materials which can harvest energy are reviewed and elaborated in depth. The paper also describes different merits and presents a methodical comparison of newly anticipated harvesters. Zhang et al. (2017) presented a rotating harvester made of PZT to harvest wind. Here, a PVDF beam produces electricity employing the impact vibration. It is noted that the wind speed may not always increase the output. A theoretical model is prepared and analyzed by FEM..

The above section screens the enormous usage of piezoelectric material as energy harvesters. Apart from energy harvesting, the piezoelectric material is extensively used in different equipment to control as well as to monitor. A good number of research publications are available based on the application of piezoelectric material in medical equipment. Therefore it is appropriate for the present thesis to include a review of a few kinds of literature based on the application of PZT in different electronic equipment.

Applications in different equipment: Lockwood et al. (1996) discussed in their literature that the majority of imaging systems based on ultrasound works in the series of frequency from 3 to Hz and determines the object size of 1 mm. During the mid of 1980s, a contemporary material (Piezoelectric) for transducers headed to the improvement for the clinical imaging of high frequency. These particular transducers can present images in microscopic resolution. Ritter et al. (2000) addressed the crystallographic demeanour of piezoelectric material linking orientation to the capability of the device. Piezoelectric materials of 1-3 composite PZT materials were chosen for experimental investigations. Subsequently, theoretical analyses were performed. This section presents the account and growth of PZT in medical aspects. Chen-Glasser et al. (2018)

described the properties of PZT materials and how it assists in treatment. It also enfolds the uses of PZT in medical grafts by clearing up how PZT be able to be used as sensors, to try to be like usual materials. Lastly, the likelihood of using PZT to model medical tools and how present designs can be enhanced by more research is suggested. Griggio et al. (2013) presented an ultrasound device made-up using a PZT transducer of diaphragm geometry with PZT films. One 1-D array consisting of 8 elements was considered and prepared by photolithography. Qiu et al. (2015) provided an outline of the recent developments of ultrasonic transducers made of PZT (PMUT) and a review of their appropriateness for miniaturized devices. The required PZT to functionalize these systems are reviewed, subsequently, the microfabrication methods used to form PMUT fundamentals are also discussed. Ghafari et al. (2019) intended a self-polarized polyvinylidene fluoride (PVDF) which banishes a post-treatment process. The possibility of using the PVDF sensor for active as well as passive sensing is investigated. Furthermore, the experimental study involves numerous parameters e.g. amplitude, and frequency of the transmitted signal using PVDF. Chen et al.(2013) presented a sensor to measure acoustic emission (AE) founded on a lead zirconate titanate (PZT) membrane. The nanofibers made of lead zirconate titanate, having diameters from 50 nm - 120 nm, are electro-spun and allied across electrodes. After encasing in a polymer structure of a thickness of 5 micro-meter, this particular AE sensor can bend to track embedded surfaces. Wu et al. (1992) introduced a robotic arm controlled by active elements of PZT ceramics. It characterizes lightweight, higher flexibility and enhanced performance owing to novel approaches in the device. Foremost, a new motor based on piezoelectricity is intended as a light, fast-responsive driving element for the manipulator.

The motor is included with a servo motor to impel the arm. To facilitate controlling the unwanted vibration of the manipulator, a control system is proposed based on piezoelectricity.

Chopra and Gravish (2019), presented a PZT actuator with an incorporated sensor to sense the position of a very small scaled mobile robot. Here the actuators are made-up using the fabrication process of composite microstructure, which uses laser machining and lamination of composites. Insulated sensing parts of the Piezoelectric material experiences the same motion as same as the actuation layers and as a result can sense the deflection at the tip by the piezoelectricity. It describes the Design aspects of strain-sensing PZT actuators which can be fabricated over a broad range of dimensions. Maillard et al. (2014) described the application of PZT actuators in space vehicles. In the case of Space vehicles, missiles etc., the committed place and electric power are very restricted. For example, a satellite of micro size usually operates with power below 100 W. Accordingly, the part of electric power for each actuator is normally in a range of 0.1 to 10W. As an account of the cost of implanted mass, space actuators require having high energy output to mass proportion. To deal with these difficulties, actuators made of piezoelectric materials are proposed to use in space vehicles.

In the last section, a brief review of the applications of piezoelectric sensors/actuators for different purposes such as medical equipment, robotics, and spacecraft are presented. Undoubtedly piezoelectric material is most suitable in such fields. Apart from that, an effective application of piezoelectric sensors/actuators is noticed in the field of crack detection and repair.

1.2.2 Crack model:

Even though a range of research works have introduced the stress field at the crack tip (Irwin (1957)), monitoring of cracked elements is introduced by Dimarogonas (1970) and thereafter in a project of General Electric Ltd by Pafelias (1974). Later in another article, Dimarogonas and Papadopoulos (1983), estimated the induced local flexibility due to crack using the Paris equation. The same had been validated by experiments. Afterwards, with these data, a cracked shaft under transverse vibration is analyzed. Gudmundson (1982) introduced an equation, where by using the perturbation method the relation between crack severity and change in natural frequency is proposed. Subsequently, FEA and experimental analysis were performed to compare the analytical results. Later on Gudmundson (1983) presented two methods to establish a flexibility matrix under static load, which in effect shows the presence of a crack. Among the two methods in one, the integration of SIF (stress intensity factor) is approached and in another method, finite element analysis is performed. The authors applied these methods to analyze a slender cantilever beam with an edge crack. Christides and Barr (1984) presented a novel model of crack which is not based on the conventional fracture mechanics approach. The hypothesis is established on an assumption that the stress decays exponentially with distance from the crack location. More than a few pragmatic mathematical formulae are available to relate the intensity of stress with the depth of crack in published books (Tada et al. (1985)). Krawczuk and Ostachowicz (1992) established a model of crack to reveal the effect of cracks on the beam while vibrating in transverse mode. Flexibility is considered a time-dependent function as it changes due to the opening and closing of cracks. For one cycle of vibration in the first half, it is considered that depth is increasing and decreasing while on the other cycle the crack depth remains constant as the beam acts as an intact beam. In an additional study, Krawczuk (1993) presented an open dormant crack in a plate structure to examine the free vibration by using the finite element method. A simple assumption is taken to simplify the model that the effect of crack only changes the stiffness and the mass does not change.

In succeeding literature, Krawczuk and Ostachowicz (1993) analyzed the effect of a crack in a beam under forced vibration. Here the dynamic response was obtained by using finite element analysis. Ostachowicz and Krawczuk (1991) presented two models of cracks; in one case a crack with a double side is considered which is proposed in cyclic loading situations, and a single-sided crack, for fluctuating loading conditions. The influence of such open cracks on the natural frequencies of beam vibration is examined. Papaeconomou and Dimarogonas (1989) described the induced flexibility due to the crack and applied this to get compliance to know the behaviour of the beam near the crack. The outcome of this compliance on the response to the vibration of the beam is studied. Krawczuk (1992) used a model by considering a dormant crack, including another matrix for the flexibility induced due to the crack, incorporating the approach of fracture mechanics. Petroski (1981 and 1984) proposed an equivalent approach to resolve vibration problems of cracked beams, by considering the effect of crack with a reduced cross-sectional area of several elements. Chondros and Dimarogonas (1998) analyzed the consequences of flaws in the dynamic response of the beam. Sinha et al. (2002) introduced an easy approach to model the beams with a crack under transverse vibration. The approach is established on the Euler-Bernoulli theory in which a modification is done by incorporating extra flexibility near the crack. This model is then utilized to find the position and depth of the crack. Nahvi and Jabbari (2005) used an analytical and experimental approach to locate the crack in the cantilever beam under vibration. In the experimental analysis, a cantilever beam with a crack is excited under impact load and the output response is captured using an accelerometer on the beam. To avoid non-linearity, instead of a breathing crack an open crack is considered. To locate the crack, contours of the frequency against depth and location of the crack are plotted. Chasalevris and Papadopoulos (2006) conducted the dynamic analysis of a beam with transverse cracks. The cracks are specified by depth, position and orientation. Lin and Chang (2006) developed a method to study the dynamic response of a cantilever beam with a crack, which is subjected to a moving load. The crack is replicated by a rotational spring. Two sub-beams are considered to track Euler-Bernoulli's theory. The compatibility is imposed to relate the sub-beams. Loya et al. (2006) estimated the natural frequencies of a cracked beam based on the Timoshenko beam theory. The model incorporates discontinuities in both deflections and rotational displacements. The equilibrium equations of free transverse vibrations are developed and then evaluated for both segments by considering different boundary conditions and the proper compatibility conditions at the crack location. Subsequently, the problem is evaluated using the perturbation method.

Sadettin (2007) analyzed a beam with a crack under the free and forced vibration to find the crack in cantilever beams. Aydin (2007) developed an analytical approach to estimate the frequencies and mode shape of Timoshenko beams under axial load with random cracks. The local flexibility induced due to the crack is replicated by a mass-less torsion spring. Several boundary conditions are considered as preliminary parameters to characterize the mode shape of the beam before the crack. Viola et al. (2007) studied the difference in the modal response induced due to the presence of a crack in Timoshenko beams under axial load. Yang et al. (2008) described an analytical study on cracked beams based on Euler Bernoulli's theory, under forced and free vibration. The beam is considered under axial compressive force as well as a moving transverse along the length. The crack is modelled by considering a torsion spring at the place of the crack. The response in forced vibration is determined by using modal superimposition. Yan and Yang (2001) had shown an analytical method for the forced bending vibration by considering open cracks. The crack is replicated by a rotational spring whose local flexibility is intended using the fracture mechanics approach. Zheng and Ji (2012) presented a beam with having crack by considering the reduced sectional area in the healthy beam at the location of the crack. Mazanoglu (2015) investigated the forward as well as backward problem of beams with cracks. In forward analysis changes in frequencies due to the cracks are analyzed. Conversely by using the relationship between crack depth, the difference in natural frequency and the location of the crack the inverse problem is conceded. Batihan and Kadioglu (2016) used a model, where the crack is simulated as a rotational spring. Using different theories of the beam, the dynamic response of cracked beams on an elastic base is studied. Capozucca and Magagnini (2017) studied the frequencies of the cracked reinforced beam via the FEM as well as the experiment. Many relevant papers are reviewed in the last section which has helped the analytical modeling of the cracked beam in this thesis. A few more articles would be discussed here i.e. under the subheading "Modeling of Crack". In the next section, a review of the available literature on different crack detection/repairing techniques with and without using piezoelectric material is presented.

1.2.3 Crack detection in structural elements (with/without aid of piezoelectric material):

Structural elements react somewhat differently if any crack or any kind of damage is present in it. The presence of cracks clearly shows some variation in the response under static and dynamic load in comparison to a healthy condition. Following the changes in output responses such as the deflection of a beam, rotational displacement, natural frequency and displacement-time response a crack is usually evaluated. Caddemi and Morassi (2007) accorded the detection of a crack in beam-type structural element by using the variation in deflection of the beam due to the presence of damage. Here the flaw such as a crack is replicated by a linear spring between the two segments of the cracked element. Adequate condition for measurements that allocate the sole identification of the flaw is proposed. Frank et al. (2001) presented a comprehensive study of the BED (boundary effect detection) method to detect damages in structural elements by the use of

ODSs (operational deflection shapes) obtained by a vibrometer. Buda and Caddemi (2007) intended a crack detection method for the structural beams acting under static loads. Euler beam theory is considered to model the problem and the damage is modelled by Dirac's delta. The study is done on both statically indeterminate and determinate beams. Rubio e al. (2015) described that the crack can be located in a rod simply by using the first and 2nd natural frequencies. Capecchi and Vestroni (1999) presented a way to assess structural health by using dynamic response history. It evaluates the problem that when should one determine the natural frequency for the detection of any kind of damage. The typical problem of locating damage in beams is analyzed. The presence of a crack is considered a change in stiffness. Panigrahi and Pohit (2016) developed a numerical method to determine the nonlinear response based on neutral surface approach on cracked FGM beams. Banerjee et al. (2016) proposed an approach for examining the dynamic response of a beam with a crack employing Timoshenko beam theory and using Ritz approximation. Xiang et al. (2012) proposed a two-step procedure for structural health monitoring employing deviation in mode shape and natural frequencies for the evolution of cracks. Kim et al. (2003) had shown a sensible non-destructive method to find a crack by using the differences in the natural frequencies of an element. Caddemi and Morassi (2007) presented the detection of a crack in beams following variations in deflection induced by damages. The crack is replicated by a linear spring which is connected between the two edges of the crack. Umesha et al. (2008) proposed a new method to locate the crack following the changes in the deflection of the cracked beams. The deflection is considered at a specific point for different locations of load. The deflection profile is given as input for wavelet transformation to detect the position of the crack. Azaheri et al. (2018) presented a simple method to reveal the changes in the elastic behaviour of beam induced by a crack. The cracked beam is simulated using a rotational spring and the solution is obtained by finite element analysis. Chinka et al. (2019) had shown a new method to detect a crack in beam-like structures where a beam had been divided into some zones following the normalized frequencies. Successfully the crack location is detected by analyzing the zones and following the vibration nodes. Filho et al. (2018) pointed out that the mode shapes of a vibrating beam are more responsive to cracks rather than the natural frequencies of a beam. It is also seen that the changes concerning the crack are relative to the depth of the crack. Bozyigit et al.(2020) proposed a new approach to locate the crack in a beam by using the transfer matrix method. Here in this study both the single variable shear deformation (SVSD) and Timoshenko beam theories are employed. Bozyigit et al.(2020) described an analytical method for finding the cracks in Euler-Bernoulli beams with multiple spans on elastic foundations. Both the forward and inverse analyses are carried out by the transfer matrix method. Bozyigit et al.(2020) proposed a way to use SVSD theory for estimating the response of cracked frames under free as well as steady state vibration. Yin et al. (1996) proposed a strain-based method for possible damage detection of composites using PVDF. Fukunaga et al. (2002) proposed a crack identification method using piezoelectric sensors and subsequently an iterative method to quantify the crack severity. Zhang (2011) studied the response in free vibration of cracked FGM beams with a piezoelectric patch.

Afshari and Inman (2013) proposed a crack beam model integrated with the piezoelectric patch considering the energy loss due to the crack. Patil et al.(2017) proposed a finite element method to detect flaws such as bolt-loosening effects on framed structures using piezoelectric materials. Vitola et al. (2017) investigated the damage identification process in structures applied to thermal loading using piezoelectric sensors. Qian et al. (1990) proposed a finite element modelling of a beam consisting of cracks where the stiffness matrix of the cracked beam is obtained based on the integration of the stress intensity factor. Here for locating the crack the association between the crack and eigenvalue is followed. Azrar and Benamar (1999) discussed on localization of nonlinearities due crack. Fernadndez et al. (1999) presented an analytical model to estimate the natural frequency of an Euler-Bernouilli beam with an open crack under bending vibrations. Panigrahi & Pohit (2018) proposed a dynamic analysis based on the nonlinear modelling of a cracked Timoshenko beam. Owolabi et al. (2003) used frequency contour lines to locate the crack and to estimate its size in an aluminium beam through experimental work. Zheng & Ji (2012) provided a method to calculate the stiffness and the natural frequency of a beam with a crack. The effect of the cracks is modelled by a stepped beam in which periodic variation of the cross-section is considered. Shuncong & Oyadiji (2011) proposed a crack detection technique using the difference of the wavelet transforms of mode shape data that communicate between the left and right half of a cracked simply supported beam. Fukunaga et al. (2002) projected a method to detect cracks by piezoelectric sensors and consequently proposed an iterative method to measure the damage severity.

Sumant and Maity (2006) presented a new approach to detect a crack in a short simply supported beam where two PZT patches are bonded on the top and bottom faces of the beam. The integrated beam patch specimen is studied with different crack depths, under gradually applied static load through the three-point bending machine. Subsequently, an effective way is approached to solve the inverse problem. Chuan and Chen (2018) presented the use of the piezoelectric material as an actuator where the piezoelectric patch is forced by a voltage to cause bending in the composite beam. An Euler beam is considered in this study, subsequently, continuous interaction between the patch and beam is considered to resolve the stresses. Ashtari and Kareem (2018) approached an effective repair technique employing a piezoelectric actuator to repair a beam under static load. Wanga et al.(2004) projected an effective repairing technique for a statically loaded cracked beam in their paper a mathematical expression is established by using which one can calculate the required voltage which can nullify the effect of the crack. Rees et al. (1992) have shown that within a range PZT can be used to monitor the development of a crack below a boron/epoxy patch. Also, the study shows that the sensitivity of the Piezo sensor is a function of its size and location. Zou & Aliabadi (2015) introduced a method to detect the position and extent of cracks in plate Structures by the use of a piezoelectric network.

Chaudhry et al. (1995) proposed a monitoring technique based on high-frequency vibration. The technique operates on the alteration of the electrical impedance of a surface-bonded PZT actuator-sensor. Subsequently, the experimental analysis of aircraft structures is presented. Tua et al. (2004) proposed a technique to detect the size of linear cracks in homogeneous plates through the analysis of the time-of-flight of Lamb wave propagation. Lee et al.(2014) had shown a method to inspect the integrity of the structure by vibration analysis where the excitation was generated and the response was monitored by a piezoelectric transducer. Beomihn & Chang (2004) developed a technique to detect the growth of cracks due to fatigue by using diagnostics signals generated by a built-in piezoelectric patch. Jiang et al. (2006) introduced a crack identification method based on frequency shift where a piezoelectric transducer is used to improve the measurement of modal frequency. Wang et al.(2007) presented an experimental work to detect a crack in concrete in which EMI of PZT is used to identify the position of the crack. It is observed from the experimental work that due to the presence of the crack the admittance curves of the piezoelectric sensor have changed near the crack. Yan et al.(2008) approached a technique based on Electrical Mechanical Impedance for Crack Detection in Beams. A tied structure system consisting of a PZT patch and Timoshenko beam model with a crack is considered. A massless torsional spring is considered to model the crack and the shear lag model is taken to consider the effect of bonding layers. Chomette et al.(2013) had shown a new diagnostic method based on active modal damping and piezoelectric material for the identification of small cracks. Here one PZT is used as an actuator and another as a sensor. Zhao et al.(2017) proposed a crack detection method for a beam with the help of piezoelectric materials. The piezoelectric patches are used to generate excitations to alter the local stiffness near the crack and increase the effect of the crack on the dynamic character of the element. Narayanan et al. (2016) approached the prospective with Electro-mechanical impedance measurements of PZT patches mounted on the surface of the structure to monitor the health of a structure. Changes in response to the Electro-mechanical conductance of PZT tied to the structure at resonance are observed due to damage. It was also proposed that damage can be detected at a very early age.

Reddy & Swarnamani (2007) analyzed the changes in power spectral magnitude of vibration due to different depths of a crack in the beam. Here PZT is used as an actuator to vibrate the beam. Butt et al. (2016) proposed a model consisting of 1 degree of freedom to obtain the voltage output through a piezoelectric material under different loading conditions. The analytically obtained results are compared with the FEA results as well as with experimental results. Zhao et al. (2016) introduced a method to locate the crack in the beam like elements based on a vibration characteristic. Lu and Li (2010) analyzed altered modes of fractures in elements made of concrete with the help of piezoelectric sensors (cement-based material). The damage detection procedure is based on 3-D localization as well as fracture index evolution. Hoon et al. (2004) presented a couple of fatigue experiments. The presented results were acquired from the progression of the fatigue experiments under high frequency. Zhu et al.(2012)

approached a study to analyze the possibility to detect cracks based on wave propagation as a non-destructive technique for structures made of concrete through embedded piezoelectric patches. Thomas et al.(2016) presented a crack detection technique in a cantilever beam by using a piezo electric sensor/ actuator. The response of strain near the fixed end of the cantilever structure was obtained through the output voltage of the sensor.The change in the natural frequency due to crack was found less than 3% due to that it was not considered as an important factor and the effect was neglected in the analysis.

1.2.4 Repair and control of damages in structural elements by using piezoelectric patch:

Another significant characteristic of the piezoelectric material is that piezoelectric material deforms due to the application of external voltage (reverse effect). This special character of PZT can be used to reduce the effect of a crack or to restore the properties of structures which have changed due to the presence of damage. Several research articles are found that piezoelectric materials are used as an actuator to repair or control damages in structural elements.

Meressi and Paden (1993) addressed how buckling can be controlled using PZT actuation by analyzing an axially loaded element. It is described that the buckling of the element can be deferred away from the normal critical load by way of piezoelectric actuation. Wang and Quek (2002) adorn that a pair of PZT can increase the critical load of a column. In his analysis, a column is considered where one end is fixed and another end is constrained by a transverse spring. The problem is mathematically modelled and solved by numerical method. Liu T. (2008) introduced a particular repair case based on two separate repair criteria discontinuity in slope and Fracture Mechanics criteria. Plane strain condition is considered near the crack tip for contact analysis. As per the obtained results, it is shown that the 2nd criterion is preferable to estimate the required input voltage. Wang et al. (2002) presented a novel way to repair the damage, like cracks, in beam types of elements. By using the inverse effect of the piezoelectric patch, a PZT patch is actuated to produce a local moment near the crack. This local moment eliminates the singularity in the slope curve induced by the crack.

Wang et al. (2004) approached another application of piezoelectric actuation to restore the dynamic response of a cracked beam. Here the PZT produces a local moment near the crack which globally alters the natural frequency**.** [Waleed and Kareem (2018)] Here the PZT patch is used to restore the changes in a beam which induces due to a crack in a statically loaded beam. A mathematical formulation is presented to design the PZT patch which can nullify the effect of the crack and subsequently the required voltage to repair the crack is estimated. [Wu and Wang (2010)] This work represented another use of the PZT patch, where a feedback control tactic is introduced to repair the crack in a delaminated structure under vibration. The electromechanical feature of the PZT is used to tempt shear force above the fault by applying voltage externally. Platz et al.(2011) intended an innovative way is proposed to save shell structures from fatigue failures. This paper shows how PZT patches can stop the propagation of fatigue cracks or can keep the crack dormant. The effort is to decrease the cyclic stress at the crack tip. By decreasing the intensity of stress the propagation rate of an existing crack can be lowered appreciably. Ariaei et al. (2010) represented an analytical study where a PZT actuator is used to restore the dynamic response of a beam under moving mass. For the analysis, the Timoshenko beam theory is considered with the dynamic effect of the moving mass. As the repair criterion, the natural frequency is restored. Where the bonded piezoelectric patch is actuated by applying external voltage. This result in the closing of the crack. Kumar et al. (2020) presented an experimental work to repair the crack in structures under mechanical and thermo-mechanical loading. At first, a plate with a crack is loaded under cyclic mechanical loading under constant temperature. A PZT sensor is used to get the feedback. The output voltage is utilized to determine the SIF in active modes. Two separate studies are performed with a single and double patch to see the effect. It is observed that the effect is more when double patches are working together.[Khalili et al. (2010)] Here in this work to reduce the impact of the stress concentrations in layers on the single Lap Adhesive Joints, a smart lap joint is developed by tying the PZT patch on the surface. The moment and force at the boundaries of the joint are adjusted by varying the applied external voltage. Hence the stresses can be controlled. For this study, an analytical model has been developed. It introduces an analytical approach to see how the mechanical properties as well as geometric properties of adhesive effects the attached PZT. A model with an adhesive layer where the adhesive layer undergoes shear deformation is anticipated to replicate the electromechanical character. Yan et al. (2014) proposed a study of time-dependent analysis of a simply supported ply laminate made of piezoelectric in a cylindrical shape with viscoelasticity. The bonding in laminates is assumed a weak conductor and compliant. [Yu and Wang (2017)] This paper presented a study of bonded PZT actuators under applied voltage that undergoes axial bending deformations. The electromechanical characteristics of the PZT are studied in several mechanical, and geometrical circumstances to assess the consequence of bending. Abuzaid et al. (2017) described how a PZT patch can be used to stop the propagation of a crack in plates. Since Mode I type of crack is most dangerous under tensile loading hence the effect of PZT actuation on this kind of crack propagation is analyzed. Here a plate with bonded PZT patch and having a crack at mid is modelled using LEFM (Linear Elastic Fracture Mechanics). As well the SIF (Stress intensity factor) was estimated by superimposing the effect of actuating voltage and stress. Subsequently, a model was prepared in ANSYS to compare the results.

1.3.1 Research Gap: Literature review shows that there is a vast scope of research in the field of crack detection for beam-type structures. The conventional technique focuses on the natural frequency, mode shape and measurement of change in rotational displacement of the cracked structure. From the deviation of these parameters, the detection of crack is identified. However, if a structural element is subjected to a low level of transverse load or the crack depth is very small, it becomes very difficult to predict the presence of a crack following these methods. The literature review indicates the importance of smart materials, like piezoelectric material in the field of crack detection as well as in repair. At the same time, it is pragmatic that the approach techniques by different researchers are not as simple as the usual modal analysis procedures. The present research work proposes a simple way to use a piezoelectric sensor to detect a crack in beam-type elements under harmonic excitation. A crack or any kind of damage considerably affects the local stiffness as a result the rotational displacement curve gets discontinued at the crack location. Though it's an effective damage index at the same time there is no conventional practical technique to check the discontinuity. In the present study, attempts are made to measure the same by employing a piezoelectric patch. Eventually, it leads to a novel way to detect the presence of a crack.

1.3.2 Objective of the present thesis:

The available pieces of literature where piezoelectric material has been used either as a sensor/ actuator or energy harvester are discussed in detail in the last section. The review of the relevant literature clearly shows the potential of piezoelectric material. At the beginning of the literature review, the importance of piezoelectric material in energy harvesting as well as the utilization of different equipment is focused on. Then in the next section how an open can be modelled has been reviewed from the literature. The later part of the literature review shows the uses of piezoelectric material are not restricted to energy harvesting and measuring equipment only. Also, it has high importance in the field of crack detection and repair.

By reviewing several pieces of literature, it is observed that the piezoelectric material has the potential to be used as an effective sensor as well as an actuator. Several researchers have used piezoelectric material either as a sensor or actuator to detect a crack or any kind of flaws in the structure. At the same time, some articles are there where piezoelectric actuators are used to control the failure or repair the damages in the structure. Undoubtedly Structural health monitoring always should be the priority of the researchers, who are working in this field.

The objective of any research work starts by defining a problem. Therefore in this section, the drawback of the earlier methods where piezoelectric patches were used to detect the crack or to repair a crack is highlighted. Thereafter new approaches have been introduced that advance the uses of piezoelectric material in the field of crack detection and repair.

In the earlier methods, it is observed that though the piezoelectric patches are used to detect or repair the crack the approaches are not easy or sometimes the damages can be detected in a very small span. By surveying the mathematical models in refereed articles two important points are marked which are mentioned below.

1) An open crack produces a discontinuity in the rotational displacement curve or slope curve (Euler's beam) in a beam under static loading or while vibrating.

2) The generated voltage in a piezoelectric patch is proportional to the difference in slope or rotational displacement at the ends of the patch when the polarization direction of the piezoelectric patch is along the thickness.

Indeed, these two key points are the backbone of the present thesis. It is to be noted that the above-mentioned points are only for beam structure. Hence the present analysis also considered only beam-type elements under different boundary conditions. The first point signifies that whenever a crack is present in a beam then at the location of the crack a sharp change in slope is observed mathematically. So it suggests that by measuring the slope or rotational displacement one can detect the location of the crack and can get an idea about the depth. But the problem is there is no conventional way to measure the same. It is even more difficult to measure when the beam is under dynamic conditions.

The second point states that a piezoelectric patch generates a voltage proportional to the difference in slope at its end when the polling direction is along the thickness. Therefore if a piezoelectric patch is bonded on the surface of the beam then it can measure the slope in form of voltage. Now if there is a crack in the beam then the difference in slope will be large as a result the patch will give a higher voltage. This way one can easily locate the crack.

In addition to it, there is one more special character of a piezoelectric material that can be strained by applying external voltage. Now if the same patch (where the polling direction is along the thickness) comes under an external electric field then the patch will bend (actuator), and as a result, it can close the crack opening. Mathematically it can be shown that by actuating the patch a local moment can be produced which can nullify the effect of the crack.

The objectives are elaborated on in the above paragraphs. However, to facilitate it more precisely, objectives are written in points.

1. Identification of cracks in statically loaded beams under different boundary conditions using the above-discussed approach i.e. using a piezoelectric patch.
2. Repair of the statically loaded beam by actuating a piezoelectric patch so that the discontinuity in slope or rotational displacement will eliminate.
3. Identification of cracks in beams under harmonic loading by using the piezoelectric sensor.
4. Restoration of the dynamic response of a vibrating cracked beam by actuating the piezoelectric patch.
5. Development of an instrument to measure the slope in a deformed beam by employing a PZT patch.

CRACK IDENTIFICATION IN BEAM TYPE STRUCTURAL ELEMENTS USING PIEZOELECTRIC SENSOR

2.1 Introduction:

Over the last few years, the detection of a crack in structures through different techniques has become a focus of attention. One such potential method is to ascertain the presence of a crack following the changes in certain parameters like natural frequencies, mode shape, damping coefficient etc. of a vibrating structure. The relevant literatures regarding the present chapter are elabororately reviewed in the Chapter 1 under the sub heading 1.2 Literature review.

The identification of crack by using piezoelectric patch by the available techniques are not much efficient to detect cracks at the preliminary stages since the effect of minute cracks on the dynamic as well as static responses of the structure are not appreciable to diagnose damage in the structure. Several research works are available where different nondestructive techniques are used to locate a crack in a structural element before it severely damages the structure. In this regard, the use of piezoelectric material is found as an efficient approach. If a piezoelectric material comes under any kind of deformation like bending, shear deformation, or compression then the free dipole moments polarize along a specific direction causing the apparent flow of electric charges. On the other hand, if an external electric field is applied to a piezoelectric material then it gets deformed. Using these properties, piezoelectric (PZT) materials recently employed for the detection and identification of cracks in structural elements (Yang et al.(2018), Howells C.A (2009), Safaei et al.(2019), Chawanda and Luhanga (2019), Elhalwagy (2017), Kumar C.N (2015)).

The present study aims to employ a PZT sensor attached to the loaded structure to follow the change in rotational displacement in a beam that occurs in the vicinity of the crack. The output voltage of the sensor would give a clear indication of the presence of a crack and a rough estimation of the crack depth also. In the present non-destructive investigation, two types of beams under different boundary conditions are considered, short and long beams. For both cases, analytical and FEA models are developed using Timoshenko beam theory to detect a crack in the beam. The results of the numerical analysis are compared with the experimental results available in the open literature. It is shown that the present study can be used to locate multiple cracks in the beam also by a single investigation.

2.2 Formulation and methodology:

Two different methods are employed for the detection of a crack in beams having different types of boundary conditions. First, an analytical model of the beam is developed. The effect of an open crack in the beam is replicated by introducing a mass-less rotational spring in place of the crack as described below. A patch of PZT is tied with the beam. A second analysis is carried out by FEA on the ABAQUS platform. In the case of a short beam, the whole beam is tied with a long PZT strip and analysis is undertaken.

The following assumptions are taken to model the cracked beam:

- The beam material is homogeneous and isotropic.
- The behaviour of the beam is linear and elastic.
- The crack remains open during the bending.

2.2.1 Analytical model of Beam:

In the present study, the mathematical model of cracked beam under static loading is considered based on Timoshenko beam theory. The Timoshenko beam theory deals with two field variables, as it includes the effect of pure bending and the shear. Therefore, two field variables namely $\Psi(x)$,$\emptyset(x)$ are considered, for the flexure and shear, respectively. The deflection of beam is represented as w(x).Since the slope of deflection curve at a point represents the rotational displacements, the relationship between $w(x)$, $\Psi(x)$and$\emptyset(x)$can be expressed as follows,

$$dw/\mathrm{dx} = \Psi(x) + \emptyset(x)$$

The total strain energy stored in a healthy beam due to bending is calculated as in equation (2.1)

$$\mathrm{U}=\frac{1}{2}EI(\frac{d\Psi(x)}{dx})^2 + \frac{1}{2}GA_s\emptyset(x)^2 \tag{2.1}$$

Where, $\mathrm{A_s} = \frac{\mathrm{A}}{\mathrm{k}}$, k is shear correction factor, I is area moment of inertia of the cross section of beam about neutral axis. E and G are modulus of elasticity and shear modulus, respectively. Work done by the external load is calculated by following fundamental definition, i.e.$\int Fdx$.Crack is modeled as a mass-less rotational spring with negligible mass as shown in Fig. 2.1. Beam is divided into two different sub beams connected by the mass less rotational spring. The stiffness of the rotational spring (K_t) is ascertained by considering the flexibility induced due to the crack (1/S). The induced flexibility due to crack can be related to the stress intensity factor K_I by adopting fracture mechanics approach, which is provided in equation 2.2(a), where M is the bending moment. Further simplifying, one can obtain the relation for flexibility as mentioned in equation 2.2(b).

$$\frac{(1-\upsilon^2)K_I^2}{E} = \frac{M^2}{2}\frac{dS}{da} \tag{2.2a}$$

$$S = \int_0^{\varsigma} \frac{72\pi(1-\nu^2)\varsigma F^2(\varsigma)}{Eh^2} d\varsigma \tag{2.2b}$$

From the equation (2b), since the expression is a function of only ς, crack depth ratio, therefore, its value depends on crack depth. From the modeling point of view, since the crack is assumed to be an open crack, and material is isotropic, magnitude of bending moment of beams doesn't get altered at the bottom or top surface of the beam. In Eq.(2.2), ν and ς are respectively the Poisson's ratioand crack depth ratio (*a/h*).

Where *a*,*h* are the crack depths and beam depth respectively. ***F (ς)*** indicates the crack correction factor and is given as follows.

$$F(\varsigma) = 1.150 - 1.662\varsigma + 21.667\varsigma^2 - 192.451\varsigma^3 + 909.375\varsigma^4 - 2124.310\varsigma^5 + 2395.83\varsigma^6 - 1.031.750\varsigma^7$$

The strain energy of a deformed Timoshenko beam consisting of an open crack is expressed in equation (2.3), which is obtained from equation (2.1) by introducing change in strain energy due to a crack at L_1

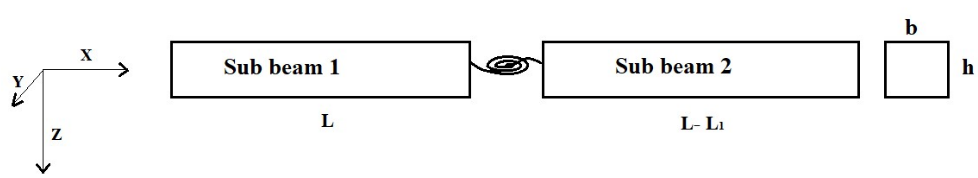


Fig 2.1: The effect of crack is represented by a mass less rotational spring.

$$U = \frac{1}{2}\int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left\{ \left\{ Ez^2 \left(\frac{\partial \psi}{\delta x}\right)^2 \right\} + \frac{kE}{2(1+\upsilon)} \left(\frac{\delta W}{\delta x} - \psi\right)^2 \right\} dxdydz + \frac{1}{2} K_t \left(\Delta \psi_{x=L_1}\right)^2 \qquad (2.3)$$

Stiffness parameters are given as

$$K_1^b = \int_{-h/2}^{h/2} Ez^2 dz, K_2^b = \int_{-h/2}^{h/2} \frac{\kappa E}{2(1+\nu)} dz \qquad (2.4)$$

Load potential (V) of concentrated load (F) can be expressed as

$$V = FW_{x=x_p} dx \tag{2.5}$$

Where x_p is the load point

The energy function thus obtained is expressed as follows

$$\pi = (U + V) \tag{2.6}$$

The above analytical model is solved by Ritz method. According to this method, firstly, trial functions for the displacement fields are assumed by considering the spatial functions multiplied by unknown coefficients. These trail functions are replaced in the strain energy and load potentials and corresponding energy functional is obtained using equation (2.6). Subsequently, energy functional is optimized by taking derivatives of the energy functional with respect to unknown coefficients and equating the resulting equations to zero. Trial functions for displacement fields w_1, w_2 and ψ_1, ψ_2 , where, subscripts denote the displacement of respective sub beams, are chosen following the compatibility at the crack location and the boundary conditions of beam, which are shown in equation 2.8(a) and 2.8(b). In order, to achieve this expression, continuity is considered in transverse (w) displacement at the crack location, since total strain energy mostly depends on first mode of fracture. However, discontinuity is assumed (between ψ_1, ψ_2)in rotational displacement at the crack location. Accordingly, two sub beams in either side of the crack are assumed. The following non dimensional scheme is introduced.

$$\xi = \frac{x}{L_1},\ \eta = \frac{x - L_1}{L - L_1}, w = \frac{W}{h}, \psi = \Psi, \alpha = \frac{L_1}{h}, \alpha_1 = \frac{L - L_1}{h}, \beta = \frac{L - L_1}{L_1}$$

$$\{k_1, k_2\} = \left\{ \frac{K_1^b}{\int_{-h/2}^{h/2} Edzh^2}, \frac{K_2^b}{\int_{-h/2}^{h/2} Edz} \right\}. \tag{2.7}$$

Admissible functions are considered as follows,as stated earlier that in the modelling of crack, only discontinuity in rotational displacement is considered by considering a mass less rotational spring at the place of crack.

Thus, $Moment_{\text{at crack}} = k_t^*(\psi_2 - \psi_1) = k_1 \frac{\partial \psi_2}{\partial \eta}$, where $k_t^* = \frac{K_t(L - L_1)}{K_1^b h^2}$

For Example, in clamped-clamped beam, kinematic boundary conditions are displacements are zero at both fixed ends and hence, by using this and considering compatibility condition discussed above, displacement fields can be derived as

$$w_1 = \sum_{j=1}^{j=N} B_j \xi^j (1-\xi) + \xi \sum_{j=1}^{j=N} b_j, w_2 = \sum_{j=1}^{j=N} b_j (1-\eta)^j$$

$$\psi_1 = \sum_{j=1}^{j=N} C_j \, \xi^j (1-\xi) + \xi \sum_{j=1}^{j=N} \left\{1 + (j) \frac{k_1}{k_t^*}\right\} c_j, \psi_2 = \sum_{j=1}^{j=N} c_j (1-\eta)^j \quad (2.8a)$$

Similarly, for cantilever beam, displacement fields can be given as

$$w_1 = \sum_{j=1}^{j=N} B_j \xi^j (1-\xi) + \xi \sum_{j=1}^{j=N} b_j, w_2 = \sum_{j=1}^{j=N} b_j (1+\eta)^j$$

$$\psi_1 = \sum_{j=1}^{j=N} C_j \, \xi^j (1-\xi) + \xi \sum_{j=1}^{j=N} \left\{1 - (j) \frac{k_1}{k_t^*}\right\} c_j, \psi_2 = \sum_{j=1}^{j=N} c_j (1+\eta)^j \quad (2.8b)$$

Here, B_j,b_j, C_j, c_j are the unknown coefficient of trial function which are evaluated by the Ritz approximation method as mentioned earlier in this section by optimizing energy functional. By taking derivation of energy functional with respect to B_j,b_j, C_j, c_j and equating to zero, one can obtain a set of algebraic equation in the form of [k]{u}={f}, where, vector {u} contains the information of unknown coefficients. By solving the above mathematical model, transverse as well as rotational displacement (w, ψ) at required points along the length of the beam are obtained. There is a PZT patch attached below the surface of the beam (Fig. 2.3) having crack. The patch is subjected to rotational displacement. The collected values of rotational displacement, Ψ are put in to the mathematical model of PZT to determine the generated voltage from the patch as shown in the following section.

2.2.2 Mathematical expression of piezoelectric patch

PZT patch can be casted as given by Zhao et. al. (2016)

$$Voltage = \frac{-e_{31} b(\delta+d)}{2C_v} \int_{l_2}^{l_1} d\,\Psi \tag{2.9}$$

Where,

b is length of patch

d is depth of beam and δ is depth of patch.

e_{31} is stress constant.

C_v is the value of capacitance

l_1 ,l_2 represents two ends of the patch

From Eq. (2.8), one may obtain the output voltage generated in the PZT patch. The voltage output confides on the difference in rotational displacement at the ends of the PZT patch. Since crack considerably changes the rotational displacement, there will be a spike in the output voltage confirming the position of the crack.

2.2.3 Finite element analysis (FEA):

In order to carry out the FEA analysis, an integrated 3D shell model of beam and PZT patch is prepared in Standard Explicit session of ABAQUS. PZT patch is defined by 8 Noded brick element. In element types the patch is defined as piezoelectric material. A Datum is created to define the polarization direction. 'EPOT' field output is requested to get the electrical voltage output. Crack is introduced using special interaction provision.

2.3 Results and discussion:

Table: 2.1 Properties of Beam and Piezoelectric Patch

Property	Beam Instance	Piezoelectric Patch
Material	Aluminum	PZT 5H
Length(mm)	600(Long)/ 200(short)	50(Long)/ 200(for short)
Depth(mm)	10(Long/short)	0.5(Long /short)
Width(mm)	50(Long/Short)	50(Long)/10(short)
E_{11}(N /mm^2)	0.69×10^5	1.13×10^5
E_{33}(N/mm^2)		1.05×10^5
ϑ	0.29	0.31
G(N/mm^2)	0.25×10^5	0.23×10^5
d_{31}(m/V)		-274×10^{-12}
d_{33}(m/V)		593×10^{-12}
d_{15}(m/V)		714×10^{-12}
Relative Dielectric constant		3100
Density(kg/m^3)	2789	7600

In order to validate the present work, at first a 3D model with integrated patch as shown in Fig.2.2(a)is prepared following the experimental work of Sumant and Maity,2006. Subsequently, a 3D model of a slender beam, tied with PZT patch is prepared as shown in Fig.2.2 (b) to identify the presence of crack. Lastly, another beam-patch model is created to detect crack in short beams as shown in Fig. 2.2(c).

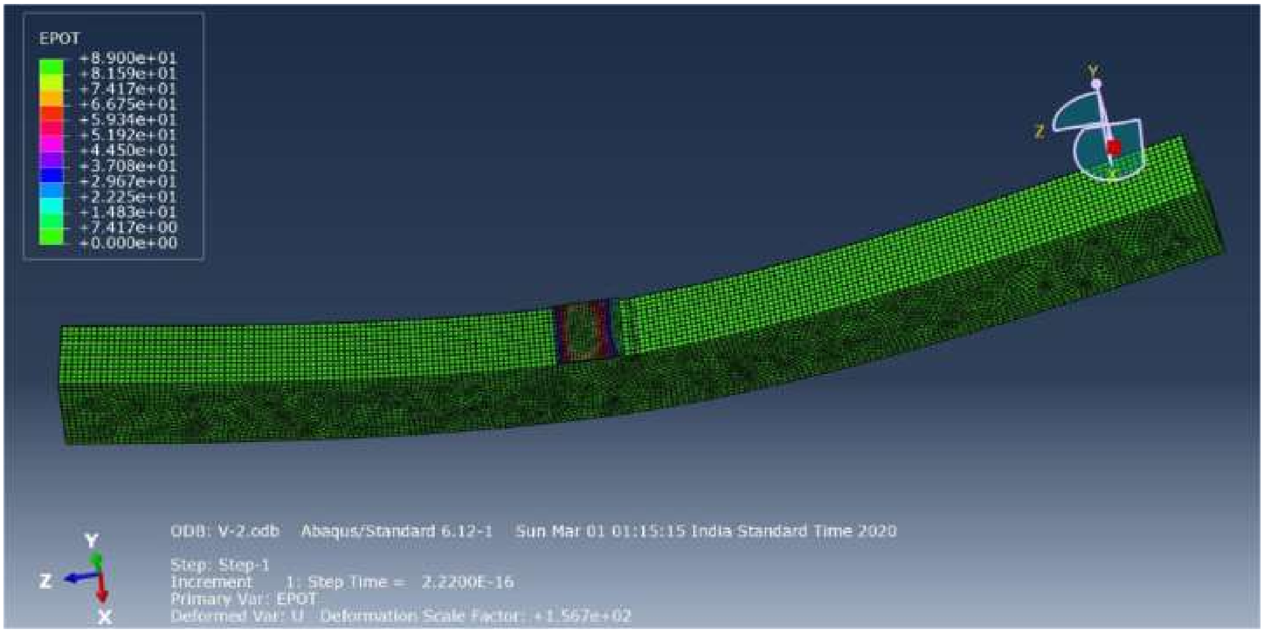


Fig.2.2 (a).3D Solid model of integrated simply supported beam and PZT patch to validate the results published by Sumant and Maity, 2006

Fig. 2.2(b). 3D solid integrated clamped-clamped beam patch model of long slender beam.

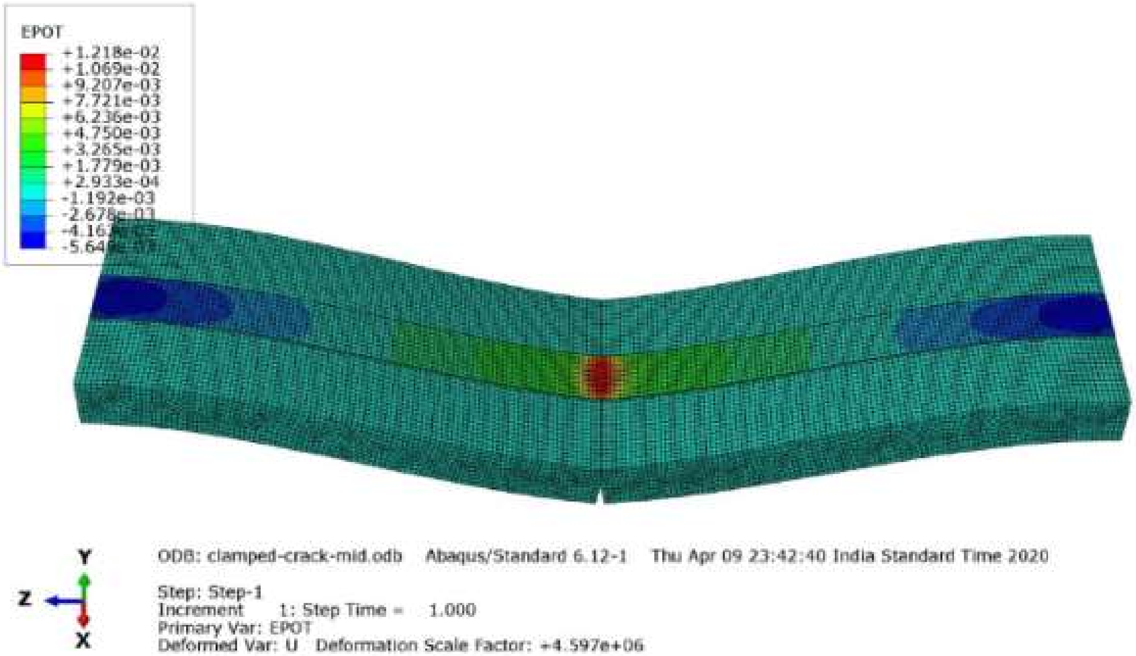


Fig.2.2(c): 3DSolid model of integrated short beam and piezoelectric strip

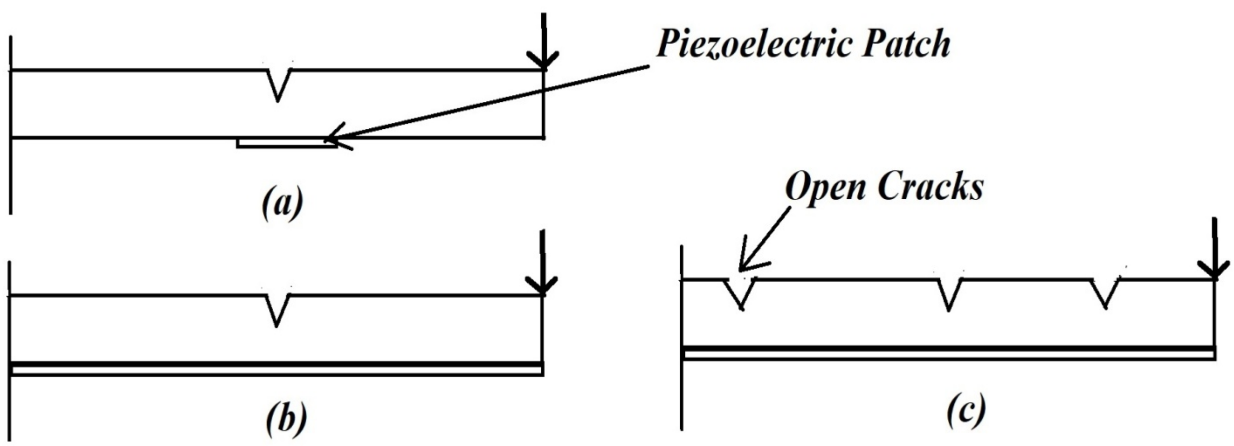


Fig.2.3 statically loaded cracked beam of different lengths.

Fig.2.3 describes beams along with PZT patch to assess the damage in beam of different lengths**.** Though the diagram is prepared for only cantilever beam, the same is followed for clamped beam also. It is to be noted that in case of short beam, crack is placed at the bottom surface while the patch is attached at the top surface. But the same does not make any difference in the analytical calculations according to the equation 2.2(b). The results are presented in three

Table:2.2 Comparison between analytical and FEA results

Beam Type	Crack position (m)	Crack depth ratio	Patch position (m)	Position of load	Voltage(v) Analytical	Voltage(v) FEA
Cantilever(Long)	0.18	0.5	0.18	Open End	10.9	10.11
		0.25			4.3	4.44
Fixed-Fixed(Long)	0.18	0.5	0.18	Middle of the beam	0.310	0.317
		0.25			0.14	0.15
Cantilever(Short)	0.1	0.5	Throughout the length	Open End	2.61	2.65
Fixed-Fixed(Short)	0.1	0.5	Throughout the length	Middle of the beam	0.56	0.58

sections. The validation of the present methodology is depicted in the first section followed by crack detection in long and short beams having different boundary conditions.

2.3.1 Validation with Experimental results:

Before implementing the proposed methodology in this work, the same is validated with the experimental work. Accordingly, an aluminum beam, with the dimensions 165mm x 10mm x 15mm, is modeled with a PZT patch (10mm x10mm x0.5mm) tied up at the bottom. Boundary conditions, loading conditions and other parameters are used as used in the experimental study in published literature

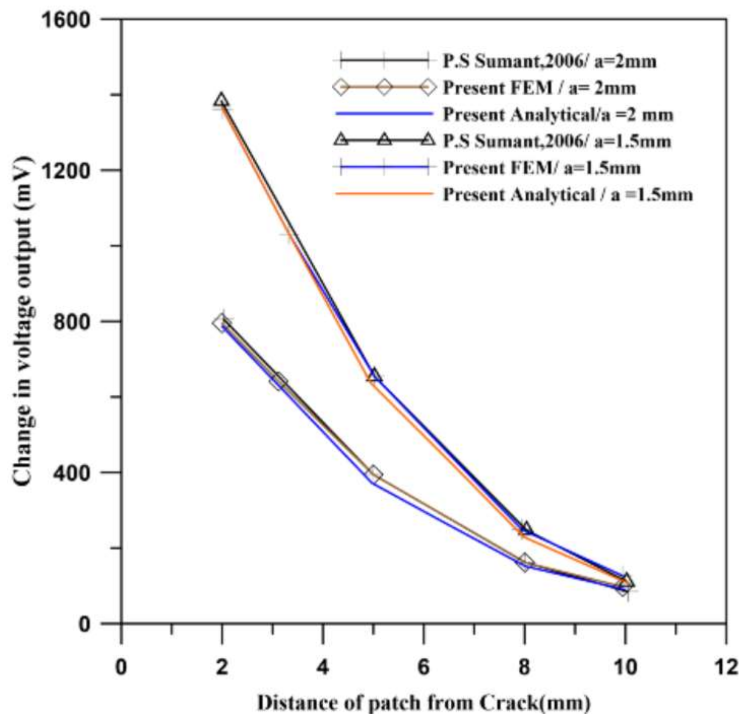


Fig.2.4. Validation of present work by comparing with experimental work published by Sumant and Maity, 2006.

In the numerical study, the distance of PZT patch from the crack is varied and corresponding change in voltage output at the patch is obtained using analytical method and FEA. Fig.2.4 depicts the comparison between the present results with the experimentally obtained result of Sumant and Maity, 2006 .Two different crack depths are considered. It is observed that the results of the present analysis match pretty well with that of the experimental results. Voltage output from the patch for cantilever and fixed-fixed beams for different crack positions and crack depths are obtained using the two methods. A comparison between the results of two techniques, shown in Table. 2.2, indicates a good agreement.

2.3.2 Crack detection in slender beams:

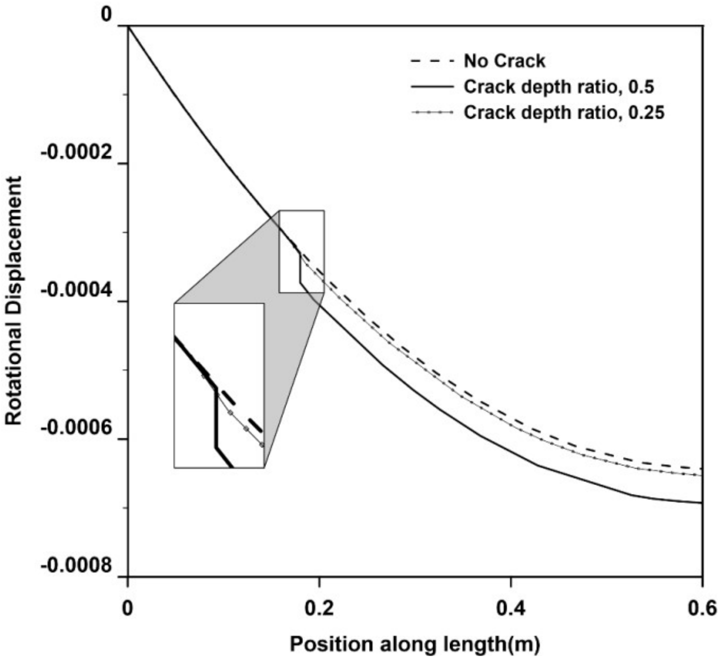


Fig.2.5(a).Change in rotational displacement of long cracked and healthy cantilever beams under a static load

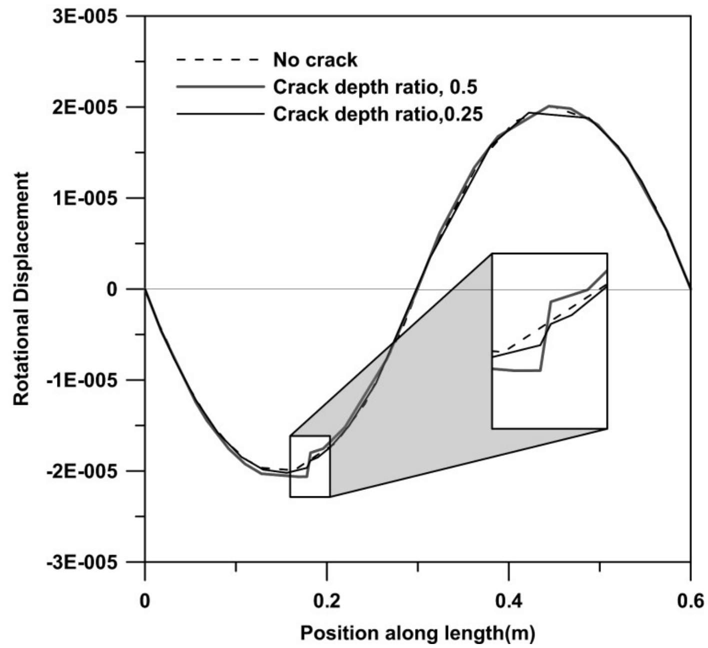


Fig.2.5(b).Change in rotational displacement of long cracked and healthy clamped beams under a static load

The changes in rotational displacements along the length of the statically loaded beams are obtained with cantilever and clamped boundary conditions. For each case, three beams are considered, one healthy (without crack) beam and two other beams with cracks having depths of 2.5mm and 5 mm, respectively. Fig.2.5 (a) and (b) depicts the change in rotational displacement for cantilever and clamped-clamped long beams (600mm in length), due to the presence of open crack under load. In case of cantilever beam a point load of 1 N is applied at the open end. The position of the crack is at 180mm from the fixed end. In case of clamped beam, similar procedure is followed with the load applied at the midpoint of the beam. In both the studies, a discontinuity in rotational displacement curve is observed at the location of crack. It is also observed that the level of discontinuity varies with crack depth. It is very obvious that this discontinuity may lead to a way for the detection of crack in beam and many researchers have already applied this technique for the detection of crack. However, it is to be highlighted that if the beam is not able to sustain high transverse load or if the crack depth is very small (2.5 mm) then it becomes very difficult to find the discontinuity to identify the location of crack. The present research work approaches a new technique to detect the discontinuity in rotational displacement of the beam from the output voltage of PZT tied to the beam near the vicinity of crack. In order to determine the presence of crack in beams with different boundary conditions, two different crack depths are considered for each type of beam. An Aluminum beam having 600mm length, 50 mm width and 10 mm depth, is tied with PZT patch. The dimension of patch is 50mm x50mm x 1mm. Material properties of the patch is taken from open literature and is shown in Table 1. Transverse load is applied on the beam and the patch position is varied along the length of the beam against a particular crack position to assess the influence of crack on the voltage output of the patch due to rotational moment produced at the crack. Fig.2.6 (a) and (b) represents the voltage output of PZT patch tied at different positions along the length of cantilever and clamped beam, respectively. For both types of beams, investigation carried out on a healthy beam having no crack, a beam having an open crack at a position of 180 mm from fixed end. However, two different crack depth ratio (a/h = 0.25 and 0.5) are considered. The beam is loaded by 1Npoint load at open end of the cantilever beam. Fig.2.6 (b) depicts the result for clamped beam under a load of 1 N at the midpoint. In both cases, a considerable change in voltage output is noticed at the location of crack. It is evident from the analysis that output voltage of PZT patch can clearly identify the position of the crack in the beams. It may be highlighted that although output voltage is proportional to the crack depth, method quite capable of identifying the position of the crack even when crack depth. is quite small or the load on the beam is not appreciable to produce large rotational moment at the crack

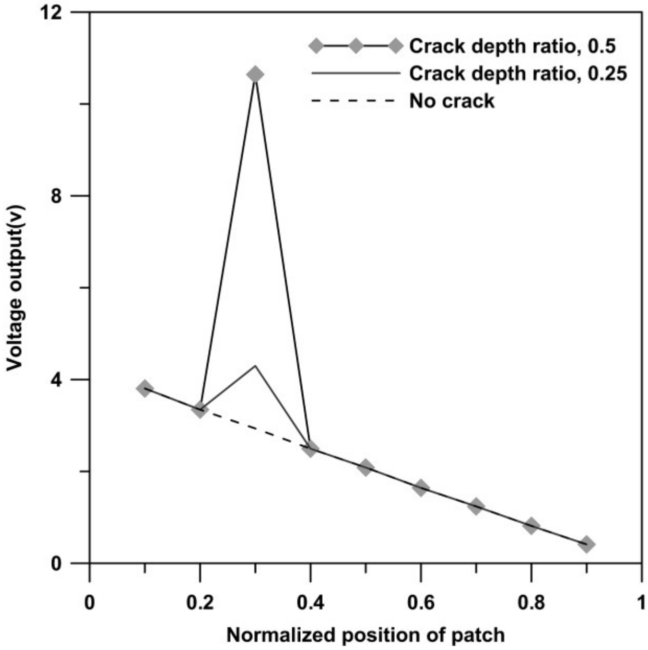


Fig.2.6 (a).Voltage output from the patch tied on a Cantilever beam.

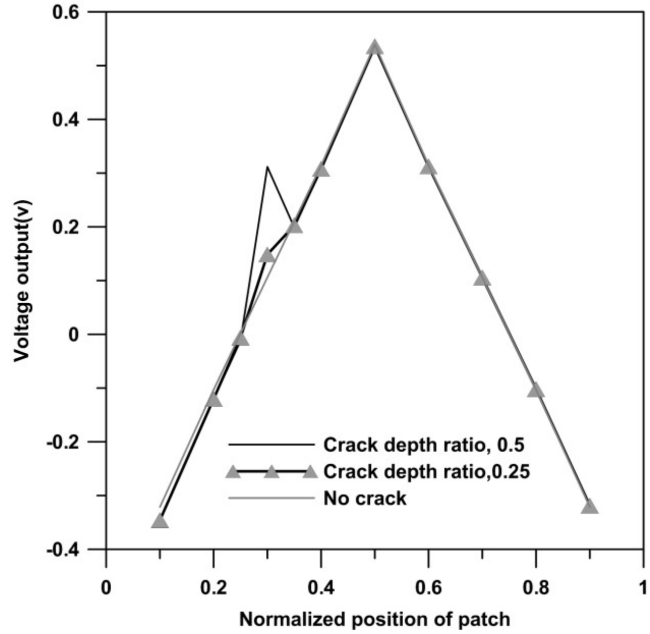


Fig.2.6 (b).Voltage output from the patch tied on Clamped beam.

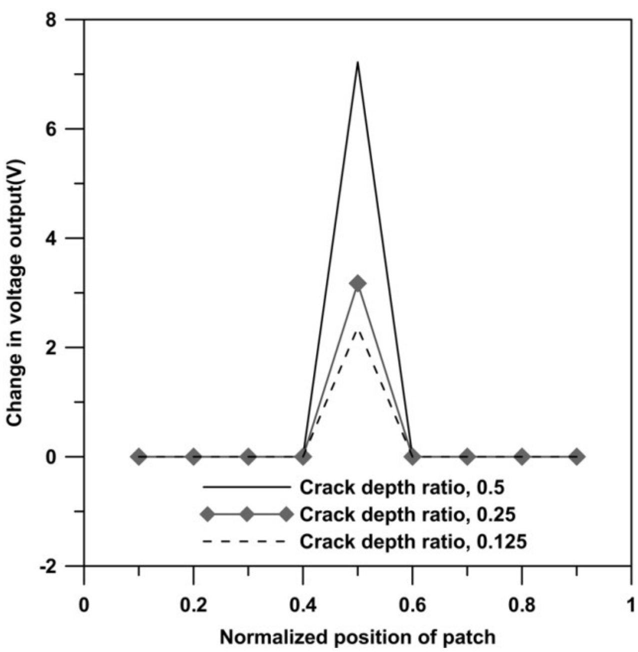


Fig.2.7 (a). Change in voltage output for different crack locations, PZT patch is at the middle of the beam (cantilever)

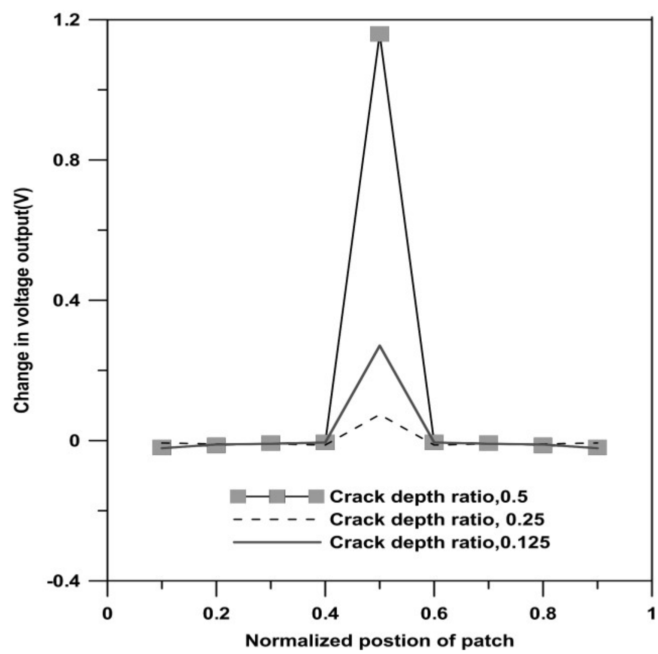


Fig.2.7 (b). Change in voltage output for different crack locations, PZT patch is at the middle of the beam (clamped)

In order to identify the influence of output voltage on the position of the patch relative to the crack position, following numerical experiment is carried out. The patch is fixed at the middle of beam and a half depth crack is introduced at different locations along the length one at a time. Fig.2.7 (a) and 2.7(b) shows the change in voltage output of the patch for cantilever and clamped beam for different crack positions. Fig.2.7 (a) shows that when the patch is just below the crack there is a sharp change in voltage output pointing out the position of the crack. Similarly, Fig.2.7 (b) shows the same behavior for a both end clamped beam. Since crack rigorously effects the rotational displacement in its locality and at the same time the output voltage of PZT patch depends on the difference of slope at its two ends, it is obvious to get this kind of huge response in output voltage of the patch in the vicinity of a crack while at other location of the patch this is negligible. This phenomenon can be utilized for the detection of crack in beam. The method is quite effective and easy to implement. Now an attempt is taken to detect multiple cracks in a cantilever beam, which is loaded under static force. Two cracks are considered at two different positions, one at 180 mm from fixed end and another at 420 mm from fixed end. Two separate voltage peaks are clearly visible indicating the location of the cracks in the cantilever beam (Fig.2.7c).Therefore, by following the peak in voltage output the crack can be easily detected. The method can be practiced very easily in practical purposes; a PZT sensor can be attached at different positions along the length of a beam, which has to be gradually loaded. Now by using a voltage measuring instrument the output voltage of the PZT patch is to be measured. At the location of crack, the voltage output shows a spike. Therefore, by following the peak in voltage, the location of crack can be predicted. In addition, from the value of the output voltage, one can have an estimate about the crack depth. It is also observed that effect of crack on output voltage is more when it is located near the root of a cantilever beam, since the difference in rotational displacement in between the two ends of the patch is more near the root of the cantilever beam.

For the same reason, whereas the effect on voltage output is more at the mid position for a clamped beam.

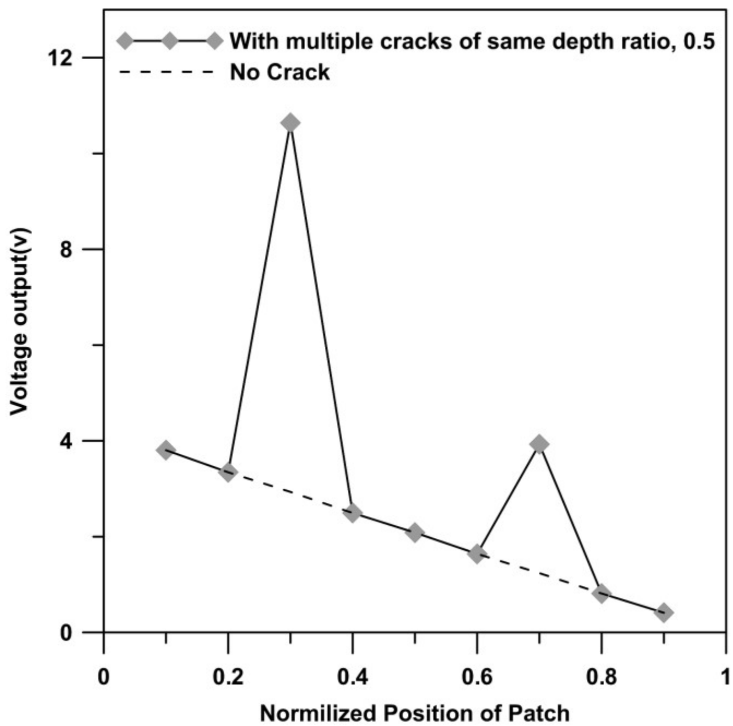


Fig.2.7(c). Voltage output through the PZT patch tied on a cantilever beam with two open cracks.

2.3.3 Crack detection in short size beams

Earlier technique which is used to detect crack in a long slender beam would be simpler if the whole length of the beam could be patched by piezoelectric sensor. Since it is a very costly material, it is not logical to cover up a long beam by piezoelectric patch to detect crack. However, for a short beam if a strip of PZT patch is attached all along the beam length, the technique will be easier to implement. At the same time, if the whole beam is patched with a single PZT strip then by following the voltage output multiple cracks can be detected in a single test.

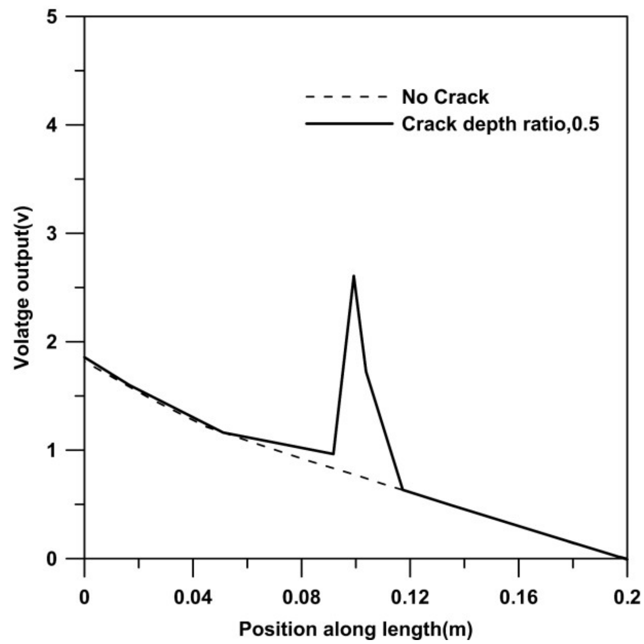


Fig.2.8 Voltage output from a PZT strip tied along the span of a cantilever beam, with a single crack

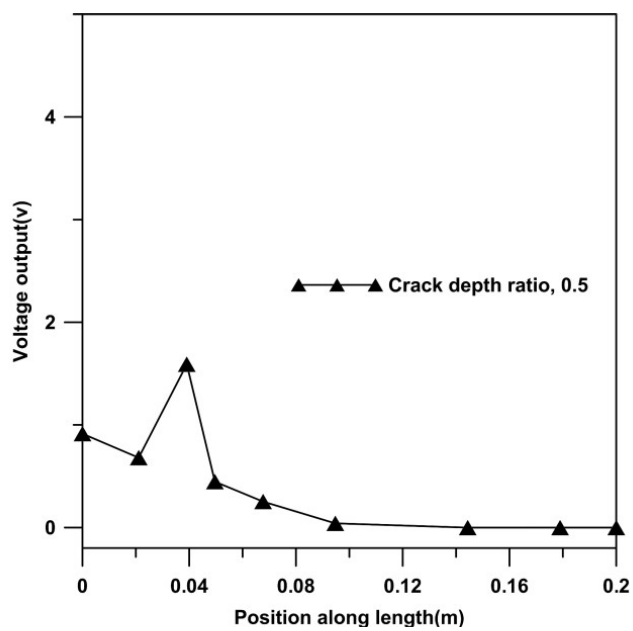


Fig.2.9(a) Voltage output from a PZT strip tied along the span of a cantilever beam, with multiple cracks(loaded at mid of beam)

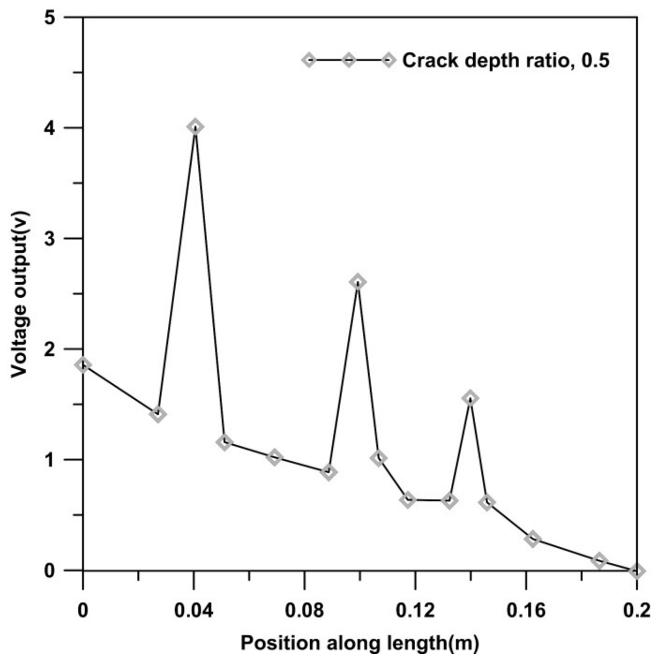


Fig.2.9 (b) Voltage output from a PZT strip tied along the span of a cantilever beam, with multiple cracks (loaded at open end)

In order to locate the position of crack, healthy and cracked cantilever beams made of aluminum are tied with a PZT strips having same length as that of short beam. Properties of the integrated beam-PZT strip model are as shown in Table 2.1. Each of the beams is loaded with a 1N point load at the open end and voltage output at equally spaced point along the length of the PZT strip is obtained. Fig. 2.8 shows a monotonous decrease in output voltage along the length of healthy beam having no crack at the same time a peak in voltage is observed at the location of crack.

Fig. 2.9(a) and (b) depicts the voltage response, obtained through the PZT strip, tied on a cantilever beam with multiple cracks with crack depth of 0.5. The positions of the cracks are at .04m, 0.1m and 0.14m, respectively from the fixed end. It is observed that when the load is acting at the mid position of the cantilever beam, the voltage response gets a peak only for the crack which is close to fixed end as can be seen from Fig.2.9(a). This is because the load is applied at mid-span; hence the cracks which are situated after mid-span do not open. Hence, the crack does not cause any discontinuity in rotational displacement and will not lead to peak in voltage output. However, when the same beam is loaded at open end, three distinct voltage peaks are observed.This is due to the fact that when the load is at middle of the beam then the rotational displacement is constant for the part of beam which is between load point and free end leading to no voltage peak even though there are cracks. Therefore, application of load point is important for the detection of crack in cantilever beam.

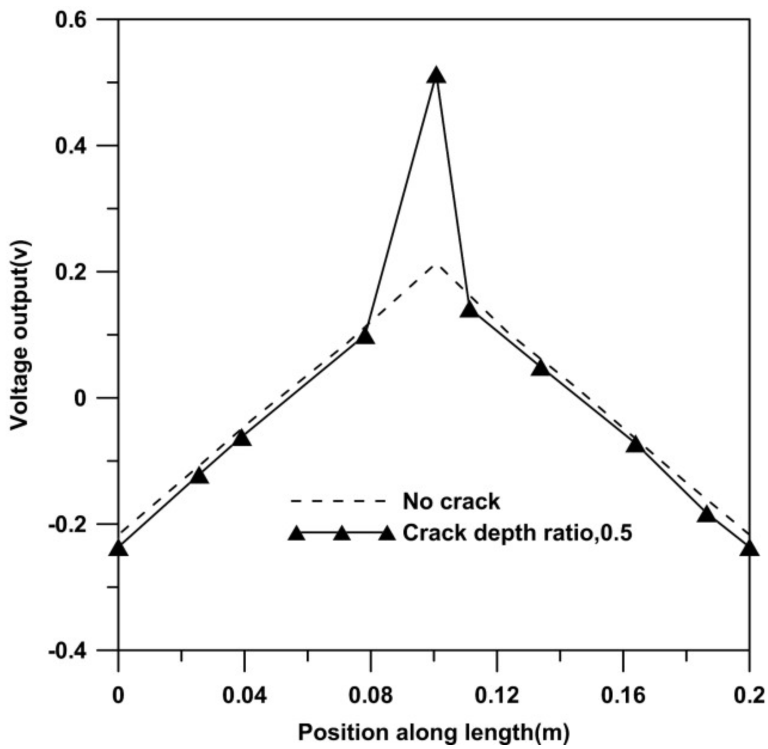


Fig.2.10 (a) Voltage output from a PZT strip tied along the span of a clamped beam, with a single crack at mid)

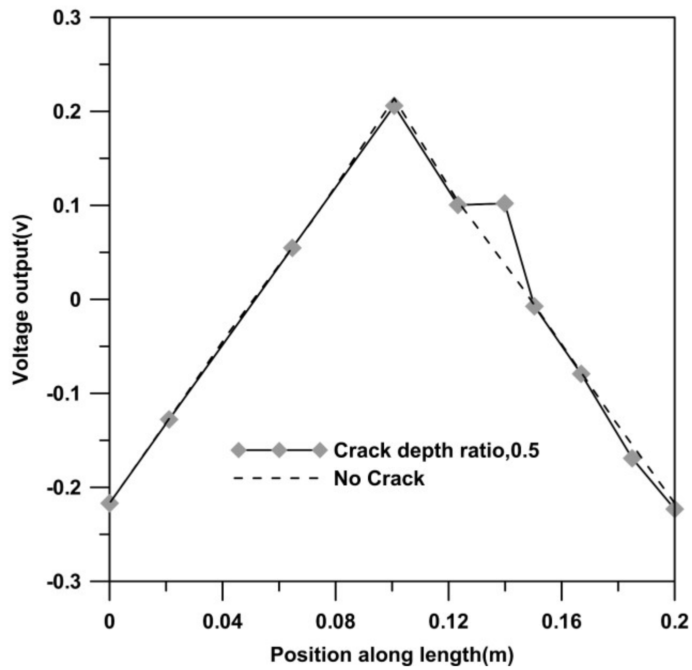


Fig.2.10 (b) Voltage output from a PZT strip tied along the span of a clamped beam, with a single crack at 180 mm from right end

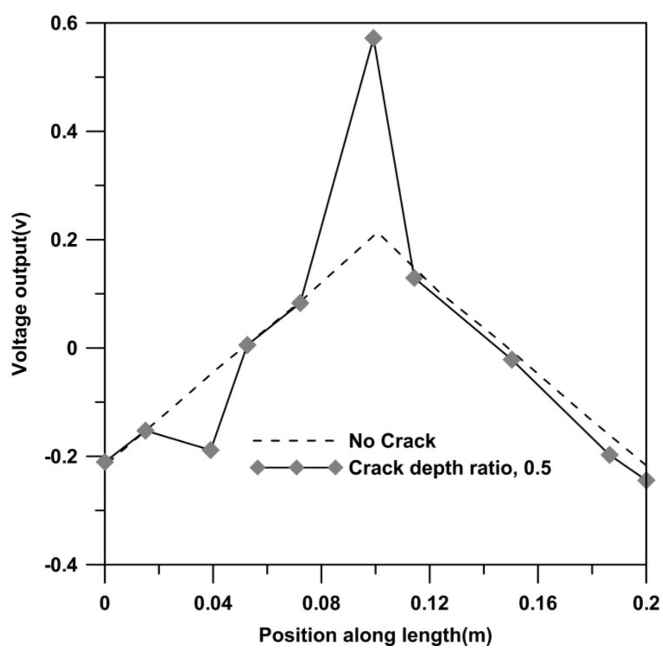


Fig.2.10 (c) Voltage output from a PZT strip tied along the span of a clamped beam, with a multiple cracks

Lastly, investigation is also carried out on short clamped beam tied with a long PZT patch with two different crack positions one at mid of the beam and another at 0.18 m far from right fixed end. In both the cases, sharp peaks in voltage are observed as depicted in Fig.2.10 (a) and

2.10(b). The advantage of using PZT patch all long the short beam can be clearly observed from Fig.2.10(c) where multiple cracks can be detected by a single investigation.

2.4 Summary

Conventional techniques are available for the detection of crack in beam like structures based on natural frequency, mode shape for dynamic loading and change in rotational displacement for the static loading. In the present paper, it is shown that if the beam is not able to sustain high transverse load or if the crack depth is very small then it is not feasible to identify the location of crack following this procedure. In this investigation, it is highlighted that application of PZT is found to be very useful for the identification of the location and size of the crack in beams with different boundary conditions. For the first time an analytical model to measure the rotational discontinuity due to crack through PZT is developed. The effect of an open crack in the beam is modeled with the introduction of a mass less rotational spring. A second analysis is carried out by FEA on ABAQUS platform. The results of the theoretical analysis are validated with the experimental results obtained by earlier research work. The technique is applied to both long and short beams. It is evident from the study that the method is very effective to identify minute crack in beam subjected to small amount of rotational displacement. The method is also capable to predict presence of multiple cracks in a single investigation. As a whole, this approach has the potential to act as a very good non destructive crack detection technique. As a future research scope undoubtedly this kind of approach can be followed to monitor the damages in frame structures. Another aspect must be considered that present work shows a scope to perform the analysis of the model considering higher order beam theory.

EVALUATION AND REPAIR OF CRACK ON STATICALLY LOADED BEAMS USING PIEZOELECTRIC ACTUATION.

3.1. Introduction:

In the last few decades, smart structures have gained tremendous popularity among researchers for their application in vibration control and damage compensation capability. These smart structures can be considered as the structure having several distributed sensors and/or actuators (Lee and Moon ,1990) made of piezoelectric materials such as lead zirconate titanate (PZT) and polyvinylidene fluoride (PVDF). However, due to the environmental perspective, efforts have now shifted towards lead-free piezoelectric materials. Nevertheless, piezo ceramics such as PZT are still being used due to their effective electromechanical properties. Energy harvesting and repair of cracks on structural elements using such piezoelectric sensors and actuators are one of the active areas of research in the last few years.

Several ideas have been proposed to shape and design portable electronic devices where battery size is a limiting factor. In this regard, the use of piezoelectric sensors has gained popularity as they can be used as electrical energy harvesters drawing energy from vibrating or deformed structural elements. There have been continuous efforts to reduce the power requirements for microelectronic devices by using environmental energy sources to meet these requirements in wearable devices. The use of piezoelectric harvesters is not limited to wireless portable electronic or small wearable devices.(Starner T, 1996; Matsuzawa and Saka , 1997 ; Kymissis J., Kendall K., Paradiso J. , Gershenfeld N ,1998) and It's uses have been extended in various civil structures as well, where the finite life cycle of batteries is being replaced by such harvesters for prolonged and continuous power supply.

The other relevant literatures in this field are elaborately discussed in the literature review subsection of the Chapter 1.

Indeed the literature review suggests that there is a huge scope to incorporate a PZT sensor/actuator to monitor as well as repair the structural elements.

3.2. Methodology:

A cracked beam with a piezoelectric patch under various applied load patterns are shown in Figure 3.1. Beam length, crack location, crack depth, beam depth, beam width, patch depth and patch lengths from cracked positions and are shown as L, L_1, a, h, b, δ, p_1 and p_2, respectively. Although in Figure 3.1, beam is shown as fixed in both ends, investigation is carried out for both clamp-clamp beam and cantilever beam under similar loading conditions. Axial and transverse direction is chosen parallel to x and z axis with z axis is considered positive downwards. Origin of the coordinate system is chosen to be at the mid plane of the beam alone. Crack is modeled as mentioned by Panigrahi and Pohit, 2016 by a mass less rotational spring at the cracked section and the stiffness of rotational spring (K_t) is obtained as inverse of the flexibility (1/S) induced due to crack. Locally induced flexibility due to crack is derived using fracture mechanics approach and is provided in Equation (3.1)

$$S = \int_0^{\varsigma} \frac{72\pi(1-\nu^2)\varsigma F^2(\varsigma)}{Eh^2} d\varsigma \qquad (3.1)$$

In, equation (1), υ, ς and E are the poison's ratio, crack depth ratio (a/h) and modulus of elasticity, respectively. F (ς) represents the crack correction factor which is given by as in equation (3.2)

$$F(\varsigma) = 1.150 - 1.662\varsigma + 21.667\varsigma^2 - 192.451\varsigma^3 + 909.375\varsigma^4 - 2124.310\varsigma^5 + 2395.83\varsigma^6 - 1.031.750\varsigma^7 \qquad (3.2)$$

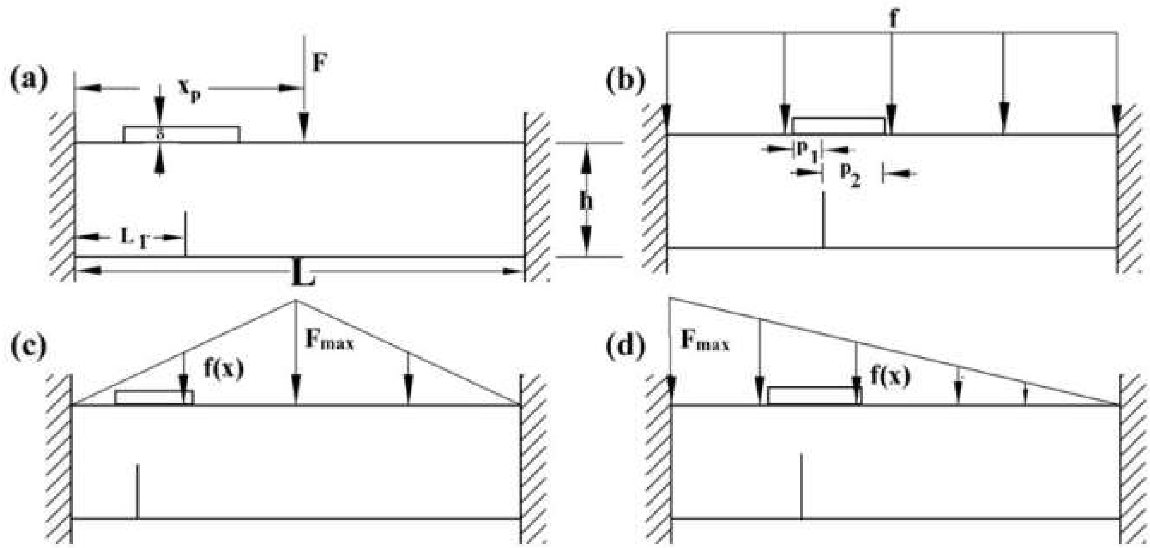


Figure (3.1): Fixed-Fixed cracked beam with a piezoelectric patch applied with different load patterns

3.2.1 *Energy formulation of pzt patched cracked beam*:

Mathematical formulation for statically loaded PZT patched cracked beam is presented in this section based on the energy method. Displacement fields are represented by U, W and Ψ for axial, transverse and rotational displacement, respectively. Energy formulation for beam is carried out considering Timoshenko beam theory and piezoelectric patch is considered as a thin patch. Strain and stress fields are expressed as in equation (3.3)

$$\varepsilon^b_{xx} = \frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x}, \gamma^b_{xz} = \frac{\partial W}{\partial x} + \Psi, \ \sigma^b_{xx} = E\left[\frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x}\right], \sigma^b_{xz} = \frac{\kappa E}{2(1+\upsilon)}\left(\frac{\partial W}{\partial x} + \Psi\right).$$
$$\sigma^p_{xx} = c^E_{11}\left[\frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x}\right] - e_{31}E_3 \quad (3.3)$$

In equation (3.3), the superscript b and p stands for the beam and patch, respectively. ε and γ are the normal and shear strain whereas σ_{xx} and σ_{xz} are normal and shear stresses. E_3 is the externally applied electric field across the electrodes of the PZT and can be written as (V_a/δ), V_a being the applied actuation voltage. Here, e_{31} and c_{11}^E are piezoelectric stress constant and Young's modulus at constant electric field. Axial effects are also considered for the completeness of the formulation. However, in the present work since there is no axial effect, it can be neglected in further processing and potential energy of the problem can be derived as mentioned in equation (4)

$$U = \frac{1}{2}\int_0^L \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} \left\{ Ez^2 \left(\frac{\partial \Psi}{\partial x}\right)^2 + \frac{\kappa E}{2(1+\nu)}\left(\frac{\partial W}{\partial x} + \Psi\right)^2 \right\} dxdydz + \frac{1}{2}\int_{L_1-p_1}^{L_1+p_2} \int_{h/2}^{h/2+\delta} \int_{-b/2}^{b2} \left\{ c_{11}^E z^2 \left(\frac{\partial \Psi}{\partial x}\right)^2 - e_{31}\frac{V_a}{\delta} z \frac{\partial \Psi}{\partial x} \right\} dxdydz + \frac{1}{2}K_t\left(\Delta \Psi_{x=L_1}\right)^2 \tag{3.4}$$

Stiffness parameters are considered as

$$K_1^b = \int_{-h/2}^{h/2} Ez^2 dz, K_2^b = \int_{-h/2}^{h/2} \frac{\kappa E}{2(1+\nu)} dz, K_1^p = \int_{h/2}^{\frac{h}{2}+\delta} c_{11}^E z^2 dz, K_2^p = \int_{h/2}^{\frac{h}{2}+\delta} e_{31}\frac{Va}{\delta} zdz \tag{3.5}$$

Potential of loads can be expressed as $V = \int_0^L fWdx$, however for concentrated load $V = FW_{x=x_p} dx$ where x_p be the load application point for point loading.

Substituting parameters from equation (3.5) to equation (3.4), equation 4 reduces to

$$U = \frac{b}{2}\int_0^L \left\{ K_1^b \left(\frac{\partial \Psi}{\partial x}\right)^2 + K_2^b \left(\frac{\partial W}{\partial x} + \Psi\right)^2 \right\} dx + \frac{b}{2}\int_{L_1-p_1}^{L_1+p_2} \left\{ K_1^p \left(\frac{\partial \Psi}{\partial x}\right)^2 - K_2^p \left(\frac{\partial W}{\partial x} + \Psi\right)^2 \right\} dx + + \frac{1}{2}K_t\left(\Delta \Psi_{x=L_1}\right)^2 \tag{3.6}$$

It has been concluded in the crack modeling by Panigrahi and Pohit, 2016 that first mode of fracture dominates the total strain energy of the system and hence it can be assumed that at the crack location, there is continuity in transverse (W) direction whereas rotational displacement is discontinuous. Therefore, two different sub beams in either side of the crack is assumed and entire span of the beam is divided into two parts. Displacement fields are named with subscript 1 and 2 for sub beams. Non dimensional scheme is followed as mentioned in equation (3.7).

$$\xi=\frac{x}{L_1},\ \eta=\frac{x-L_1}{L-L_1},w=\frac{W}{h},\psi=\Psi,\alpha=\frac{L_1}{h},\alpha_1=\frac{L-L_1}{h},\beta=\frac{L-L_1}{L_1}$$

$$\left\{k_1^b,k_2^b,k_1^p,k_2^p\right\}=\left\{\frac{K_1^b}{\int_{-h/2}^{h/2}Edzh^2},\frac{K_2^b}{\int_{-h/2}^{h/2}Edz},\frac{K_1^p}{\int_{-h/2}^{h/2}Edzh^2},\frac{K_2^p}{\int_{-h/2}^{h/2}Edzh}\right\}. \tag{3.7}$$

Dividing equation (3.6) and load potential (V) by $\int_{-h/2}^{h/2}Edzh^2 \Big/ L-L_1$ non-dimensional form of energy equations (U^* and V^*) for different sub beams can be rewritten as provided in equations (3.8) and (3.9).

$$\begin{aligned}
U^* &= \frac{\beta}{2}\int_0^1\left\{k_1^b\left(\frac{\partial\psi_1}{\partial\xi}\right)^2+k_2^b\left(\frac{\partial w_1}{\partial\xi}\right)^2+k_2^b\alpha^2\psi_1^2+k_2^b\alpha\frac{\partial w_1}{\partial\xi}\psi_1\right\}d\xi+\\
&\frac{1}{2}\int_0^1\left\{k_1^b\left(\frac{\partial\psi_2}{\partial\xi}\right)^2+k_2^b\left(\frac{\partial w_2}{\partial\xi}\right)^2+k_2^b\alpha^2\psi_2^2+k_2^b\alpha\frac{\partial w_1}{\partial\xi}\psi_2\right\}d\eta+\\
&\frac{\beta}{2}\int_{1-\frac{L_1}{p_1}}^{1}\left\{k_1^p\left(\frac{\partial\psi_2}{\partial\xi}\right)^2-k_2^p\left(\frac{\partial\psi_2}{\partial\xi}\right)\right\}d\xi+\frac{1}{2}\int_0^{\frac{p_2}{L-L_1}}\left\{k_1^p\left(\frac{\partial\psi_2}{\partial\xi}\right)^2-k_2^p\left(\frac{\partial\psi_2}{\partial\xi}\right)\right\}d\eta+\frac{1}{2}k_t^*(\Delta\psi)^2
\end{aligned} \tag{3.8}$$

$$\text{where } k_t^*=\frac{L-L_1}{\int_{-h/2}^{h/2}Edzh^2}K_t$$

Load potential is given as

$$V^*(Concentrated) = \frac{Fw_{\xi p}\left(L-L_1\right)}{\int_{-h/2}^{h/2} Edzbh}, \quad V^*(UDL) = \frac{f\, L_1\left(L-L_1\right)}{\int_{-h/2}^{h/2} Edzhb}\left\{\int_0^1 w_1 d\xi\right\} + \frac{f\left(L-L_1\right)^2}{\int_{-h/2}^{h/2} Edzhb}\left\{\int_0^1 w_2 d\eta\right\}$$

$$V^*(Triangular) = \frac{L_1\left(L-L_1\right)}{\int_{-h/2}^{h/2} Edzhb}\left\{\int_0^1 f(\xi) w_1 d\xi\right\} + \frac{\left(L-L_1\right)^2}{\int_{-h/2}^{h/2} Edzhb}\left\{\int_0^1 f(\eta) w_2 d\eta\right\}, \tag{3.9}$$

$$V^*(Hat) = \frac{L_1\left(L-L_1\right)}{\int_{-h/2}^{h/2} Edzhb}\left\{\int_0^1 f(\xi) w_1 d\xi\right\} + \frac{\left(L-L_1\right)^2}{\int_{-h/2}^{h/2} Edzhb}\left\{\int_0^1 f(\eta) w_2 d\eta\right\},$$

Energy functional can be obtained as

$$\pi = (U^* + V^*) \tag{3.10}$$

3.2.2 Selection of the trial functions:

Displacement fields are defined by suitably choosing the admissible coordinate functions. These functions are chosen in such a way that it should satisfy the kinematic boundary conditions as well as the compatibility condition at the crack location. Trial functions for the transverse displacement field satisfy the continuity condition at crack location whereas for rotational displacement, it should have a discontinuity at the crack section and should satisfy the moment compatibility at crack location.[7] Here, the trial functions for clamped-clamped beam are derived and for the other boundary conditions, the trial function is simply mentioned. For a clamped-clamped beam moment (M_x) can be derived as

$$M_x = \int_{-h/2}^{h/2} \sigma_{xx}^b zdz + \int_{h/2}^{\frac{h}{2}+\delta} \sigma_{xx}^p zdz = \int_{-h/2}^{h/2} Ez^2\left(\frac{\partial \Psi}{\partial x}\right)dz + \int_{h/2}^{\frac{h}{2}+\delta}\left\{c_{11}^E z^2\left(\frac{\partial \Psi}{\partial x}\right) - e_{31}\frac{V_a}{\delta}\right\}dz = \left(K_1^b + K_1^p\right)\frac{\partial \Psi}{\partial x} - K_2^p \tag{3.11}$$

Bending moment at the crack location can also be defined as the stiffness of rotational spring multiplied by the discontinuity in rotation.

$$M_{|x=L_1} = K_t\left(\Psi_{2|x=L_1} - \Psi_{1|x=L_1}\right) \tag{3.12}$$

Equating value of equation (3.11) at crack location with that of equation (3.12) and writing the expression in its non dimensional form, one may deduce the expression as

$$k_t^*\left(\psi_{2|\eta=0} - \psi_{1|\xi=1}\right) = \left(k_1^b + k_1^p\right)\frac{\partial\psi}{\partial\eta}|_{\eta=0} - k_2^p \tag{3.13}$$

Using equation (3.13) and considering the kinematic boundary conditions, trial functions for clamped-clamped beam are derived and is shown in equations 3.14(a). Following the steps mentioned above, trial functions for cantilever boundaries are obtained as shown in equation 3.14(b).

$$\begin{aligned} w_1 &= \sum_{j=1}^{j=N} B_j \xi^{j+1}(1-\xi) + \xi\sum_{j=1}^{j=N} b_j,\ w_2 = \sum_{j=1}^{j=N} b_j (1-\eta)^{j+1} \\ \psi_1 &= \sum_{j=1}^{j=N} C_j \xi^{j+1}(1-\xi) + \xi\sum_{j=1}^{j=N}\left\{1+(j+1)\frac{k_1^b + k_1^p}{k_t^*}\right\}c_j + k_2^p, \psi_2 = \sum_{j=1}^{j=N} c_j (1-\eta)^{j+1} \end{aligned} \tag{3.14a}$$

$$\begin{aligned} w_1 &= \sum_{j=1}^{j=N} B_j \xi^{j+1}(1-\xi) + \xi\sum_{j=1}^{j=N} b_j,\ w_2 = \sum_{j=1}^{j=N} b_j (1+\eta)^{j+1} \\ \psi_1 &= \sum_{j=1}^{j=N} C_j \xi^{j+1}(1-\xi) + \xi\sum_{j=1}^{j=N}\left\{1-(j+1)\frac{k_1^b + k_1^p}{k_t^*}\right\}c_j + k_2^p, \psi_2 = \sum_{j=1}^{j=N} c_j (1-\eta)^{j+1} \end{aligned} \tag{3.14b}$$

Replacing the trial functions from equations (3.14a) or (3.14b) into equation (3.10) and optimizing the energy functional with respect to unknown coefficients (B_j, b_j, C_j and c_j), set of algebraic equations can be derived as

$$[K]\{c\} = \{f_v\} \tag{3.15}$$

In equation (3.15), K is stiffness matrix; c and f_v are vectors of unknown coefficient and load vector, respectively. Equation (3.15) is solved for unknown coefficients using matrix inverse technique and displacement fields is obtained by post processing the data using equation (3.14).

3.2.3 Evaluation and repair of crack:

It can be stated that presence of crack in a beam under load causes rotational discontinuity at the crack location whereas piezoelectric patch attached to the beam and functioning as an actuator can induce a local moment due to application of voltage from outside. This will give an idea of extent of damage created by the crack also. If PZT patch is utilized in such a fashion that the direction of the induced moment opposes the effect of rotational discontinuity induced by the external applied load at crack location, then by controlling the external voltage of the PZT patch, rotational discontinuity can be removed and repair of the crack is achieved. The beam thus behaves as if no crack is present in the beam.

It has been shown by researchers (Panigrahi and Pohit, 2016) that the influence of the crack is negligible if it is located at any of the node in case of vibration or at the point of contra flexure. Severity of the crack in fact depends on the bending moment at the crack location. In the present work, this fact is used and eventually PZT is used to minimize or eliminate the severity of crack. Moment induced at the crack location due to externally applied load is balanced by an additional reactive moment inducing due to the externally applied voltage V_a. This reactive moment is locally induced where the PZT patch is attached. Therefore, it is necessary to first locate the crack examining the displacement or by the voltage generated due to PZT as a sensor. Once the crack is located, the PZT actuator is placed in such a manner that restoring moment due to the externally applied voltage at PZT completely neutralizes the bending moment due to external loading at the crack location. Therefore, the severity of the crack is completely neutralized and crack is said to be repaired. When the crack location is moment free, left hand side of equation (3.13) will be equal to zero and the condition for crack repair can be written as

$$\left(k_1^b + k_1^p\right)\frac{\partial \psi}{\partial \eta}|_{\eta=0} = k_2^p \qquad (3.16)$$

Equation (3.16) is solved to determine the actuation voltage required for repair of the crack.

3.2.4 Finite element analysis by ABAQUS:

In order to show the precision of present work a subsequent finite element analysis is done using ABAQUS 6.12. In ABAQUS a 3D aluminum beam tied with piezoelectric patch is modeled with identical dimensions and properties as that of numerical work. Seam crack is introduced using special interaction; healthy as well as cracked both models are run under steady state dynamic state to obtain the results.

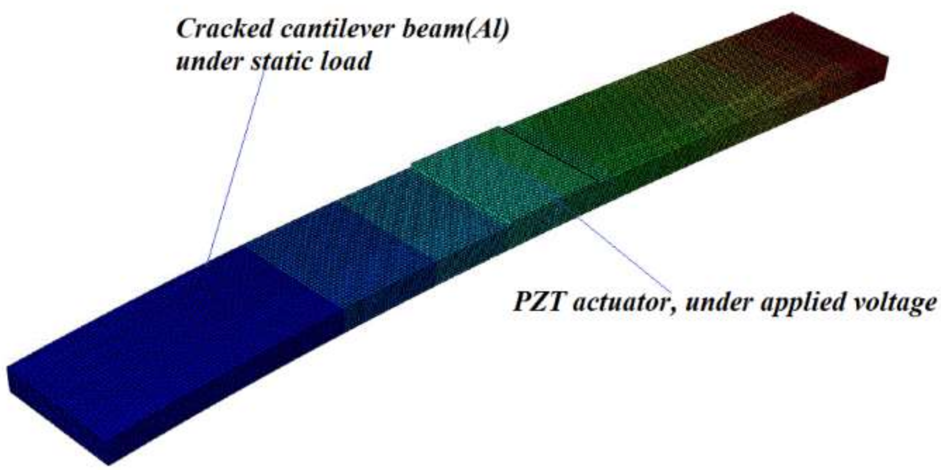


Figure (3.2). 3D Solid model of beam with tied piezoelectric patch

3.3. Results and discussions: A computational code is developed in MATLAB® in order to solve Equations (3.15) and (3.16). Actuation voltages are provided to the PZT patch to completely nullify the effect of crack on the beam. They are obtained for different crack locations and crack sizes. The accuracy of this numerical analysis is compared with the results obtained from Finite Element Analysis in ABAQUS platform. To represent the axial coordinates, gauss points are generated using suitable normalized coordinates. A convergence study is performed on the number of gauss pints required to describe the solution accurately. For any arbitrary number of polynomial functions (N), it is observed that the solutions (displacements) are converged when gauss points are more than or equal to 22. Therefore, 24 numbers of gauss points are chosen for all subsequent study. Another convergence study is performed in order to insure that minimum number of terms required for representing the trial functions without altering the efficacy of the solution. Figure 3.3(a) shows the convergence of the tip displacement in transverse direction with respect to number of terms in trial functions for different magnitude of loading (F = 10, 15 and 20N) at the tip of a cantilever beam. Similar type of analysis is carried out in ABACUS also in respect to global element size. Figure 3.3(b) shows the convergence with respect to element size for ABAQUS. It is evident from Figs. 3.3 and 3.4 that results are converged when number of terms in the trial functions is more than eight in numerical analysis and the finite element analysis converges when the element size is less than 0.004 . Therefore, eight numbers of terms is considered throughout the numerical study and 0.0015 element size is considered for the finite element analysis. Material and geometric properties of the beam are considered as L=0.4m, h=0.01m, b=0.05m, E_b=69GPa. Properties of piezoelectric patches are taken as c_{11}^{E}= 63GPa, e_{31}=-11.27, δ=0.15h, p_1 and p_2=0.02 and 0.025, respectively. These properties are used for all subsequent analysis except for the cases in validations, where appropriate system parameters are taken similar as in the referred results.

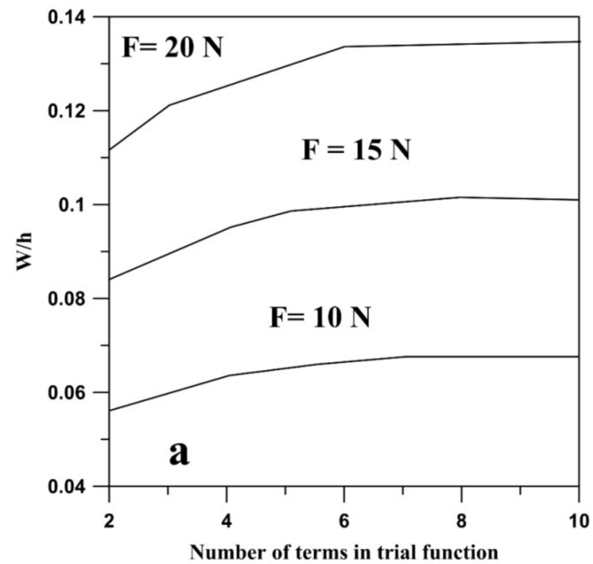


Figure (3.3): (a) Convergence study for the displacements at tip of a cantilever beam in terms of number of considered polynomial terms

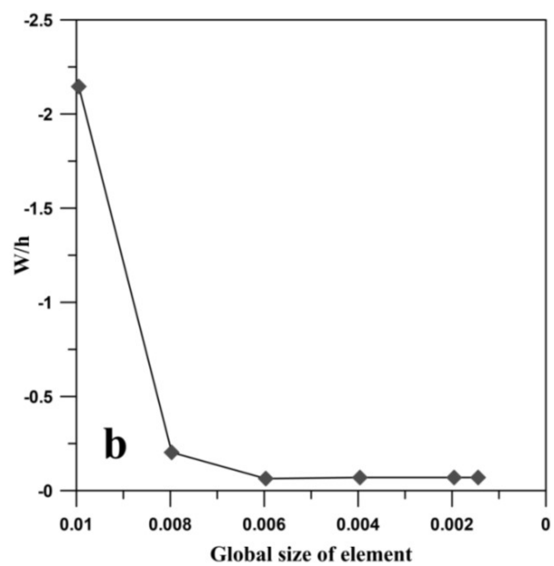


Figure (3.3) (b) Convergence study for displacements at tip of a cantilever beam for required global element size (ABAQUS)

3.3.1 Validation:

In order to ascertain the accuracy of the methodology, results obtained using present methods are compared with the result already available in literatures. As mentioned earlier, PZT patch is provided with an actuation voltage for a simply supported beam subjected to a static loading of 100 N. Crack depth is kept at 0.5 for all locations of crack along the length of the beam. The magnitude of the voltage is maintained at a level corresponding to which the moment at crack section is completely neutralized by the local reactive moment generated by PZT Patch. Using equation (3.16), required actuation voltage is obtained for different crack locations. An analysis is also carried out in ABAQUS 6.12 to determine the repair voltage under similar condition. The repair voltages thus obtained are compared to that of (Wang et al.(2002)) for different crack locations of the beam and is shown in Figure 3.4(a). Comparison shows that the results obtained by the present analysis match fairly well with that of the referred result.

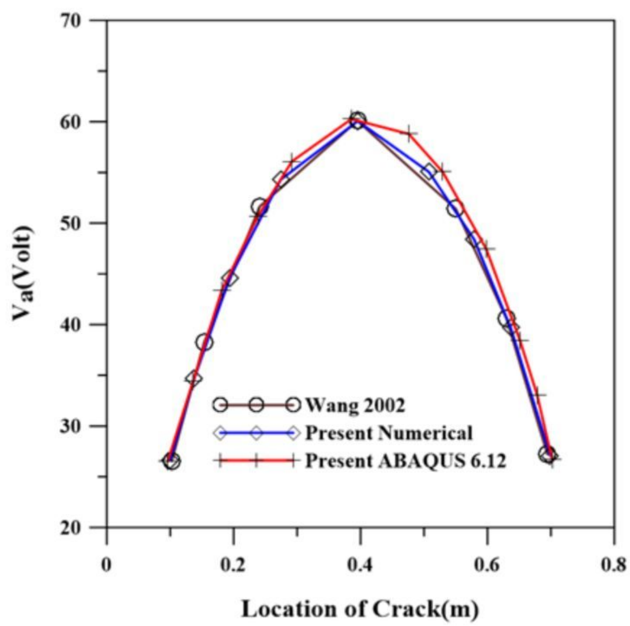


Figure 3.4(a): Actuation voltage required to repair crack at different locations.

The results obtained by the FEA and numerical analysis are validated with the experimental result obtained by Sumant and Maiti, 2006. Step load is applied on an Aluminum specimen having crack with different depth and at different location under three point bending condition. The change in voltage output in PZT sensor is plotted against crack position as well as for crack depths. The same study is carried out using FEA and numerical analysis. It is evident from Figure. 3.4(b) that results obtained by our methods matches well to that of experimental results.

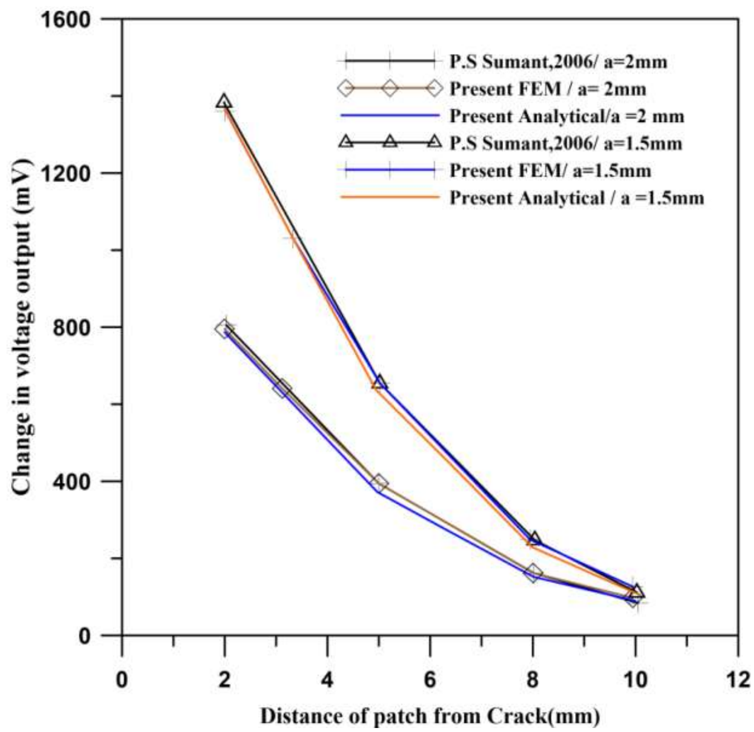


Fig 3.4(b): Comparison between the present procedures with the experimental study by Sumant and Maiti, 2006

3.3.2 Analysis of results: Transverse displacements of a mid cracked (L_1= 0.2 m) cantilever beam for various crack depth ratios (0.5 and 0.7) are obtained by numerical analysis and ABACUS. No actuation voltage (V_a = 0) is applied at the piezoelectric patch. Load is considered to be point load at the free end with magnitude of 10 N. Transverse deflection of the cantilever beam is shown in Figure.(3.5). It has been shown by researchers (Panigrahi and Pohit ,2016; .

Panigrahi and Pohit, 2019, Sinyeob et al., 2014) earlier in various articles that the crack introduces a local flexibility in the beam and hence the displacement of the cracked beam is higher compared to the intact beam. This statement can be verified from Figure.(3.5) where beam with higher crack depth ratio is deflected more in the transverse direction compared to the lower crack depth. A similar behavior of the beam is obtained from ABAQUS. Next, actuation voltage is provided to neutralize the effect of crack on deflection. The numerical value of actuation voltage is obtained using equation (3.16) of numerical analysis. It can be clearly observed that for a particular value of actuation voltage deformed profile of cracked beam is matching with the deformed profile of intact beam without any actuation voltage. The corresponding actuation voltages are found to be 618.86 and 600 respectively for numerical analysis and ABAQUS analysis. This phenomenon is presented in Fig. 3.6(a) for a crack depth ratio of 0.7 under a tip load of 10 N. Here, the deformed deflection profile of piezoelectric patched cracked beam with is plotted with the deformed deflection of the intact beam. Figure 3.6(b) shows a similar comparison for a crack depth ratio of 0.5 where the required voltages are 390V and 370V respectively for numerical and finite element analysis. Therefore, It can be culminated by saying that crack have been repaired.

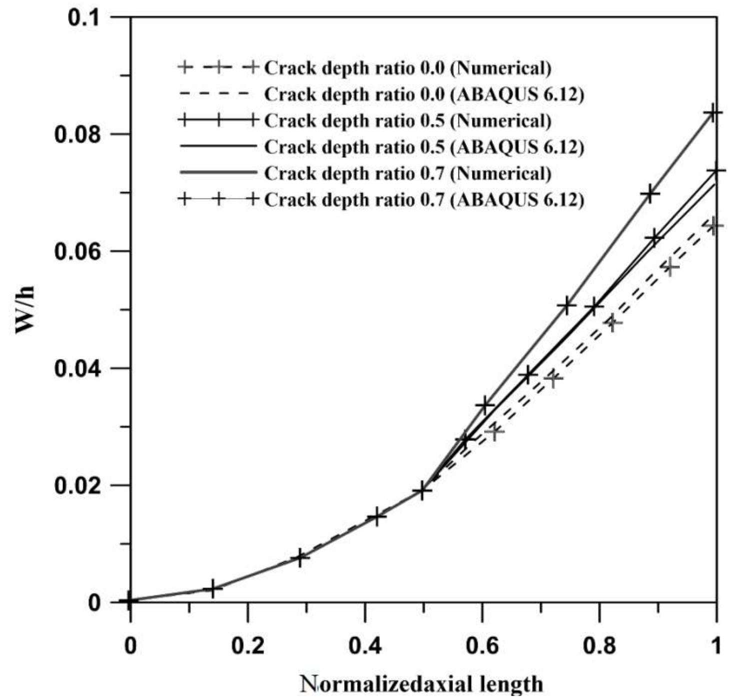


Figure (3.5): Transverse deflection of piezoelectric patched cantilever cracked beams with crack at mid.

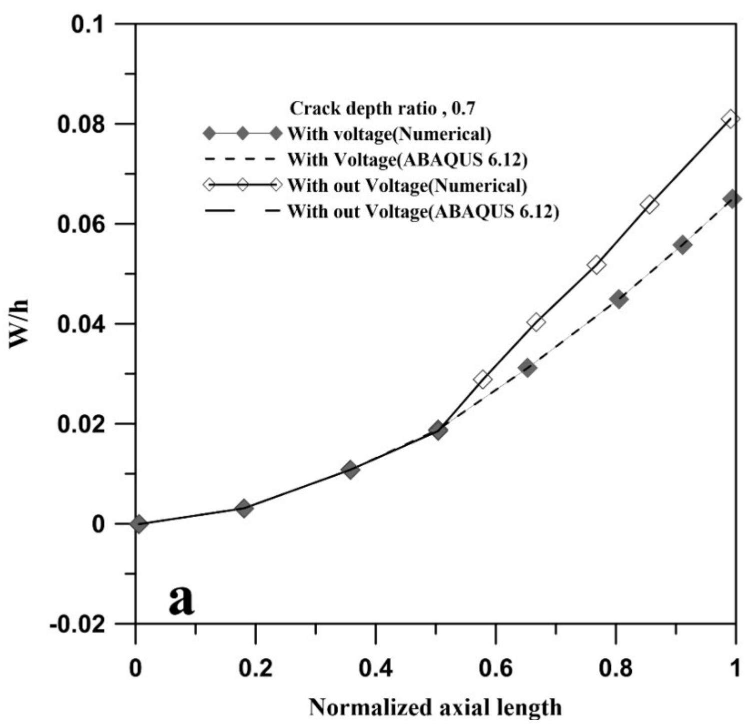


Figure (3.6(a)): Transverse deflection of piezoelectric patched Beam with crack depth ratio of 0.7

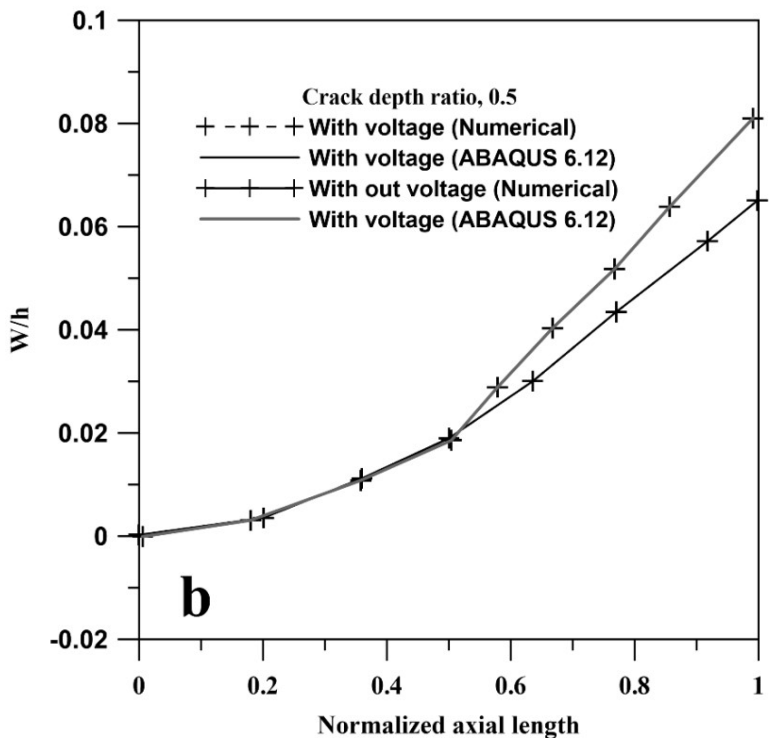


Figure (3.6(b)):Transverse deflection of piezoelectric patched Beam with crack depth ratio of 0.5

Fig.3.7(a) shows the rotational displacement profile of piezoelectric patched cracked cantilever beam for different crack depth ratios keeping the load same. A discontinuity in the rotational displacement can be clearly visible at the crack location. However, after applying the actuation voltage of 616.86 volts in case of numerical analysis and 600 volts in ABAQUS analysis for aforesaid discontinuity disappears for a crack depth ratio of 0.7 (Fig. 3.7b). When crack depth ratio is 0.5, corresponding actuation voltages are found to 390 volts in numerical analysis and 370 volts in ABAQUS (Fig.:3.7c). This observation supports the statement that crack have been repaired.

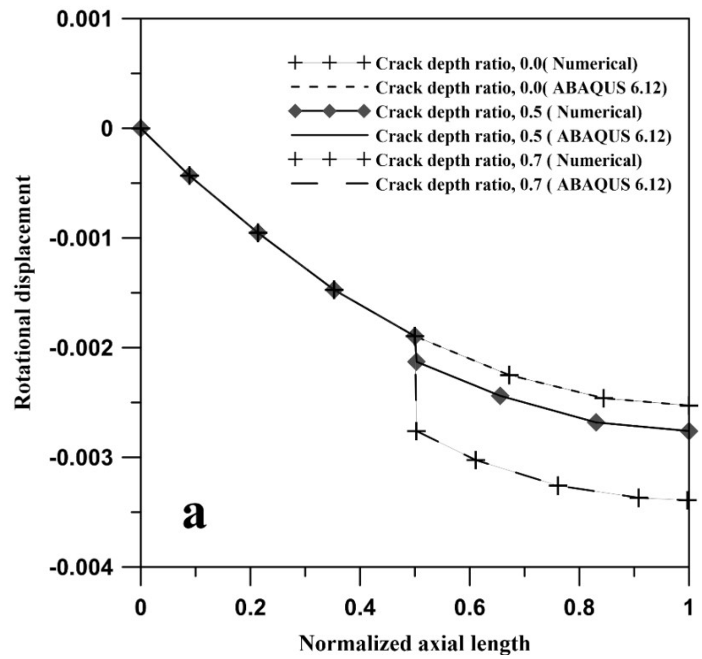


Figure (3.7(a)): Rotational displacement along the span of piezoelectric Patched cantilever beam having crack at mid span and cantilever beam without crack

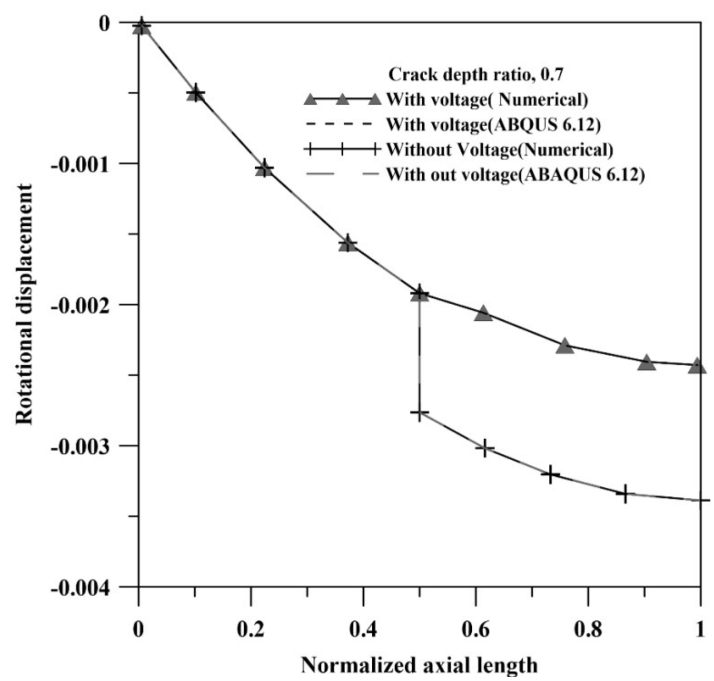


Figure (3.7(b)): Rotational displacement with actuation voltage of 616.86 (Numerical) and 600 (ABAQUS) for 0.7 depth ratio of crack.

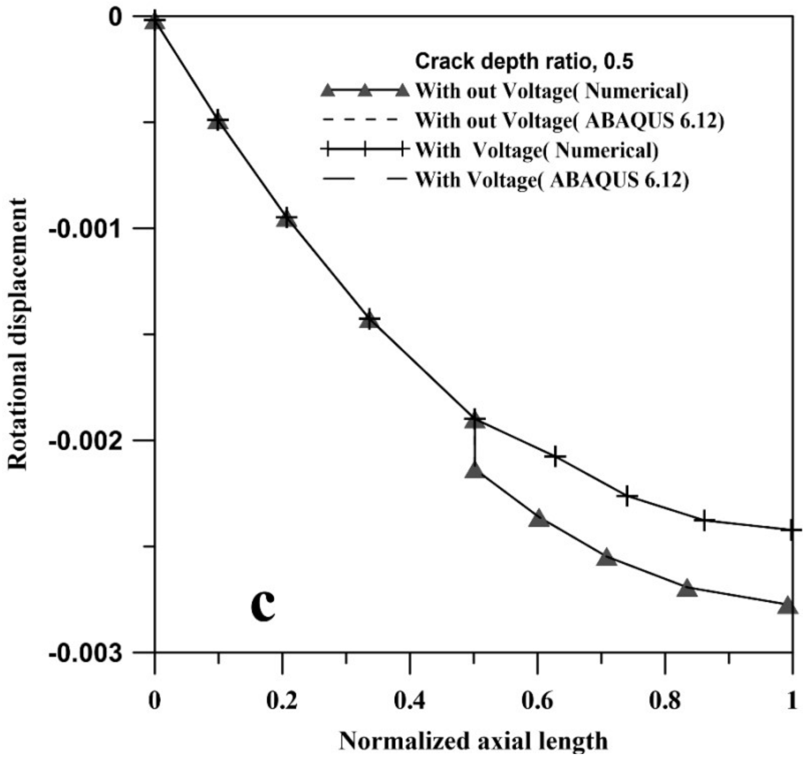


Figure (3.7(c)): Rotational displacement with actuation voltage of 390 (Numerical) and 370 (ABAQUS) for depth ratio of crack is 0.5.

Fig. (3.8) (a), variation in actuation voltage (v_a) required to repair the crack at various crack locations are reported. For point loading at the free end for a cantilever beam, variation in required actuation voltage varies linearly and approaches to zero when the crack is located near the free end. In the present repairing method the purpose is to diminish the discontinuity in rotational displacement which has arisen due to crack. Since the crack does not induce any discontinuity in rotational displacement curve near the open end of a cantilever and at the points of contra-flexures for a clamped-clamped beam, so, the required actuation voltages are zero for these two conditions.

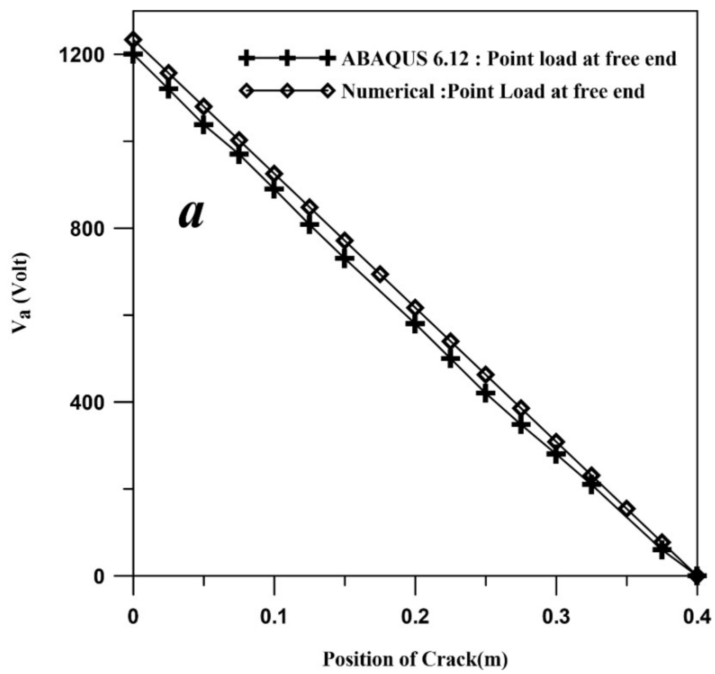


Figure (3.8(a)): Repaired actuation voltage (V_a) v/s crack location for cantilever beam.

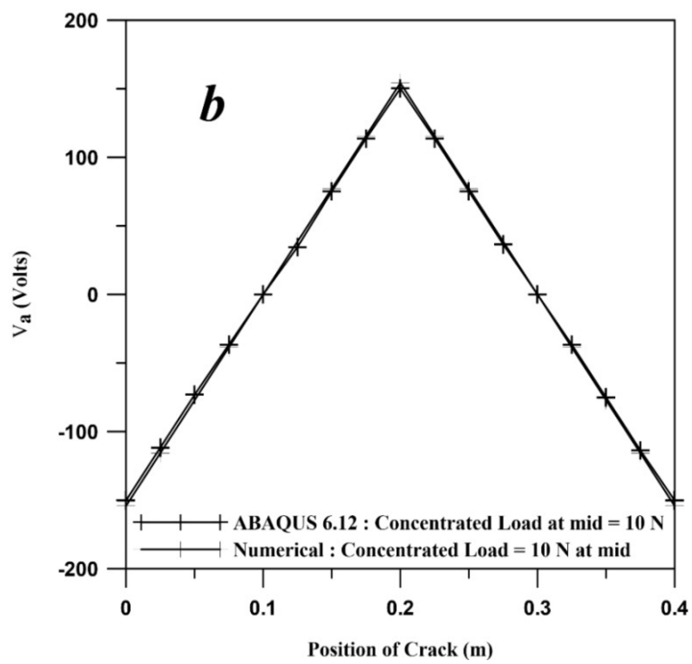


Figure (3.8(b)): Repaired actuation voltage (V_a) v/s crack location for clamp-clamp beam.

A similar plot is shown in Figure 3.8(b) for a beam with clamped-clamped boundary condition. Variation in actuation voltage to repair the crack follows the pattern of flexural moment. At the point of contra-flexure, required actuation voltage (V_a), becomes zero and afterwards it changes its sign. It has been shown by (Panigrahi and Pohit ,2016), that crack has insignificant influence on system response when it is located at any of the node or at point of contra-flexure. Present result justifies this statement by showing zero required voltage for crack repair at these locations. Other loading arrangements on a PZT patched cracked cantilever and clamp-clamp beams are also considered in Figure 3.9. Since the flexural moment is zero at free boundaries, for all the loading arrangements required voltage vanishes towards free ends of cantilever beam. However, variation in actuation voltage is parabolic for UDL, triangular and hat type of loadings. In case of clamp-clamp beam they follow the pattern of flexural moment as shown in Fig. 3.9(b). A study is carried out to determine the influence of patch thickness on actuation voltage. Variation of the required actuation voltage for various patch thickness ratio (δ/h), is shown in Figure 3.10(a) and (b), for cantilever and clamped-clamped boundary conditions, respectively.

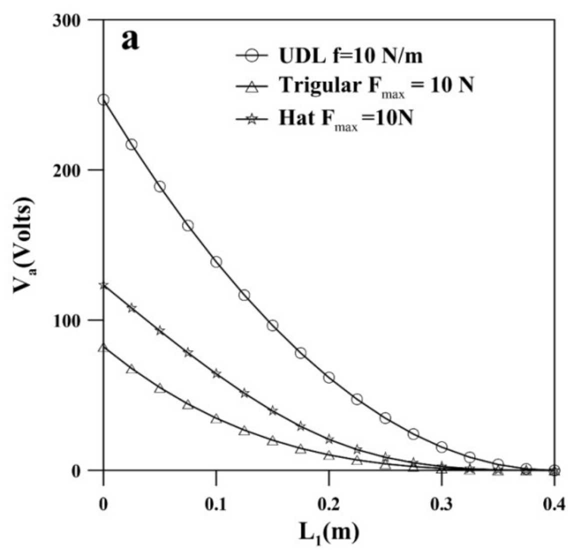


Figure (3.9(a)): Variation in actuation voltage (V_a) with crack location for cantilever cracked beam

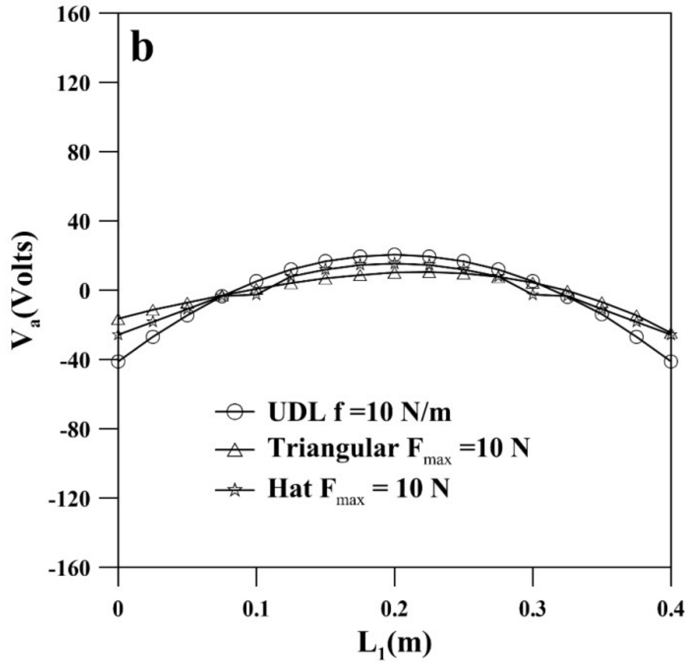


Figure (3.9(b)): Variation in actuation voltage (V_a) with crack location for clamped-clamped cracked beam.

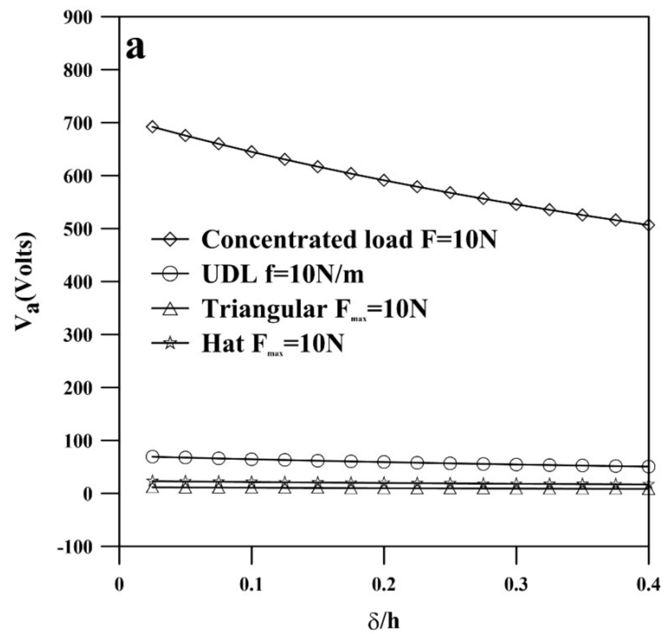


Figure (3.10(a)): Variation in actuation voltage (V_a) vs. patch thickness for cantilever beams with crack at the middle

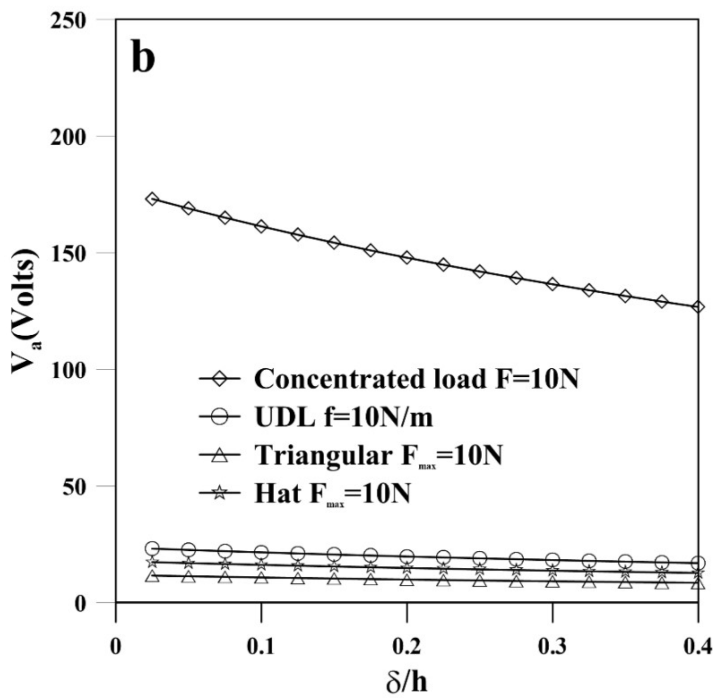


Figure (3.10(b)): Variation in actuation voltage (V_a) vs. patch thickness for clamped-clamped beams with crack at the middle.

3.4. Summary:

An integrated static model for PZT patched cracked beam is developed using Ritz method and rotational spring model to illustrate a methodology for the evaluation of damage due to crack in a beam and its repair of using PZT patch. An equation is proposed to compute the actuation voltage required for the repair of a crack. Initially, influence of the crack on the response of the beam is observed and subsequently, repair of crack is carried out by the actuation voltage to PZT patch. This patches induces an additional local reactive moments which are used to counter the crack influence. It is observed that the amount of actuation voltage required to repair the crack completely depends on the location of the crack. Actuation voltage for crack repair is maximum when crack is located in such locations where severity of the crack is maximum. There are some specific locations in the statically loaded beams where crack has insignificant influence on the systems response, for examples, point of contra flexure, free boundaries or any node of the deformed shapes. Therefore, there is no requirement of actuation voltage at such locations. The results obtained from the numerical analysis are compared with ABAQUS results. They are found to be matching fairly well. Finally, a detail parametric study on required actuation voltage for different loading arrangements and boundary conditions is provided, which will definitely be useful for practicing engineers and researchers working in the field of structural health monitoring. Variation of the required actuation voltage for various patch thickness ratio is also investigated. Here in the present research work the repairing is done by diminishing the discontinuity of rotational displacement but the method is not bothering about the recovery of dynamic response after repair. Hence, it is a drawback of the presently used repairing method.

IDENTIFICATION OF CRACK BY VIBRATION ANALYSIS AND RESTORATION OF DYNAMIC RESPONSE IN BEAMS USING PZT SENSOR/ACTUATOR

4.1. Introduction:

Machine elements weaken over time because of its frequent usage. Weakening of machine elements unfavorably affects the performance of the machine. Premature detection of such defects is essential for its treatment, reinforcement and restoration. This leads a great scope of research i.e. Crack or Damage detection in Machine Elements as well as in structural elements. Numerous sensible and stout non-destructive (NDE) techniques for crack detection at the primary stage have been proposed in published literatures, few of them are discussed as follows.

Free vibration analysis of cracked beam and healthy beam is frequently proposed as an effective non-destructive testing by researcher. The relevant literatures to the present chapter viz. Crack modeling, Energy Harvestation by Piezo electric patch, Apllication of piezoelectric patch for detection and repair of crack etc. are discussed in the Chapter 1 under 1.2 section.

Literature review clearly indicates the importance of smart material, like piezoelectric material in the field of crack detection as well as in repair. At the same time, it is pragmatic that the approached techniques by different researchers are not as simple as the usual modal analysis procedures. The present research work proposes a simple way to use a piezoelectric sensor to detect crack in beam type elements under harmonic excitation. A crack or any kind of damage considerably affects the local stiffness as a result the rotational displacement curve gets discontinued at the crack location. Though it’s an effective damage index but at the same time there is no conventional practical technique to check the discontinuity. In the present study, attempts are made to measure the same by employing a piezoelectric patch. Eventually it leads a novel way to detect the presence of crack. Here in the present study at first a mathematical model of a cracked beam based on Euler Bernoulli's beam theory is developed then the mathematical model of piezoelectric patch is tied with it. To compare the obtained results from the analytical analysis a finite element model of the same is developed in ABAQUS 6.12. Subsequently to validate the theoretical results an experimental setup is developed. Simultaneously another theoretical analysis is carried to reduce the effect of crack in a vibrating beam by actuating a piezoelectric patch.

4.2. Mathematical modeling, theoretical Simulation by ABAQUS and Experimental verification:

It is evident from the earlier researches that a crack causes a discontinuity of the rotational displacement curve at its location while the beam is under either static or dynamic loading. Hence measure of discontinuity in rotational displacement can be used to identify the location of the crack in beam type structures. It is found that for this purpose a piezoelectric patch is an efficient tool. When a piezoelectric (PZT) patch (where the polarization direction is along the thickness) is bent it gives a voltage output which is proportional to the difference in rotational displacement at its two ends. In case of a vibrating beam, difference in rotational displacement will be large at the location of crack. The piezoelectric patch attached to the beam just below the crack will produce considerably high voltage. So by attaching the piezoelectric patch at different position along the beam the voltage output can be observed. When the voltage output is considerably high, one can conclude that the crack is present at that location.

It is also observed that due to presence of crack the dynamic response of crack changes considerably. In the 2^{nd} part of the analysis, an attempt is made to reduce the effect of crack in a vibrating beam. It is observed that the presence of a crack in a beam considerably alters the dynamic response under harmonic excitation. Hence the purpose of the analysis is to see how effectively the dynamic response can be restored of a cracked beam can be restored to the healthy beam by actuating a piezoelectric patch properly. At first the response of a cracked beam under harmonic excitation is obtained. Subsequently, voltage output is also measured and compared with the data which is obtained for the healthy beam. Then to restore the dynamic response an external time varying voltage is applied on the attached piezoelectric patch.

4.2.1 Detection of crack:

Mathematical modeling of a cracked beam with attached piezoelectric patch, vibrating under harmonic excitation:

An analytical model of a beam with and without crack is developed and then the model is analyzed under external harmonic excitation. From the analysis the rotational displacement at different points are obtained and then by using Eq.4.12, the voltage output for the patch is calculated.In Figure 4.1, a cracked slender cantilever beam with attached piezoelectric patch is shown where L, h and Q are the length, depth and width of the beam, respectively. In the modeling, a crack of depth 'a' at a distance 'b' from the left support is considered. The incidence of the crack causes a discontinuity in the slope curve of beam and it is proportional to the bending moment at the location of crack .The position of patch is defined by the distances, x_{p1} and$x_{p2}$from left support (Fig.4. 1) of the beam. The depth of the patch is denoted byδ.

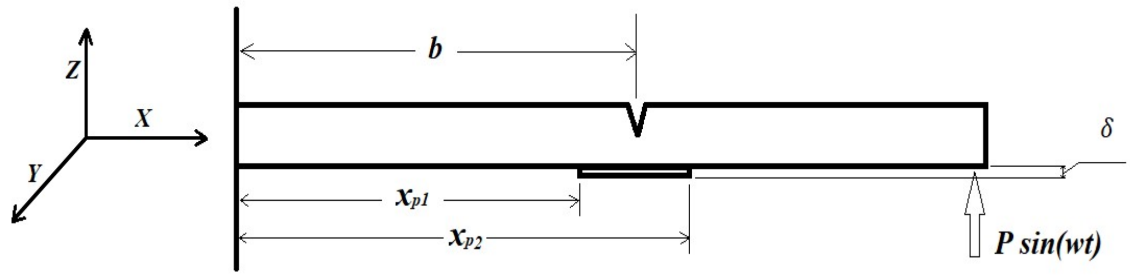


Fig.4.1: A beam with crack and attached with piezoelectric patch

Here, θ(x) denotes the slope, and the discontinuity is expressed as

$$\Delta\theta = SM \tag{4.1a}$$

$$\mathrm{S} = \frac{h}{\mathrm{EI}}\mathrm{F}(^{\mathrm{a}}/_{h}) \tag{4.1b}$$

$$F\left(^{a}/_{h}\right) = 2\left(\frac{^{a}/_{h}}{1-^{a}/_{h}}\right)^{2}\left[5.93 - 19.69\left(^{a}/_{h}\right) + 37.14\left(^{a}/_{h_b}\right)^{2} - 35.84\left(^{a}/_{h}\right)^{3} + 13.12\left(^{a}/_{h}\right)^{4}\right] \tag{4.1c}$$

Where S , $\mathrm{F}(^{\mathrm{a}}/_{h})$, M are the flexibility induced due to crack, Crack correction factor, Bending Moment respectively.

Beam is considered as two sub beams considering left and right side of the crack individually. The transverse displacements of the beam at time t, is presumed to be

$$w_1(\mathrm{x},\mathrm{t}) = w_1(\mathrm{x})\sin(\omega\mathrm{t})$$

$$w_2(\mathrm{x},\mathrm{t}) = w_2(\mathrm{x})\sin(\omega\mathrm{t})(2)$$

Where ω is the exciting frequency and $\{w_1(x,t), w_2(x,t)\}$are the transverse displacement function for left and right part of beam from the crack position.

Spatial function w_1(x) and w_2(x) are chosen as mentioned in equation 4.3(a) and 4.3(b), respectively.

$$w_1(x) = u_{nc}(x) + B_0 + B_1 x + B_2 x^2 + B_3 x^3\ ,\ 0 \le x \le b, \tag{4.3a}$$

$$w_2(x) = u_{nc}(x) + C_0 + C_1 x + C_2 x^2 + C_3 x^3\ ,\ b \le x \le L, \tag{4.3b}$$

In the above equation, u_{nsc} is the natural mode for healthy beam. Constants B_i and C_i are obtained using kinematic boundary conditions and compatibility conditions.

Since the beam is integrated with piezoelectric patch, expression of strain energy (U) can be obtained as mentioned below.

$$U = \frac{1}{2}\iiint_{V_b} \sigma_{xx}\varepsilon_{xx} dV + \frac{1}{2}\iiint_{V_p} \sigma^p{}_{xx}\varepsilon^p{}_{xx} dV + \frac{1}{2}\Delta\theta\ M \tag{4.4}$$

where $\sigma_{xx} = E_b\varepsilon_{xx}$, $\sigma_{xx}{}^p = C_{11}^E \varepsilon_{xx}{}^p + e_{31}E_3$

$E_b$is the modulus of elasticity of beam material, C_{11}^EYoung's modulus of patch material, e_{31} Stress constant at constant electric field. The electric field, E_3 can be given by

$E_3 = \frac{V_0}{\delta}$, where V_0 is the output voltage.

$\varepsilon_{xx} = z\frac{\partial^2 w}{\partial x^2}$, where z is considered along thickness and w represents transverse displacement.

Thus, the expressions for maximum strain energy can be obtained as

$$U_{max} =$$

$$\frac{1}{2}E_b I_b \int_0^b \left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 dx + \frac{1}{2}E_b I_b \int_b^l \left(\frac{\partial^2 w_2}{\partial x^2}\right)^2 dx + \frac{1}{2}C_{11}^E I_p \int_{x_{p1}}^b \left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 dx +$$

$$\frac{1}{2}C_{11}^E I_p \int_b^{x_{p2}} \left(\frac{\partial^2 w_2}{\partial x^2}\right)^2 dx + \frac{e_{31}V_0}{\delta} J_p \int_{x_{p1}}^b \frac{\partial^2 w_1}{\partial x^2} dx + \frac{e_{31}V_0}{\delta} J_p \int_b^{x_{p2}} \frac{\partial^2 w_2}{\partial x^2} dx \tag{4.5a}$$

Where $J_p = w\int_{-\frac{h}{2}}^{-\left(\frac{h}{2}+\delta\right)} zdz$, Q is the width of both the patch and beam. Kinetic energy expression can be obtained as

$$T = \int_0^Q \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^b \rho_b \left(\frac{\partial w_1(x,t)}{\partial t}\right)^2 dxdydz + \int_0^Q \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_b^l \rho_b \left(\frac{\partial w_2(x,t)}{\partial t}\right)^2 dxdydz + \int_0^Q \int_{-\frac{h}{2}}^{-\left(\frac{h}{2}+\delta\right)} \int_0^b \rho_p \left(\frac{\partial w_1(x,t)}{\partial t}\right)^2 dxdydz + \int_0^Q \int_{-\frac{h}{2}}^{-\left(\frac{h}{2}+\delta\right)} \int_b^l \rho_p \left(\frac{\partial w_2(x,t)}{\partial t}\right)^2 dxdydz \tag{4.5b}$$

where ρ_b, ρ_p are the densities respectively for the beam and patch material

Potential of harmonic excitation can be considered as $V = P \sin(\omega t) u$ (4.6)

The bending vibration of a uniform healthy Euler Bernoulli beam can be expressed as

$$u_n''''(x) - \beta_n^4 u_n(x) = 0 \tag{4.7}$$

The general solution of the equation (7) is considered as

$$u_n(x) = D_1 Sin\,(\beta_n x) + D_2 Cos\,(\beta_n x) + D_3\, Sinh\,(\beta_n x) + D_4\, Cos\, h(\beta_n x) \tag{4.8}$$

Mode shape of healthy cantilever beam and Clamped-clamped beam can be obtained by applying the known boundary conditions in to the equation (4.8)

They are presented in equations 4.9(a) and 4.9(b).

$$u_{n(x)} = D_n \left[Sin(\beta_n x) - Sinh(\beta_n x) - \frac{Sin(\beta_n L) + Sinh(\beta_n L)}{Cos(\beta_n L) + Cosh(\beta_n L)} \{Cos(\beta_n x) - Cosh(\beta_n x)\} \right]$$

(4.9a)

$$u_{n(x)} = D_n \left[Cosh(\beta_n x) - Cos(\beta_n x) + \frac{Cosh(\beta_n L) - Cos(\beta_n L)}{Sin(\beta_n L) - Sinh(\beta_n L)} \{Sinh(\beta_n x) - Sin(\beta_n x)\} \right]$$

(4.9b)

The natural frequencies are $\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}}$

The eight unknown constants for each type of beam in equations (4.3a) and (4.3b) are evaluated by imposing boundary conditions and compatibility conditions as follows.

For cantilever beam:

At the fixed end, $x = 0, w_1(0) = 0 , w_1'(0) = 0$

At the open end, $x = L,\ w_2''(L) = 0 , w_2'''(L) = 0$

For clamped beam:

At the fixed end, $x = 0, w_1(0) = 0 , w_1'(0) = 0$

At the another fixed end, $x = L,\ w_2(L) = 0 , w_2'(L) = 0]$

For both the beams:

At the location of crack, $x = b, w_1(b) = w_2(b),\ w_1''(b) = w_2''(b) , w_1'''(b) = w_2'''(b)$

Considering the extra flexibility which induces at location of crack as mentioned earlier,

$$w_2'(b) - w_1'(b) = hF\left({}^{a}/_{h}\right) w_2''(b)$$

Thus the spatial functions are obtained satisfying the boundary conditions at the ends and compatibility condition at the location of crack.

For cantilever beam,

$w_1(x) = u_n(x) + [L u_n'''(L) - u_n''(L)]x^2 - \frac{u_n'''(L)}{6} x^3$ (4.10a)

$$w_2(x) = u_n(x) + \left[\left(\frac{5L}{2}b^2 u_n'''(L) - \frac{3}{2}b^2 u_n''(L)\right) - bQF\left({}^{a}/_{h}\right)\{u_n''(b) + Lu_n'''(L) - u_n''(L)\}\right]$$

$$+ \left[hF\left({}^{a}/_{h}\right)\{u_n''(b) + Lu_n'''(L) - u_n''(L)\} - 3bLu_n'''(L) + 2bu_n''(L)\right]x + \left[\frac{Lu_n'''(L) - u_n''(L)}{2}\right]x^2$$

$$- \frac{u_n'''(L)}{6}x^3$$

(4.10b)

For clamped beam

$B_0 = 0\ , B_1 = 0, B_2 = C_2\ , B_3 = C_3\ , C_0 = -C_1 b$

$$C_1 = hF\left({}^{a}/_{h}\right)\left[u_n''(b) - 2L\left\{\frac{hF\left({}^{a}/_{h}\right)u_n''(b) + u_n'(L)}{2hF\left({}^{a}/_{h}\right) + L}\right\} - \left\{\frac{2L\left(6hF\left({}^{a}/_{h}\right)b + 3L^2\right)}{2hF\left({}^{a}/_{h}\right) + L} - 6b\right\}C_3\right]$$

$$C_2 = -\frac{hF\left({}^{a}/_{h}\right)u_n''(b) + u_n'(L) + \left\{6hF\left({}^{a}/_{h}\right)b + 3L^2\right\}}{2hF\left({}^{a}/_{h}\right) + L}C_3$$

(4.10c)

Functions obtained in equations (4.10a),(4.10b) and (4.10c) are substituted in the expression of Potential and Kinetic energy expressions given in equation 4.5(a) and 4.5(b) respectively and optimizing the energy functional (U_{max}-T_{max}) following classical Ritz method, one may obtain the algebraic equation in the form of

$$\left(K - \omega^2 M\right)\{UC\} = \{F\} \qquad (4.11)$$

In equation (4.11), K and M are stiffness and mass matrices and UC is unknown vector and F is force vector. Since in the considered trial function for cantilever beam, all the unknown constants are already evaluated by imposing the boundary conditions and the kinematic conditions, therefore only the constant 'D_n'(used in equation 4.9(a),4.9(b)) which has association with the Eigen function of the beam, requires to be determined and can be obtained by solving equation (4.11). But for the clamped-clamped beam two unknowns 'D' and 'C_3' there, which are determined by optimizing (U_{max}-T_{max}) and by solving equation (4.11)

Once all the unknown coefficients are determined for a certain crack location and depth, the same can be used to get the slope at different locations along the length of the vibrating beam.

Subsequently, the estimated values of slope are put in the mathematical model of a piezoelectric patch to determine the voltage output which is shown in equation (4.12)[Zhao et. al. (2016)]

$$V_o = \frac{-e_{31}(x_{p2}-x_{p1})(\delta+h_b)}{2C_v}\int_{x_{p1}}^{x_{p2}}\frac{d^2u}{dx^2} \qquad (4.12)$$

In equation (12), $C_v$is the capacitance.

Hence by using this analytical model the voltage output from the piezoelectric patch at different point along the length of the vibrating beam can be determined. Then by observing the peak in voltage output one can locate the position of crack.

Finite element analysis by using ABAQUS:

In this section, a similar analysis is carried out in ABAQUS platform to compare the results obtained by both methods *i.e* analytical and finite element analysis (FEA).Linear dynamic analysis under harmonic excitation is undertaken.First, an aluminum beam having the properties as mentioned in Table1 is modeled in ABAQUS. For that, a 3D solid model consists of 8 Noded brick element based on linear elasticity is considered. Crack is introduced in the model by assigning Seam from special interaction in ABAQUS. Then a PZT 5H patch is assembled with the beam by Tie interaction. The properties of the patch are also given in Table 4.1.

The model is simulated considering cantilever as well as clamped boundary conditions. In the both cases, a fixed boundary condition is used to fix the end. Another electrical boundary condition is used on the mating surface of beam and patch. The polarization direction is defined by creating datum. Output voltage from the patch due to deformation of the beam is obtained by field output request in ABAQUS platform. A modal analysis is performed with a harmonic excitation $F = 0.6\,Sin(\omega t)$ to get the voltage v/s time characteristic during steady state vibration.

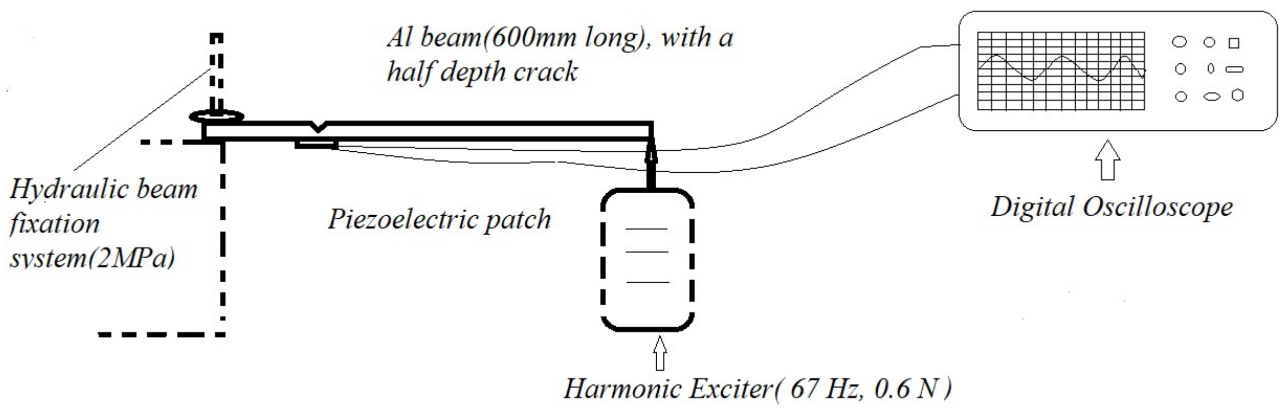
The rotational displacement along the length of the beam is obtained for healthy as well as cracked beam under different boundary conditions and different exciting frequencies. The comparison between the rotational displacement for healthy and cracked beam is presented in the result section. At the same time, voltage output from the patch placed at different positions along the length under different boundary conditions and different exciting frequencies are obtained. Here it is observed that the peak in voltage curve comes at the same point where the discontinuity is in the rotational displacement curve. The discontinuity in slope curve comes due to the presence of crack hence the peak in voltage is observed at that point. This peak in voltage indicates the location of the crack.

Table.4.1 Property table

Length(mm)	600
Depth(mm)	10
Width(mm)	50
E_{11}(N /mm^2)	0.69×10^5
E_{33}(N/mm^2)	
ϑ	0.29
G(N/mm^2)	
d_{31}(m/V)	

Experimental Verification:

The theoretical results are validated with experiment to ascertain the efficacy of the numerical procedure adopted in the present investigation. Accordingly, an experimental setup is developed consisting of Digital oscilloscope, Exciter, Piezoelectric patch and aluminium beam (with crack/ without crack).The arrangement is shown in Figures 4.1(a) and (b).



(a)

Fig.4.2 (a) A graphical representation of the experimental setup

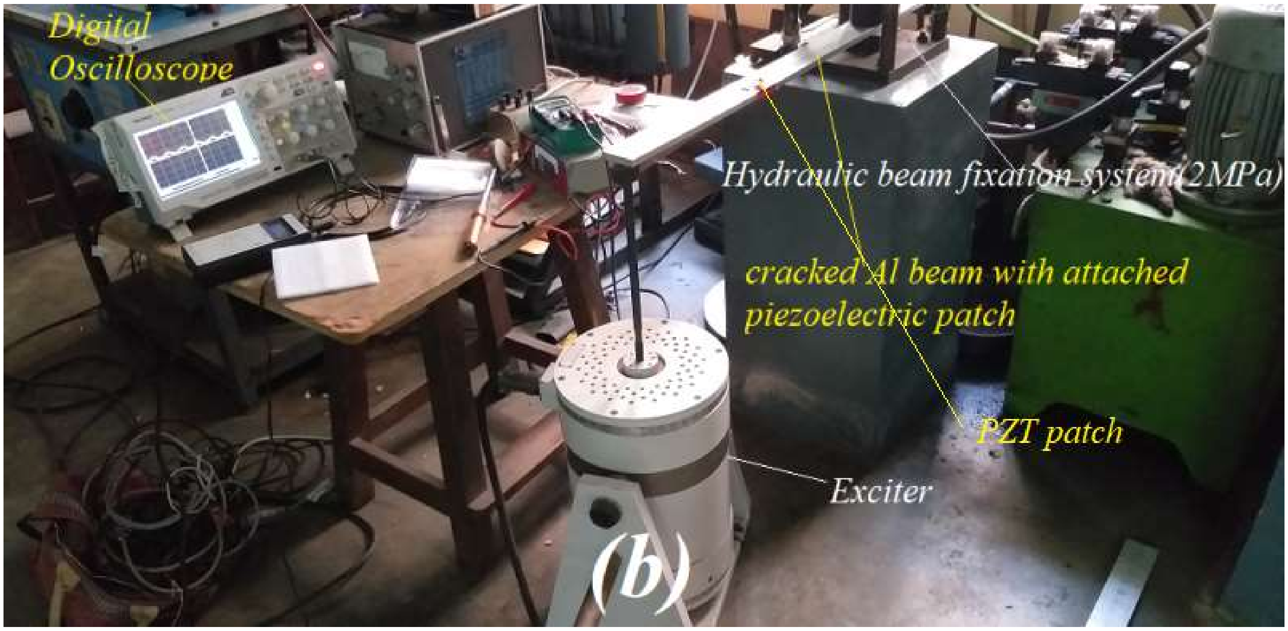


Fig.4.2 (b) Photograph of the experimental setup

Four numbers of aluminium bars are taken with the dimensions as mentioned in Table.4.1. Three cracks are cut at a distance of 50 mm (0.25 normalized distance), 300 mm (0.50 normalized distance) and 420 mm (0.7 normalized distances) from one end of the bar. At first one of the ends of a healthy beam (un cracked beam) is fixed by the hydraulic ram under 2MPa and the open end is excited with 0.6 N at67 Hz. A wired piezoelectric patch (Table.4.1) is bonded at the bottom of the beam by glue at locations where the cracks are cut in other beams. For the three different locations of patch, the voltage is obtained from the oscilloscope. Similarly the patch is bonded below the crack on three cracked beams and correspondingly the voltage output is measured through the oscilloscope.

4.2.2 Restoration of dynamic response of a vibrating beam by actuating a piezoelectric patch:

As mentioned earlier, an attempt is taken to repair a cracked cantilever beam in addition to crack identification. It has been observed that due to the presence of crack the response of the cantilever beam to dynamic loading changes significantly. If somehow the response of the cracked beam is restored, *i.e.* the response of the cracked beam is brought back to that of a healthy beam, then it can be inferred that the beam element has been repaired.

As before, an aluminium beam element with a PZT patch is considered for analysis. The dimensions and geometric properties are considered as same as that of previous case (Table 4.1). The analyses are performed by using the step: dynamic Implicit in ABAQUS CAE. At first a healthy cantilever beam is with a tied PZT patch at 150 mm from fixed end. It is subjected to a harmonic loading of 5N at 25.45 Hz at open end. The response analysis is carried out in and the acceleration of the tip of the beam in the transverse direction against time is obtained. At the same time, voltage output versus time plot is also captured. Next a cracked beam (crack at 150 mm from fixed end) is subjected to the same procedure. As in the both cases the beam is vibrating near to its natural frequency (22.23 Hz), a beat phenomenon is observed. Now to repair the beam a trial and error procedure is followed, where at first the difference between voltage outputs obtained from the PZT patch for the above two analyses are calculated and the differences are multiplied by a factor, which is termed as voltage factor. The obtained time varying voltage is applied on the PZT patch to actuate it and the response of the cracked beam is obtained.

4.3. Results and discussion:

The results of the present study are described in the following section. The details of the experimental analysis are presented in Section 3.1. Subsequent sections deals with the different aspects of crack identification and repair.

4.3.1 Experimental verification and comparison of few results between analytical, finite element analyses, experimental:

The purpose of this experimental work to find the difference in voltage output from the piezoelectric patch attached to the vibrating cantilever beam at different points along the length of the beam. This change in voltage will be considerably high in magnitude near the crack due to the discontinuity in rotational displacement. In the theoretical study the voltage is collected for several numbers of positions of crack. However, in case of experimental work it is limited to only 3 positions of crack.

Three cracked beams having crack at a distance from 150 mm, 300 mm and 420 mm from the fixed end are considered. The depth of the crack is half of the beam thickness. For each beam, a piezoelectric patch is attached just below the crack. Voltage is measured through digital oscilloscope while beam is vibrating at 67 Hz harmonic excitation with 0.6 N forces. Next, the same experiment is carried out with the healthy cantilever beam. Three sets of output voltages are obtained for the healthy beam with piezoelectric patch placed at 150 mm, 300 mm and 420 mm from the fixed end. The healthy beam is also allowed to vibrate at 67 Hz harmonic excitation with 0.6 N forces. Similar sets of results are captured using numerical analysis and ABAQUS. They are shown in Table 4.2 along with experimental results.

It is observed that the results from numerical model and ABAQUS model match pretty well with that of experiment. It is observed that at some points the theoretical voltage is negative while the experimental result shows positive value. In experimental study, the output voltage is considered as the peak of sinusoidal voltage v/s time graph and hence only the magnitude is in focus. But in theoretical analysis the voltage is obtained following the difference in rotational displacement. As a result when the difference in rotational displacement at the two ends of patch is negative, the voltage will also be negative.

Table.4.2 Comparison of results in between FEA, Analytical and Experimental

Crack location (mm)	Patch Location (mm)	Excitation Frequency (Hz)	ABAQUS (V)	Analytical Model (V)	Experiment (V)
No Crack	150	67	0.21451	0.215	0.2
150	150	67	0.595	0.598	0.5
No Crack	300	67	-0.27412	-0.276	0.25
300	300	67	-0.967413	-0.968	0.9
No Crack	420	67	-0.38341	-0.385	0.3
420	420	67	-1.19293	-1.21	1

From the above investigation, it can be inferred that numerical model and ABAQUS model are capable of simulating crack beam with piezoelectric patch for further studies.

As seen from Fig.4.3 (a), the voltage output of the PZT patch corresponding to the peak deformed condition is 0.5V200 mV. Subsequently a cantilever beam having a half depth crack at 150 mm from the root is studied by attaching the patch below the crack and corresponding voltage output is measured approximately 0.8V500 mV (Fig. 4.3(b)). In order to compare the obtained voltage response from oscilloscope, the same is obtained for the steady state analysis in ABAQUS. Fig.4.4 (a) & 4.4(b) show that the voltage outputs from the PZT patch are0.45V and 0.75V, respectively for healthy and a cracked beam when the analysis is carried out in ABAQUS under the same conditions of experimental study. Thus, the analysis methodology is validated with the experimental study.

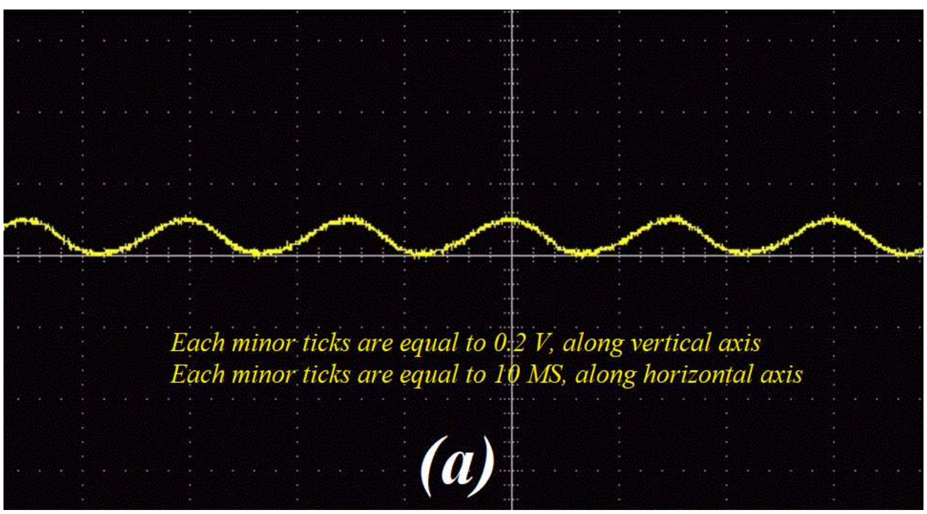


Fig. 4.3(a) Voltage response of healthy beam

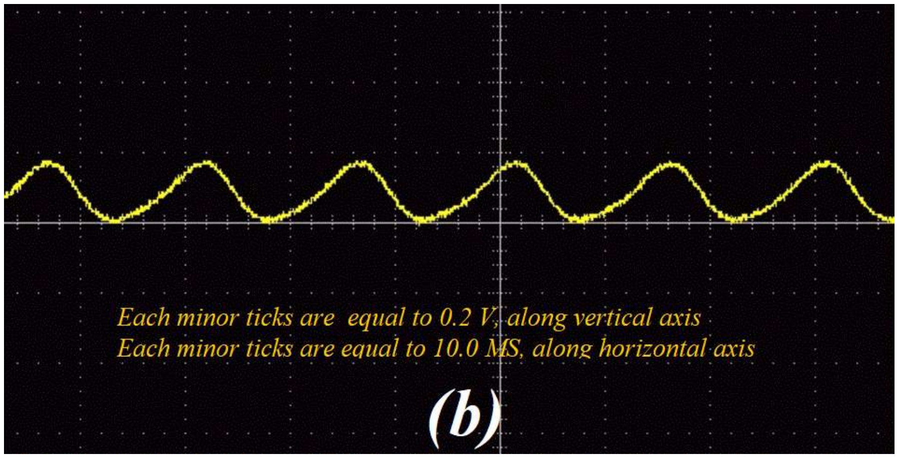


Fig. 4.3(b) Voltage response of cracked beam

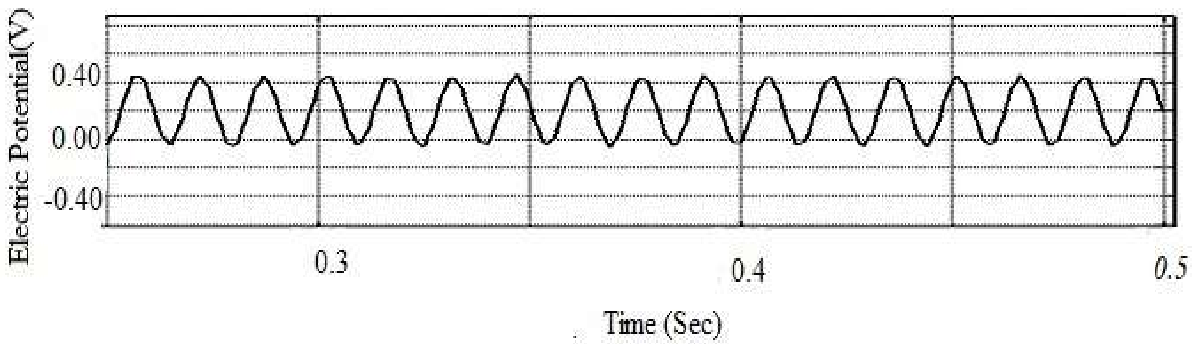


Fig. 4.4(a) Voltage response of healthy beam (ABAQUS 6.12)

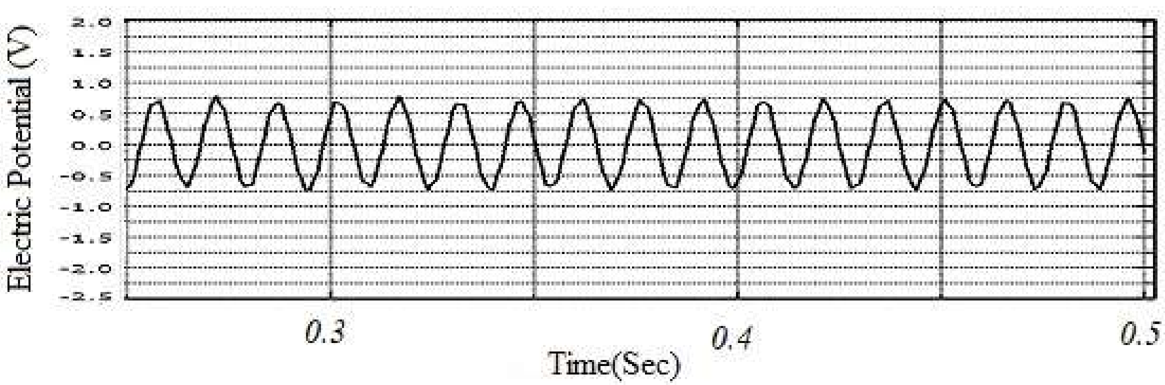


Fig. 4.4(b) Voltage response of Cracked beam (ABAQUS 6.12)

4.3.2 A study of change in rotational displacement and an approach to measure the same in a cracked beam under harmonic excitation:

This section presents how effectively a piezoelectric patch can be used to find the discontinuity in rotational displacement curve of a beam type structure leading to identification of the location of crack.

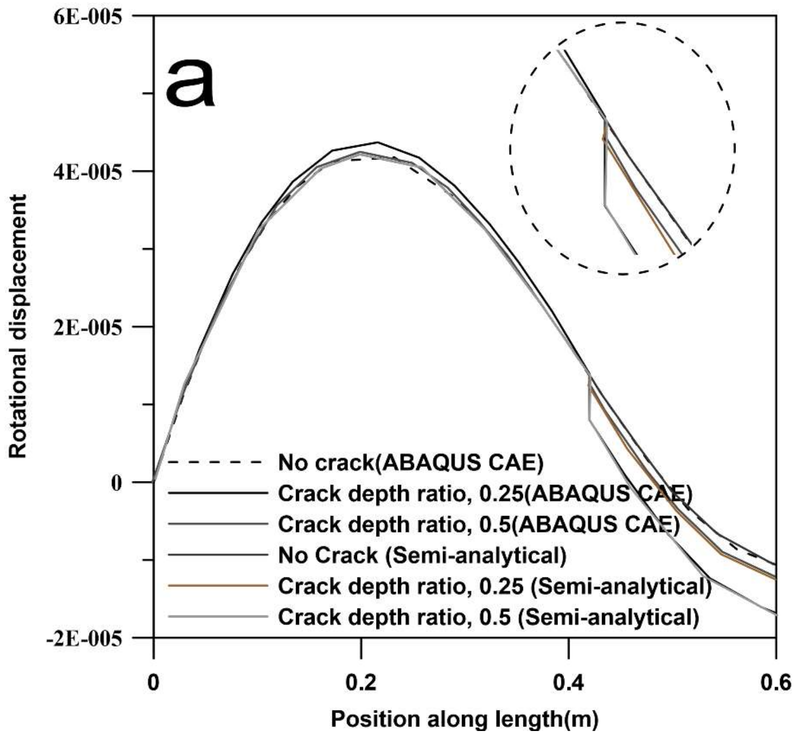


Fig. 4.5(a) Discontinuity in rotational displacement (cantilever)

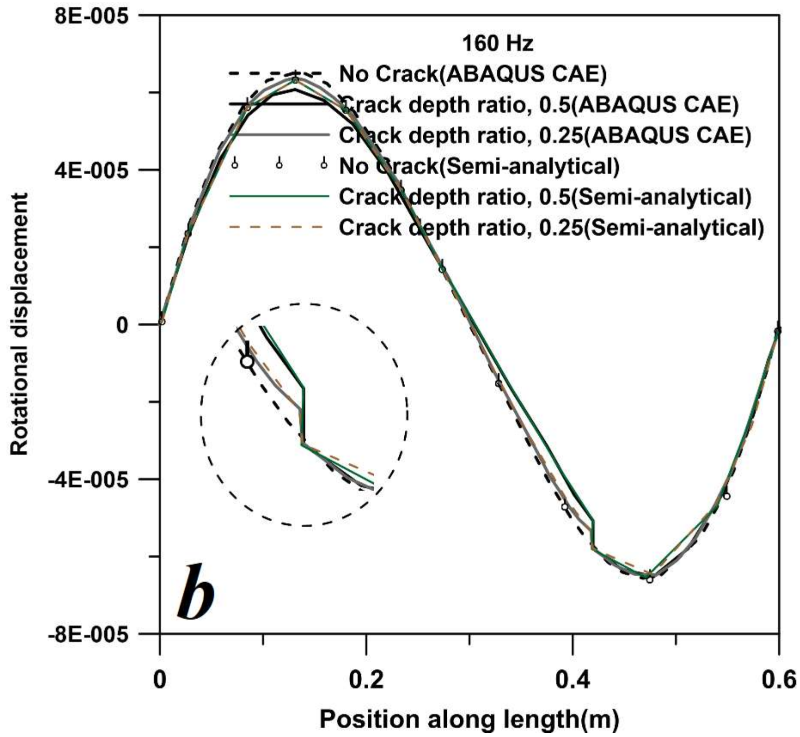


Fig. 4.5(b) Discontinuity in rotational displacement (clamped beam)

At first a healthy beam under both the boundary conditions i.e. cantilevers as well as clamped-clamped is excited under an external force excitation. The rotational displacements at different points along the length of the beam are obtained. In case of cantilever beam, the crack is considered at a distance 420 mm from the root and the open end of the beam is excited at 67 Hz harmonic excitation with 0.6 N forces. Another beam is also analysed under both end fixed

condition while vibrating at 160 Hz with 0.6 N forces, acting at the middle of the beam. The crack is located 420 mm from left end. In both the cases two different crack depths viz. 5mm, 2.5 mm, are considered. It is observed from Figures 4.5(a) and 4.5(b) that there is a sharp discontinuity in the curve of rotational displacement proportional to crack depth at the location of crack. Nevertheless, rotational displacement is an efficient index to detect the crack. However, there is no conventional way to check the same in a vibrating beam.

It is known that the voltage output from the piezoelectric patch (polarization direction along thickness) under bending depends only on the difference in rotational displacement at its two ends. As a result this property of the piezoelectric patch can be easily utilised to find the discontinuity in rotational displacement of a vibrating beam. With this notion, the same cracked and healthy beams are again studied with an attached piezoelectric patch at the locations at a time where rotational displacements are measured earlier. The corresponding output voltages from the patch are measured for different patch locations for the cracked and healthy beams.

As depicted in Figures 4.6 (a) and (b), there is a peak in voltage near the crack, which is at 420 mm from the root of cantilever beam or the left end of clamped-clamped beam. This peak in voltage clearly indicates the location of crack. It is observed in case of cantilever beam that the voltage output starts to be negative after the point from where the slope of the Fig. 4.5(a) turns negative. The same nature is also seen in case of clamped beam and it is very obvious as per the characteristic of the piezoelectric patch.

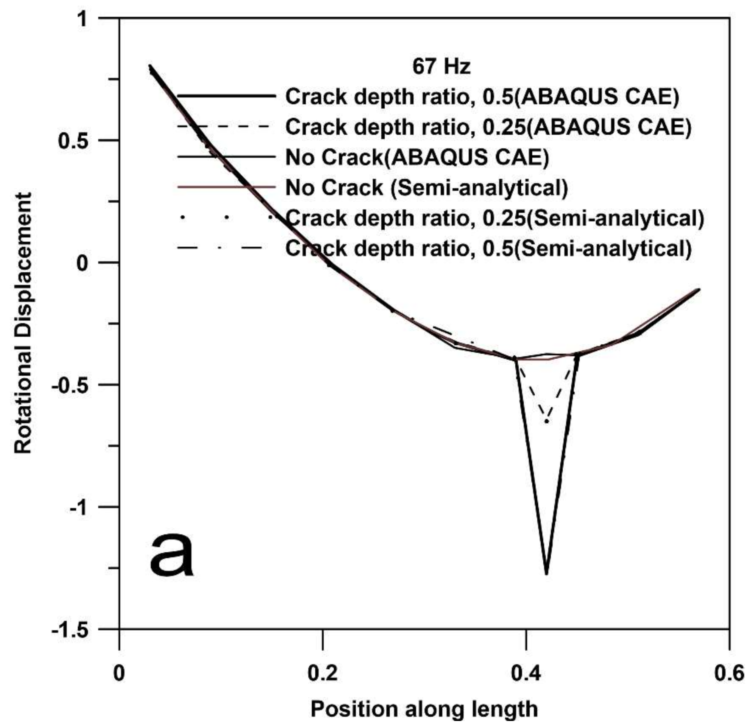


Fig.4.6 (a) Voltage output for Cantilever beams

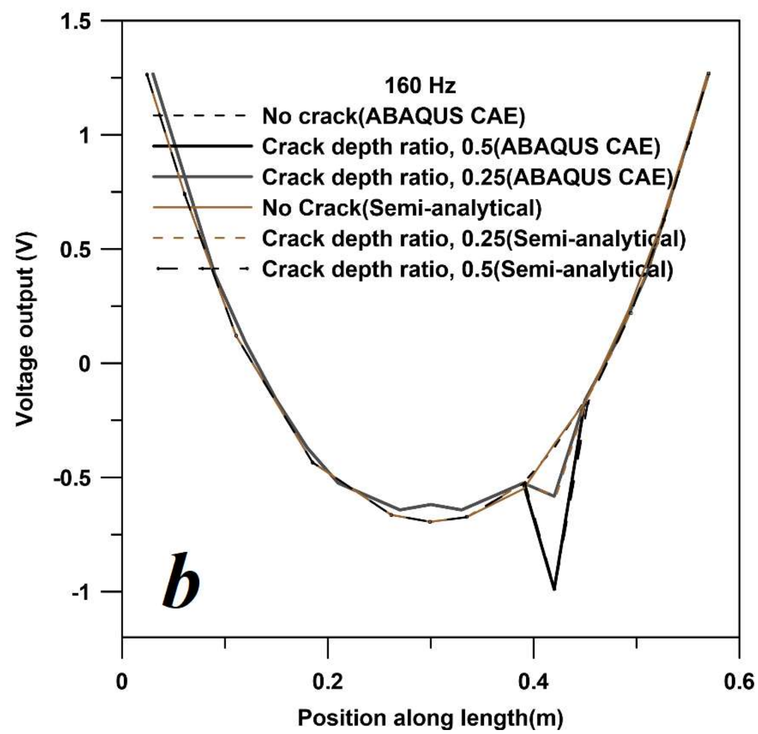


Fig. 4.6(b) Voltage output for clamped beams

Though Fig.4.6 clearly indicates a way to identify crack of different depths, it may not be true always as shown in the investigation presented in the next section. In fact a thorough study on the difference in voltage output of the patch, between cracked and healthy beam for different locations of crack is very much essential.

4.3.3 A study of change in voltage output of the patch, between healthy and cracked beam, considering different crack locations and depths:

It is not certain that the peak at a particular location of crack will be observed for other locations as well. To make it clear, behaviour of beams are studied for different crack locations along the length of the beam, to see whether the peak in PZT voltage is occurring for all the crack locations. Here crack depth of 2.5 mm and 5mm are considered at a gap of 30 mm starting from fixed end of the beam. For each of the crack position, PZT patch is attached just below the crack locations and the voltage is obtained while the beam is vibrating under the constant external excitation. One crack is considered at a time and the voltage is obtained for the both conditions i.e. for healthy beam and after the cracked beam. The difference between these two voltages is plotted against the normalized crack locations to see the response of crack at different locations. The modulus of output voltage differences between cracked and healthy beam is plotted against normalized axial length to see the response of crack at different locations under three different harmonic excitations having frequencies 35 Hz, 67 Hz and 1200 Hz. From the experimental results shown in Fig.4.3, it is clearly observed that the magnitude of difference in voltage is significant in the present case rather than the sign. Therefore, modulus of change in voltage output is focused in the present study. When the beam is vibrating at 35 Hz, Fig.4.7 (a) depicts that there is hardly any change in voltage for the crack located at 0.65 from fixed end regardless the depth of crack. A similar behaviour is noticed in Fig.4.7 (b) for the crack located at 0.35 from fixed end and the beam is vibrating at 67 Hz.Fig.4.7(c) represents a different nature. There are three locations at which the effect of cracks is almost zero. However, considerable voltage changes in the form of peaks are observed at other crack locations including the cases of Fig. 4.7(a) and (b) *i.e.* 0.65 and 0.35 from the fixed end.

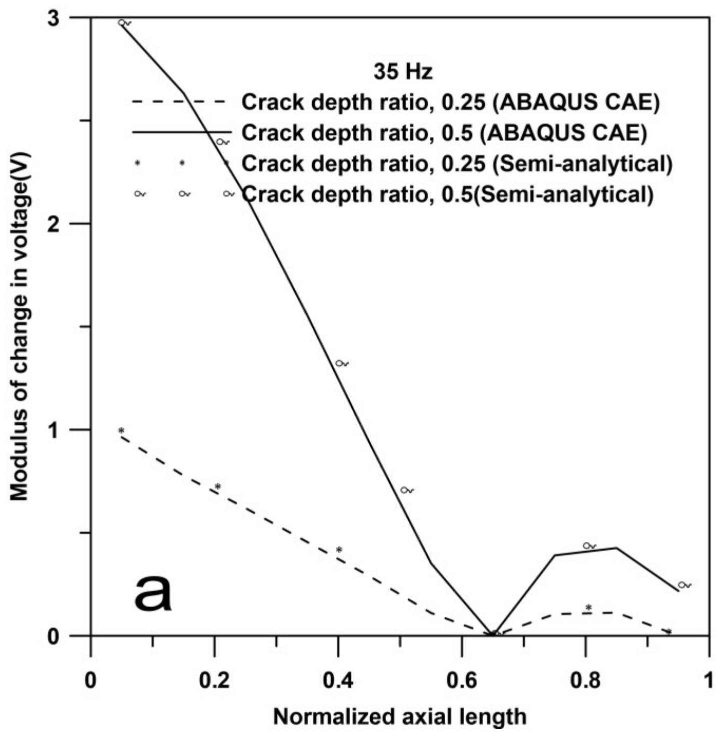


Fig.4.7 (a) Modulus of difference in voltage output for cantilever beam (35Hz)

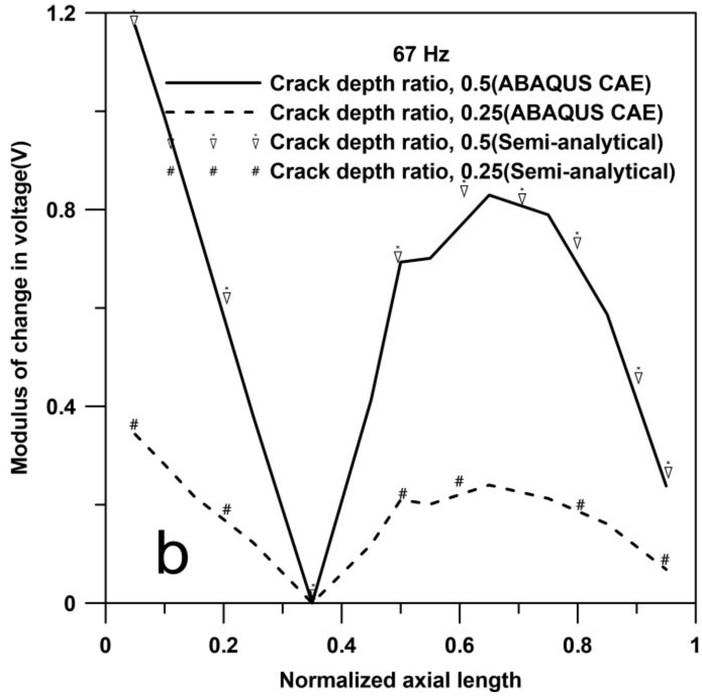


Fig.4.7 (b) Modulus of difference in voltage output for cantilever beam (67Hz)

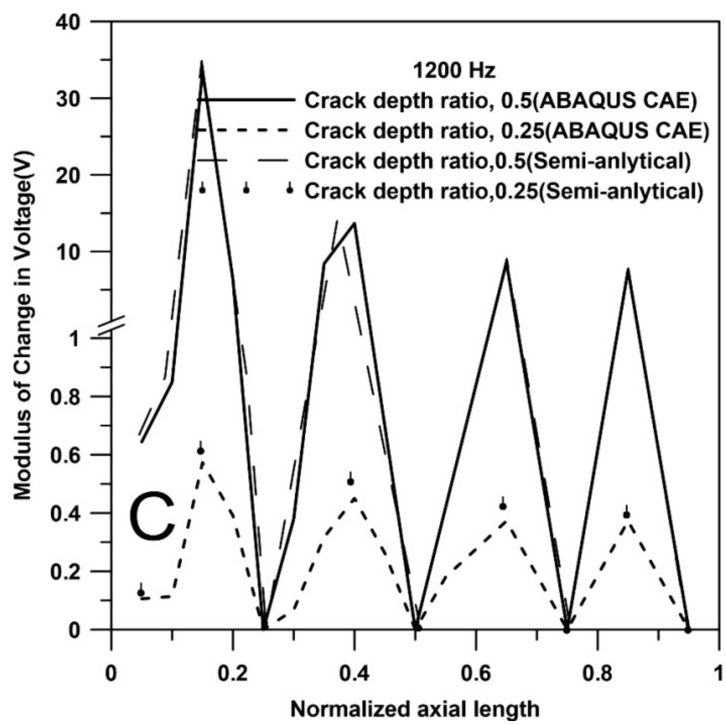


Fig.4.7(c) Modulus of difference in voltage output for cantilever beam (1200Hz)

This kind of behaviour of zero voltage change occurs due to the fact that at these points slope of the beam becomes zero and hence effect of crack on rotational displacement is negligible. This is evident from the Fig. 4.8 (a) to (c).

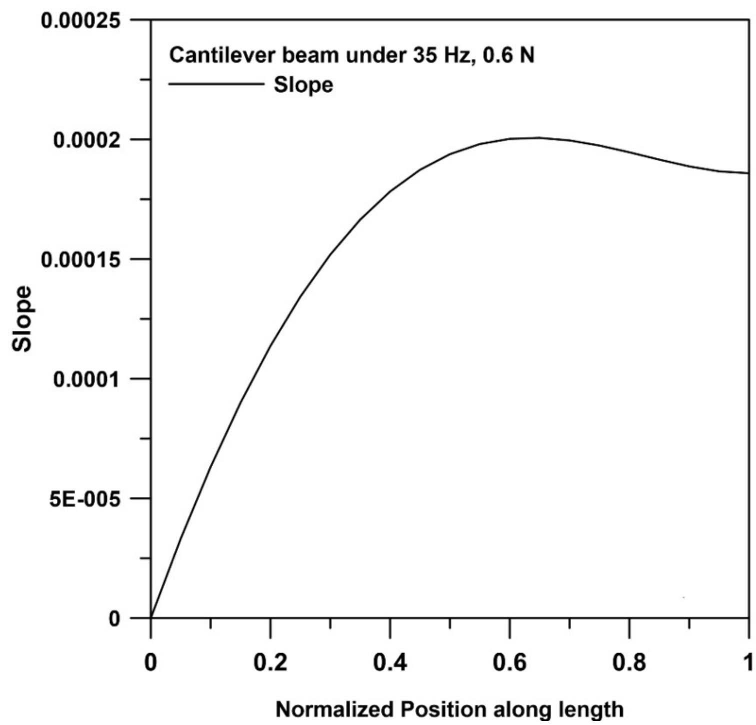


Fig. 4.8(a) Slope of the cantilever beam vibrating at 35 Hz

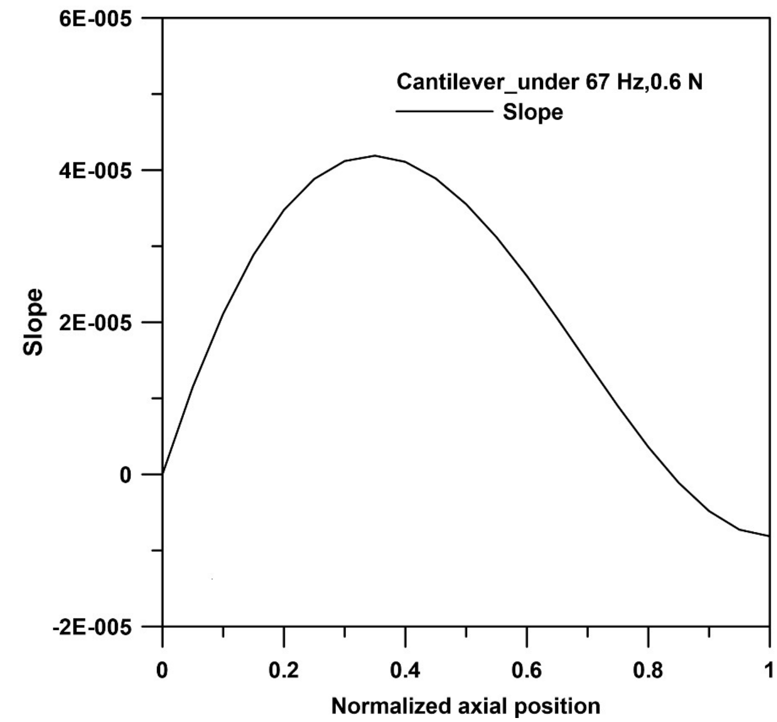


Fig. 4.8(b) Slope of the cantilever beam vibrating at 67 Hz

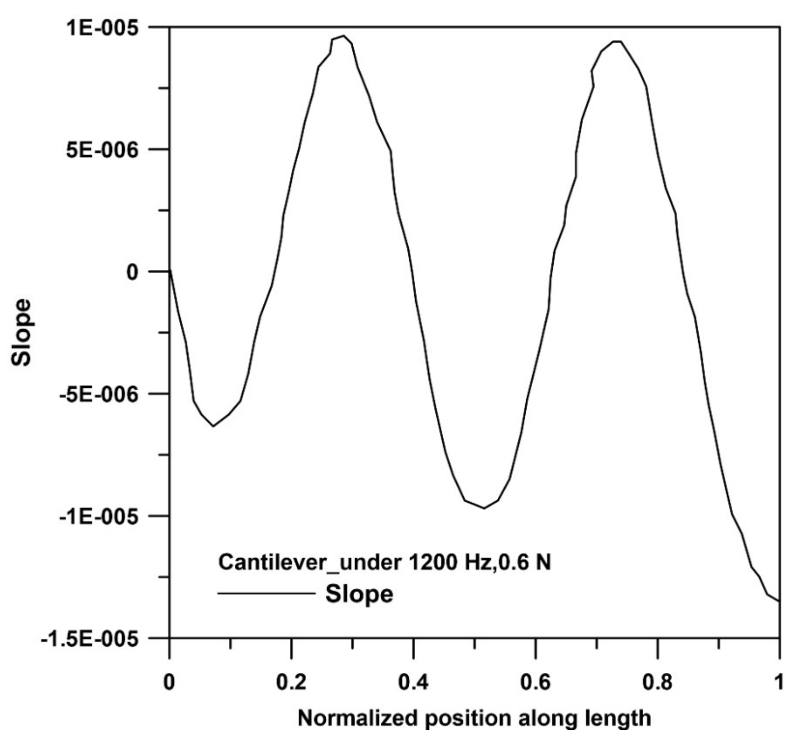


Fig. 4.8(c) Slope of the cantilever beam vibrating at 1200 Hz.

As the output voltage through the considered PZT depends solely on the difference in rotational discontinuity, so when the difference in rotational displacement is zero the voltage difference for that respective crack position is also zero.

Therefore, in order to detect the location of a crack in a beam by vibration, the beam should be allowed to vibrate at least two different frequencies which are considerably far from each other. By chance if one crack position exhibits negligible change in voltage between cracked and healthy beam (zero effect) for a particular vibrating frequency, the same crack position will display a peak value of change in voltage output while vibrating at the other frequency. This analysis strongly clears the doubts for cantilever beam, which aroused at the end of section 4.3.2.

A similar study is also carried for clamped beam where the beam is vibrating at three different frequencies, say160 Hz, 35 Hz and 500 Hz, respectively. The change in voltage output between cracked and healthy beams are obtained for different crack locations havingcrack depths of 2.5 mm and 5mm. When the vibrating frequency is 160 Hz, the difference in voltage output is almost zero as observed from Fig. 4.9(a) for the crack locations of 0.2 and 0.8 from the fixed ends. However, when the beam is vibrated at other excitation frequencies (35 Hz and 500 Hz), considerable change in voltage is observed for the same crack locations(Fig.4.9b and c).Again it can be interpreted that a crack which has no effect at a certain exciting frequency can be dangerous while vibrating at any other frequency. At the same time it is also observed that if the clamped-clamped beam is analysed under two different frequencies then the crack can be identified without any confusion regardless of its position.

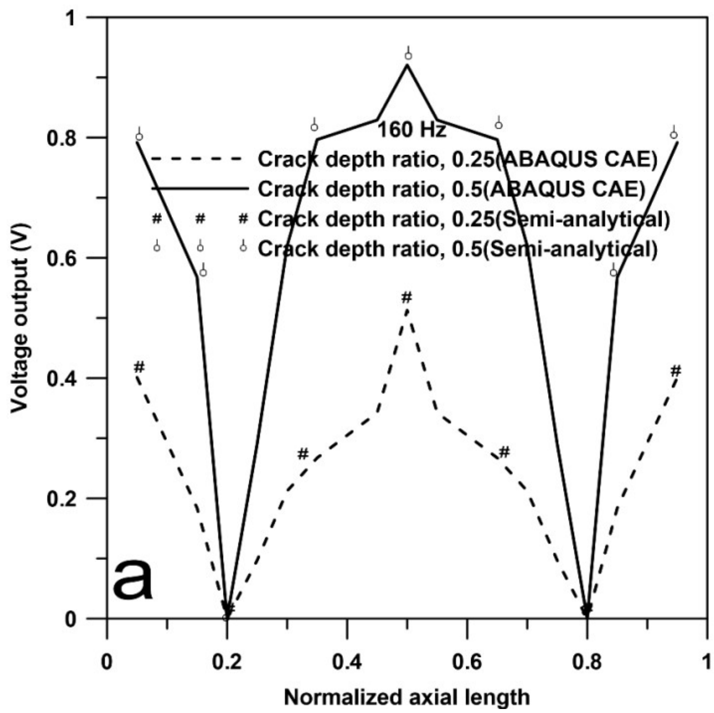


Fig.4.9 (a) Modulus of difference in voltage output for clamped beam (160Hz)

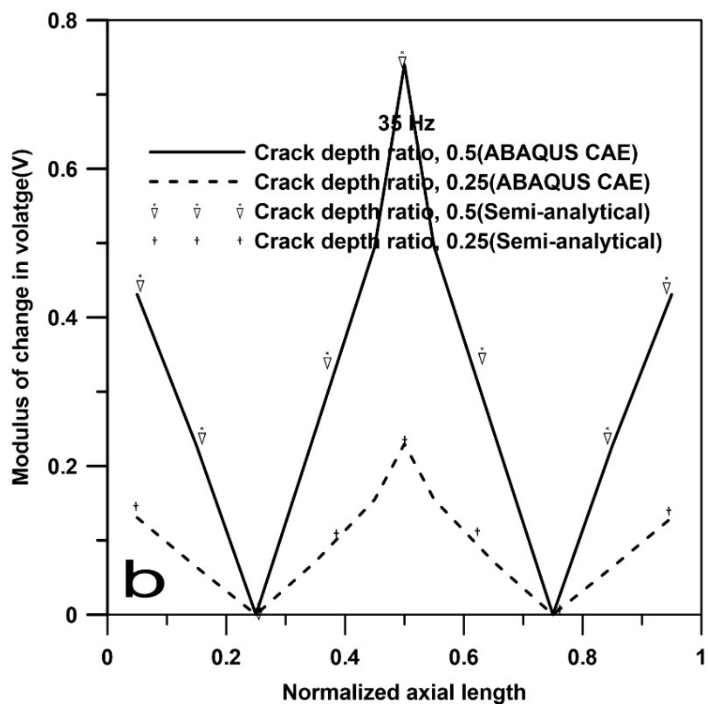


Fig.4.9 (b) Modulus of difference in voltage output for clamped beam (35Hz

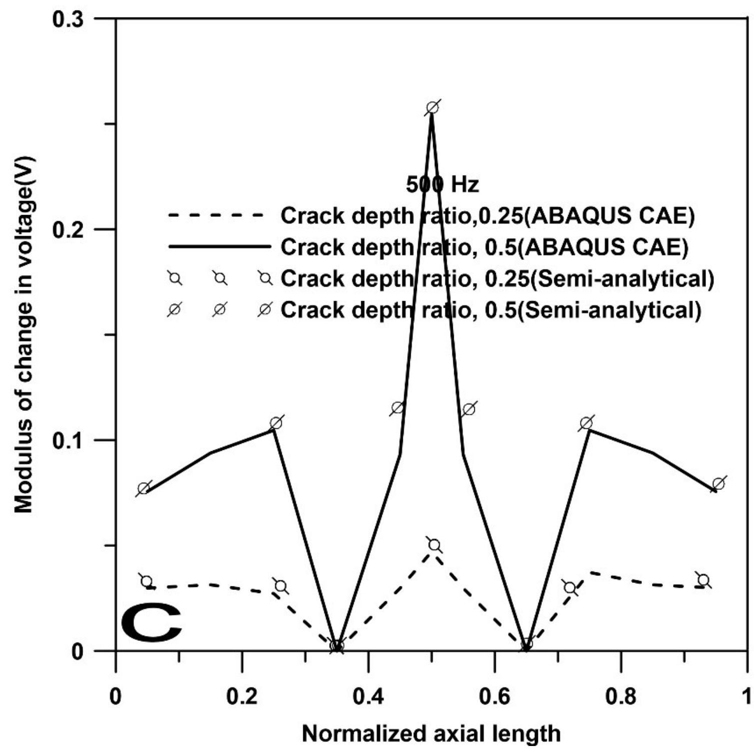


Fig.4.9 (c) Modulus of difference in voltage output for clamped beam (500Hz)

4.3.4 Selection of frequencies at which the beam has to be excited to detect the location of crack:

As discussed earlier a crack regardless the position of it in a cantilever as well as in clamped-clamped beam can be identified by vibrating at two different frequencies. But the question is how to select the vibrating frequencies? In regard of that Fig. 4.10 represents the steady state deformed shapes of beams. It suggests that when the deformed shapes are similar to each other, the points where the voltage changes are zero also be close to each other as for clamped beam at 35 Hz and 160 Hz. So one should select the frequencies such a way, which will deform the beam in dissimilar shapes as in case of clamped 160 or 35 Hz and 500 Hz. So the two points where the crack will so zero effect in voltage response will be considerably far from each other.

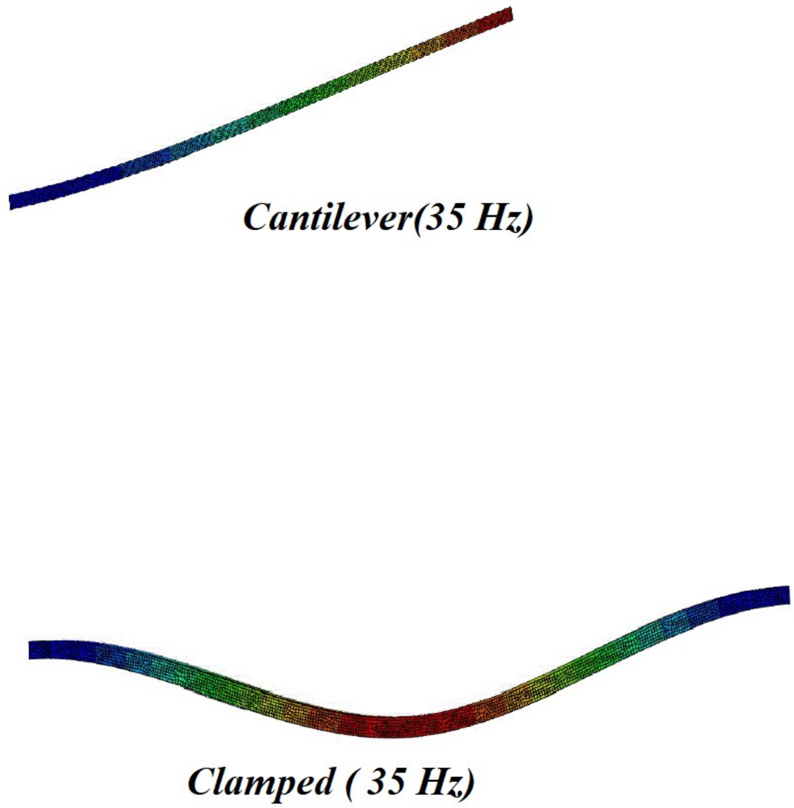


Figure 4.10(a) Steady state deformed shapes of cantilever and clamped beams under 35 Hz harmonic excitation

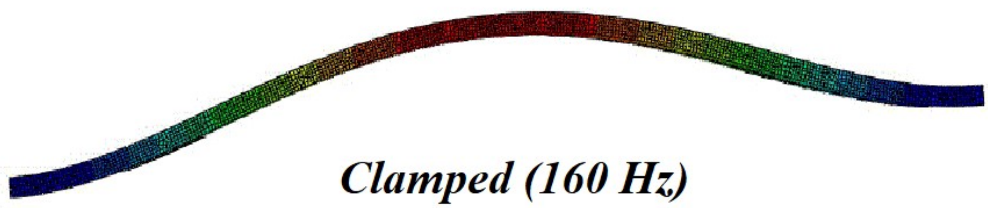


Fig.4.10 (b) Steady state deformed shapes under 67 Hz and 160 Hz harmonic excitation respectively for cantilever and clamped beams .

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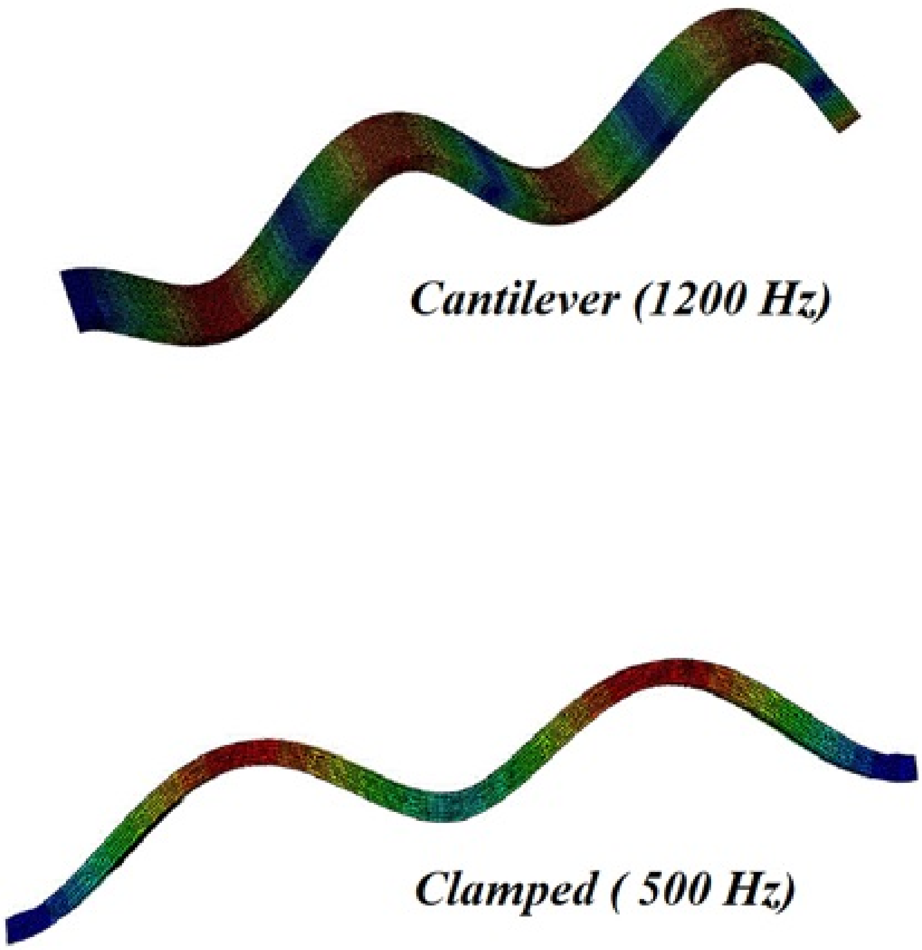


Fig.4.10(c) Steady state deformed shapes under 1200 Hz and 500 Hz harmonic excitation respectively for cantilever and clamped beams .

4.3.5 Crack repair by external voltage at the PZT patch:

In this section, an attempt is made to restore the response of a cracked beam to that of a healthy beam or more clearly to repair the crack to get the same response from the beam under a dynamic excitation as it is displaying in a healthy condition. At first a PZT patch is bonded to a healthy cantilever beam and the voltage output v/s time response plot is obtained for the patch, while the beam is vibrating under an externally applied dynamic load. Now a half depth crack is introduced in the beam and the same result is again obtained i.e. voltage output v/s time response. Now as a trial, the difference between voltage responses of healthy and cracked beam is calculated. This voltage difference is applied as external field on the PZT patch so that the PZT patch can produce a local moment which minimizes the effect of crack.

Fig.10 shows the acceleration responses of tip of a healthy and a cracked cantilever beam (crack at 150 mm from fixed end). It is to be noted that in each case the beam is under a forced excitation having amplitude of 5N and frequency of 25.45 Hz. It is clearly observed from Fig. 4.10(b) that due to the presence of a crack, there is a significant change in the dynamic response of the beam in comparison to that of the healthy beam (Fig.4.10a). As mentioned earlier in section, an attempt is taken to restore the response an external time varying voltage is applied to the PZT patch which is attached just below to the crack.

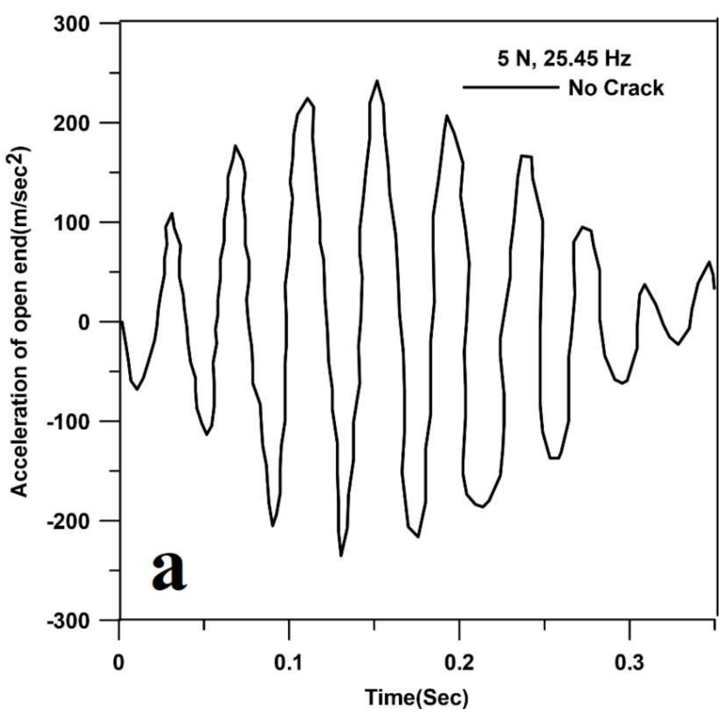


Fig. 4.11(a) Acceleration response of tip of the healthy cantilever beam

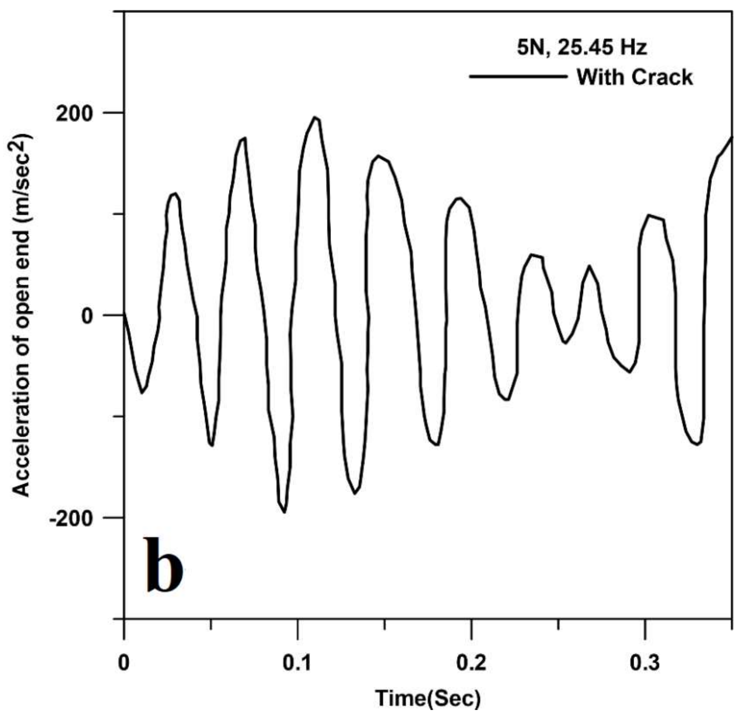


Fig.4.11 (b) Acceleration response of tip of the cracked cantilever beam with a half depth crack at 150 mm from fixed end

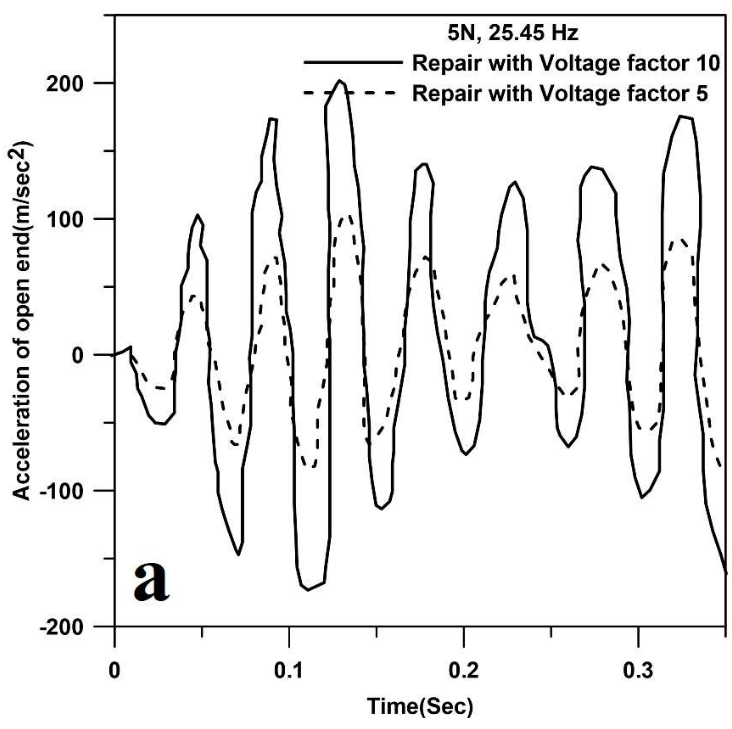
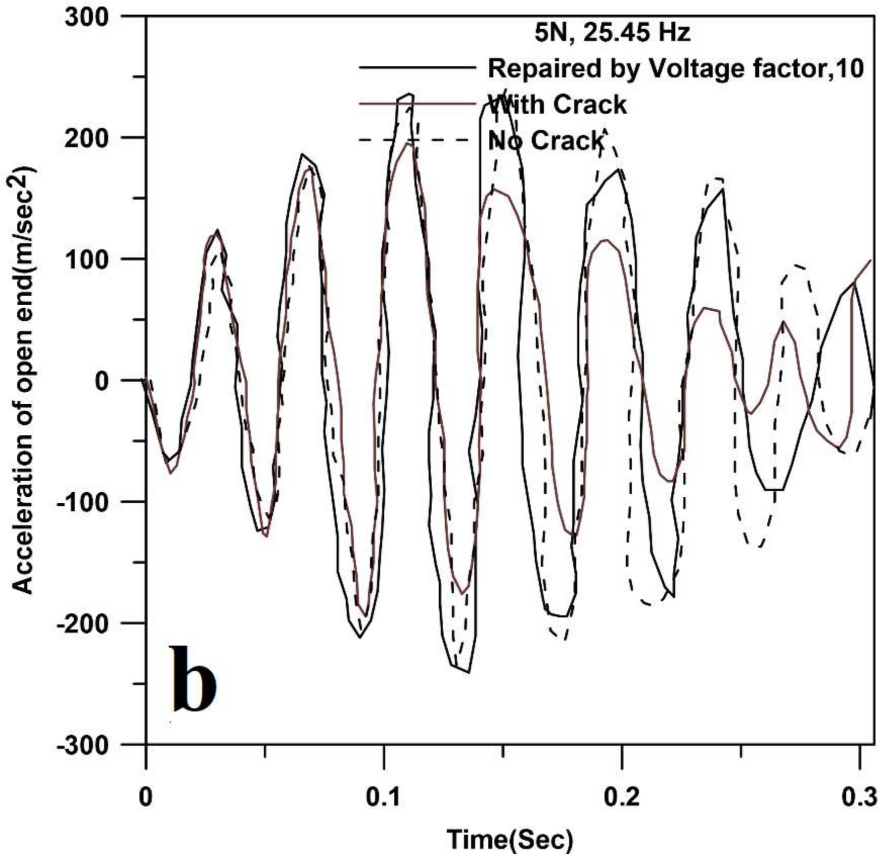


Fig.4.12(a) Comparison of acceleration response of the cracked beam under two different voltage



4.12 (b) Comparison between the acceleration responses in between healthy, cracked and repaired.

As mentioned earlier, a trial method is followed to restore the response. At first the voltage responses of the healthy as well as for cracked beam at the present vibrating condition are obtained and the difference between the two voltages is multiplied by a factor. This way the external voltage/time input is obtained and applied on the patch. Fig.4.11 (a) shows a comparison between the responses of the cracked beam when the patch is excited with voltage factor 5 and 10, respectively. It is to be noted that the beams are subjected to same forced excitation.Fig.4.11 (b) depicts the comparison between the responses of beam in healthy, cracked and repaired condition. It is seen that due to the crack the beat response of the cracked beam comes before than that of healthy beam. At the same time, amplitude of the response also decreases compared to healthy case. But by applying the external voltage field at the PZT patch, the position of beat comes at the same position as it is for healthy condition, with restoration of amplitude to some extent as well.

4.4 Summary:

A novel crack detection method in beam type structures by employing piezoelectric sensor is proposed. At the same time an effort to restore the dynamic response of a cracked cantilever beam is proposed. It is shown that discontinuity in rotational displacement is an efficient index to identify the crack but at the same time there is no conventional way to measure the same in a beam under dynamic condition. A simple use of piezoelectric material is proposed to measure the discontinuity in rotational displacement. It is clearly observed at few locations the crack is not dangerous for a certain exciting frequency but the same may be severe for other frequencies. A clear interpretation is given to detect the crack by vibrating at two different frequencies in cantilever as well as in clamped beam structures. Also a simple way is described to select the required vibrating frequencies. An analytical model is developed as well as Finite element analysis is performed in ASBAQUS 6.12 platform to obtain the results. An experimental setup is built up to verify the theoretical results. Indeed the proposed method is a novel efficient way to monitor the cracks in different beam type structures sooner than the crack severely damages the whole structure. Simultaneously a new scope of research is proposed in the field of crack repair by piezoelectric patch.

DESIGNING OF AN INSTRUMENT TO MEASURE ROTATIONAL DISPLACEMENT AT A POINT OF A DEFORMED BEAM USING PIEZOELECTRIC MATERIAL

5.1 Introduction:

Structural health monitoring is a notable part of present research areas. Early detection of any kind of flaws or crack in structural elements can prevent sudden failure. To monitor structural dynamics as well as to follow the behavior under static load different parameters are taken in to consideration e.g. displacement response, slope, natural frequency, deflection. Simultaneously researches are going on to develop different instruments by the help of which direct measurement of the said parameters can be done. Existing published literatures divulges that piezoelectric material is very efficient in designing of this kind of instruments due to their special characteristics. The characteristics of piezoelectric material are, it gets polarized when strained and can be deformed by applying Voltage. Wen et al. (2010) discussed in his review paper that there are some materials in nature that shows piezoelectricity, e.g. few ceramics (PZT or Lead Zirocondate Titanate), few polymers (Polyvinylidene Fluoride), the composites of PZT and PVDF. These are used as either Sensor or Actuator in different Engineering purposes. Due to this special property that a piezoelectric material can generate electricity when it is vibrated, it is extensively used in energy harvestation also.

The important published literatures in this field are carefully discussed in the Chapter 1 under the subsection 1.2 Literature Review.It is noted from earlier researches that there are different types small size instruments by using which one can study different parameters of a structure, e.g accelerometer to measure natural frequency. At the same time it is also noted that there are few parameters though which are very effective to study the behavior of structure e.g. slope, but there is no conventional instrument to measure the same. The present research work proposes the designing of a new instrument by using piezoelectric material, which can directly measure the slope.Here in the present work a Finite Element model of the beam-PZT Patch is prepared in ABAQUS. Subsequently a mathematical model of the same is also presented. The model is simulated under both the static and dynamic loading conditions. The slope at different points along the length of the beam is measured. Subsequently the voltage from the piezoelectric patch is measured by attaching on different points along the length. It is clearly presented that the difference in slope in between the two end points of PZT patch is proportional to voltage output. Therefore if the slope at one end is zero as an example at the root of cantilever then the voltage output can directly measure the slope at the other end of the patch. Based on the above discussion a novel instrument is designed to measure the slope.

5.2. Modeling and simulation:

As stated earlier here in this present work the purpose is to design an instrument which can directly measure the slope at a point. The concept that a piezoelectric material can serve the purpose can be interpreted using the equation (5.1) [Zhao et al. (2016)]

$$Voltage = \frac{-e_{31}b(h+d)}{2C_v}\int_{l_2}^{l_1} d\ \Psi \qquad (5.1)$$

Where, Ψ Represents the slope, b is length of the patch, d,h are respectively depth of the beam and patch.,e_{31} is constant (stress).,C_v is capacitance,l_1 ,l_2 are the two ends of the patch.Here according to the equation (5.1) the voltage output from a piezoelectric patch solely depend on the difference in slope at its two ends. Therefore if a piezoelectric patch is bonded on a surface of beam, then it will give a voltage which will be proportional to the difference in slope. In case of a cantilever beam the slope at the root is zero, when the patch is attached at the root then the voltage output is proportional to the slope at l_2, since l_1 is zero in this case. Now if the patch is attached to the next position where the starting location of patch is same as the end location in earlier case then the voltage outcome will be proportional to the difference in slope at its two ends. This time slope at l_1 in present case i.e. l_2 in the 1st case is known, therefore adding it with the difference in slope one can get the slope at new l_2.The primary target is to find the slope at different point on a deformed beam either under static loading or a dynamic excitation. Then the difference between slopes at the two ends of the patch is calculated by positioning the patch at different location along the length. The difference in slope is put in to the equation [5.1] and the required voltage is obtained. Then it is interpreted that if the voltage is known then by reversing the process the slope can be determined.

Modeling of the problem under static loading:

At first a mathematical model of Euler beam is prepared which is solved by Ritz method to obtain the slopes at different points along the length. To apply the Ritz method the total potential energy in a statically loaded Euler beam is determined and expressed in the equation (5.2)

$$U = \frac{1}{2} EI \int_0^L \left(\frac{d^2Z}{dx^2}\right)^2 dx + F.Z_{x=L} \tag{5.2}$$

Where U is the total potential energy,E is Young modulus,I is area moment of inertia,L is length of the beam,Z is deflection,F is point load ,x is coordinate along length

To solve the equation (5.2) a 6th order polynomial is taken as a trial function as given below,

$$Z= \sum_{i=1}^{5} C_i x^i$$

For the cantilever beam by putting one of the general boundary condition that $\frac{dz}{dx} = 0\ at\ x = 0$, the coefficient of x i.e C_1 and the other constants are determined by optimizing the energy function by each of the constants. It is found the $C_5, C_6\ are\ zero$. So the trial function has converged. This way the function Z(x) is obtained and the same is differentiated with respect to x to obtain the slope.

Modeling of the problem under dynamic loading:

Here a mathematical model of an Euler Bernoulli beam is prepared to study under harmonic excitation. When the beam vibrates under an excitation then one more energy form i.e Kinetic energy adds in the energy calculation. Therefore the equation (5.2) is modified for dynamic analysis by adding the kinetic energy term and by replacing the load potential of static loading by the harmonic loading.

$$U = \frac{1}{2} EI \int_0^L \left(\frac{d^2 Z}{dx^2}\right)^2 dx + F\sin(\omega t) Z \tag{5.3}$$

$$T = \int_0^L \int_0^d \int_0^b \rho \left(\frac{\partial Z(x,t)}{\partial t}\right)^2 dx dy dz \tag{5.4}$$

Where T is kinetic energy,d is depth of the beam,b breadth of the beam,ρ is the density ,Z is transverse displacement,t is time in second,ω is exciting frequency,Here transverse displacement is a function of space coordinate(x) as well as time (t) so the trail function is also chosen which depends on both the x and t.

$$Z(\mathrm{x}, \mathrm{t}) = Z(\mathrm{x}) \sin(\omega \mathrm{t})$$

Where spatial function Z(x) is considered as expressed below

$Z(x) = Z_n(x) + \sum_{i=1}^{5} C_i x^i$

Where $Z_n(x)$ are the natural modes of vibration.

Now by optimizing the energy function the unknown constants of the spatial function is determined and from there the slope is calculated at different points along the length. Finally which are put into the equation (5.1) to obtain the required voltage.

FEA Simulation by ABAQUS CAE: In the present study a cantilever beam and a piezoelectric patch is modeled in ABAQUS 6.12. Piezoelectric patch is tied on the beam using tie interaction and polarization direction along the thickness of patch is defined by material orientation. A zero voltage electrical boundary condition is applied on the face of the patch which is in contact with beam surface. One of the ends of the beam is made fixed by imposing required boundary condition (rotation and displacements are ceased) on it to make it a cantilever. In the first case the beam is loaded by a static load with 10 N magnitudes in two positions at the middle and at open end, one at a time. Then the slope at different positions along the length is measured in both conditions. Subsequently the voltage for the corresponding points is obtained. In the next study the same steps are followed only instead of 10 N static loads, a 10 N harmonic load at 40 Hz frequency is applied.

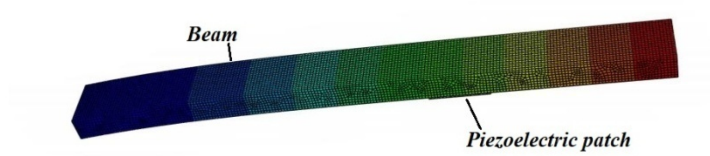


Fig: 5.1 3D Model of cantilever beam with attached piezoelectric patch

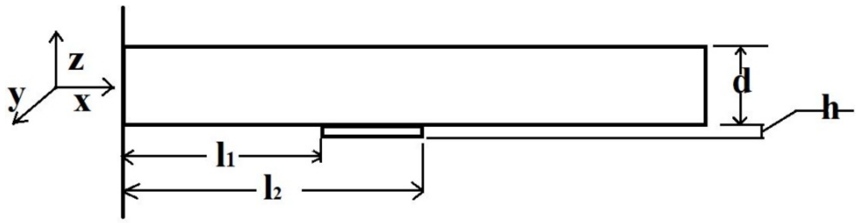


Fig: 5.2 Schematic diagram of beam patch

At the output EPOT is requested as field out. Where EPOT is the notation of voltage in ABAQUS CAE. The EPOT is measured at a mid node of the patch. The EPOT i.e voltage output from the patch is measured for all the positions of patch along the length.

5.3. Result and discussion:

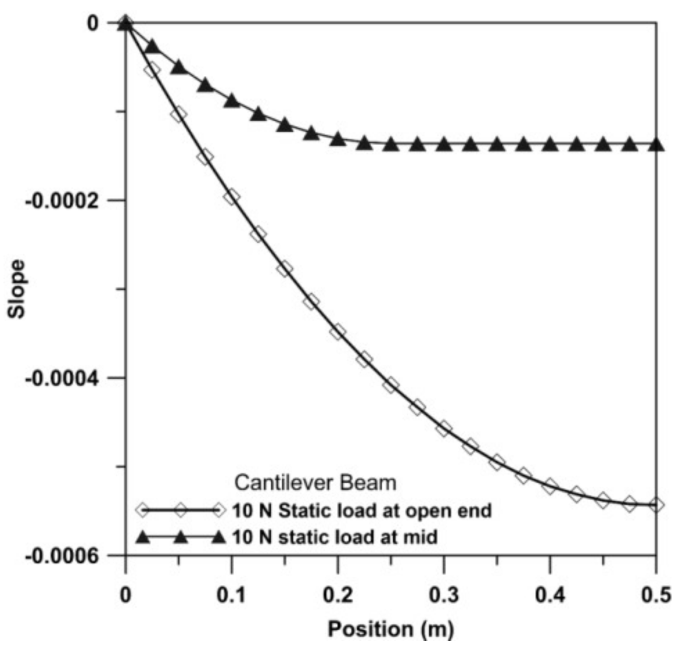


Fig: 5.3(a) Cantilever beam under 10 N static load at open end and at mid, one at a time

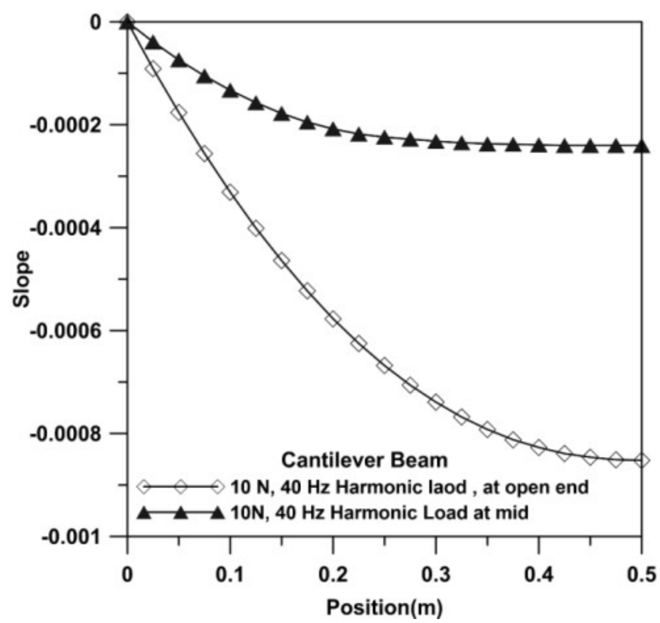


Fig:5.3(b) Cantilever beam under 10N, 40 Hz Harmonic load at open end and mid, one at a time

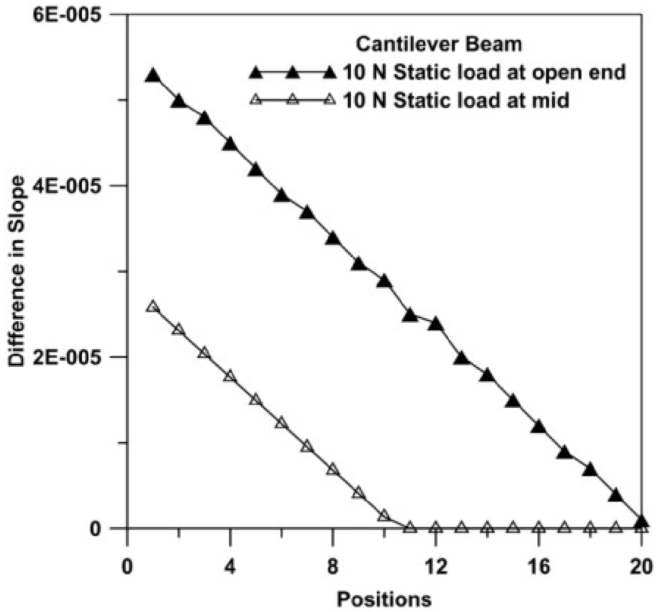


Fig :5.4(a) Difference in slope for Cantilever beam under 10 N static load at open end and at mid, one at a time

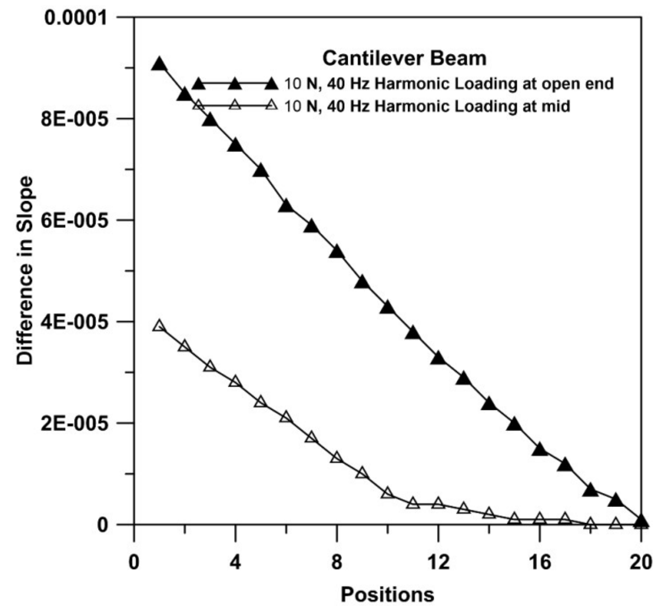


Fig:5.4(b) Difference in slope for Cantilever beam under 10N, 40 Hz Harmonic load at open end and mid, one at a time

In Fig.5.4 (a) and (b), the slope at different positions on the length of a beam (cantilever) is plotted against the different positions, under static and harmonic loading respectively. It is observed that since the beam is vibrated near its 1st natural frequency as a result the nature of slope curve is approximately same in both cases. It is to be noted that when the load is at mid then for the rest part of beam the slope is approximately zero. In Fig.5.5 (a) and (b) the difference in slope between two consecutive values are taken and plotted against the number. Where number '1' represents the difference for the 1st two slope values of the earlier figures (Fig.5.4 (a) and 5.4(b)).

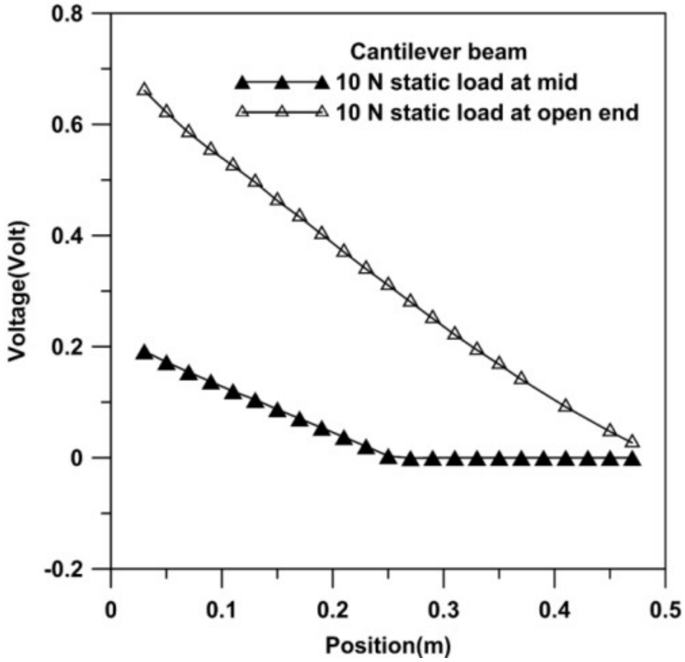


Fig:5.5(a) Voltage at different point along length of Cantilever beam under 10 N static load at open end and at mid, one at a time.

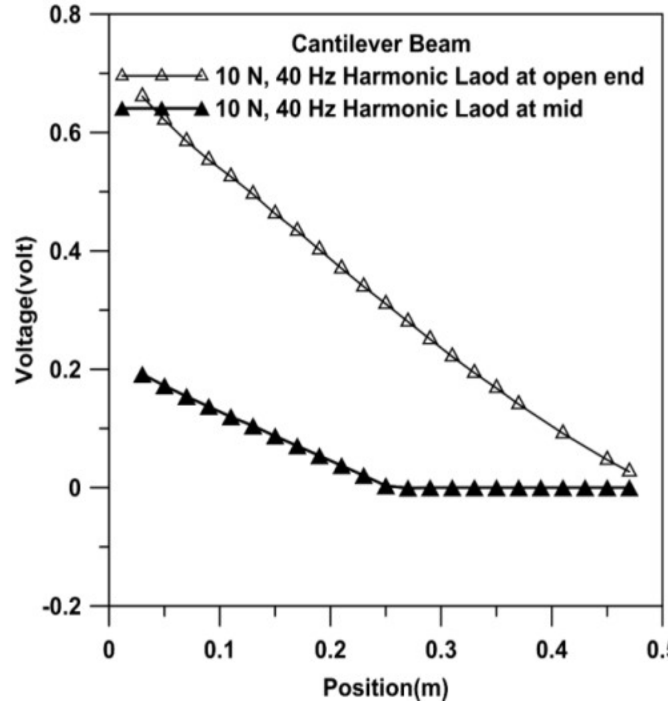
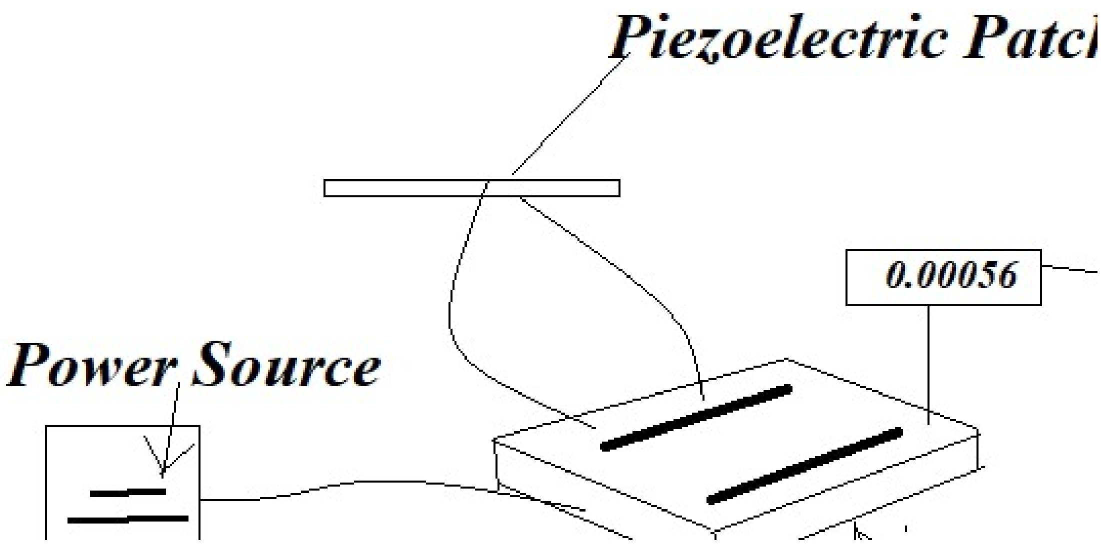


Fig:5.5(b) Voltage at different point along length of Cantilever beam under harmonic loading with 10 N,40 HZ open end and at mid, one at a time

Here in the Fig .5.6(a) and (b) the measured voltage from the patch at different positions along the length of the beam is plotted. It is clearly observed that Fig.5.5 (a), (b) and 5.6(a), (b) are exactly same in nature. It makes clear that the voltage output is proportional to the difference in slope at the two ends of the patch. Therefore, it is undoubtedly clear that how an effective instrument can be made by using the piezoelectric patch, which can measure the slope directly.

A simple programmed (to read the voltage) microcontroller can be used with a display board which can be wired with the patch. According to the equation (5.1), if the voltage output is divided by the remaining constant from the right side, then the slope can be determined directly as discussed in section (5.2).



5.6 Arrangement to use the piezoelectric patch

Summary:

A novel way to measure the slope in beam like structural element is approached. A 3D Beam patch model is prepared in ABAQUS 6.12 platform, which is analyzed under static as well as harmonic loading. Subsequently the mathematical models for the same are presented. The model is validated by the experimental work published in refereed journal. Difference in slope is plotted against positions to compare with the obtained output voltage graph. It is observed that both the difference in slope and voltage graphs are adjacent. It is interpreted that the said observation means the proportional relationship between the difference in slope and voltage output, by virtue of which the slope can be measured directly by using the PZT patch. It is clearly represented how a simple piezoelectric patch in combination with small microcontroller and display board can be used to measure the slope. As a future scope of the study, piezoelectric patch can be used to develop the mode shapes of vibrating beam.

CLOSURE

6.1 Conclusions:

The present thesis work is based on the evaluation and repairing of crack in beams using piezoelectric actuator /sensor. Finite Element Analysis (ABAQUS), numerical modeling as well as experimental works have been performed to observe the behavior of piezoelectric patch under different situation regarding the above mentioned studies. To detect the crack in beam type elements, the cracked beams are studied under different boundary conditions with attached piezoelectric patch. The considered cracked beams are analyzed under static concentrated load as well as external harmonic excitation. In case of static analysis, short and long beams are studied separately. For static modeling, a simple Timoshenko beam is considered with a torsional spring to replicate the effect of crack. Since piezoelectric bending sensor gives a voltage output proportional to the difference in rotational displacements at its end hence a peak in voltage output is observed near the crack location from the attached PZT bending sensor. This peak in voltage is the key point of crack detection in statically loaded beams. Subsequently another mathematical as well as FEA model is established to so how a piezoelectric patch can nullify the effect of crack. It is shown that by applying an external voltage to the piezoelectric patch one can produce a local moment at the location of crack. This local moment nullifies the effect of crack by eliminating the discontinuity in rotational displacement and the beam is supposed to be repaired. The repairing method has been applied on beams under different boundary conditions as well as under different loading situations. For the mathematical modeling in both the cases crack detection and repairing under static load the energy function is obtained by taking the difference between strain energy of the beam and external work. The equation is then solved by Ritz approximation. Simultaneously Finite Element models are developed in ABAQUS 6.12 and analyzed. In another part of the present research work, the dynamic behavior of a cracked beam is studied by attaching a PZT patch on the beam while vibrating under external harmonic excitation. In this case also a clear peak in voltage is observed at the location of crack. It is also studied whether the peak is observed at any location of crack for a vibrating beam. It is observed that for each exciting frequencies there are some locations where the difference in voltage in between healthy and cracked beam are negligible. Therefore, it is suggested to do the analysis at least at two different frequencies. A simple method is prescribed to select the frequencies in which beam has to be excited. In addition another attempt is taken to restore the dynamic response of a cracked beam. The response of a cracked beam under harmonic excitation is obtained, and then compared with the response of a healthy beam. A PZT patch is attached at the location of crack and an external time varying voltage is applied on it. It is observed that by applying a certain voltage, which is a function of time, one can get the same kind of response from a cracked beam as that is observed for healthy beam.

In the mathematical modeling of crack detection in beam under dynamic loading, the beam is divided in to two sub beams at the location of crack. Then for each sub beams two functions of x and t are considered to represent the transverse displacement at time'‘t’ and distance ‘x’. Where the transverse displacement is considered as a product of a sinusoidal function with another spatial function. Since the beam is excited under harmonic force, it is obvious to have a sinusoidal nature in transverse displacement. The spatial function is sum of two parts; one is mode shape function and another 4^{th} order polynomial. Where most of the values of unknown constants are determined by putting end conditions and the kinematic conditions at crack location. Finally the energy function is obtained by subtracting maximum potential energy to maximum kinetic energy, which is optimized and sets of equations are obtained. By solving those equations the transverse displacement functions are obtained. Then this solution is put in the mathematical model of patch, which gives the voltage output for respective conditions. Subsequently Finite Element analysis is also performed to validate the results obtained from the numerical work. To validate the theoretical results, an experimental setup is developed, which consists of a digital oscilloscope, exciter, beam holding setup, wired piezoelectric patch. It is observed that experimental results matches well with the theoretical one.At the last part of the present research, a novel way to measure the slope in a deformed beam is shown by using piezoelectric bending sensor. It is proposed how simply an instrument can be developed which can directly measure the slope at a point on a deformed beam.

6.2 Main Findings:

In the above paragraphs the whole thesis is briefly concluded. Here the major observations are mentioned point wise.

1. It is observed that a bending piezoelectric sensor can be used to find the discontinuity in slope/rotational displacement in a statically loaded cracked beam. This way the crack is detected in statically loaded beams.
2. It is observed that a piezoelectric actuator with polarization direction along thickness can produce a local moment at the desired location if external voltage field is applied on it. This local moment can nullify the effect of crack. This way the crack is repaired in beams under static load.

3. It is observed that the crack in beam can be detected by attaching the piezoelectric patch on a beam vibrating under harmonic excitation. There are some locations for each exciting frequencies for a beam where the piezoelectric patch does not show any considerable change in voltage output due to the crack. Therefore a technique is prescribed to select the suitable exciting frequency for the detection of crack.
4. It is observed that a PZT actuator restores the dynamic response of a cracked beam if a required external voltage field is applied. Hence the beam is repaired by attaching piezoelectric patch on a beam vibrating under dynamic loading.
5. It is found that an instrument can be developed by employing piezoelectric sensor in combination with a microcontroller to measure slope at a point on a beam. Hence the modeling of an instrument to measure the slope with mathematical expression is presented.

6.3 Practical applications:

In traditional practice in the field of structural and machine part related work, technocrats have two main aims– either development of a new element or monitoring of an existing one. In case of the assessment of structures, NDT viz. ultrasonic testing are very helpful. But, to facilitate monitoring to the machine parts, an assessment procedure is in preference which will not hamper the scheduled job of the machine. In this purpose, a few methods already have been proposed viz. vibration analysis, deviation in slope, change in natural frequency and changes in deflection. The present research proposes a new approach by using piezoelectric patch that can detect the crack at very early stage as well as can repair or can reduce the effect of crack in the beam type elements. Indeed it will be a new weapon in the hand of technocrats.

6.4 Future scopes:

It is needless to say, the importance of detection of crack in a element at very early stage, when it can be repaired. Day by day researchers are focusing more on more to develop effective techniques to detect any kind of damage in a structure at early stage. At the same time research works are going on to develop new techniques to repair the cracks to prevent failure of the whole element. Subsequently, new instruments are also developing which can help in practical field to monitor the structural health easily. In the present research, PZT patches are used to evaluate as well as to repair the crack. In another approach it is shown how an instrument can be developed to measure the slope. Therefore by extending the present work a few future scope of researches can be opened, which are listed below:

1) In the present research, only beam in transverse loading is analyzed with integrated PZT patch. As a future scope, the same kind of analysis can be performed on rotating shafts with integrated piezoelectric ring.

2) In the present thesis, simple beams are considered. Instead of that beam can be modeled as any actual component of machine, aircraft viz. aircraft wing. Then the same kind of analysis can be performed.

3) In the present thesis the main focus of PZT actuator was to repair the crack, instead of this the PZT patches can be used to make smart structures. Where if the structure comes under ground acceleration due to earth quake then the PZT will actuate and will balance the disturbing forces.

4) In this work, an attempt is taken to develop an instrument to measure slope. As a future scope, PZT patches can be used to develop a strain gauge or any other kind of instrument.

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