4. a) Find the Fourier Transformation of $e^{-|t|}$.

b) Find Fourier series of the function $f(x) = x + x^2$ in $-\pi < x < \pi$.

Deduce also $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + ...$ 4+6

5. a) Using Z-transformation, solve the equation:

$f(n+2) - 5f(n+1) + 6f(n) = 2^n$

given: $f(0) = 1$ and $f(1) = 0$

b) If $Z(f(n)) = F(z)$, then show that

$Z\left(\frac{f(n)}{n}\right) = -\int_0^z \left(\frac{F(x)}{x}\right) dx$ 5+5

6. a) Solve the equation using Laplace Transformation

$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 4$ given : $x(0) = -3,\ x'(0) = 5$

b) Find the Fourier Transformation of Ne^{-ax^2}. 5+5

Ex/BS/PRN/MTH/T/122/2022

Bachelor of Engineering in Printing Engineering Examination, 2022

(1st Year, 2nd Semester)

Mathematics II

Time : Three hours Full Marks : 100

(50 Marks for each Part)

(Symbols and notations have their usual meanings)

(Use separate answer script for each Part)

PART – I (50 Marks)

Answer any ***five*** questions.

1. a) Test the convergence of

i) $\int_0^\infty \frac{\cos x}{1+x^2} dx$ ii) $\int_0^1 \frac{\log x}{\sqrt{x}} dx$

b) Find $\frac{d}{dt}\int_1^2 \frac{x^2}{(1-tx)^2} dx$.

c) State fundamental theorem of Integral Calculus.

3+3+2+2

2. a) Evaluate (i) $\int_0^\infty \sqrt{x} e^{-x^3} dx$ (ii) $\int_0^{\pi/2} \sin^4 x \cos^4 x dx$.

b) Prove that $\sqrt{\pi}\Gamma(2n) = 2^{2n-1}\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)$, for $n > 0$. 3+3+4

3. a) Show that $\iint_R e^{\frac{y-x}{y+x}} dxdy$ over the triangle with vertices at (0, 0), (0, 1), (1, 0) is $\frac{1}{4}\left(e-\frac{1}{e}\right)$.

b) Prove that $\iiint \left(x^2+y^2+z^2\right) dxdydz$, taken throughout the sphere $x^2+y^2+z^2 \le 1$ is $\frac{4}{5}\pi$. 4+6

4. a) Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x dx$ and hence find $\int_0^{\pi/2} \sin^9 x dx$.

b) Prove that $u=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic. Find the harmonic conjugate of u. 3+2+5

5. Show that the function

$$f(z)=\begin{cases} \dfrac{x^3-y^3+i\left(x^3+y^3\right)}{x^2+y^2} & x^2+y^2 \ne 0 \\ 0 & x^2+y^2=0 \end{cases}$$

satisfies Cauchy-Riemann equations at origin but the function is not differentiable at that point. 10

6. Evaluate (i) $\int_c \frac{dz}{z^2(z+1)(z-1)}$ where $C:|z|=3$

ii) $\int_C \frac{ze^z}{(z-a)^3} dz$, where $c:|z|=a+1,\ a>0$ 5+5

PART – II (50 Marks)

Answer any ***Five*** questions. 10×5=50

1. Test for convergence of the following series.

a) $\frac{5}{1.2.4}+\frac{7}{2.3.5}+\frac{9}{3.4.6}+\frac{11}{4.5.7}+\ldots\ldots\ldots\ldots$

b) $\frac{1+2}{2^3}+\frac{1+2+3}{3^3}+\frac{1+2+3+4}{4^3}+\ldots\ldots\ldots\ldots$ 5+5

2. a) Show that $\lim_{n\to\infty} n^{\frac{1}{n}}=1$

b) Verify whether the sequence is convergent or divergent :

$x_n=\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots\ldots+\frac{1}{n+n}$ 5+5

3. a) Solve the equation using Z-Transformation

$f(n+1)+f(n)=n$ given : $f(0)=1$

b) Find Laplace Transformation of second derivative of a function i.e., $L\left[F''(t)\right]$ 5+5