B.E. MECHANICAL ENGINEERING FOURTH YEAR SECOND SEMESTER - 2022

Subject: **INTRODUCTION TO MODERN CONTROL THEORY**

Time : Four hours **Full Marks: 70**

Answer any **TWO** *questions from* **Group A** and *any* **THREE** *questions from* **Group B**.

Different parts of the same question should be answered together.

Assume any relevant data if necessary.

GROUP A

Answer **any 2** questions. Each question carries **20 marks**.

[1] (a) A control system with 2 inputs and 2 outputs are shown in Fig. P1 below where the controls are shown as r_1 and r_2, which are the valve openings in hot and and cold supply lines; while y_1 and y_2 are outputs of the system representing flow rate and temperature of liquid. The states are shown as x_1, x_2, x_3 and x_4. Construct a state space model for the system.

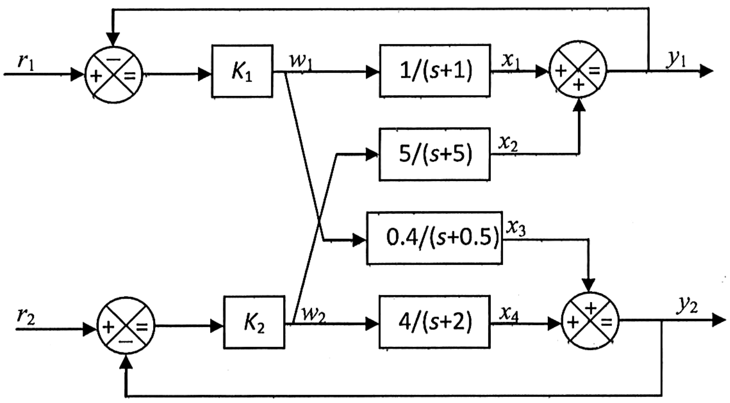

Fig. P1

(b) For a SISO system with transfer function $G(s) = 2/(s^2+4s+5)$, design a state feedback control $u = -k'x$ using the method of pole assignment for companion matrix, to place the closed-loop eigenvalues at $-3\pm2j$. **[12+8]**

[2] (a) Design a Mamdani type Fuzzy Logic Controller (FLC) for a typical electro-hydraulic actuation system with actuator position error and velocity error as inputs to the controller. The output of the FLC is the control voltage to the proportional valve. Choose 3 membership functions for each of the inputs as well as for the output. Show the triangular membership functions, rule tables and the membership function combinations as per the rules.

(b) (i) What is meant by learning of an Artificial Neural Network?

(ii) What is meant by a feedforward neural network?

(iii) Explain with an example the difference between crisp set and fuzzy set in defining a physical variable.

[10+10]

[3] (a) Consider:

$$\dot{x}_1 = 2x_2$$
$$\dot{x}_2 = -4x_1 - 9x_2 + 5u$$

If the system lumped uncertainty can be expressed as $|e(\mathbf{X}, \mathbf{U}, \mathbf{V}, t)| \leq 3$ and a sliding surface is defined as, $\sigma = 4x_1 + x_2$, then obtain the sliding mode control u in terms of x_1 and x_2.

(b) Consider a state model $\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} + \Delta\mathbf{AX} + \Delta\mathbf{BU} + \mathbf{GV}$ where $\Delta\mathbf{A}$ and $\Delta\mathbf{B}$ are the uncertainties associated with the state matrix **A** and the input matrix **B**; **V** are the external disturbances. With suitable assumptions, identify the *lumped uncertainty* **e(X,U,V,t)**. **[14+6]**

[4] (a) Obtain a state space model for a passive car suspension system using a *quarter car model*. Assume any symbols necessary with justification.

(b) For a general state space model $\mathbf{X} = \mathbf{AX} + \dot{\mathbf{B}}\mathbf{U}; \mathbf{Y} = \mathbf{CX}$, obtain an expression for the transfer function matrix **G**(s). Take all initial conditions to be zero. What is the condition of stability of the system? **[10+10]**

[Turn over

GROUP B

Answer **any 3** questions. Each question carries **10 marks**.

[5] (a) Determine the output response of the system to the initial conditions and a unit step input [$u(t) = 1$].

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}u\ ;\ \mathrm{y} = \begin{bmatrix} 3 & 1 \end{bmatrix}\mathbf{x}\ ;\ \mathbf{x}_0 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(b) How can one obtain the *transition matrix* from the state model? **[7+3]**

[6] (a) (i) For a state space model $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, explain how a dynamic observer can be designed. What is the difference between a *full-order observer* and a *reduced-order observer*?

(ii) Design the observer matrix **L** to estimate the states of the system $\dot{\mathbf{x}} = \begin{bmatrix} -1 & 1 \\ 0 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}\mathbf{u};\ y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$ from the output y. Place the observer eigenvalues at $-10 \pm 10j$. **[(4+2)+4]**

[7] (a) For a nonlinear second order system given by $\ddot{x} + g(x, \dot{x})\dot{x} + h(x, \dot{x})x = 0$, obtain the *phase-plane equation* and the *isocline equation*.
(b) Draw the *phase-plane* portrait of a linear underdamped second order system.
(c) Explain 4 behaviours in system responses that indicate presence of nonlinearity. **[4+3+3]**

[8] (a) For a *Liapunov* function $V(\mathbf{X})$, explain the condition(s) for which the function is *postive definite*, *positive semidefinite* and *indefinite*.
(b) For a quadratic function $Q = x_1^2 + 12x_2^2 + 16x_3^2 + 2x_1x_2 + 6x_1x_3$, check using *Sylvester's Theorem*, whether Q is positive definite.
(c) For an n^{th} nonlinear system, suggest a possible *Liapunov* function $V(\mathbf{X})$. **[4+4+2]**

[9] (a) For a state space model $\mathbf{X} = \mathbf{AX} + \dot{\mathbf{B}}\mathbf{U}; \mathbf{Y} = \mathbf{CX}$, explain the terms *controllability*, *onservability* and *stabilizability*.
(b) What is meant by the *companion form* of the state matrix? Explain with a typical third order system example. **[6+4]**