Ex/ME(M2)/PE/B/T/325A/2022

B.E. MECHANICAL ENGINEERING THIRD YEAR SECOND SEMESTER – 2022

Subject: MECHANICAL VIBRATION ANALYSIS **Time: Three Hours** **Full Marks: 100**

The value of the acceleration due to gravity (g) can be taken as 10 m/s², if it is not specified.

Any missing information may be suitably assumed with appropriate justification.

Group A (*Answer any two questions from this group*)

QA1. Explain with neat figure the concept of the '**sharpness at resonance**'. How will it be used for measuring the damping present in a vibrating system? **[10]**

QA2. Consider a spring-mass-damper system which is excited by a harmonic motion ($y = Y \sin \omega t$) at its supporting base as shown in **Fig. QA2**. Show that the displacement transmissibility of the system is given by

$$T_R = \left|\frac{X}{Y}\right| = \left[\frac{1+(2\varsigma r)^2}{(1-r^2)^2+(2\varsigma r)^2}\right]^{1/2},$$

where, X is the amplitude of the absolute displacement response of the mass m , ς is the damping ratio and $r = \dfrac{\omega}{\sqrt{k/m}}$. **[10]**

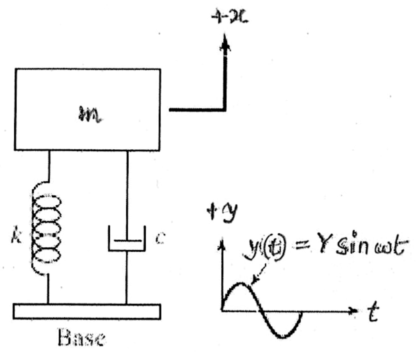

Fig. QA2

QA3. Consider a single degree-of-freedom system, as shown in **Fig. QA3a** subject to a periodic force $f(t)$, which can be approximately expressed as a saw-tooth pattern force as shown in **Fig. QA3b**. Show that the force $f(t)$ can be expressed as $f(t) = \dfrac{2F}{\pi}\displaystyle\sum_{r=1}^{\infty}\frac{(-1)^{r+1}}{r}\sin\left(\frac{2\pi r t}{T}\right)$. **[10]**

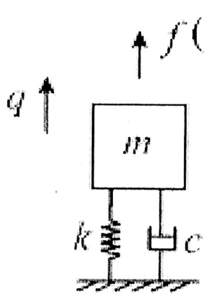

Fig. QA2a

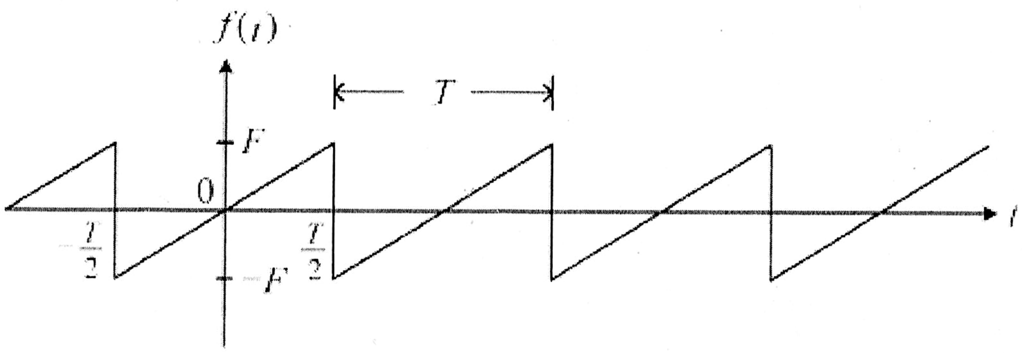

Fig. QA2b

[Turn over

Group B (*Answer any three questions from this group*)

QB1. Consider the spring-mass system as shown in **Fig. QB1**. Obtain the equations of motion using Newton's Law. Determine the natural frequencies and the corresponding mode-shape vectors of the system with $m_1 = 1$ kg, $m_2 = 4$ kg, $k_1 = k_3 = 10$ N/m and $k_2 = 2$ N/m. Normalize the mode-shapes with respect to mass matrix. Find the free vibration response of the system due to the following initial conditions: $x_1 = 1$, $x_2 = 0$, $\dot{x}_1 = \dot{x}_2 = 0$. **[20]**

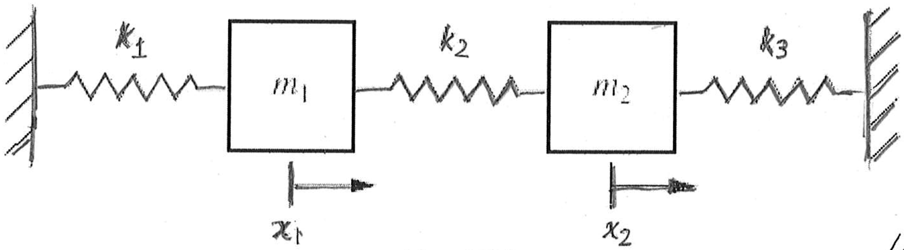

Fig. QB1

QB2. (a) Define the flexibility influence coefficient. Prove Maxwell's Reciprocity Theorem in relation to flexibility influence coefficients. **[10]**

(b) Find the flexibility influence coefficients for the double pendulum as shown in **Fig. QB2**. Consider the bars connecting masses are massless and the amplitudes of oscillation are small. **[10]**

Fig. QB2

QB3. Consider the two-degrees-of-freedom system shown in **Fig. QB3** that is subject to a harmonic force $F(t) = F_0 \sin(\omega t)$ applied on the mass m_1. Obtain the equations of motion for the forced vibration of the system about the static equilibrium point. With $m_1 = m$, $m_2 = 2m$, $k_1 = k$ and $k_2 = 2k$ derive an expression of frequency response function matrix. Obtain the steady-state response amplitude of each mass in terms of F_0, m, k and ω. Plot the steady-state response amplitude of each mass as a function of ω. **[20]**

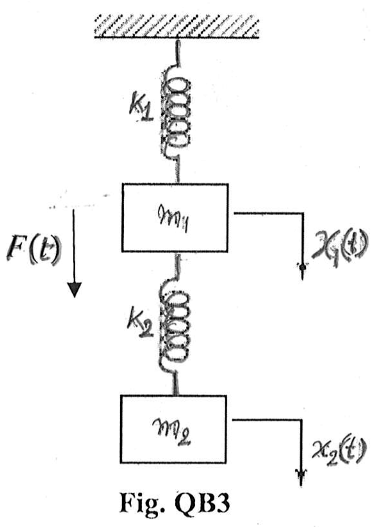

Fig. QB3

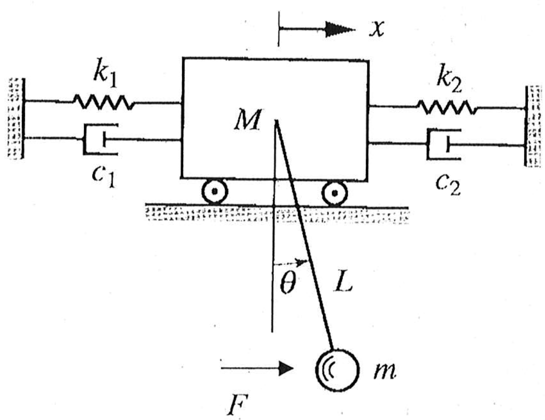

Fig. QB4

QB4. Derive the differential equations of motion of the system shown in **Fig. QB4** using Lagrange's method. Consider x and θ as two generalized coordinates. In the process show the expressions of the total kinetic energy, potential energy, dissipation function and generalized forces. Assume that the pendulum mass m is connected to the cart mass M with a rigid massless rod of length L and angle of oscillation of the pendulum is small. Express the equations of motion in state space selecting suitable state variables. **[20]**

Group C (*Answer any one question from this group*)

QC1. With neat sketches derive the governing differential equation for the free torsional vibration of a circular shaft. Obtain an expression of the angle of twist of the shaft as function of position x of the shaft cross-section along the axis and time t. **[10]**

QC2. Consider the governing differential equation of a uniform, prismatic beam for its free transverse vibration as $EI\frac{\partial^4 y}{\partial x^4} = \rho A\frac{\partial^2 y}{\partial t^2}$, where $y(x,t)$ is the deflection of any generic point on the beam elastic line at any instant t during vibration, EI is the flexural rigidity, A is the cross section area of the beam and ρ is the density of the beam material. Show that if the beam is simply supported the mode shape of the beam during free vibration in n^{th} normal mode is given by $Y(x) = \sin\left(\frac{n\pi x}{l}\right)$ where l is the length of the beam between supports. **[10]**

Group D (*Answer any one question from this group*)

QD1. Explain with neat sketches the principle of active isolation system of a quarter car model using PI (proportional-plus-integral) control law. Derive the necessary equations of motion. Obtain the final state-space matrix of the controlled system. **[10]**

QD2. Explain with neat sketches the principle of operation of an undamped vibration absorber. Deduce the necessary equations of motion in this process. **[10]**