ME(M2)/BS/B/MATH/T/221

B.Mechanical. Examination, 2022

(2nd YR, 2nd SEM)

MATHEMATICS - IV

Full Marks : **Time: Three hours**

Use Separate Answer Script for each Part

Part - I (50 Marks)

Answer any 5 questions $10 \times 5 = 50$

1. Find the Z-Transformations of the following functions:

$$(i) f(n) = \cos nx \qquad (ii)\ f(n) = na^n$$

2. Solve the equation using Z-Transformation

$$(i).\ f(n+2) - 3f(n+1) + 2f(n) = 0,\ given:\ f(0) = 1,\ f(1) = 2.$$

$$(ii).\ f(n+1) + 2f(n) = n,\ given:\ f(0) = 1.$$

3. Find the Laplace Transformations of the following functions:

$$(i)\ f(t) = \frac{t}{T},\quad 0 < t < 2T$$
$$= 1,\quad t > 2T$$
$$(ii)\quad f(t) = t^{100}$$
$$(iii)\ f(t) = \sin\sqrt{t}$$

(4) Solve the equation using Laplace Transformation:

$$(i)\ y'' + 9y = 0,\quad given:\ y(0) = 0,\ y'(0) = 2$$

$$(ii)\ \frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = 0,\quad given:\ x(0) = x'(0) = 1$$

[Turn over

5. Find the Fourier Transformations of the following functions

$$(i)\ \ e^{-|t|} \quad (ii)\ \ f(t) = 5e^{-10t^2}$$

$$(iii)\ \ f(t) = 1, \quad when\ |t| \leq t_0$$
$$= 0\ , \quad otherwise.$$

6. State Dirichlet's conditions for convergence of a Fourier series. Find the Fourier series of the function

$$f(x) = 0, \quad when \quad -\pi < x \leq 0$$
$$= \frac{\pi x}{4}\ , \quad when\ \ 0 \leq x \leq \pi$$

Deduce also

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots.$$

7. Find the Fourier series of the function

$$f(x) = x\sin x, \quad when \quad -\pi < x < \pi$$

Deduce also

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots.$$

Mechanical Engineering Second year Second semester Examination, 2022
Subject : Mathematics - IV
Subject Code : ME(M2)/BS/B/MATH/T/221

(Use a separate Answer script for each part.)

PART II (50 Marks)

(Notations and Symbols have their usual meanings)
ANSWER ANY FIVE QUESTIONS:
(All questions carry equal marks)

$10 \times 5 = 50$

1. The following numbers give the weights of 55 students of a class. Prepare a frequency table with starting weights 40 units and length of each class interval is 10 units :

$$42, 74, 40, 60, 82, 115, 41, 61, 75, 83, 63,$$
$$53, 110, 76, 84, 50, 67, 65, 78, 77, 56, 95,$$
$$68, 69, 104, 80, 79, 79, 54, 73, 59, 81, 100,$$
$$66, 49, 77, 90, 84, 76, 42, 54, 69, 70, 80,$$
$$72, 50, 79, 52, 103, 96, 51, 86, 78, 94, 71.$$

(i) Draw the histogram and frequency polygon of the above data.
(ii) Find the mean, median and mode from the constructed frequency table.

$4 + 6$

2. a) Define

(i) arithemtic mean,
(ii) geometric mean,
(iii) harmonic mean

of the grouped and ungrouped data.
b) If A, G and H are respectively the arithmetic mean, geometric mean and harmonic mean of a frequency distribution, show that $A \geq G \geq H$, mentioning the case when equality holds.

$4 + 6$

[Turn over

3. a) Find out the mean deviation from mean, standard deviation and quartile deviation from the following table :

Wages in Rs.	No. of labourers
Above 0	685
Above 100	500
Above 200	423
Above 300	389
Above 400	209
Above 500	73
Above 600	50
Above 700	0

b) Show that the sum of absolute deviations about median is least.

$6 + 4$

4. a) Define correlation coefficients between two random variables. Show that this coefficient always lies between -1 and 1. Deduce the case when equality holds.
b) Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y) :

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

$4 + 6$

5.(a) Explain what are regression lines, why are there are two such lines. Also, derive their equations.
(b) Fit a parabola of second degree to the following data :

X :	0	1	2	3	4
Y :	1	1.8	1.3	2.5	6.3

$4 + 6$

6. (a) State the axioms of probability. Derive the classical definition of probability from it.
(b) For n events $A_1, A_2, \ldots, A_n$, show that

$$i)\ P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1),$$

$$ii)\ P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i).$$

$4 + 6$

7. a) 'n' different objects $1, 2, \ldots, n$ are distributed at random in n places marked $1, 2, \ldots, n$. Find the probability that none of the objects occupies the place corresponding to its number.

b) If n letters are randomly placed in correctly addressed envelopes, prove that the probability that exactly r letters are placed in correct envelopes is given by

$$\frac{1}{r!}\sum_{k=0}^{n-r}(-1)^k\frac{1}{k!};\ r = 1, 2, \ldots, n.$$

5 + 5

8. a) State and prove Bayes' theorem.

b) There are three urns having the following compositions of black and white balls :

Urn 1 : 7 white, 3 black balls
Urn 2 : 4 white, 6 black balls
Urn 3 : 2 white, 8 black balls.

One of these urns is chosen at random with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn two balls are drawn at random without replacement. Calculate the probability that both these balls are white.

5 + 5