BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2022

(1st Year, 1st Semester)

MATHEMATICS I

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use Separate Answer scripts For each Part

Part - I

Answer any 5 questions $10 \times 5 = 50$

1. Test for convergence of the following series. 5+5

a.

$$\frac{5}{1.2.4} + \frac{7}{2.3.5} + \frac{9}{3.4.6} + \frac{11}{4.5.7} + \ldots\ldots\ldots\ldots$$

b.

$$\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \ldots\ldots\ldots\ldots$$

2. a. Show that 5+5

$$\lim_{n\to\infty} n^{\frac{1}{n}} = 1$$

b. Verify whether the sequence is convergent or divergent:

$$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots\ldots\ldots + \frac{1}{n+n}$$

3. a. Use Lagrange M.V.T, prove that $\sqrt{101}$ lies in between 10 and 10.05. 5+5

b. Find the value of

$$\lim_{x\to 0} \sin x^{\,2\tan x}.$$

[Turn over

4. a. Show that the function 5+5

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}} \quad when\ (x,y) \neq 0$$

$$= 0, \quad when\ (x,y) = 0$$

is continuous at (0,0).

b. Find the few terms of Maclaurin's expansion of

$$e^{ax} \cos by$$

5. a. Show that the point of inflexion of 5+5

$$y^2 = (x-a)^2(x-b)$$

lies on line

$$3x + a = 4b$$

b. Find the asymptote of the curve

$$y = \frac{ax}{x-b} + ax$$

6. State and prove Euler theorem for homogeneous function. 10

Using it prove that if

$$V = \log[x^3 + y^3 + z^3 - 3xyz],$$

then

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) V = \frac{3}{x+y+z}$$

7. a. Find the maximum value of 5+5

$$x^2 + y^2 + z^2$$

subject to

$$ax + by + cz = p$$

b. Find the radius of curvature at its maximum point.

$$y = xe^{-x}$$

EX/ME(M2)/BS/B/MATH/T/111/2022

B.MECHANICAL. EXAMINATION, 2022

(1ST YR, 1ST SEM)

MATHEMATICS-I

FULL MARKS : TIME: 3 HOURS

ANSWER ANY 5 QUESTIONS $10 \times 5 = 50$

PART - II

1.

(a) Define upper and lower integrals for a bounded function f defined on [a, b]. Prove that for a bounded function f defined on $[a, b]$, [2+3=5]

$$\int_{\underline{a}}^{b} f(x)dx \leq \int_{a}^{\bar{b}} f(x)dx.$$

(b) A function $f : [0, 1] \to \mathbb{R}$ is defined by $f(x) = e^{x/2022}$. Find $\int_{\underline{0}}^{1} f(x)dx$ and $\int_{0}^{\bar{1}} f(x)dx$. Deduce that f is Riemann integrable and find $\int_{a}^{b} f(x)dx$.

2.

(a) Using Riemann integration theory, show that [5]

$$\frac{1}{2} < \int_0^1 \frac{dx}{\sqrt{4 - x^2 + x^3}} < \frac{\pi}{6}.$$

(b) Show that $f(x) = [x^2]$ is Riemann integrable on $[0, 2.5]$ and hence show that [3]

$$\int_0^{2.5} f(x)dx = 12 - \sqrt{2} - \sqrt{3} - \sqrt{5} - \sqrt{6}.$$

(c) Give an example of a function $f : [a, b] \to \mathbb{R}$ which is not Riemann integrable but $|f|$ is Riemann integrable on $[a, b]$. [2]

3.

(a) Discuss the convergence of the following integral [5]

$$B(m, n) = \int_0^1 x^{m-1}(1 - x)^{n-1}dx.$$

[Turn over

(b) Prove that [3]

$$\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}dx = \frac{\pi}{4\sqrt{2}}.$$

(c) Prove that $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$. [2]

4.

(a) Show that the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the axis of x is $\frac{4}{3}\pi ab^2$. [5]

(b) Evaluate the integration $\int_0^{\pi/2} \sqrt{\sin x}dx$ by correct up to four significant figures, taking 8 intervals by Simpson's 1/3 Rule. Discuss the geometrical interpretation of Simpson's 1/3 Rule. [4+1=5]

5.

(a) Compute the value of $\iint_{\mathcal{R}} ydxdy$, where $\mathcal{R}$ is the region of the first quadrant bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. [5]

(b) Evaluate the integration $\int_0^1 \cos x dx$ by correct up to four significant figures, taking 5 intervals by Trapezoidal Rule. Discuss the geometrical interpretation of Trapezoidal Rule. [4+1=5]

6.

(a) Change the order of integration

$$I = \int_0^1 \int_{\sqrt{x}}^1 e^{\frac{x}{y}}dxdy$$

and hence find the value. [5]

(b) Show that the volume of the solid obtained by revolving the cardioid $x = a(\theta + \sin\theta)$ and $y = a(1+\cos\theta)$ is $\frac{3}{2}\pi a^3(\pi^2-1)$. [5]

7.

(a) Show that the function

$$f(x) = \begin{cases} 0, & x \in [0,1]\cap\mathbb{Q} \\ 1, & x \in [0,1]\cap\mathbb{Q}^c \end{cases}$$

is not Riemann integrable. [3]

(b) Prove that $\Gamma\left(\frac{1}{9}\right)\Gamma\left(\frac{2}{9}\right)\times\cdots\times\Gamma\left(\frac{8}{9}\right) = \frac{16\pi^4}{3}$. [3]

(c) Show that $\int_0^1 \frac{1}{(1+x)(2+x)\sqrt{x(1-x)}}dx$ is convergent. [4]