Ex/ME/(M2)/BS/B/MATH/T/121/2022

Bachelor of Engineering In MechanicalEngineering Examination, 2022

(1st Year, 2nd Semester)

Mathematics II

Time : Three hours   Full Marks : 100

(50 Marks for each Part)

(Use separate answer script for each Part)

PART I

Notations and symbols have their usual meaning.

Answer any ten questions from the followings.   **5x10= 50**

1. Find the inverse of the matrix $\begin{pmatrix} 8 & 4 & 2 \\ 2 & 8 & 4 \\ 1 & 2 & 8 \end{pmatrix}$.

2. Show that if A and B be invertible matrices of the same order then AB is invertible and $(AB)^{-1} = B^{-1} A^{-1}$.

3. Reduce the following matrix to row echelon matrix and determine its rank.

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

4. Show that the system of equations

$$x + 2y - z = 10,$$
$$-x + y + 2z = 2,$$
$$2x + y - 3z = 2,$$

is inconsistent.

5. Verify that the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}$$

satisfies Caley- Hamilton theorem. Hence find A^{-1}

6. Find the eigen values and corresponding eigen vectors (any two) of the matrix.

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$$\begin{pmatrix} 1 & 3 & 5 \\ 0 & -7 & 2 \\ 0 & 0 & 4 \end{pmatrix}$$

7. A line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube; show that

$Cos^2\alpha + Cos^2\beta + Cos^2\gamma + Cos^2\delta = \frac{4}{3}$.

8. A point P moves on the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, which is fixed and the plane through P perpendicular to OP meets the axes in A, B, C. If the planes through A, B, C parallel to the co-ordinate planes meet in a point Q then show that the locus of Q is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}$.

9. Show that the equation of the plane through the intersection the planes

$x + 2y + 3z = 4$ and $2x + y - z + 5 = 0$ and which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ is $51x + 15y - 50z + 173 = 0$.

10. Find the image of the straight line $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$

in the plane $2x - y + z + 3 = 0$.

11. Prove that the locus of a line which meets the lines $y = mx, z = c$ and

$y = -mx, z = -c$, and which intersects the hyperbola $xy = c^2, z = 0$ is $(cmx - yz)(mxz - cy) + m(c^2 - z^2)^2 = 0$.

12. Show that only one tangent plane can be drawn to the sphere

$x^2 + y^2 + z^2 - 2x + 6y + 2z + 8 = 0$ through the straight line

$3x - 4y - 8 = 0 = y - 3z + 2$. Find equation of the plane.

13. A sphere S has points (0, 1, 0) and (3,-5, 2) has opposite ends of a diameter. Show that the equation of the sphere, on which the intersection of the planes

5x - 2y + 4z+7=0 with the given sphere S is a great circle, is $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$.

Ex/ME/(M2)/BS/B/MATH/T/121/2022

BACHELOR OF ENGINEERING IN MECHANICALENGINEERING EXAMINATION, 2022

(1st Year, 2nd Semester)

MATHEMATICS II

Time : Three hours Full Marks : 100

(50 Marks for each Part)

(Use separate answer script for each Part)

PART II

Answer any 5 questions $10 \times 5 = 50$

1. (a) Show that the necessary and sufficient condition for a vector function $\overrightarrow{F}(t)$ to have constant direction is

$$\overrightarrow{F}(t) \times \frac{d\overrightarrow{F}(t)}{dt} = 0$$

(b) Find the directional derivative of a scalar point function $f(x, y, z)$ along any line whose direction cosines are l, m, n.

2. (a) Find the constants a, b so that the surfaces

$$ax^2 - byz = (a+2)x$$

will be orthogonal to the surface

$$4x^2y + z^3 = 4, \quad at \quad (1, -1, 2).$$

(b) State Gauss Divergence theorem.
Verify Gauss Divergence theorem for

$$\overrightarrow{F} = 4x\overrightarrow{i} - 2y^2\overrightarrow{j} + z^2\overrightarrow{k}$$

taken over the region bounded by

$$x^2 + y^2 = 4, \ z = 0 \text{ and } z = 3$$

[Turn over

3. (a) If $\vec{A}$ and $\vec{B}$ are irrotational , then prove that $\vec{A}\times\vec{B}$ are solenoidal.

(b) Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative field and find a function ϕ such that $\vec{\nabla}\phi = \vec{F}$.

4. (a) If

$$\vec{f} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k},$$

evaluate

$$\int_S (\vec{\nabla} \times \vec{f}).\vec{n}\, dS,$$

where, S is the surface

$$x^2 + y^2 + z^2 = a^2,$$

above xy plane.

(b) If $\vec{r} \times d\vec{r} = 0$, show that $\vec{r}$ is a constant vector.

5. (a) Define solenoidal vector. Find a so that the vector

$$\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$$

is solenoidal.

(b) Show that $\vec{F} = (\sin y + z)\vec{i} + (x\cos y - z)\vec{j} + (x - y)\vec{k}$ is a conservative field and find a function ϕ such that $\vec{\nabla}\phi = \vec{F}$.

6. (a) Find the angle between two surfaces

$$xy^2z = 3x + z^2 \quad and \quad 3x^2 - y^2 + 2z = 1 \quad at \ (1, -2, 1).$$

(b) Evaluate

$$\int_C \vec{F} \cdot d\vec{r},$$

where

$$\vec{F} = (x^2 + y^2)\vec{i} - 2xy\,\vec{j}$$

and the curve C is the rectangle in the xy plane bounded by

$$y = 0, \ x = a, \ y = b, \ x = 0.$$