BACHELOR OF ENGINEERING IN INSTRUMENTATION AND ELECTRONICS ENGINEERING EXAMINATION, 2022

(1st Year, 2nd Semester)

MATHEMATICS II

Time : Three hours Full Marks : 100

(50 Marks for each Part)

(Use separate answer script for each Part)

Symbols / Notations have their usual meanings.

PART - I (50 Marks)

Answer any five questions ($5 \times 10 = 50$)

1. (a) State first Mean value theorem of integral calculus.

(b) Use first Mean value theorem to prove that

$$\frac{\pi}{6} \leq \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \leq \frac{\pi}{6}.\frac{1}{\sqrt{1-\frac{k^2}{4}}}, \quad \text{where } k^2 < 1.$$

(c) Show that the improper integral

$$\int_0^{\infty} \frac{\cos x}{1+x^2}\, dx$$

is absolutely convergent. [2+5+3]

2. (a) Prove that

$$\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \ \cos^{2n-1}\theta \ d\theta, m > 0, n > 0.$$

(b) Prove that

$$\int_0^1 \frac{1}{(1-x^n)^{\frac{1}{n}}}\, dx = \frac{\pi}{n} \text{ cosec } \frac{\pi}{n}, n > 1.$$

(c) Assuming the validity of differentiation under integral sign, show that

$$\int_0^{\infty} e^{-x^2} \cos(ax)\, dx = \frac{1}{2}\sqrt{\pi}e^{-\frac{1}{4}a^2}.$$

[3+3+4]

3. (a) Find the area of the region which is bounded by

$$x^2 = 4ay \ and \ y^2 = 4ax.$$

(b) Evaluate

$$\int\int\int_V \frac{dx\ dy\ dz}{(x+y+z+1)^3}$$

where V is the tetrahedron bounded by the planes $x=0, y=0, z=0, x+y+z=1$.

[5+5]

4. (a) Evaluate the integral

$$\iint_R (x-y)^2 \cos^2(x+y)\ dx\ dy,$$

where R is the rhombus with successive vertices at $(\pi, 0), (2\pi, \pi), (\pi, 2\pi)$ and $(0, \pi)$.

(b) Find the reduction formula for

$$\int \sin^m x \cos^n x\ dx,$$

m, n being positive integers, greater than 1. [6+4]

5. (a) If $\sum\limits_{n=1}^{\infty} u_n$ be a convergent series of positive real numbers, then prove that the series $\sum\limits_{n=1}^{\infty} \frac{u_n}{1+u_n}$ is convergent.

(b) Test the convergence of the series

$$\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \cdots .$$

[5+5]

6. (a) If $\{a_1, a_2, a_3, \ldots\}$ be the collection of those natural numbers that end with 1, prove that the series $\sum\limits_{n=1}^{\infty} \frac{1}{a_n}$ is divergent.

(b) Test the convergence of the series $\sum\limits_{n=1}^{\infty} (1 + \frac{1}{\sqrt{n}})^{-n^{\frac{3}{2}}}$.

[5+5]

7. (a) Show that the series $\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \cdots$ is convergent.

(b) Find the radius of convergence of the power series $\sum\limits_{n=1}^{\infty} (1+1/n)^{n^2} x^n$.

(c) Give an example of two divergent sequences $\{u_n\}$ and $\{v_n\}$ such that the sequence $\{u_n v_n\}$ is convergent. [4+3+3]

Ex/BS/IEE/MTH/T/122/2022

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PART II

(50 Marks)

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Answer any FIVE questions

1) Obtain Fourier series in $[-\pi, \pi]$ for $f(x) = x^2$.
Hence show that i) $\Sigma_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ ii) $\Sigma_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$

(4 + 3 + 3)

2) Obtain Fourier series for

$$f(x) = \begin{cases} x, & \text{for} \quad 0 \leq x \leq \pi, \\ 2\pi - x, & \text{for} \quad \pi < x \leq 2\pi. \end{cases}$$

Hence show that $\Sigma_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

(10)

3a) Show that $\mathcal{F}[f(x)] = 2\sqrt{\frac{2}{\pi}} \frac{\sin^2 \frac{s}{2}}{s^2}$ where

$$f(x) = \begin{cases} 1 - |x|, & |x| < 1, \\ 0, & \text{otherwise} \end{cases}$$

Hence show that

$$\int_{-\infty}^{\infty} \left(\frac{\sin x}{x} \right)^2 dx = \pi.$$

3b) If $\mathcal{F}[f(x)] = F(s)$ and $\mathcal{F}[g(x)] = G(s)$, then prove that

$$\mathcal{F}^{-1}[F(s)G(s)] = (f * g)(x)$$

where $(f * g)(x)$ is the convolution of $f(x)$ and $g(x)$.

(3 + 3) + 4

4a) i) Find $\mathcal{L}[\int_0^t \frac{\sin x}{x} dx]$ ii) Use convolution theorem to find $\mathcal{L}^{-1}[\frac{s}{s^2+1}]$.

4.b) Solve by using Laplace transform

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t},\ 0 < t < 1,\ \ y(0) = -3,\ \ \frac{dy(0)}{dt} = 5.$$

(2 + 3) + 5

5) Suppose $\mathcal{Z}[f(n)] = F(z)$. Then prove that
(a) If $f(n) = 0$ for $n < 0$, then $\lim_{|z|\to\infty} F(z) = f(0)$.
(b) If $F(z)$ is defined for $r_1 < |z|$ and for some integer m, $\lim\limits_{|z|\to\infty} z^m F(z) = A$, then $f(m) = A$ and $f(n) = 0$ for $n < m$.

5 + 5

6a) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$, where $u(x, y) = y^3 - 3x^2y$, $f(0) = 0$, then find $f(z)$.
6b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series for $1 < |z| < 3$.

6 + 4

7) Evaluate i) $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{z^2(z^2+2z+2)}dz$ where $C : |z| = 3$ ii) $\int_C \bar{z}dz$ from $z = 0$ to $z = 4 + 2i$ along the curve $C : z = t^2 + it$.

6 + 4