Ex/BS/IT/MTH/T/122/2022

Bachelor of Engineering in Information Technology Examination, 2022

(1st Year, 2nd Semester)

Mathematics II

Time : Three hours | Full Marks : 100

(50 Marks for each Part)

(Use separate answer script for each Part)

No. of questions	Part-I	Marks
	Answer any five questions.	
1.	a. Show that the sequence $\{x_n\}_n$ where $$x_n = \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right]$$ converges to 1. b. Define monotone sequence. Show that $\{x_n\}_n$ where $x_n = \left(1+\frac{1}{n}\right)^n$ is monotone non-decreasing.	5+5
2.	Test for convergence of the following series a. $\sum \frac{n^2-1}{n^2+1} x^n$ b. $\sum \frac{3\cdot6\cdot9\ldots.3n}{7\cdot10\cdot13\ldots.(3n+4)} x^n, x > 0$	5+5
3.	a. State the first mean-value theorem for integrals and hence show that $$\frac{\pi}{6} \leq \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \leq \frac{\pi}{6} \frac{1}{\sqrt{1-\frac{k^2}{4}}}$$ b. Find the radius of convergence of the series $$x + \frac{x^2}{2^2} + \frac{2!\,x^3}{3^3} + \frac{3!\,x^4}{4^4} + \cdots$$	(2+5)+3
4.	a. If $I_n = \int_0^{\frac{\pi}{4}} tan^n x\, dx$, show that $I_{n+1} + I_{n-1} = \frac{1}{n}$; use this relation to evaluate $\int_0^{\frac{\pi}{4}} tan^8 x\, dx$. b. Show that the following series $$\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots$$ converges for p>0.	7+3

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Q.No		Marks
5.	a. Test the convergence of $$\int_1^\infty \frac{x^2}{(1+x^2)^2}dx$$ b. State the relation between Beta ad Gamma functions and use it to show that $$\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}\,dx = \frac{3\pi}{128}$$	5+5
6.	a. Using transformation $x+y=u$ and $y=uv$ show that $$\int_0^1\int_0^{1-x} e^{\left(\frac{y}{x+y}\right)}\,dx\,dy = \frac{1}{2}(e-1)$$ b. Show that $\int_0^\infty e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$	7+3
7.	a. Find the volume bounded by the *xy* plane, the cylinder $x^2+y^2=1$ and the plane $x+y+z=3$. b. Evaluate $$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$$ by changing the order of the integration.	5+5

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PART - II (50 Marks)

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer **any five** questions.

1. (a) Let $w = f(z) = u + iv$ be differentiable at the point $z_0 = x_0 + iy_0$. Show that

$$u_x = v_y \ , \ u_y = -v_x \ \text{ at the point } (x_0, \ y_0) \ .$$

(b) Show that $\overline{z}$ is a nowhere differentiable function.

(7 + 3)

2. (a) Evaluate $\int_{(0,3)}^{(2,4)} \left[(2y + x^2)dx + (3x - y)dy\right]$ along the parabola $x = 2t$, $y = t^2 + 3$.

(b) State Cauchy's Integral formula. Use this formula to evaluate the following integral

$$\int_C \frac{z+4}{(z^2+2z+5)}dz,$$

where $C : |z + 1 - i| = 2$.

(5 + 5)

3. (a) Evaluate $\int_{|z|=3} \frac{e^z}{(z+2)^5}dz$.

(b) Find the Laurent series expansions in power of z of the function $f(z) = \frac{1}{z(1+z^2)}$ for $0 < |z| < 1$.

(5 + 5)

4. (a) If $u = e^x(x\cos y - y\sin y)$, then find v such that $f(z) = u + iv$ is analytic. Then find $f'(z)$.

(b) If $f(z) = u + iv$ is analytic in a domain D of the complex plane $\mathbb{C}$ then show that u and v both are harmonic in D.

(6 + 4)

5. (a) Use unilateral Z transformation to evaluate $Z\{a^n\}$.
Use this result to find $Z\{r^n \cos n\theta\}$ and $Z\{r^n \sin n\theta\}$.

(b) Use Laplace transformation to find the solution of the following ODE :

$$y' + 2y = 10e^{3t}$$

subject to the initial condition $y(0) = 6$.

(7 + 3)

6. (a) Evaluate : $L^{-1}\left\{\frac{s+4}{s^2+4s+13}\right\}$

(b) Find the Fourier Sine and Cosine transform of e^{-x} and hence show that

$$\int_0^\infty \frac{\cos mx}{1+x^2}dx = \int_0^\infty \frac{x\sin mx}{1+x^2}dx = \frac{\pi}{2}e^{-m}$$

(3 + 7)

7. Obtain the Fourier series expansion of the function $f(x) = x\sin x$ on $[-\pi, \pi]$, and show that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + \ldots$

(7 + 3)

8. (a) Using Parseval's identity prove that $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$

(b) Evaluate: $F^{-1}\left\{\frac{1}{(9+\lambda^2)(4+\lambda^2)}\right\}$

(6 + 4)