BACHELOR OF ELECTRONICS AND TELECOMMUNICATION ENGINEERING EXAMINATION, 2022

(3rd Year, 2nd Semester)

DIGITAL SIGNAL PROCESSING

Time: Three Hours **Full Marks: 100**

(Answer Any Five Questions)

1. (a) Determine whether the following signal is periodic or not. If periodic, find its fundamental period

$$x(n) = sin\left(\left(\frac{\pi}{8}\right).n^2\right)$$

(b) The input to a linear shift-invariant system is the unit step sequence and the corresponding response is the unit sample sequence. Find the unit sample response of the system.

(c) Check the linearity and time-invariance of the following system.

$$T[x(n)] = x(n)x(n-1)$$

(d) Find the convolution of the following.

$$x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$$

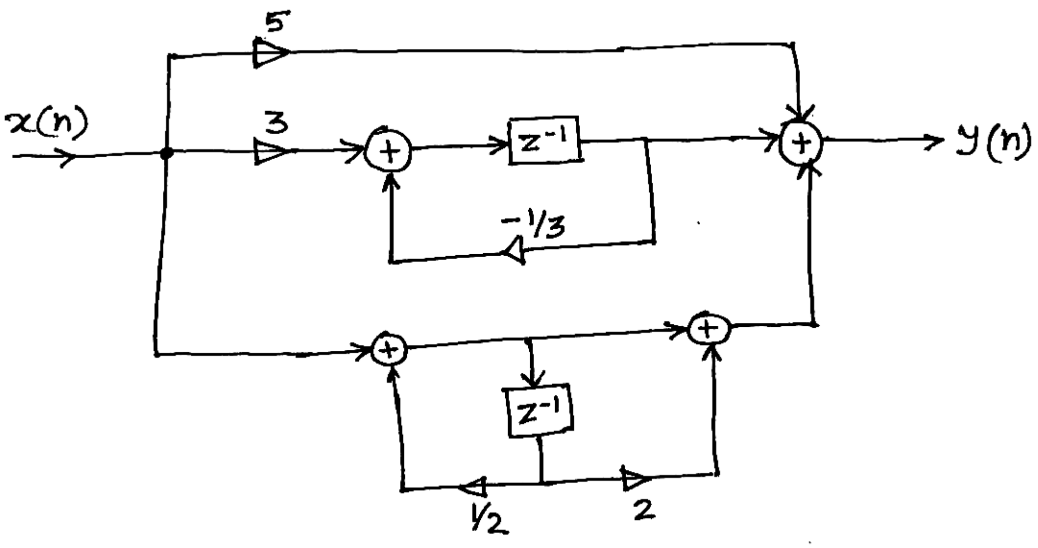
$$h(n)=\{1, -2, -3, \underset{\uparrow}{4}\}$$

(e) Check the causality and BIBO stability for the following.

$$h(n)=\left(\frac{1}{2}\right)^n u(n+1)$$ (5×4)

[Turn over

2. (a) Determine the system function, difference equation and the impulse response of the following system.



(b) Compute the impulse response and the step response of a DT LTI system described by the following difference equation.

$$y(n) = x(n) + 2\,x(n-1) + 0.5\,y(n-1)$$

Assume that the system is initially at rest.

(10+10)

3. (a) A 3-point moving average filter is described by the difference equation

$$y(n) = (1/3)\,.\,[x(n+1) + x(n) + x(n-1)]$$

Compute and sketch its magnitude and phase response.

(b) Consider the filter.

$$y(n) = 0.9\,y(n-1) + b\,x(n)$$

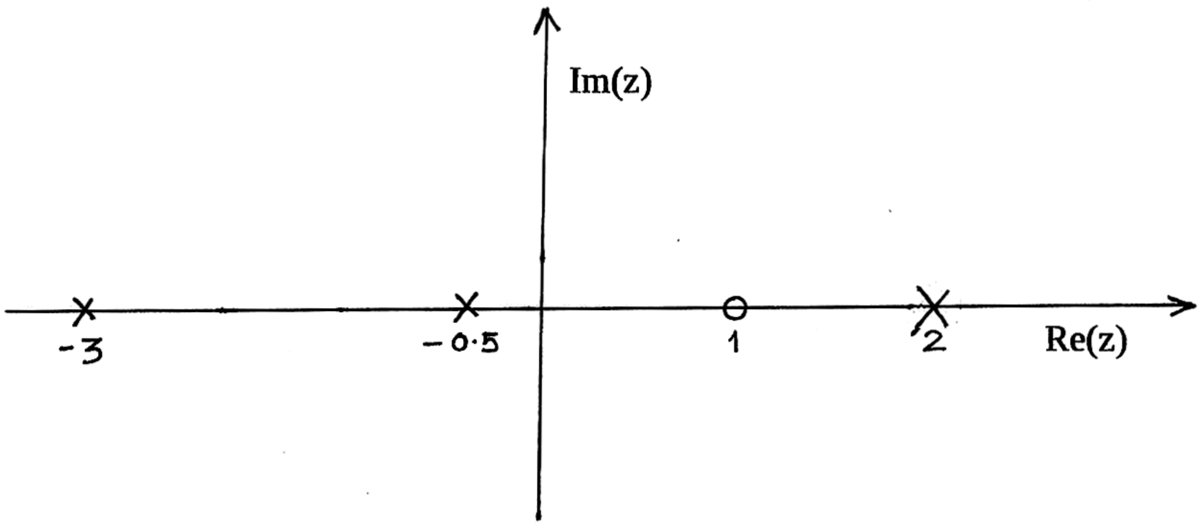
(i) Determine b so that $H(e^{j\omega})|_{\omega=0} = 1$.
(ii) Determine the 3dB cut-off frequency.
(iii) Is this filter lowpass, bandpass or highpass?
(iv) Determine the output of the filter to the input signal

$$x(n) = 5 + 12\sin\left(\frac{\pi}{2}\right)n - 20\cos(\pi . n + \frac{\pi}{4})$$

(10+10)

4. (a) Consider an LTI discrete-time system whose pole-zero pattern is shown below.

(i) Determine the ROC of the system function H(z), if the system is known to be stable.
(ii) Is it possible for the given pole-zero plot to correspond to a causal and stable system? If so, what is the appropriate ROC?
(iii) How many possible systems can be associated with this pole-zero pattern?



(b) For the transfer function, $H(z) = z^{-1} + z^{-6}$ of an FIR linear-phase filter

(i) sketch the impulse response.
(ii) what is the type of the filter (I, II, III, or IV)?
(iii) sketch the pole-zero diagram.
(iv) sketch the magnitude frequency response.

(10+10)

5. (a) Design a 5-tap FIR bandpass filter with a lower cut-off frequency of 2000 Hz and an upper cut-off frequency of 2400 Hz at a sampling rate of 8000 Hz.

(b) A second-order system has a double pole at $p_{1,2} = 0.5$ and two zeros at

$$z_{1,2} = e^{\pm j3\pi/4}$$

Using geometric arguments, choose the gain of the filter so that $|H(e^{j\omega})|_{\omega=0} = 1$.

(14+6)

6. (a) Consider the following system:

$$y(n)=a\,y(n-1) - a\,x(n) + x(n-1).$$

(i) Show that it is allpass.
(ii) Obtain the direct form - I and direct form-II realizations of the system.

(b) Derive and sketch the cascade and parallel structures for the system with the following system function.

$$H(z) = \frac{(1-0.5z^{-1})}{(1-0.25z^{-1})(1+0.25z^{-1})}$$

(10+10)

7. (a) Use the four-point DFT and IDFT to determine the sequence
$x_3(n)= x_1(n) \textcircled{N} x_2(n)$ (Circular Convolution)
where $x_1(n)$ and $x_2(n)$ are the sequences given below.

$$x_1(n) =\{1, \underset{\uparrow}{2}, 3, 1\}$$

$$x_2(n)=\{4, \underset{\uparrow}{3}, 2, 2\}$$

(b) Derive the Butterfly structure for 8-point FFT following decimation-in-time method.

(8+12)

8. (a) A two-pole lowpass filter has the system function

$$H(z) = k / (1 - pz^{-1})^2$$

Determine the values of k and p such that the filter gain is normalized at dc and the cut-off frequency occurs at $\pi/4$ rad.

(b) Design a two pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristic at $\omega = 0$ and $\omega = \pi$ and a magnitude response of $1/\sqrt{2}$ at $\omega = 4\pi/9$.

(10+10)

9. Discuss in details any practical application of Digital Signal Processing.

(20)