Ref. No.: Ex/ET/PC/B/T/323/2022

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING THIRD YEAR SECOND SEMESTER - 2022

Subject: DIGITAL CONTROL SYSTEMS Time: 3 Hours Full Marks: 100

All parts of the same question must be answered at one place only.

PART-A: Answer any ONE

- 1. State and prove Nyquist sampling theorem.
- 2. For an open loop digital control system, derive the expression of the spectra of the flat- 10 top sampled error signal.

PART-B: Answer any TWO

- 3. What is a fractional order hold circuit? How it overcomes the limitation of a first order 10 hold?
- 4. (a) A sampler cannot be represented by transfer function. Justify.
 - (b) Derive the transfer function of a polygonal hold circuit.
- 5. (a) Explain how a fast sampler with sampling period T/N can be realized by a slow 3 sampler of sampling period T.
 - (b) Determine the output response of a fast-slow sampling system using the model of the fast sampler derived in part (b).

PART-C: Answer any TWO

6. Determine C(z) of the following system.

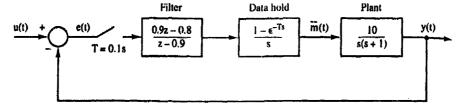
E(s)

T

Samples at 0,
T, 2T, 3T, ... $G_1(s)$ T

samples at hT, T + hT, 2T + hT, 3T + hT, ...

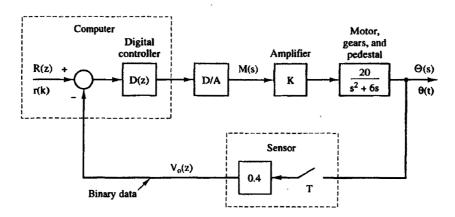
7. Derive the state-space representation of the following closed loop digital control 15 system.



8. Derive the expression of maximum overshoot of a second-order closed loop digital 15 control system.

15

9. Evaluate the closed loop transfer function of the following antenna control system with D(z) = 1, T = 0.05 s, K = 20.



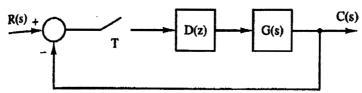
PART-D: Answer any TWO

- 10. (a) What is bilinear transform?
 - b) Design a digital controller D(z) for the following system to attain a steady state error less than 0.01 for unit ramp input and to ensure stability of the entire system with

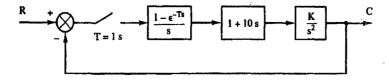
4

10

$$G(s) = \frac{1 - \exp(-Ts)}{s(s+1)}$$
 and $T=0.1$ sec.



Find the range of K for stability of the system from its root locus. Also determine the oscillating frequency for the marginal stability.



- 12. State and prove Nyquist stability criterion for digital control system.
- 13. (a) Discuss how the stability of a closed loop digital control system is influenced by the 5 addition of poles to an open loop transfer function using root locus.
 - (b) Using Nyquist stability criteria, comment on stability of a closed loop system with 5 open loop transfer function $\overline{GH}(z) = \frac{0.632Kz}{(z-1)(z-0.368)}$.

PART-E: Answer any TWO

14. For a plant described by

$$\vec{x}(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

10

10

10

10

find the gain matrix K required to realize the closed loop characteristic equation with zeros providing a damping ratio of 0.46 and a time constant of 0.5 s.

- 15. Derive the state dynamics and hence the transfer function of a state observer.
- 16. Determine the control law u(k) that minimizes

$$J_2 = \sum_{k=0}^{2} \left(x^2(k) + u^2(k) \right)$$

for the plant given by x(k + 1) = 2x(k) + u(k).

17. Consider a linear digital control system described by

$$\vec{x}(k+1) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k).$$

Find the optimal control $u^o(k)$ so that the Lyapunov function $V(\vec{x}) = \vec{x}^T(k)P\vec{x}(k)$ is minimized where **P** is a positive definite solution of $\mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} = -\mathbf{I}$.