Ex/BS/ET/MTH/T/122/2022

Bachelor of Engineering in Electronics and Tele-Communication Engineering Examination, 2022

(1st Year, 2nd Semester)

Mathematics II

Time : Three hours Full Marks : 100

(50 Marks for each Part)

(Use separate answer script for each Part)

PART – I

Group – I

Answer ***any five*** questions. Each question carry 5 marks.

1. Show that the sequence $\{x_n\}$ defined by $x_n = \left(1+\frac{1}{n}\right)^n$ is convergent and the limit lies between 2 and 3.

2. Show that for any $a > 0$, $\lim_{n\to\infty} \sqrt[n]{a} = 1$.

3. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ is conditionally convergent.

4. Define the radius of convergence of a real power series. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!(5x+3)^n}{(n+1)^2+4n}$.

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5. Define the limit superior and the limit inferior of a sequence. Find the limit superior and limit inferior of the sequence $\left\{1+\frac{(-1)^n}{n}\right\}$.

6. Let $x_n = \frac{1+2+3+...+n}{n^2}$. Show that $\{x_n\}$ is bounded monotone sequence and $\lim x_n = \frac{1}{2}$.

7. Examine the convergence of the following series:

i) $\sum_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ ii) $\sum_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{n^{3/2}}$

8. Examine the convergence of the infinite series $\sum_{n=1}^{\infty}\frac{n!}{n^n}$.

Group – II

Answer ***any five*** questions. Each question carry 5 marks.

9. Evaluate $\iint_R \sqrt{x^2+y^2}\,dxdy$, the region of integration being *R*, the region of the *xy*-plane bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$.

10. Compute the surface area of the sphere $x^2+y^2+z^2=a^2$.

11. Evaluate $\iiint xyzdxdydz$, over the region bounded by $x \geq 0$, $y \geq 0$, $z \geq 0$ and $x+y+z \leq 1$.

5. a) Evaluate Laplace inverse transform of $\log\left(\frac{s+3}{s+2}\right)$. 4

b) Solve by Laplace transform method the following simultaneous differential equations : 6

$$\frac{dx}{dt} = 2x - 3y$$

$$\frac{dy}{dt} = y - 2x$$

6. a) Find the Fourier Transform of $e^{-|x|}$. Hence show that $\int_0^{\infty}\frac{\cos mx}{1+x^2}dx = \frac{\pi}{2}e^{-m}$, $m > 0$ 5

b) State the convolution theorem in Fourier transform. Using this theorem evaluate the integral $\int_{-\infty}^{\infty}\frac{dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$. 5

7. a) Define Z-transform of a function undefined for discrete values of $n(=0,1,2,...)$ and $u_n = \theta$ for $n < \theta$. Find Z-transform of $\cos\theta$ and $\sin n\theta$. 4

b) Solve the differenc equations by Z-transform.

$y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = y_1 = 0$. 6

2. a) If $0 < |z-1| < 2$ then express $f(z) = \dfrac{z}{(z-1)(z-3)}$ in a series of positive and negative powers of $(z-1)$. 5

b) Show that $\omega = \dfrac{i-z}{i+z}$ maps the real axis of z-plane into the circle $|\omega| = 1$. 5

3. a) If $f(z)$ is a regular functon of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ 4

b) Determine the poles of the following functions and the residue of each pole : 6

i) $\dfrac{z^2 - 2z}{(z+1)^2(z^2+1)}$ ii) $\dfrac{1-e^{2z}}{z^4}$

4. a) Expand in Fourier series $f(x) = x + x^2$, $-x < x < \pi$.

Hence prove that $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots..$ 6

b) Determine the half-range Fourier cosine series for

$$f(x) = \begin{cases} x, & 0 < x < \dfrac{\pi}{2} \\ \pi - x, & \dfrac{\pi}{2} < x < \pi \end{cases}$$ 4

12. Show that the Improper integral $\int_0^\infty e^{-x} x^{n-1} dx$ exists if $n > 0$.

13. Change the order of the integration and evaluate the integral $\int_0^1 dy \int_y^1 e^{x^2} dx$.

14. If $I_n = \int_0^{\pi/4} \tan^n x dx$, then show that $I_{n+1} + I_{n-1} = \dfrac{1}{n}$.

15. By applying MVT of Integral calculus, show that $\dfrac{1}{3\sqrt{2}} \le \int_0^1 \dfrac{x^2}{\sqrt{1+x}} dx \le \dfrac{1}{3}$.

16. Assuming the validity of differentiation under the integral sign, show that $\int_0^{\pi/2} \dfrac{\log(1 + y\sin^2 x)}{\sin^2 x} dx = \pi\left(\sqrt{1+y} - 1\right)$, where $y > -1$.

PART – II (50 marks)

Answer ***any five*** questions.

1. a) Show that the function $u = \dfrac{1}{2}\log(x^2 + y^2)$ is Harmonic and find its Harmonic conjugate. 6

b) Evaluate $\int_C \dfrac{z^2 - z + 1}{z - 1} dz$, where C is the circle

(i) $|z| = \dfrac{1}{2}$ (ii) $|z| = 2$. 4