Ref. No : *Ex/EE/PE/B/T/421A/2022*

Bachelor of Electrical Engineering Examination, 2022
4th Year, 2nd Semester

Advanced Control Theory

Full Marks: 70

Answer both parts on the same answer script

Part-I

Answer any *three* questions from this part (all questions carry equal marks)

Two marks for neat and well-organized answerscript

1. a) Discuss the common sources of nonlinearity in plants. **4+4+3**
   b) What is static non-linearity? Give two examples of static nonlinearity.
   c) What is hysteresis? Explain why it is called a nonlinearity with memory.

2. a) Explain what is meant by "Equilibrium Point" of a nonlinear dynamic system. **2+3+6**
   b) A nonlinear system is expressed as follows:

$$\dot{x}_1 = -x_1 + x_2$$

$$\dot{x}_2 = 2x_1 - 8x_2 - 5x_1^2 - x_1^3 .$$

   (i). Determine the equilibrium points of the above system.
   (ii). Linearize the above system about ALL its equilibrium points.

3. a) State Lyapunov's 2nd theorem. What are its limitations? **3+8**
   b) The dynamics of an unforced nonlinear system is described by $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 - x_1\left(x_1^2 + x_2^2\right) \\ -x_1 - x_2\left(x_1^2 + x_2^2\right) \end{bmatrix}$. Using the function $V = \frac{1}{2}\left(x_2^2 + x_1^2\right)$ as the Lyapunov function, investigate the stability of the system about its equilibrium point at the origin.

4. a) Enumerate the advantages and disadvantages of on-off control. **6+5**
   b) With schematic diagrams explain how an on-off type temperature control system functions. Sketch the necessary controller characteristics.

[Turn over

5. a) What is a phase plane plot? **3+8**
   b) With suitable phase plane diagrams discuss how the stability of standard second order systems with different pole locations may be analyzed by their phase portraits.

6. A satellite attitude control system has forward-reverse type of thrusters and a controller with proportional plus derivative control with dead zone. **4+2+5**
   a) Draw the block diagram of the above system.
   b) Sketch the controller characteristics.
   c) With the help of a phase plane plot investigate the stability of the system.

Part II

Answer any *three* questions from this part (all questions carry equal marks)

Two marks for neat and well-organized answerscript

7. a) Explain the difference between the terms 'Structured uncertainty' and 'Unstructured uncertainty. **6+5**
   b) Check for the robust stability of the system whose characteristic polynomial is given by

$$p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 = 0,$$

   where

$$p_5 \in [1, 1],\ p_4 \in [35, 40],\ p_3 \in [61, 64],\ p_2 \in [35, 36],$$
$$p_1 \in [11, 15] \text{ and } p_0 \in [52, 58].$$

8. A process plant given by $G_1(s) = \dfrac{2}{(s+1)(0.01s+1)}$ is modeled by using the transfer function $G_2(s) = \dfrac{2}{s+1}$. Compare (i) the open loop unit step responses, (ii) the closed loop unit step responses and (iii) the frequency responses of the plant and its model. **3+5+3**

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9. a) What is the physical significance of H_2 norm of a signal? **2+4+5**

b) Given a transfer function $G(s)=\dfrac{16}{(s+1)(s+2)^2(s+4)}$. Find $\|G\|_2$.

c) For the system with transfer function $G(s)=\dfrac{0.2s+1}{s+1}$, find $\|G\|_\infty$.

10. a) What is an observer? What are its uses? Explain with the help of a block diagram. **6+5**

b) Design a full order observer for observing the second state variable for the following continuous time system so that the observer poles are located at -5, -5

$$\dot{\mathbf{x}}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\mathbf{x}+\begin{bmatrix}0\\ 0.01\end{bmatrix}u\ ;\quad y=[1\ \ 0]\,\mathbf{x}\ .$$

11. a) Explain the meaning of the term 'quadratic performance index'. **1+10**

b) Explain what is meant by the following terms giving an example in each case:

(i) The tracking control problem

(ii) The regulator control problem

(iii)The terminal control problem

(iv) The minimum-time control problem

(v) The minimum energy control problem

12. A regulator contains a plant described by **6+2+3**

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u\ ;\quad y=\begin{bmatrix}1 & 0\end{bmatrix}x$$

and has the performance index

$$J=\int_0^\infty\left[x^T\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}x+u^2\right]dt\ .$$

Determine

a) the Riccati matrix P

b) the optimal control law

c) the closed loop eigenvalues.