Ref No: Ex/EE/ES/H/T/325/2022

B.E. ELECTRICAL ENGINEERING THIRD YEAR SECOND SEMESTER EXAMINATION, 2022

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS (HONS.)

Full Marks 100

Time: Three hours (50 marks for each part)

Use a separate Answer-Script for each part

Question No. | PART- I | Marks

Answer any *THREE* questions

Two marks reserved for neatness and well organized answers.

1. (a) The probability density function of a random variable X has the form $f(x) = 4e^{-k|x|}$; $-\infty \le x \le +\infty$. **8**

   Find the
   (i) value of k.
   (ii) probability that $X > 1$.
   (iii) probability that $X \le -0.5$.
   (iv) variance of X.
   (v) characteristic function of X.

   (b) The random input voltage Y (mV) to an electrical circuit has a cumulative distribution function given by **8**
   $F(y) = \left[1 - e^{-2y}\right] u(y)$; where $u(y)$ is a unit step.
   Y is first amplified by a gain =5 and then added to a constant offset voltage of 3 mV. Thus, the output is a random variable $Z = 5Y+3$. Determine the expectation, the mean squared value and the third moment of Z. *Do not use any transform method.*

2. (a) Determine the first four central moments of a random variable with the standard normal distribution, using any appropriate transform method. **6**

   (b) Let X be a Poisson random variable with parameter $\nu = ½$. Bound the probability of $X \ge 3$, using the Markov's inequality, the Chebyshev's inequality and the Chernoff bound. Compare these bounds with the exact probability value. **6**

[Turn over

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(c) Prove that the variance of the sum of statistically independent random variables is equal to the sum of their variances. **4**

3. (a) Define covariance of two random variables.

X and Y are two statistically independent random variables, such that X has a Laplacian distribution with parameter $\lambda=2$ and Y has a Gamma distribution. If $Z = X+Y$, determine the value of the covariance of X and Z. **2+ 6**

(b) It is claimed that the joint probability density functions involving two continuous random variables X and Y is

$$f(x,y)=\frac{6}{5}\left(x+y^2\right)\ ;\quad 0\le x\le 1\ ,\ 0\le y\le 1$$
$$=0 \quad \text{otherwise.}$$

(i) Prove that $f(x,y)$ is a legitimate probability density function.

(ii) Determine the expressions for the marginal probability density functions.

(iii) Determine the correlation of X and Y. **8**

4.(a) Two statistically independent stationary random processes having sample functions of $X(t)$ and $Y(t)$ have autocorrelation functions of **10**

$R_x(\tau)=25e^{-10|\tau|}\,Cos(100\pi\tau)$ and

$R_y(\tau)=16\dfrac{Sin(50\pi\tau)}{50\pi\tau}$ respectively.

Find the following.

(i) Autocorrelation function of $X(t) + Y(t)$

(ii) Autocorrelation function of $X(t) - Y(t)$

(iii) Both cross-correlation functions involving $X(t)$ and $Y(t)$.

(b) An LTI system is excited by a WSS random process. Which property of the system relates the cross power spectral density of the output and the input to the power spectral density of the input.

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Give mathematical derivations starting from the convolution representation of the system. **6**

5. Write short notes on <u>any two</u> of the following. **8+8**

(a) White noise.

(b) Probability generating function of random variables and its applications.

(c) Bivariate Gaussian Distribution.

(d) Power spectral density of random processes.

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B. Electrical Engineering, 3rd Year, 2nd Semester 2021-22

Introduction to Statistical & Probabilistic Methods

Part II

Answer Any Three Questions.

Two marks for well-organized answers.

Time : 3 hrs Full marks 50

1. (a) Discuss Chi-square distribution and the t-distribution. Also explain, how in case of the latter, $-t_{\alpha,n} = t_{1-\alpha,n}$. 10

   (b) Show analytically how the sample mean and the sample variance are related to the population mean and the population variance. 06

2. (a) State and explain the Central Limit Theorem. 06

   (b) Comment on the distribution of the Sample Mean 04

(c) An insurance company has 25,000 automobile policy holders. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8.3 million. 06

3. (a) In a sample from a normal population having unknown mean and known variance, explain how an interval can be specified, for which we can have a certain degree of confidence, for the mean to lie within. 06

(b) What is maximum likelihood estimator ? 04

(c) An astronomer wants to measure the distance from her observatory to a distant star. However, due to atmospheric disturbances, any measurement will not yield the exact distance *d*. As a result, the astronomer has decided to make a series of measurements and then use their average value as an estimate of the actual distance. If the astronomer believes that the values of the successive measurements are independent random variables with a mean of *d* light years and a standard deviation of two light years, how many measurements need she take to be at least 95 per cent certain that her estimate is accurate to within $\pm$ 0.5 light years? 06

4. What is linear regression equation? Discuss the least square estimators. Also establish analytically why the least square estimators are unbiased estimators? 3+5+8

5. Write short notes on any two : 08+08
(a) The F - Distribution ; (b)) The Bayes Estimators ; (c) One-sided tests of the null hypothesis.