Ex/BS/EE/MTH/T122/2022

Bachelor of Engineering in Electical Engineering
Examination, 2022

(1st Year, 2nd Semester)

Mathematics II

Time : Three hours Full Marks : 100

(50 Marks for each Part)

(Use separate answer script for each Part)

(Notations / Symbols have their usual meanings.)

PART – I

Answer ***any five*** questions.

1. a) State fundamental theorem of integral calculus.

   b) Find the integral $\int_0^1 x^3\left(1-x^2\right)^{\frac{5}{2}} dx$

   c) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta d\theta$, then prove that $n\left(I_{n-1}+I_{n+1}\right)=1$

2+3+5

2. a) Find the length of the arc of the curve $y = \log\left(\sec x\right)$ between $x=0$ to $x=\frac{\pi}{6}$.

   b) If $U_n = \int_0^1 x^n + \tan^{-1} x\ dn$, then prove that $\left(n++1\right)U_n + \left(n-1\right)U_{n-2} = \frac{\pi}{2}-\frac{1}{n}$.

[Turn over

c) Find the reduction formula of

$$I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos nx du$$ 2+4+4

3. Examine the convergence of

i) $\int_0^1 x^{n-1} \log x du$

ii) $\int_0^\infty e^{-nx} x^{m-1} du$

iii) $\int_0^\infty \left(\frac{1}{1+x} - e^{-x} \right) \frac{du}{x}$ 10

4. Define Beta function and Gamma function. Also prove that $\beta_{m,n} = \frac{\gamma(m).\gamma(n)}{\gamma(m+n)}$ 10

5. a) Give an example of a sequence which is bounded but not convergent.

b) Use Cauchy's criterion to prove that $\{x_n\}$ does not convergent, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

c) If the series $\sum_{n=1}^{\infty} a_n$ converges then $\lim_{n \to \infty} a_n = 0$. Is the converse true? Justify your answer with an example.

3+4+3

b) Give the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{when } |x| \le 1 \\ 0 & \text{when } |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. 6

13. a) Show that $\int_0^\infty \frac{f(t)}{t} dt = \int_0^\infty F(p) dp$,

where $F(p) = L\{f(t) : p\}$.

Hence evaluate the integral $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t}$. 6

b) Find $L^{-1}\left\{\log \frac{p+b}{p+a}\right\}$, $a > 0,\ b > 0$. 4

14. a) If $Z(u_n) = U(z)$, then show that $Z(a^n u_n) = U\left(\frac{z}{a}\right)$.

Hence show that $Z(\cos n\theta) = \frac{z(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}$,

given that $Z(1) = \frac{z}{z-1}$. 5

b) Given that $U(z) = Z(u_n) = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find u_2 and u_3. 5

9. a) Show that an analytic function with constant modulus is constant. 5

b) If C be a closed contour containing the origin inside it, then show that $\frac{a^n}{n!} = \frac{1}{2\pi i}\int_C \frac{e^{az}}{z^{n+1}} dz$. 5

10. a) Evaluate: $\int_C |z|^2 dz$, around the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1). 10

Or

b) Evaluate: $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2}$. 10

11. Find the Fourier series expansion of $f(x) = x^2$, $-\pi \le x \le \pi$. Hence show that

i) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots\ldots = \frac{\pi^2}{6}$

ii) $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\ldots = \frac{\pi^2}{12}$

iii) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\ldots\ldots = \frac{\pi^2}{8}$ 10

12. a) Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^{\infty} \frac{x\sin mx}{1+x^2} dx$. 4

6. Test the convergence of the following series

i) $1+\frac{1}{2^3}+\frac{1}{2^2}+\frac{1}{2^5}+\frac{1}{2^4}+\ldots\ldots\ldots$

ii) $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\ldots\ldots\ldots$

iii) $\frac{1}{1\cdot 3}+\frac{2}{3\cdot 5}+\frac{3}{5\cdot 7}+\ldots\ldots\ldots$ 10

7. a) Evaluate the double integration of x^2+y^2 over the region bounded by $x=0,\ y=0,\ x+y=1$.

b) State the Dirichlet's integral associate to triple integral and using it, find the volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. 4+6

PART – II (50 marks)

Answer ***any five*** questions.

8. a) Evaluate: $\lim_{z\to 3i} \frac{z^3+z^2+9z+9}{(z-3i)(z+1)}$ 2

b) Show that $\lim_{z\to 0} \frac{\overline{z}}{z}$ does not exists. 3

c) Show that $u(x,y) = x^3-3xy^2+3x^2-3y^2+1$ is a harmonic function and determine the corresponding analytic function $f(z) = u+iv$. 5