Bachelor of Engineering in Contruction Engineering Examination, 2022

(1st Year, 2nd Semester)

Mathematics II

Time : Three hours Full Marks : 100

(50 Marks for each Part)

(Use separate answer script for each Part)

PART I

1.

(a). Proved that $\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx$. Dose the double integral $\int\int_E \frac{x-y}{(x+y)^3} dxdy$ exist if $E = R[0,1;0,1]$? justify your answer. [5]

(b). Let $I_n = \int_0^{\frac{\pi}{2}} x \sin^n x \ dx$, $n > 1$, show that $I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$. Hence evaluate $\int_0^{\frac{\pi}{2}} x \sin^5 x \ dx$. [5]

2.

(a). Evaluate $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}}$ by changing the order of integration. [5]

(b). If $I_{m,n} = \int \cos^m x \cos nx \ dx$ then prove that $I_{m,n} = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n} I_{m-1,n-1}$. [5]

[Turn over

3.

(a). Show that $\int_0^1 \frac{1}{(x+1)(x+2)\sqrt{x(1-x)}}dx$ is convergent. [5]

(b). Using the differentiation under the sign of integration, show that $\int_0^{\theta} \ln(1+\tan\theta\tan x)dx = \theta\ln\sec\theta \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$. [5]

4.

(a). Show that the entire volume bounded by the positive side of the three coordinate planes and the surface $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} + \sqrt{\frac{z}{c}} = 1$ is $\frac{abc}{90}$. [5]

(b). Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ is convergent if and only if m, n are both positive. [5]

5.

(a). Prove that every convergent sequence is bounded. Is the converse is true? [5]

(b). Test the convergence of the series $\sum_{n=1}^{\infty} u_n$, where $u_n = \sqrt{n^4+1} - \sqrt{n^4-1}$. [5]

6.

(a). Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{7}$ and $u_{n+1} = \sqrt{7+u_n}$ for all $n \geq 1$ converges to the positive root of the equation $x^2 - x - 7 = 0$. [5]

(b). Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$, is convergent if $p > 1$ and divergent $p \leq 1$. [5]

7.

(a). Test the converges of the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n-1}$. [5]

(b). Find the sum of the series indicating the range of validity: $\Sigma_{n=0}^{\infty}(n+3)x^n$. [5]

Ex/BS/CON/MTH/T/122/2022

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PART - II (50 Marks)

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer **any five** questions.

1. (a) Use Parseval's identity to prove $\int_0^{\infty} \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$

   (b) Evaluate: $F^{-1}\left\{\frac{1}{(9+\lambda^2)(4+\lambda^2)}\right\}$

   $(6+4)$

2. Obtain the Fourier series expansion of the function $f(x) = x \sin x$ on $[-\pi, \pi]$, and show that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + ...$

   $(7+3)$

3. (a) Let $w = f(z) = u + iv$ be differentiable at the point $z_0 = x_0 + iy_0$. Show that

   $$u_x = v_y \ , \ u_y = -v_x \ \text{ at the point } (x_0, \ y_0) \ .$$

   (b) Show that $\overline{z}$ is a nowhere differentiable function.

   $(7+3)$

4. (a) Evaluate : $L^{-1}\left\{\frac{s+4}{s^2+4s+13}\right\}$

   (b) Find the Fourier Sine and Cosine transform of e^{-x} and hence show that

   $$\int_0^{\infty} \frac{\cos mx}{1+x^2} dx = \int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$$

   (3 + 7)

[Turn over

5. (a) Evaluate $\int_{|z|=3} \frac{e^z}{(z+2)^5} dz$.

(b) Find the Laurent series expansions in power of z of the function $f(z) = \frac{1}{z(1+z^2)}$ for $0 < |z| < 1$.

(5 + 5)

6. (a) If $u = e^x(x \cos y - y \sin y)$, then find v such that $f(z) = u + iv$ is analytic. Then find $f'(z)$.

(b) If $f(z) = u + iv$ is analytic in a domain D of the complex plane $\mathbb{C}$ then show that u and v both are harmonic in D.

(6 + 4)

7. (a) Evaluate $\int_{(0,3)}^{(2,4)} [(2y + x^2)dx + (3x - y)dy]$ along the parabola $x = 2t$, $y = t^2 + 3$.

(b) State Cauchy's Integral formula. Use this formula to evaluate the following integral

$$\int_C \frac{z+4}{(z^2 + 2z + 5)} dz,$$

where $C : |z + 1 - i| = 2$.

(5 + 5)

8. (a) Use unilateral Z transformation to evaluate $Z\{a^n\}$.
Use this result to find $Z\{r^n \cos n\theta\}$ and $Z\{r^n \sin n\theta\}$.

(b) Use Laplace transformation to find the solution of the following ODE :

$$y' + 2y = 10e^{3t}$$

subject to the initial condition $y(0) = 6$.

(7 + 3)