Ref No: Ex/CSE/PC/B/T/225/2022

Bachelor in Computer Science and Engineering
2^{nd} Year, 2^{nd} Semester Exam 2022
Graph Theory and Combinatorics

Time : 3 Hrs

Full Marks : 100

Write answers to the point. Make and state all the assumptions(wherever made).
ALL PARTS OF A QUESTION SHOULD BE ANSWERED TOGETHER

Section A Answer all questions [$10 \times 2.5 = 25$]

(1) Which of the following sequences can not be the degree sequence of any graph? Give reasons

(I) 7, 6, 5, 4, 4, 3, 2, 1
(II) 6, 6, 6, 6, 3, 3, 2, 2
(III) 7, 6, 6, 4, 4, 3, 2, 2
(IV) 8, 7, 7, 6, 4, 2, 1, 1

(2) Prove that in finite graph, the number of vertices of odd degree is always even.

(3) Let G be a simple connected planar graph with 13 vertices and 19 edges. State the number of faces of the graph

(4) Construct a 8-vertex tree whose Prüfer code is: $(1, 8, 1, 5, 2, 5)$.

(5) How many edges can there be in a forest with p components having n vertices in all ?

(6) Let $G(x) = \dfrac{1}{(1-x)^2} = \sum_{i=0}^{\infty} g(i)x^i$, where $|x| < 1$. What is $g(i)$?

(7) n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. What are the number of different gatherings possible at the party ?

(8) How many different strings can be made by reordering the letters of the word "S U C C E S S "?

(9) What is the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$?

(10) What is the generating function for the sequence 1, 1, 1, 1, 1, 1?

Section B Answer any Five(5) Questions. You can keep your answer in the "Combinatorial Form" [$5 \times 6 = 30$]

(1) We have Rs n. Every day we buy exactly one of the following products: candy (Rs 1), chocolate (Rs 2), ice-cream (Rs 2). What is the number M_n of possible ways of spending <u>all</u> the money?

(2) Prove by Induction that every third element in a Fibonacci sequence is an even number.

(3) Show that the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs (a, b) and (c, d) in the chosen set such that, $a \equiv c \mod 3$ and $b \equiv d \mod 5$ is 16.

(4) What is the closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$?

(5) In how many ways can a given positive integer $n \geq 2$ be expressed as the sum of 2 positive integers (which are not necessarily distinct). For example, for $n = 3$, the number of ways is 2 , i.e., $1 + 2$, $2 + 1$.

(6) In how many ways can a given positive integer $n \geq 3$ be expressed as the sum of 3 positive integers (which are not necessarily distinct). For example, for $n = 4$, the number of ways is 3 , i.e., $1 + 2 + 1$, $2 + 1 + 1$ and $1 + 1 + 2$.

Section C Answer any Five(5) Questions [$5 \times 6 = 30$]

(1) Prove or disprove (i) K_5 is non-planar (ii) $K_{3,3}$ is planar.

(2) A graph is *d–regular* if every vertex has degree d. For a *d–regular* graph on n vertices, show that at least one of d and n is even

(3) Prove that in a connected graph, any two longest paths have at least one vertex in common.

(4) Prove or disprove: No digraph contains an odd number of vertices of odd outdegree or an odd number of vertices of odd indegree.

(5) Let F and H be two disjoint connected non-Eulerian regular graphs and let $G = (F + H).K_1$; that is, G is obtained from F and H by adding a new vertex v and joining v to each vertex in F and H. Prove that G is Eulerian.

(6) Show that the Petersen graph does not have a Hamilton circuit, but the subgraph obtained by deleting a vertex v has a Hamilton circuit.

Section D Answer any one question [15]

(1) In an undirected graph G with n vertices, vertex 1 has degree 1 , while each vertex $2, \ldots, n-1$ has degree 10 and the degree of vertex n is unknown. Give reasons why the following statements are TRUE or FALSE

(a) There is a path from vertex 1 to vertex n :- TRUE

(b) There is a path from vertex 1 to each vertex $2, \ldots, n-1$. :- FALSE

(c) Vertex n has degree 1 :- FALSE

(d) The diameter of the graph is at most $\frac{n}{10}$. :- FALSE

(2) Let v be a vertex in a connected graph $G(V, E)$. Prove that there exists a spanning tree $T(V, E')$ of $G(V, E)$ such that the distance of every vertex from v is the same in $G(V, E)$ and in $T(V, E')$