Bachelor of Computer Science Examinations, 2022

(First Year, Second Semester)

MATHEMATICS

Time: Three hours　　　　Full Marks:100

(50 marks for each part)

Use separate answer script for each part

Symbols/Notations have their usual meanings

Part-I

Answer ***any five*** questions.

1. (a) Examine the nature of the series $(\frac{2^2}{1^2}-\frac{2}{1})^{-1}+(\frac{3^3}{2^3}-\frac{3}{2})^{-2}+(\frac{4^4}{3^4}-\frac{4}{3})^{-3}+......\infty$. (5)

   (b) Show that the sequence $\{x_n\}$, where $x_n = 1+\frac{1}{3}+\frac{1}{3^2}+......+\frac{1}{3^{n-1}}$ converges. Find $lim x_n$. (5)

2. (a) Prove that $\int\limits_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}; a > 0$. Hence using differentiation under integral sign, show that $\int\limits_0^\infty x^{2n}e^{-ax^2}dx = \frac{\sqrt{\pi}}{2}\frac{1.3.5...(2n-1)}{2^n a^{(n+1)/2}}$. (5)

   (b) Discuss the convergence of the integral $\int\limits_1^\infty \frac{\sin x}{x^2}dx$. (5)

3. (a) Given $x_n = (a^n + b^n)^{\frac{1}{n}}$, where $0 < a < b$. Show that the sequence $\{x_n\}$ converges to b. (3)

   (b) Discuss the convergence of the series $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + ...\infty$. (3)

   (c) Prove that $\int\limits_0^1 dx \int\limits_0^1 \frac{x-y}{(x+y)^3}dy \neq \int\limits_0^1 dy \int\limits_0^1 \frac{x-y}{(x+y)^3}dx$. Does the double integral $\iint\limits_E \frac{x-y}{(x+y)^3}dxdy$ exist if $E = R[0,1;0,1]$. Justify your answer. (4)

4. (a) Show that $\beta(m,n) = 2\int\limits_0^{\frac{\pi}{2}} \sin^{(2m-1)}\theta\cos^{(2n-1)}\theta\ d\theta$. Using it prove that (5)

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$\int_0^{\frac{\pi}{2}} \sin^p\theta \cos^q\theta d\theta = \frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{q+1}{2})}{2\Gamma(\frac{p+q+2}{2})}$, where $p, q > -1$.

(b) Prove that $\Gamma(a)\Gamma(1-a) = \pi cosec(a\pi),\ 0 < a < 1$. (5)

5. (a) Evaluate $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}}$ by changing the order of integration. (3)

(b) Using the transformation $x + y = u$, $y = uv$ show that $\int_0^1 dx \int_0^{1-x} e^{\frac{y}{x+y}} dy = \frac{1}{2}(e-1)$. (4)

(c) Evaluate by Green's theorem $\int_\Gamma (x^2y\ dx + xy^2\ dy)$ where Γ denotes the closed path formed by $y = x$ and $y^3 = x^2$ in the first quadrant described in the positive sense. (3)

6. (a) Evaluate $\iint_R (x^2 + y^2) dxdy$ over the region R bounded by $xy = 1$, $y = 0$, $y = x$, $x = 2$. (5)

(b) Use Stoke's theorem to find the line integral $\int_C (x^2y^3dx + dy + zdz)$, where C is the circle $x^2 + y^2 = a^2$, $z = 0$. (5)

7. (a) Find the reduction formula for $\int \sin^m x \cos^n x\ dx$, where m and n are both positive integers. Hence evaluate $\int_0^{\frac{\pi}{4}} \sin^4 x \cos^3 x dx$. (5)

(b) Prove that the volume common to the sphere $x^2 + y^2 + z^2 = a^2$ and the cylinder $x^2 + y^2 = ay$ is $2(3\pi - 4)\frac{a^3}{9}$. (5)

Ex/BS/CSE/MTH/T/122/2022

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MATHEMATICS II

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Part-II

Answer ***any five*** questions.

1. (a) Find the inverse Z-transform of (5)

$$F(z) = \frac{4z^2 - z}{(z-3)(z+2)^2}.$$

(b) Using Z-transform, solve the difference equation (5)

$$f(n+2) + 4f(n+1) + 3f(n) = 0,\ f(0) = 1, f(1) = 1.$$

2. (a) Let $f : \mathbb{C} \to \mathbb{C}$ be an analytic function $f(x+iy) = u + iv$ such that $u = constant$. Prove that $f = constant$. (3)

(b) Show that the real and imaginary parts of the function (3)

$$f(z) = z^2 - iz + 3 + i$$

satisfy Cauchy-Riemann equations.

(c) Evaluate $\oint_C f(z)dz$ using Cauchy's Integral Formula, where $f(z) = \dfrac{sin(z)}{(z - \frac{\pi}{2})^3}$ and C is the circle $|z - \frac{\pi}{2}| = 1$. (4)

3. (a) Find the points of isolated singularities of the function $f(z) = \dfrac{z^2}{sin(z)}$. If any of them is a pole, find its order. (6)

(b) Evaluate $\oint_C f(z)dz$, where $f(z) = \frac{z-1}{(z^2+2)z^2}$ and C is the circle $|z| = 2$. (4)

4. (a) Find the Fourier series of the function $f(x) = x + x^2, \ -\pi < x < \pi$. Hence prove that (6)

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + ...$$

(b) Let $f(x)$ be an odd function in $[-l, l]$ with period $2l$. If the Fourier series expansion of $f(x)$ is (4)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n cos(\frac{n\pi x}{l}) + b_n sin(\frac{n\pi x}{l})),$$

then find $a_0, a_n, b_n, \ n = 1, 2, 3.....$

5. (a) State the Fourier integral expansion theorem. Are the conditions in the theorem necessary and sufficient? (4)

(b) Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & -a < x < a \\ 0, & \text{elsewhere} \end{cases}$. (6)
Hence, using Parseval's identity for Fourier transform, prove that

$$\int_0^{\infty} \frac{sin^2 ax}{x^2} dx = \frac{\pi a}{2}.$$

6. (a) Find the Laplace transform of $(t^2 - 3t + 2)e^{5t}$. (3)

(b) If $F(t)$ is a periodic function of 2π then find its Laplace transform, where (3)

$$F(t) = \begin{cases} \sin t & 0 \leq t < \pi, \\ 0 & \pi \leq t \leq 2\pi. \end{cases}$$

(c) Using Laplace transform and convolution theorem, show that (4)

$$L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{t \sin at}{2a}.$$

7. (a) Using Laplace transform, solve the following differential equation: (8)

$$(D^2 + 2D + 5)y = e^{-t}sint, \quad y(0) = 0, \ y'(0) = 1,$$

where $D = \frac{d}{dt}$

(b) State the sufficient conditions for Laplace transform of a function. (2)