B.E. CHEMICAL ENGINEERING SECOND YEAR SECOND SEMESTER 2022

2nd Year, 2nd Semester

INTRODUCTION TO TRANSPORT PHENOMENA

Time: Three hours Full Marks: 100

Use separate answer scripts for part I and part II

Part I

Answer all the questions, assume any missing data

Full Marks 50

Question no	CO no	Question	Marks
1(a)	5	A thin flat plate 0.4 m × 0.4 m is exposed to an air stream at 40 m/s and 30° C. When air flows over both sides of the plate, the total drag experienced by the plate is 0.35 N. Obtain an estimate of the rate of heat transfer from the plate, if the plate temperature is 55° C. Following air properties may be used: ρ =1.15 kg/m^3, C_p = 1007 J/kg K, Pr = 0.707	8+6
1(b)		You want to visualize the flow over a spinning baseball. You have decided to perform a laboratory experiment in a water tunnel by injecting multi-coloured dye streaklines. The actual baseball moves through the air at 100 km/hr and spins at 250 rpm. Identify the dimensionless numbers which are important in this situation. Find the speed at which you should run the water in the water tunnel and the rpm at which you should spin the baseball in the tunnel. The kinematic viscosity of water: 1×10^{-6} m^2/s and air : 1.6×10^{-5} m^2/s.	
2.	2	Biofilms which are colonies of bacteria that can cling to living or inert surfaces, can cause a wide array of human infections. Consider a biofilm that is associated with a skin infection. An antibiotic (species A) is applied to the surface of a biofilm (species B) so that a fixed concentration of medication C_{A0} (kmol/m^3) is maintained at the top surface of the biofilm. The diffusion coefficient of the medication within the biofilm is known. The antibiotic is consumed within the film by a first order homogeneous biochemical reaction $\dot{N}_A = -k_1 C_A$ Obtain the governing equation and the appropriate boundary conditions (do not Solve) which has to be solved to obtain the concentration profile of the medication within the biofilm at steady state. Consider the bottom of the biofilm to be impermeable to the antibiotic.	6

[Turn over

3.	4	In Siberian regions, the soil with its moisture content freezes over in the winters. In the summer time when the temperatures rise above freezing, the ground thaws and turns to slush posing danger to the foundations, if not deep enough. The region that does not thaw even during the summer is termed as *permafrost* and all constructions must have their foundations anchored in this region. If at the onset of summer the ground can be assumed to be at a uniform temperature of -32°C, and if the summer lasts for 120 days when the temperature at the surface of the earth is 8°C find the depth to which the ground thaws. Following properties of the soil may be considered: Thermal conductivity : 0.52 W/m.K, Specific heat:1840 J/kg.K, Density: 2050 kg/m^3	14
4.	3	A cylindrical beaker filled with an incompressible fluid is being rotated about its own axis which is oriented vertically. The radius and the angular velocity of rotation of the beaker are R and Ω_0 respectively. Derive the expressions for the steady state velocity distribution and the pressure distribution in the fluid clearly mentioning all the assumptions.	14+2
	1	Discuss the mechanism of momentum transport in this situation.	

w	erf w	w	erf w	w	erf w
0.00	0.00000	0.36	0.38933	1.04	0.85865
0.02	0.02256	0.38	0.40901	1.08	0.87333
0.04	0.04511	0.40	0.42839	1.12	0.88679
0.06	0.06762	0.44	0.46622	1.16	0.89910
0.08	0.09008	0.48	0.50275	1.20	0.91031
0.10	0.11246	0.52	0.53790	1.30	0.93401
0.12	0.13476	0.56	0.57162	1.40	0.95228
0.14	0.15695	0.60	0.60386	1.50	0.96611
0.16	0.17901	0.64	0.63459	1.60	0.97635
0.18	0.20094	0.68	0.66378	1.70	0.98379
0.20	0.22270	0.72	0.69143	1.80	0.98909
0.22	0.24430	0.76	0.71754	1.90	0.99279
0.24	0.26570	0.80	0.74210	2.00	0.99532
0.26	0.28690	0.84	0.76514	2.20	0.99814
0.28	0.30788	0.88	0.78669	2.40	0.99931
0.30	0.32863	0.92	0.80677	2.60	0.99976
0.32	0.34913	0.96	0.82542	2.80	0.99992
0.34	0.36936	1.00	0.84270	3.00	0.99998

The Navier–Stokes equations in cylindrical coordinates for constant density and viscosity are

r component:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$

$$= \rho g_r - \frac{\partial p}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}[r v_r]\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right\}$$

θ component:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$

$$= \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}[r v_\theta]\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right\}$$

z component:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$

$$= \rho g_z - \frac{\partial p}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right\}$$

The continuity equation in cylindrical coordinates for constant density is

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

CO1: Identify the transport properties and describe different mechanisms of momentum, energy and mass transport.
CO2: Develop the governing conservation equations and boundary conditions for the steady state and transient momentum, heat and mass transport.
CO3: Analytically solve and analyze a variety of steady state and transient momentum, heat and mass transport problems with appropriate assumptions and approximation.
CO4: Analyze and solve a practical real life problem applying the momentum, heat and mass transport equations and appropriate solution techniques.
CO5: Non-dimensionalize the transport equations, identify the dimensionless numbers and apply analogies between momentum, heat and mass transport to scale up.

Ex/Che/PC/B/T/224/2022

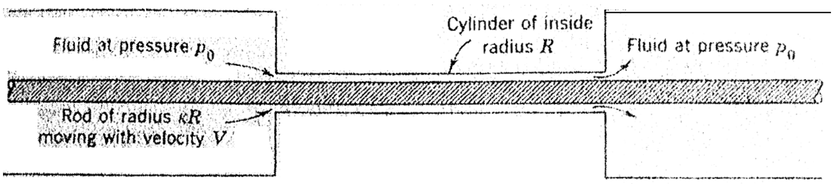
B.E. CHEMICAL ENGINEERING SECOND YEAR SECOND SEMESTER - 2022
INTRODUCTION TO TRANSPORT PHENOMENA

Time: 3 Hours Full Marks: 100

Use Separate Answer Scripts for Part I and Part II

PART II

State all the assumptions. Assume missing data (if any)

Q.No.	CO No		Marks
1(i)	1	Wet paints often exhibits yield stress and it can be approximated as a Binghum plastic fluid. Consider a layer of wet paint on a vertical wall with a uniform thickness H and a yield stress ζ_0. Derive an expression for the maximum value of H which can be sustained without having the paint 'run' (i.e. flow down the wall).	(3)
1(ii)	1	Differentiate between natural (free) convection and forced convection. Consider Couette flow of a fluid confined between two parallel horizontally mounted plates maintained at different temperatures (T1 and T2, T1>T2) where the top plate is moving at a constant velocity U and the bottom plate is stationary. Is it an example of free convection/forced convection or both (mixed)? Justify your answer.	(2+1)
2.	2	Consider cardiovascular flow of blood through the artery (cylindrical pipe) of diameter d and length L from heart to kidney. The pressure differential oscillates with time $\Delta P = \Delta P_{av} + \varepsilon \cos(\omega t)$, where $\omega = (72/60) s^{-1}$. (i)Assuming that blood behaves as a Newtonian incompressible fluid, derive the governing equations and boundary conditions for the flow through artery. <u>DO NOT SOLVE.</u> (ii) Non-dimensionalize the governing equations and derive the dimensionless numbers.	(4+3)
3	3	Consider annular flow of a viscous, Newtonian, incompressible fluid with inner cylinder moving axially (refer to Fig.1). The cylindrical rod is being moved with a velocity V. The rod and the cylinder are coaxial. Derive the expressions for steady state velocity distribution and volumetric flow rate.  FIG. 1	(8)

4	4	Pulverized coal pellets, which may be approximated as pure carbon spheres of radius ro=1 mm, is burned in pure oxygen at 1450 K and 1 atm. Oxygen is transferred to the particle surface by diffusion, where it is consumed in a reaction of the form $C+O_2 \longrightarrow CO_2$. Assuming instantaneous surface reaction and neglecting the change in ro (pseudo-steady state) derive the expressions for radial distribution of the CO_2 and O_2 concentrations. What is the O_2 molar consumption rate? The diffusion coefficient of oxygen through gas mixture at reaction temperature and pressure is $1.5x10^{-4}$ m^2/s. Show the detailed derivation and solution.	(8)
5.(i)	4	Steel balls 12 mm in diameter are annealed by heating to 1150K and then slowly cooling to 400K in an air environment for which Tα=325 K and h=20 W/m2K. Assuming the properties of the steel to be k=40 W/m K, ρ=7800 kg/m3 and Cp=600 J/kg K, estimate the time required for the cooling process. Consider that the characteristic length scale Lc= V/ As, V is the volume, As is the effective surface area.	(4)
5(ii)	4	A furnace wall is fabricated from fireclay brick (α=$7x10^{-7}$ m^2/s) and its inner surface is maintained at 1200 K during furnace operation. The wall is designed according to the criterion that for an initial temperature of 303K, its midpoint temperature will not exceed 350 K after 4 hrs of furnace operation. What is the minimum allowable wall thickness? Show the detailed derivation and solution. Hints. Consider semi-infinite medium.	(7)
6. (i)	5	Consider the hydrodynamic (velocity), thermal and concentration boundary layers over a flat plate. State the conditions of Reynolds analogy. Under what conditions the thickness of all three boundary layers (δ_h,δ_t and δ_c) are same?	(3+2)
6(ii)	5	An object of irregular shape has a characteristic length of L=1 m and is maintained at a uniform surface temperature of Ts =325 K. It is suspended in an airstream that is at atmospheric pressure (p=1 atm) and has a free stream velocity U=100 m/s and a temperature of Tα=275K. The average heat flux from the surface to the air is 12000 W/m^2. Referring to the foregoing situation as case 1, consider the following case 2 and determine whether conditions are analogous to those of case 1. If analogues behavior does exist, determine the corresponding value of the average mass transfer (convection) coefficient and the average mass flux from surface to dry air. Case 2. Consider the object of the same shape (that of case 1) with characteristic length L2=2 m, which is suspended in an airstream in the same manner. The surface is coated with a liquid film that evaporates into the air. The entire system is at 300 K. The free stream velocity U2=50 m/s and p=1 atm. Data given : Air at 300 K, γ=$16x10^{-6}$ m^2/s, α=$22x10^{-6}$ m^2/s. Saturated water vapor (300K) ρ_{Asat}=0.025 Kg/m^3, water vapor-air (at 300K) D_{AB} ~$0.22x10^{-4}m^2/s$.	(5)

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NAVIER STOKES EQUATION IN CYLINDRICAL COORDINATE

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} = \tag{21}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left\{-\frac{u_r}{r^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right\} + g_r.$$

In a similar way, we can derive the momentum equations for the θ-component of the velocity

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z \frac{\partial u_\theta}{\partial z} = \tag{22}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial \theta} + \frac{\mu}{\rho}\left\{-\frac{u_\theta}{r^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right\} + g_\theta,$$

and, for the z-component:

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} = \tag{23}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right\} + g_z.$$

Complementary Error Function Table

x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)
0	1.000000	0.5	0.479500	1	0.157299	1.5	0.033895	2	0.004678	2.5	0.000407	3	0.00002209
0.01	0.988717	0.51	0.470756	1.01	0.153190	1.51	0.032723	2.01	0.004475	2.51	0.000386	3.01	0.00002074
0.02	0.977435	0.52	0.462101	1.02	0.149162	1.52	0.031587	2.02	0.004281	2.52	0.000365	3.02	0.00001947
0.03	0.966159	0.53	0.453536	1.03	0.145216	1.53	0.030484	2.03	0.004094	2.53	0.000346	3.03	0.00001827
0.04	0.954889	0.54	0.445061	1.04	0.141350	1.54	0.029414	2.04	0.003914	2.54	0.000328	3.04	0.00001714
0.05	0.943628	0.55	0.436677	1.05	0.137564	1.55	0.028377	2.05	0.003742	2.55	0.000311	3.05	0.00001608
0.06	0.932378	0.56	0.428384	1.06	0.133856	1.56	0.027372	2.06	0.003577	2.56	0.000294	3.06	0.00001508
0.07	0.921142	0.57	0.420184	1.07	0.130227	1.57	0.026397	2.07	0.003418	2.57	0.000278	3.07	0.00001414
0.08	0.909922	0.58	0.412077	1.08	0.126674	1.58	0.025453	2.08	0.003266	2.58	0.000264	3.08	0.00001326
0.09	0.898719	0.59	0.404064	1.09	0.123197	1.59	0.024538	2.09	0.003120	2.59	0.000249	3.09	0.00001243
0.1	0.887537	0.6	0.396144	1.1	0.119795	1.6	0.023652	2.1	0.002979	2.6	0.000236	3.1	0.00001165
0.11	0.876377	0.61	0.388319	1.11	0.116467	1.61	0.022793	2.11	0.002845	2.61	0.000223	3.11	0.00001092
0.12	0.865242	0.62	0.380589	1.12	0.113212	1.62	0.021962	2.12	0.002716	2.62	0.000211	3.12	0.00001023
0.13	0.854133	0.63	0.372954	1.13	0.110029	1.63	0.021157	2.13	0.002593	2.63	0.000200	3.13	0.00000958
0.14	0.843053	0.64	0.365414	1.14	0.106918	1.64	0.020378	2.14	0.002475	2.64	0.000189	3.14	0.00000897
0.15	0.832004	0.65	0.357971	1.15	0.103876	1.65	0.019624	2.15	0.002361	2.65	0.000178	3.15	0.00000840
0.16	0.820988	0.66	0.350623	1.16	0.100904	1.66	0.018895	2.16	0.002253	2.66	0.000169	3.16	0.00000786
0.17	0.810008	0.67	0.343372	1.17	0.098000	1.67	0.018190	2.17	0.002149	2.67	0.000159	3.17	0.00000736
0.18	0.799064	0.68	0.336218	1.18	0.095163	1.68	0.017507	2.18	0.002049	2.68	0.000151	3.18	0.00000689
0.19	0.788160	0.69	0.329160	1.19	0.092392	1.69	0.016847	2.19	0.001954	2.69	0.000142	3.19	0.00000644
0.2	0.777297	0.7	0.322199	1.2	0.089686	1.7	0.016210	2.2	0.001863	2.7	0.000134	3.2	0.00000603
0.21	0.766478	0.71	0.315335	1.21	0.087045	1.71	0.015593	2.21	0.001776	2.71	0.000127	3.21	0.00000564
0.22	0.755704	0.72	0.308567	1.22	0.084466	1.72	0.014997	2.22	0.001692	2.72	0.000120	3.22	0.00000527
0.23	0.744977	0.73	0.301896	1.23	0.081950	1.73	0.014422	2.23	0.001612	2.73	0.000113	3.23	0.00000493
0.24	0.734300	0.74	0.295322	1.24	0.079495	1.74	0.013865	2.24	0.001536	2.74	0.000107	3.24	0.00000460
0.25	0.723674	0.75	0.288845	1.25	0.077100	1.75	0.013328	2.25	0.001463	2.75	0.000101	3.25	0.00000430
0.26	0.713100	0.76	0.282463	1.26	0.074764	1.76	0.012810	2.26	0.001393	2.76	0.000095	3.26	0.00000402
0.27	0.702582	0.77	0.276179	1.27	0.072486	1.77	0.012309	2.27	0.001326	2.77	0.000090	3.27	0.00000376
0.28	0.692120	0.78	0.269990	1.28	0.070266	1.78	0.011826	2.28	0.001262	2.78	0.000084	3.28	0.00000351
0.29	0.681717	0.79	0.263897	1.29	0.068101	1.79	0.011359	2.29	0.001201	2.79	0.000080	3.29	0.00000328
0.3	0.671373	0.8	0.257899	1.3	0.065992	1.8	0.010909	2.3	0.001143	2.8	0.000075	3.3	0.00000306
0.31	0.661092	0.81	0.251997	1.31	0.063937	1.81	0.010475	2.31	0.001088	2.81	0.000071	3.31	0.00000285
0.32	0.650874	0.82	0.246189	1.32	0.061935	1.82	0.010057	2.32	0.001034	2.82	0.000067	3.32	0.00000266
0.33	0.640721	0.83	0.240476	1.33	0.059985	1.83	0.009653	2.33	0.000984	2.83	0.000063	3.33	0.00000249
0.34	0.630635	0.84	0.234857	1.34	0.058086	1.84	0.009264	2.34	0.000935	2.84	0.000059	3.34	0.00000232
0.35	0.620618	0.85	0.229332	1.35	0.056238	1.85	0.008889	2.35	0.000889	2.85	0.000056	3.35	0.00000216
0.36	0.610670	0.86	0.223900	1.36	0.054439	1.86	0.008528	2.36	0.000845	2.86	0.000052	3.36	0.00000202
0.37	0.600794	0.87	0.218560	1.37	0.052688	1.87	0.008179	2.37	0.000803	2.87	0.000049	3.37	0.00000188
0.38	0.590991	0.88	0.213313	1.38	0.050984	1.88	0.007844	2.38	0.000763	2.88	0.000046	3.38	0.00000175
0.39	0.581261	0.89	0.208157	1.39	0.049327	1.89	0.007521	2.39	0.000725	2.89	0.000044	3.39	0.00000163
0.4	0.571608	0.9	0.203092	1.4	0.047715	1.9	0.007210	2.4	0.000689	2.9	0.000041	3.4	0.00000152
0.41	0.562031	0.91	0.198117	1.41	0.046148	1.91	0.006910	2.41	0.000654	2.91	0.000039	3.41	0.00000142
0.42	0.552532	0.92	0.193232	1.42	0.044624	1.92	0.006622	2.42	0.000621	2.92	0.000036	3.42	0.00000132
0.43	0.543113	0.93	0.188437	1.43	0.043143	1.93	0.006344	2.43	0.000589	2.93	0.000034	3.43	0.00000123
0.44	0.533775	0.94	0.183729	1.44	0.041703	1.94	0.006077	2.44	0.000559	2.94	0.000032	3.44	0.00000115
0.45	0.524518	0.95	0.179109	1.45	0.040305	1.95	0.005821	2.45	0.000531	2.95	0.000030	3.45	0.00000107
0.46	0.515345	0.96	0.174576	1.46	0.038946	1.96	0.005574	2.46	0.000503	2.96	0.000028	3.46	0.00000099
0.47	0.506255	0.97	0.170130	1.47	0.037627	1.97	0.005336	2.47	0.000477	2.97	0.000027	3.47	0.00000092
0.48	0.497250	0.98	0.165769	1.48	0.036346	1.98	0.005108	2.48	0.000453	2.98	0.000025	3.48	0.00000086
0.49	0.488332	0.99	0.161492	1.49	0.035102	1.99	0.004889	2.49	0.000429	2.99	0.000024	3.49	0.00000080