Ref. No.: Ex/Arch/Math/T/114/2022

Bachelor of Architecture

First Year, First Semester Exam. 2022

Subject - Mathematics - 1

Part - I (50 Marks)

Answer any five questions:

1. (a) If $y = (ax+b)^m$, m is any positive integer, then show that

$$y_n = \frac{m!}{(m-n)!}(ax+b)^{m-n} a^n, \quad \text{when } m > n$$

$$= n!\, a^n. \quad \text{when } m = n$$

(b) If $y = \sin(m \sin^{-1} x)$, then show that

$$(1-x^2)y_2 - 2xy_1 + m^2 y = 0$$ 6+4

2. (a) State the Rolle's theorem.

(b) Verify the Rolle's theorem for the function

$f(x) = \ln(x^2+3) - \ln 4$ in $[-1, 1]$. 2+8

3. (a) State the Cauchy's mean value theorem.

Hence obtain the Lagrange's mean value theorem.

(b) Use mean value theorem to show that $\sqrt{101}$ lies between 10 and 10·5 $\quad 5+5$

4. (a) Find Taylor's series expansion for

$$f(x) = \ln(1+x) \quad , -1 < x < \infty$$

about $x = 2$, with Lagrange's form of remainder.

(b) Evaluate :

$$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}. \qquad 6+4$$

5. (a) State the Euler's theorem on homogeneous functions of two variables.

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^2}$ $\quad 2+8$

6. (a) Evaluate :

$$\int \frac{(x+7)\,dx}{x^2+4x+13}$$

(b) If $f(x) = a + bx + cx^2$, then show that θ is independent of x in the mean value theorem

$$f(x+h) = f(x) + h f'(x+\theta h).$$ 5+5

7. Find for what values of x

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 5$$

is maximum and minimum respectively. Also, find the maximum and minimum values of $f(x)$. 10

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BACHELOR OF ENGINEERING IN ARCHITECTURE
ENGINEERINGEXAMINATION, 2022
(1^{st} Year, 1^{st} Semester)
Mathematics-IA

Time: Three hours   Full Marks: 100

(50 marks for each Part)
(Symbols and notations have their usual meanings)
Use a separate Answer-Script for each Part

PART-II (50 Marks)

Answer *Q. No.1* and any ***three*** from the rest

Answer the following questions:

1. Evaluate $\int_0^a x^9 \sqrt[3]{a^6 - x^6}\, dx$, where a (>0) is a constant. **5**

2. Examine the convergence of the following integrals:

   a) $\int_2^\infty \frac{dx}{\log x}$

   b) $\int_0^1 \frac{dx}{(x+1)(x+2)\sqrt{x(1-x)}}$

   c) $\int_0^2 \frac{1}{\sqrt{x(2-x)}} dx$ **15**

3. a) Find the area of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

   b) Calculate the value of $\int_0^1 \frac{1}{x+1} dx$using Simpson's $\frac{1}{3}$ rule by taking six intervals.

   c) What do you mean by the convergence of improper integral $\int_{-\infty}^{\infty} f(x)\ dx$. **6+6+3**

4. a) Using a double integral, prove that $\mathrm{B}(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)};\ a, b > 0$.

   b) Find the whole length of the loop of the curve $(x-2a)(x-5a)^2 - 9ay^2 = 0$.

   c) Let f:[-3, 3]→ R be define by $f(x) = \begin{cases} 2x\sin\frac{\pi}{x} - \pi\cos\frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$.

   Examine whether f is Riemann integral in [-3, 3] and hence find $\int_{-3}^{3} f\, dx$.

   5+5+5

5. a) Evaluate $\iint_D \{2a^2 - 2a(x+y) - (x^2+y^2)\}\, dxdy$, where the region D bounded by the circle $x^2 + y^2 + 2a(x+y) = 2a^2$.

b) Find the surface area of the solid obtained by revolving one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about x-axis.

c) State condition for convergence of beta function. **6+7+2**

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