Ex/Arch/MATH/T/124/2022

BACHELOR OF ARCHITECTURE EXAMINATION, 2022

(1st Year, 2nd Semester)

MATHEMATICS II

Time : Three hours Full Marks : 100

(50 Marks for each Part)

Use Separate Answer Scripts for each Part.
Part - I (50 Marks)
Answer any five questions.

1.(a) Define symetric and skew-symetric matrix. Show that a square matrix of order n can be expressed uniquely as a sum of a symetric matrix and a skew-symetric matrix. 2+3

(b) Define nilpotent and idempotent matrix. If A is a nilpotent matrix of order n and I_n is the identity matrix of order n then show that $I_n - A$ is an invertible matrix. Give an example of an idempotent matrix other than zero and identity matrix. 2+2+1

2.(a) What do you mean by consistent and inconsistent system of linear equations? If the system of linear equations $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ have a nonzero solution then show that $a^2 + b^2 + c^2 + 2abc = 1$. 2+3

(b) If $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{pmatrix}$ then show that $AB = 6I_3$.
Utilize this result to solve the following system of linear equations :
$2x + y + z = 5$, $x - y = 0$ and $2x + y - z = 1$. 2+3

3.(a) If a square matrix A is invertible then show that A is a nonsingular matrix. Is the sum of two singular matrices always a singular matrix? Justify. Show that the product of a singular matrix and a nonsingular matrix of the same order is a singular matrix. 2+2+1

(b) Show that the value of the determinant of an orthogonal matrix is either $+1$ or -1. Let A and B be two orthogonal matrices of the same order and $detA + detB = 0$. Show that $A + B$ is a singular matrix. 2+3

4.(a) What is the length of the subtangent and subnormal of a curve $y = f(x)$ at any point ? Prove that for the parabola $y^2 = 4ax$, the subnormal is constant at any point and the subtangent varies as abscissa of the point of contact. 1+4 = 5

(b) If the two curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ cut orthogonally, then prove that $1/b - 1/b' = 1/a - 1/a'$. 5

5.(a) What is the formula for the radius of curvature of a curve $y = f(x)$ in Cartesian co-ordinate system? Find the radius of curvature at any point of the curve $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$. 1+4 = 5

(b) What is the maximum number of asymptotes of an algebraic curve of nth degree? Find the equation of the oblique asymptote of the curve $y = xe^{1/x^2}$. 1+4 = 5

[Turn over

6.(*a*) If the line $x\cos\alpha + y\sin\alpha = p$ touches the curve $(x/a)^{n/n-1} + (y/b)^{n/n-1} = 1$, then prove that $(a\cos\alpha)^n + (b\sin\alpha)^n = p^n$. 5

(*b*) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with the X-axis, then show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$. 5

7.(*a*) Find the radius of curvature of the curve $y = a\cosh(x/a)$ at any point. 5

(*b*) Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is 125/16. 5

Part – II

Answer ***any five*** questions.

8. a) Find the angle between the lines whose direction cosines are proportional to 1, 2, 1 and 2, –3, 6.
   b) A, B, C are three points on the axis of x, y and z respectively at distances a, b, c from the origin O. Find the co-ordinates of the point which is equidistant from A, B, C and O. 4+6

9. a) Find the general equation of a plane through a point (α, β, γ).
   b) Find the equation of the plane through the point (1, 2, 3) and parallel to the plane $4x + 5y - 3z = 7$. 5+5

10. a) Show that the line given by the equations $3x - 4y + z + 1 = 0$, $x - 2y + z + 2 = 0$ is equally inclined to the axes.
    b) Find in symmetrical form the equations of the line of intersection of the planes $2x - y - 1 = 0$ and $2y - z + 1 = 0$. 4+6

11. Prove that the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

are coplanar and find the equation of the plane containing them. 10

12. Find the magnitude and the equation of the line of shortest distance between the lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{5}.$$ 10

13. a) Find the equation of a sphere which passes through the points (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c).
    b) Show that the general equation of the cone of 2nd degree which passes through the co-ordinate axes is of the form $fyz + gzx + hxy = 0$. 4+6

14. Prove that the circles

$$x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0,\quad 5y + 6z + 1 = 0$$

and

$$x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0,\quad x + 2y - 7z = 0$$

lie on the same sphere and find its equation. 10