M. Sc. Physics Examination, 2022

(3rd Year, 2nd Semester)

Cosmology

Paper – 407

Time : Two hours | Full Marks : 40

Answer ***any four*** questions. 4×10=40

1. Show that in the region **II** of the Kruskal manifold, one may regard r as a time-coordinate and introduce a new spatial coordinate x such that:

$$ds^2 = -\frac{dr^2}{\left(\frac{2M}{r}-1\right)} + \left(\frac{2M}{r}-1\right)dx^2 + r^2 d\Omega^2$$

Hence show that *every timelike* curve in region II intersects the singularity at $r=0$ within a proper time no greater than πM. For which curves is this bound achieved?

10

2. Obtain/Determine the Penrose diagram for the *de Sitter spacetime*, with the metric

$$ds^2 = -dt^2 + \frac{1}{H^2}\cosh^2(Ht)\left(d\chi^2 + \sin^2\chi d\Omega^2\right)$$

where $H>0$ is a constant and $0 \le \chi \le \pi$. Here (χ,θ,ϕ) parametrizes the 3-sphere of radius χ and Ω denotes the solid angle. 10

3. The fundamental equations of relativistic hydrodynamics are the Continuity equation, the Euler equation, the Energy equation, together with the equation of state (EOS). Taking the energy momentum tensor of a perfect fluid, derive the relativistic form of those fundamental equations. Further, find the expressions for the isotropic pressure and total energy density of the fluid. 6+4

4. i) Show that the scale factor (a) is proportional to $t^{2/3}$ in the matter dominated universe and compare this with the scale factor for the empty universe. t is the cosmic time.

   ii) The dimensionless deceleration parameter at cosmic time t is defined as $q(t) = -\left(\frac{\ddot{a}}{a}\right) \Big/ \left(\frac{\dot{a}^2}{a^2}\right)$. Show that $q(t) = 0{\cdot}5$ in the matter dominated universe.

   iii) Calculate the present age of the universe (in Gyr) if it is matter dominated, where the Hubble parameter is $H_0 = 70$ Km/s/Mpc. 4+3+3

5. i) Calculate the comoving distance (in Mpc unit) between redshift $z = 0$ and 1 in the matter dominated universe. [assume Hubble parameter $H_0 = 70$ km/s/Mpc]

   ii) Estimate the CMBR temperature of the universe when the matter and radiation densities were the same. [assume the matter and radiation density parameters are, at present, $\Omega_{m0} = 0{\cdot}3$ and $\Omega_{r0} = 10^{-4}$ respectively.]

   iii) Briefly discuss how the CMBR was produced during the recombination epoch. Mention three major properties of the CMBR. 4+3+3