# M. Sc. Mathematics Examination, 2022

( 2nd Year, 2nd Semester )

## Probability and Stochastic Processes II

## Paper – 4.4 (B-2.32)

Time : 2 hours | Full Marks : 50

Answer ***any five*** questions.

Each question carries Ten marks.

Symbols/notations have usual meanings.

1. Consider the Birth and Death chain on the state space {0, 1, 2, ....} defined by $p_x = \frac{x+2}{2x+2}$, $q_x = \frac{x}{2x+2}$, $\forall x \in \{0,\ 1,\ 2,\ ....\}$

   Find whether the chain is positive recurrent, null recurrent or transient.

2. Check if the Markov chain on the state space {0, 1, 2} which has the transition matrix

$$\begin{array}{c} \\ \\ P = \end{array}\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0{\cdot}4 & 0{\cdot}4 & 0.2 \\ 0{\cdot}3 & 0{\cdot}4 & 0{\cdot}3 \\ 0{\cdot}2 & 0{\cdot}4 & 0{\cdot}4 \end{bmatrix} \end{array}$$

   has a stationary distribution. If it has, check whether the stationary distribution is unique and then find it.

3. Let $X_n,\ n \geq 0$ be the Ehrenfest chain on the state space $\{0,\ 1,\ ...,\ d\}$.

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i) Show that $\sum_{y \in \varphi} yP(x,y) = Ax + B,\ \forall x \in \varphi$, for some constant A and B.

ii) Using (i), find $E_x(X_n)$.

4. Consider the Markov Chain on $S = \{0,\ 1, 2, 3, 4, 5\}$ which has the transition matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \end{bmatrix} \end{array}$$

Find all the recurrent and transient classes.

5. A square matrix P is called doubly stochastic

if a) P is a non-negative matrix (entrywise)

and b) Each row sum and each column sum of the matrix P is unity.

If the transition matrix of a Marthov Chain with $d$ states is doubly stochastic, find a stationary distribution of this Markov Chain. Is it unique?

6. Let $\{X_n\}$ be the queuing chain.



a) Show that if $f(0) = 0$ or $f(0) + f(1) = 1$, then $\{X_n\}$ is irreducible.

b) If $f(0) > 0$ AND $f(0) + f(1) < 1$, show that $\{X_n\}$ is irreducible too. Here $f$ is the pmf of no of customers arriving during service time.