M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Modelling of Biological Events - II

Paper – 4.3 (A 2.5)

Time : Two hours   Full Marks : 50

Use a separate answer script for each Part.

Symbols and notations have their usual meanings.

Part – I (25 Marks)

Answer question **no. 1** and

answer *any two* from remaining three questions.

1. a) Consider an ordinary differential equation having the form

$$\begin{cases} x(t) = f(x(t)), \\ x(0) = x^0 \end{cases} \quad t > 0$$

Discuss the controlled dynamics of the system.

b) Construct a mathematical model by setting amount of output produced and fraction of output reinvested at time $t \geq 0$. Hence, discuss the control of production and consumption.   4+5=9

2. a) Define feedback control.

b) Formulate two different mathematical models expressing the definition of positive feedback control and negative feedback control for gaining

insight of Biological Avenue with suitable assumptions.

c) Find out the disease-free equilibrium point from any of your formulated model.   1+4+3=8

3. a) Using mass transfer rate constant along with explicit dependency of the reaction rate constant of the temperature, formulate a mathematical model on biodiesel production with suitable assumptions.

b) Obtain the optimal control induced system of the previous model for maximization of production of biodiesel and finally, evaluate the Hamiltonian.

4+4=8

4. Define M-M equation based on the definition of enzyme kinetics.

What is co-operative phenomena with considering allosteric effect? Formulate the reaction mechanism of allosteric behaviour and using two control parameters u_1 and u_2, formulate the controlled system.   2+2+4=8

Part – II (25 Marks)

Answer question **no. 4** and *any two* from the rest.

1. a) What is meant by a chemostat?

b) Formulate the model of microbial populations in a chemostat and discuss their stability properties.

2+10=12

2. a) How are the characteristics of an individual determined?

b) When two individuals mate, what are the possible genotypes of the offspring?

c) Derive the stochastic matrices for different genotypes of the offspring.

d) State and prove Hardy-Weinberg law. 2+2+3+5=12

3. A randomly moving bacteria concentration $C(x, t)$ in a semi-infinite capillary tube is governed by one dimensional diffusion equation. With suitable initial and boundary condition, show that the diffusion coefficient D can be determined from the number of bacteria in the tube and the cross-sectional area of the tube.   12

4. What is Fick's law?   1