M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Epidemiology and Eco-epidemiology

Paper – 4.5 (B-2.36)

Time : Two hours Full Marks : 50

Use a separate answer script for each Part.

Symbols and notations have their usual meanings.

Part – I (30 Marks)

Answer ***any Three*** questions.

1. a) Formulate a SIR epidemic model with necessary assumptions.

   b) What is the Kermack and McKendric threshold phenomenon in relation to your model?

   c) Show for your model system that the spread of disease will not stop for lack of susceptible. 2+2+6

2. With proper assumptions, propose a SIRS model with horizontal and perfect vertical transmission. Determine the equilibrium points of the proposed model and write the stability conditions of the endemic equilibrium point. 3+2+5

3. a) Describe a basic SEIS epidemic model with proper assumptions and give its flow diagram.

   b) Find the basic reproduction number of your epidemic model. Prescribe the conditions for the stability of disease-free equilibrium.

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c) Using Bendixon-Dulac criterion, prove that the endemic equilibrium is global asymptotically stability. 3+4+3

4. a) Construct a basic in-host HIV model with necessary assumptions.

b) Find all possible equilibrium points of your model system.

c) Determine the local stability conditions of the endemic equilibrium point. 3+2+5

Part – II (20 Marks)

Answer ***any One*** question.

1. a) Formulate an eco-epidemiological model with disease in predator population with necessary assumptions.

b) Determine different equilibrium points of your model.

c) Discuss about the local stability of interior equilibrium point of your proposed model. Give the biological interpretation of your results.

d) Using Bendixon-Dulac criterion, show that the disease-free planar equilibrium global asymptotically stable. 5+4+7+4

2. Consider the following eco-epidemic model:

$$\frac{ds}{dt} = rS\left(1 - \frac{S+I}{K}\right) - \lambda SI - \alpha PS,$$

$$\frac{dI}{dt} = \lambda SI - \beta PI - \mu I,$$

$$\frac{dP}{dt} = \theta\alpha PS - \theta\beta PI - \delta P.$$

(S=Susceptible Prey, I=Infected Prey, P=Predator, all parameters are positive.)

a) Describe the underlying assumptions of the above model. Give a flow diagram of the model.

b) Write dimensionless form of the model.

c) Find predator free planar steady state of model. Prescribe the conditions for stability of predator free planar steady state.

d) Show that the existence of positive interior steady state of the model implies its local stability.

6+4+4+6