Also, show that $Q_B \le Q_0$, where Q_0 is the optimal order quantity in no backorder case. 5+3

b) Investigate the effect of probabilistic demand on the cost of order level inventory system. 8

5. a) A company 'Electro' uses resin in its manufacturing process at the rate of 1000 gallons per month. It costs Electro \$100 to place an order. The holding cost per gallon per month is \$2, and the shortage cost per gallon is \$10. Historical data show that the demand during lead time is uniform in the range (0, 200) gallons. Develop a mathematical model and use it to determine the optimal ordering policy for the company Electro. 5+3

b) Consider an inventory system in which a quantity Q is ordered whenever inventory drops to the level R. Assuming that the demand during the lead time is random and shortages are allowed and completely backlogged, establish a stochastic continuous review inventory model and determine its optimal (R, Q) policy. 8

6. The average monthly consumption for an item is 300 units and the normal lead time is one month. If the maximum consumption is 370 units per month and maximum lead time is 1.5 months, what should be the buffer stock for the item? 2

M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Production and Inventory Control - II

Paper – 4.5 (B 2.33)

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Answer **Q. No. 6** and ***any three*** questions from the rest.

1. a) Derive the expected cost function for a stochastic order-level inventory system with uniform demand and no set up cost, when the stock level S and demand x are restricted to discrete units. Also show that the optimal value of S can be obtained by satisfying the inequality:

$$\sum_{x=0}^{S-1} p(x)+\left(S-\frac{1}{2}\right)\sum_{x=S}^{\infty}\frac{p(x)}{x}<\frac{c_2}{c_1+c_2}<\sum_{x=0}^{S} p(x)+\left(S+\frac{1}{2}\right)\sum_{x=S+1}^{\infty}\frac{p(x)}{x}$$

where $p(x)$ = p.m.f. of x, c_1 = holding cost per unit per unit time, c_2 = shortage cost per unit per unit time. 4+4

b) A newspaper boy buys papers for Rs. c_1 and sells for Rs. c_2 each. He cannot retain unsold newspapers. The probability that the demand x on a randomly selected day is $p(x)$. If each day's demand is independent of the previous day's, find the maximum number of newspapers that could be ordered each day. 8

2. a) Show that the probabilistic order-level inventory systems with instantaneous and uniform demands are equivalent.

Suppose that, in a probabilistic order level system with uniform demand, the density of demand is the following:

$$f(x)=\frac{1}{b^2}xe^{-x/b},\ x\geq 0$$

Find the optimal order level of the above system considering the equivalent system of instantaneous demand. 5+3

b) A baking company sells cake by kg. weight. It makes a profit of Rs. 50 per kg. on every kg. sold on the date it is baked. It disposes all the cakes not sold on the date it is baked at a loss of Rs. 10 per kg. If the demand is known to be rectangular between 1000 kg. and 2000 kg., determine the optimal daily amount baked. 8

3. a) Show that the expression for the expected gain for a purchase of quantity $q(>0)$ just before time T_0 when there is a random price change k is

$$G(q)=q\left[\bar{k}+\sqrt{\frac{2c_3dp}{R}}\left(1+\frac{\bar{k}}{2d}\right)\right]-\frac{dp}{2R}q^2-c_3$$

where $\bar{k}$ is the expected value of k. The other symbols d, p, R and c_3 are the purchase cost of the

commodity before time T_0, the carrying cost fraction per unit time, demand rate and the ordering cost, respectively.

Deduce that when $\bar{k}<0$, the optimal order quantity just before time T_0 is zero. 5+3

b) Suppose that, for an inventory system, an item is ordered per week. The demand for the item during the week is uniform with the probabilities: P(0)=0.04, P(5)=0.2, P(10)=0.37, P(15)=0.30, P(20)=0.09. The carrying cost is Rs. 2/unit/week. The shortage cost is Rs. 24/unit/week. Develop a mathematical model and use it to determine the optimal order level at the beginning of each week. 5+3

4. a) In a newsvendor model, if backorder option is triggered when stock out occurs then show that the optimal order quantity is

$$Q_B=F^{-1}\left(\frac{(1-b)(p-c+c_l)}{(1-b)(p+c_l)+bc-s}\right)$$

where c = unit cost, p = unit price, s = unit salvage value, c_l = shortage cost per unit item, b = percentage of unsatisfied demand that can be meet up from the next regular order, $F(\cdot)$ = c.d.f. of the stochastic demand.