6. If F(s) and G(s) are the Mellin transform of the functions f(x) and g(x), then prove that

$$M^{-1}\left[F(s)G(s)\right]=\int_0^\infty f\left(\frac{x}{t}\right)g(t)\frac{dt}{t}.$$ 5

7. a) If F(z) is Z-transform of a sequence $\{f(n)\}_{-\infty}^{\infty}$, then prove the following:

If $f(n)=0$ for $n<0$, then $\lim_{|z|\to\infty} F(z)=f(0)$.

b) Prove that $Z\left[nf(n)\right]=-z\frac{dF(z)}{dz}$. 3+2

M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Integral Equation and Integral Transform

Paper – 4.2

Time : Two hours Full Marks : 50

Use a separate answer script for each Part.

Symbols and notations have their usual meanings.

Part – I (25 Marks)

Answer **Q. No. 1** and ***any two*** from the rest.

1. Define degenerate kernel of an integral equation. 1

2. a) If f is a continuous function on [a , b] and $k(x,t)(\neq 0)$ is a continuous function on $R=\{(x,t);\ a\le x,\ t\le b\}$ and $\phi_0(x)$ is any function continuous on [a , b] and for $x\in[a\ ,\ b]$,

$$\phi_n(x)=f(x)+\lambda\int_a^x k(x.t)\phi_{n-1}(t)\,dt \quad (n=1,2,3,...),$$

then show that the sequence $\{\phi_n(x)\}$ converges uniquely to the unique continuous solution of the integral equation $u(x)=f(x)+\lambda\int_a^x k(x,t)u(t)\,dt$, for any finite value of λ.

b) Convert the differential equation $\phi''(x)=F(x,\phi(x))$, $0<x<1$ with $\phi(0)=\phi_0$, $\phi(1)=\phi_1$ to an integral equation. 8+4

3. a) Using Hilbert-Schmidt theorem, to solve the following integral equation

$$\phi(x)=1+\lambda\int_0^{\pi}\cos(x+t)\phi(t)\,dt$$

b) Show that the eigen functions of a symmetric kernel corresponding to different eigen values are orthogonal. 7+5

4. a) Define Fredholm's first minor $D(x,y;\lambda)$ and discuss its convergence.

b) Solve the integral equation

$$\phi(x)=\cos x+(x-2)+\int_0^x (t-x)\phi(t)\,dt$$

c) If $k(x,t)$ is a continuous on $R=\{(x,t);\ a\le x,\ t\le b\}$, then show that

$$\int_a^b..\int_a^b D\begin{pmatrix} x_1\ x_2\ ..\ x_p \\ x_1\ x_2\ ..\ x_p \end{pmatrix}\lambda\, dx_1\, dx_2\, ..\, d_{xp}=(-\lambda)^p D^{(p)}(\lambda)$$

$3\frac{1}{2}+3\frac{1}{2}+5$

Part – II

(Integral Transform)

(Marks : 25)

Answer ***any Five*** questions.

1. Derive Fourier Transform from Fourier series. 5

2. Solve $t\dfrac{d^2f}{dt^2}-\dfrac{df}{dt}-tf(t)=0$ with $f(0)=0$.

Given that $L^{-1}\left[\dfrac{1}{(p^2-1)^{\frac{3}{2}}}\right]=tI_1(t)$. 5

3. a) Does Fourier transform of the function $f(x)=1$, $-\infty<x<\infty$ exist? Justify your answer.

b) If F(s) and G(s) are the Fourier transforms of the functions f(x) and g(x) then prove that $\int_{-\infty}^{\infty} f(x)\overline{g(x)}dx=\int_{-\infty}^{\infty}F(s)\overline{G(s)}ds$. 2+3=5

4. If $F(p)$, $G(p)$ are Laplace transforms of two functions defined on $[0,\infty]$, then prove that the inverse Laplace transform of the product $F(p)G(p)$ is the Laplace convolution integral $f(t)$ and $g(t)$. Hence find the Laplace inverse of $\dfrac{p}{(p^2+a^2)^2}$. 3+2

5. Write down Hankel transform of order zero of a function f(r) together with its inverse transform. Find Hankel transform of order zero of the function $F(r)=e^{-ar^2}$, $a>0$. 2+3

[Turn over