6. What is a *matching* in a graph? State and prove Hall's Theorem for matching in a bipartite graph.
7. Define a *perfect matching*. Prove that every k-regular bipartite graph has a perfect matching.

M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Graph Theory II (Theory)

Paper – 4.5 (B 2.19)

Time : $1\frac{1}{2}$ hours Full Marks : 30

Answer ***any five*** questions. 5×6

1. Define a *planar graph*. Let G be a simple bipartite planar graph with n vertices and e edges. Show that $e \leq 2n-4$. Determine all r, s such that $K_{r,s}$ is planar.
2. Define a *maximal planar graph.* Let G be a simple graph with n vertices. Prove that G is a maximal planar graph if and only if $e = 3n-6$.
3. What is a *Kuratowski subgraph*? If a graph $G = (V, E)$ has no Kuratowski subgraph, then show that $G \cdot e$ has no Kuratowski subgraph for any $e \in E$, where $G \cdot e$ denotes the graph obtained from G by contracting the edge e.
4. Show that the Petersen graph is not planar. Find the crossing number of the Petersen graph.
5. Define a *flow function* of a (single source, single sink) transport network. Find a maximum flow for the following network :